Foundation of demand estimation

Theoretical and Empirical Study

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Outline

1.Introduction: Why estimate demand?

- Measuring Market Power
  Accurate demand elasticities are essential for computing markups and identifying firms' pricing power.
- Estimating Marginal Costs and Markups
  Structural demand estimates are required to back out marginal costs using firms' first-order conditions in equilibrium models.
- Simulating Counterfactual Policies
  Demand models enable simulations of mergers, taxes, price regulations, and other policy interventions on market outcomes and consumer welfare.

1.Introduction: Challenges

- The endogeneity of price

$$Q = D(X, P, U) \tag{1.1}$$
$$P = C(W, Q, V). \tag{1.2}$$

- Demand for any one good generally depends on more than one latent demand shocks

1.Introduction: Challenges

- LATE(Local average treatment effect)

$$Q = S(W, P, V), \frac{\partial P^*(X, W, U, V, \tau)}{\partial \tau} = \frac{\frac{\partial S(W,P,V)}{\partial P}}{\frac{\partial S(W,P,V)}{\partial P} + \left|\frac{\partial D(X,P,U)}{\partial P}\right|}. \tag{1.3}$$

- Fixed effect and control function

1.Introduction: Solution

- Use structural instrumental variables (e.g., BLP instruments, cost shifters) and moment conditions like
$$\mathbb{E}[\xi_{jt} \cdot Z_{jt}] = 0 \tag{1.4}$$
- Estimate a system-wide discrete choice model (e.g., multinomial logit, random coefficients logit) with full substitution structure

2.Models: RUM

Random utility model

$$u_{ij} = V_{ij} + \varepsilon_{ij}$$

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$$q_{ij} = \mathbf{1}\left\{u_{ij} \geq u_{ik}\ \forall k \in \{0, 1, \ldots, J_i\}\right\}. \tag{2.1}$$

$$s_{ij} = \mathbb{E}\left[q_{ij} \mid J_i, \chi_i\right] \tag{2.2}$$

$$= \int_{A_{ij}} dF_u\left(u_{i0}, u_{i1}, \ldots, u_{iJ_i} \mid J_i, \chi_i\right). \tag{2.2}$$

$$A_{ij} = \left\{\left(u_{i0}, u_{i1}, \ldots, u_{iJ_i}\right) \in \mathbb{R}^{J_i+1} : u_{ij} \geq u_{ik}\ \forall k\right\}. \tag{2.3}$$

2.Models: Canonical Model

$$u_{ijt} = x_{jt}\beta_{it} - \alpha_{it}p_{jt} + \xi_{jt} + \epsilon_{ijt}, \qquad \text{for } j > 0 \tag{2.4}$$

$$u_{i0t} = \epsilon_{i0t} \tag{2.5}$$

$$s_{jt} = \int \frac{e^{x_{jt}\beta_{it} - \alpha_{it}p_{jt} + \xi_{jt}}}{\sum_{k=0}^{J_t} e^{x_{kt}\beta_{it} - \alpha_{it}p_{kt} + \xi_{kt}}} \, dF(\alpha_{it}, \beta_{it}; t) \tag{2.6}$$

$$\beta_{it}^{(k)} = \beta_0^{(k)} + \beta_\nu^{(k)}\nu_{it}^{(k)} + \sum_{\ell=1}^{L} \beta_d^{(\ell,k)} d_{i\ell t} \tag{2.7}$$

$$\ln(\alpha_{it}) = \alpha_0 + \alpha_y y_{it} + \alpha_\nu \nu_{it}^{(0)}, \tag{2.8}$$

2.Models:Canonical Model

- Random coefficients are introduced in discrete choice models to address the limitations of models with only fixed coefficients.

$$u_{ijt} = x_{jt}\beta_0 - \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt} \tag{2.9}$$

$$\delta_{jt} = x_{jt}\beta_0 - \alpha p_{jt} + \xi_{jt} \tag{2.10}$$

$$u_{ijt} = \delta_{jt} + \epsilon_{ijt} \tag{2.11}$$

3.Market level:Setting

In many applications, the key data are observed at the market level. Typically, one observes:

- The number of goods J_t available to consumers in each market t;
- Their prices and other observable characteristics p_t, x_t;
- Their observed market shares $\tilde{s}_{jt}$, typically measured as the total quantity of good j sold in market t divided by the number of consumers (e.g., households) in that market;
- The distribution of consumer characteristics (d_{it}, y_{it}) in each market;
- Possibly, additional variables w_t (e.g., cost shifters) that might serve as appropriate instruments.

3.Market level: BLP

- **Goal:** Estimate demand parameters $\theta = (\theta_1, \theta_2)$
- $\theta_1 = (\alpha_0, \beta_0)$: linear parameters
- $\theta_2 = (\beta_d, \beta_\nu)$: nonlinear parameters in random coefficients
- **Key steps:**
  1. Fix a trial θ
  2. Invert market shares $\tilde{s}_{jt}$ to recover $\delta_{jt}(\theta_2)$
  3. Recover unobserved $\xi_{jt}(\theta)$
  4. Evaluate GMM objective

3.Market level: BLP

GMM Objective:

$$\min_{\theta} \quad g(\xi(\theta))'\Omega g(\xi(\theta)) \tag{3.1}$$

Moment condition:

$$g(\xi(\theta)) = \frac{1}{N}\sum_{j,t} \xi_{jt}(\theta) z_{jt} \tag{3.2}$$

Constructing $\xi_{jt}(\theta)$:

$$\xi_{jt}(\theta) = \delta_{jt}(\theta_2) - x'_{jt}\beta + \alpha p_{jt} \tag{3.3}$$

3.Market level: BLP

Matching predicted to observed shares:

$$\log(\tilde{s}_{jt}) = \log(\sigma_j(\delta_t, x_t, \theta_2, J_t)) \tag{3.4}$$

Predicted share function:

$$\sigma_j(\cdot) = \int \frac{\exp[\delta_{jt}(\theta_2) + x'_{jt}\tilde{\beta}]}{1 + \sum_k \exp[\delta_{kt}(\theta_2) + x'_{kt}\tilde{\beta}]} f_{\tilde{\beta}}(\tilde{\beta}|\theta_2) d\tilde{\beta} \tag{3.5}$$

Solve for δ_{jt} such that:

$$\sigma_j(\delta_t, x_t, \theta_2, J_t) = \tilde{s}_{jt}$$

3.Market level: BLP

- **Cost Shifters**
  - *Definition:* Supply-side variables like input costs, taxes, or tariffs.
  - *Advantage:* Clearly exogenous; commonly used in structural models.
  - *Limitation:* Limited variation or measurement error possible.
- **BLP Instruments**
  - *Definition:* Characteristics of competing products (e.g., rival's features).
  - *Advantage:* Exploits substitution across products; widely accepted.
  - *Limitation:* Requires independence from product-specific unobservables (ξ_{jt}).

3.Market level: BLP

- **Waldfogel-Fan Instruments**
  - *Definition:* Demographics from nearby markets (e.g., average income).
  - *Advantage:* Capture spatial spillovers; useful in zone pricing contexts.
  - *Limitation:* Depend on pricing geography and data resolution.
- **Exogenous Market Structure Shifts**
  - *Definition:* Firm entry/exit, mergers, or ownership changes.
  - *Advantage:* Change competitive intensity; often impactful.
  - *Limitation:* Need exogeneity and observable events.
- **Optimal Instruments**
  - *Definition:* Function of observables maximizing moment efficiency.
  - *Form:*

$$z_{jt}^* = \psi_{jt}^{-1}\mathbb{E}\left[\frac{\partial \xi_{jt}(\theta)}{\partial \theta}\middle| z_{jt}\right]$$

  - *Limitation:* Infeasible without full distributional knowledge of ξ.

3.Motivation for Adding Supply Side

- Demand-only estimation can identify θ_1, θ_2, but precision may be low.
- Instrumental variables may be weak or hard to justify.
- Supply-side modeling adds additional moment conditions.
- Enables interpretation of marginal cost and markup.
- Supports policy counterfactuals (mergers, taxes, pricing rules).

3.How to Add Supply Side Estimation

1. Estimate demand parameters $\theta = (\theta_1, \theta_2)$ using observed shares and instruments.
2. Recover mean utility δ_{jt} via share inversion.
3. Compute implied markups from demand elasticities:

$$mc_{jt} = p_{jt} - \text{markup}_{jt}(\theta)$$

4. Assume a marginal cost function:

$$mc_{jt} = w_{jt}'\gamma + \omega_{jt}$$

5. Construct additional GMM moments:

$$\mathbb{E}[\omega_{jt}(\theta, \gamma) \cdot w_{jt}] = 0$$

3.What Does Supply Side Estimation Give Us?

- **Improved identification of θ_1**:
  - Exploits cost-side variation.
  - Less dependent on demand-side instruments.
- **Estimation of cost structure parameters γ**:
  - Provides insight into marginal cost behavior.
  - Useful for merger simulations and policy evaluation.
- **Full structural interpretation**:
  - Enables joint analysis of demand and supply.
  - Supports realistic counterfactuals.

4.Working paper

Identification in Differentiated Products Markets Using Market Level Data

Steven T. Berry
Philip A. Haile

Abstract

This paper studies *nonparametric identification* in differentiated products markets using only market-level data. It allows for *random utility models*, endogenous product characteristics, and unobserved heterogeneity on both demand and supply sides.

Two identification strategies are proposed: one based on *instrumental variables*, and another using a *change-of-variables approach*.

The results formalize **Bresnahan's (1982)** test for distinguishing between oligopoly models.

4.Assumption 1a and 1b: Structure of Utility

Assumption 1a: For all $\tilde{u}t \in \mathcal{U}$, $\quad \tilde{u}it(x_{jt}, p_{jt}, \xi_{jt}) = \mu_t(\delta_{jt}, x_{jt}^{(2)}, p_{jt}) \quad$ where μ_t is strictly increasing in δ_{jt}.

Assumption 1b: There exists a monotonic function Γ_t such that $\Gamma_t(\tilde{u}it(xjt, p_{jt}, \xi_{jt})) = \mu_t(\delta_{jt}, x_{jt}^{(2)}, \omega_{it}) - p_{jt}$

Motivation: These assumptions impose quasilinearity and index restrictions to simplify the indirect utility specification and facilitate identification through monotonic transformations.

4.Assumptions 2 and 3: Market Behavior and Substitution

Assumption 2: Market shares s_j are strictly positive for all j at any observed $x_{kt}, p_{kt}, \xi_{kt}{}_{k \in \mathcal{J}}$.

Assumption 3 (Connected Substitutes): The directed graph of substitutes $\Sigma(\mathcal{J})$ is strongly connected: for all $x_{jt}, p_{jt}, \xi_{jt}{}_{j \in \mathcal{J}}$, there exists a substitution path between any pair j, j'.

Motivation: These ensure observability of all products and interconnectedness of substitution patterns, avoiding isolated or unused products.

4.Assumptions 4 and 5: Instrument Validity and Completeness

Assumption 4: $\mathbb{E}[\xi_{jt} \mid \tilde{z}_t, x_t] = 0$ for all j (instrument exogeneity)

Assumption 5: For any $B(s_t, p_t)$ with finite expectation,
n$\mathbb{E}[B(s_t, p_t) \mid \tilde{z}_t, x_t] = 0 \Rightarrow B(s_t, p_t) = 0$ almost surely

Motivation: These conditions validate IV strategies and ensure the model is rich enough (completeness) to identify structural parameters from observable variation.

4.Proof of Theorem 1 (Identification of ξ_{jt})

Step-by-step:

- Equation: $x_{jt} + \xi_{jt} = \sigma_j^{-1}(s_t, p_t)$
- Take conditional expectation: $\mathbb{E}[\xi_{jt}|\tilde{z}_t, x_t] = \mathbb{E}[\sigma_j^{-1}(s_t, p_t)|\tilde{z}_t, x_t] - x_{jt}$
- By Assumption 4: $\mathbb{E}[\xi_{jt}|\tilde{z}_t, x_t] = 0$, so:

$$\mathbb{E}[\sigma_j^{-1}(s_t, p_t)|\tilde{z}_t, x_t] - x_{jt} = 0$$

- Suppose an alternative function $\bar{\sigma}_j^{-1}$ satisfies:

$$\mathbb{E}[\bar{\sigma}_j^{-1}(s_t, p_t) - x_{jt}|\tilde{z}_t, x_t] = 0$$

- Define $B(s_t, p_t) = \sigma_j^{-1}(s_t, p_t) - \bar{\sigma}_j^{-1}(s_t, p_t)$, so:

$$\mathbb{E}[B(s_t, p_t)|\tilde{z}_t, x_t] = 0$$

- By Assumption 5 (completeness): $B = 0$ a.s. $\Rightarrow \bar{\sigma}_j^{-1} = \sigma_j^{-1}$

4.Assumptions 6–8 (Supply Side Identification)

Assumption 6. *Support Condition ("Covering Condition")*

$$\text{supp } p_t \mid \{x_{jt}, \xi_{jt}\}_{j\in\mathcal{J}} \supseteq \text{supp}\left(\mu_1(\delta_{1t}, \omega_{it}), \ldots, \mu_J(\delta_{Jt}, \omega_{it})\right)_t \mid \{x_{jt}, \xi_{jt}\}_{j\in\mathcal{J}}$$

This condition ensures that observed prices are rich enough to "cover" variation in consumers' willingness to pay.

Assumption 7. *Differentiability of Market Shares*

$\sigma_j(\delta_t, p_t)$ is continuously differentiable w.r.t. p_k for all $j, k \in \mathcal{J}$.

This is needed to apply the first-order conditions and identify supply-side behavior.

Assumption 8. *Structure of Marginal Costs and Residual Revenue Function*

(i) $mc_j(q_{jt}, \zeta_{jt})$ is strictly increasing in ζ_{jt}

(ii) $u_j(\delta_{jt}, p_{jt}, \omega_{it})$ is strictly decreasing in p_{jt}

(iii) There exists a function ψ_j such that:

$$mc_j(M_t s_{jt}, \zeta_{jt}) = \psi_j(s_t, M_t, D_t(s_t, p_t), p_t)$$

4.Theorems

Theorem 2. *Under Assumptions 1b and 2–6, the joint distribution of*

$(v_{i1t}, \ldots, v_{iJt})$ conditional on any $\{x_{kt}, p_{kt}, \xi_{kt}\}_{k \in \mathcal{J}} \in \chi^{\mathcal{J}}$ is identified.

Theorem 3. *Suppose that Assumptions 1a, 2–5, 7 and 8 hold. Then for all j*

(i) η_{jt} is identified for all t, and

(ii) if ψ_j is known, the function $mc_j(q_{jt}, \zeta_{jt})$ is identified on the support of (q_{jt}, ζ_{jt}).

4.Assumptions 9–13 for Identification

Assumption 9. There is a unique vector of equilibrium prices associated with any (δ, ζ).

Assumption 10. The random variables $(\xi_1, \ldots, \xi_J, \eta_1, \ldots, \eta_J)$ have a positive joint density $f_{\xi,\eta}$ on $\mathbb{R}^{2J}$.

Assumption 11. The vector function $\left(\sigma_1^{-1}, \ldots, \sigma_J^{-1}, \pi_1^{-1}, \ldots, \pi_J^{-1}\right)'$ has continuous partial derivatives and nonzero Jacobian determinant.

Assumption 12. $(x_t, z_t) \perp\!\!\!\perp (\xi_t, \eta_t)$.

Assumption 13. $\text{supp}(x_t, z_t) = \mathbb{R}^{2J}$.

4.Theorem 4 and 5: Identification in Differentiated Product Markets

Theorem 4. *Suppose Assumptions 1a, 2, 3, and 8–13 hold. Then for all j*

(i) ξ_{jt} *is identified for all t, and*

(ii) *the function* $s_j\left(\{x_{kt}, p_{kt}, \xi_{kt}\}_{k\in\mathcal{J}}\right)$ *is identified on* $\mathcal{X}^{\mathcal{J}}$.

Theorem 5. *Suppose Assumptions 1b, 2, 3, 6, and 8–13 hold. Then the joint distribution of* $(v_{i1t}, \ldots, v_{iJt})$ *conditional on any* $\{x_{kt}, p_{kt}, \xi_{kt}\}_{k\in\mathcal{J}} \in \mathcal{X}^{\mathcal{J}}$ *is identified.*

4.Discriminating Between Oligopoly Models

Motivation: Can different models of firm conduct (e.g., marginal cost pricing vs. monopolistic pricing) be distinguished empirically?
Main Idea: Use variations in the residual marginal revenue function ψ_j to test supply models.
Key Setup:

- Under Assumption 8, marginal cost satisfies:

$$mc_j(q_{jt}, \zeta_{jt}) = \psi_j(s_t, D_t(s_t, p_t), p_t)$$

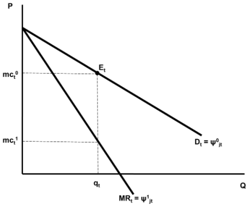
- Let ψ_j^0 and ψ_j^1 denote marginal revenue functions under the null and alternative hypotheses, respectively.

Identification via Rotations:

- Suppose market conditions change such that ψ_j rotates (e.g., due to instrument variation).
- If null hypothesis is true: implied marginal cost mc_j^0 should remain invariant.
- If under rotation, marginal cost is instead $mc_j^1 \neq mc_j^0$: null is falsified.

4.Discriminating Between Oligopoly Models (Visual)

Illustration of Model Testing:



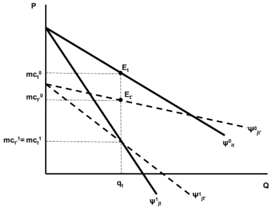


Figure 2: Mapping of E_t to marginal cost; Figure 3: Rotation rejects false null.