Two-sided Markets, Pricing, and Network Effects

Theoretical and Empirical Study

Dan (Daisy) Song Jing You

Hong Kong University of Science and Technology

April 17, 2025

Outline

Introduction

- A central aspect of platform is the role of network effects:
  - **Network effect** refers to any situation in which the value of a product, service, or platform depends on the number of buyers, sellers, or users who leverage it
  - **Direct network effects** occur when the value of a product, service, or platform increases simply because the number of users increases, causing the network itself to grow
  - **Indirect network effects** occur when a platform or service depends on two or more user groups. As more people from one group join the platform, the other group receives a greater value.

Introduction

- Indirect network leads to feedback loops between the two sides of the market, thus increasing efficiencies and also potential market power
- Pricing is one of the key tools that platforms can manage to success
- Seesaw effect describes how pricing decisions on one side of a platform affect participation on the other side — like a seesaw: push down on one side, and the other goes up

Main Focus: **indirect network effects and pricing strategies in two-sided markets**

Introduction

- Two-sided market: at least two distinct sets of agents interact through an intermediary and in which the behavior of each set of agents directly impacts the utility, or the profit of the other agents
- **Two-sided market** vs **Two-sided strategies**
- Treating two-sidedness as a market concept

Now let's proceed to the benchmark model in Armstrong(2006)

Armstrong(2006): Monopoly Pricing

$$u_j^i = \alpha_j^i n^i + \xi_j^i$$

Basic Setting

1. Two groups of agents: j = 1, 2
2. Only one platform in the market: i = 1
3. No fixed benefits from using the platform: $\xi_j^i = 0$
4. One group cares about the number of agents in the other group but not itself
5. Interaction benefits are different across groups: $\alpha_j^i \neq \alpha_k^i$
6. The platform can charge a per-agent fee to each group of agents

Armstrong(2006): Monopoly Pricing

Net Payoff of Agents

$$u_1 = \alpha_1 n_2 - p_1 \qquad u_2 = \alpha_2 n_1 - p_2 \tag{1}$$

- α_1: Interaction benefit a group-1 agent gets from each group-2 agent
- α_2: Interaction benefit a group-2 agent gets from each group-1 agent
- n_1: number of group-1 agents on the platform
- n_2: number of group-2 agents on the platform
- p_1, p_2: prices charged by the platform to groups 1 and 2

Armstrong (2006): Monopoly Pricing

Demand Functions

$$n_1 = \phi_1(u_1) \qquad n_2 = \phi_2(u_2) \tag{2}$$

- n_1, n_2: numbers of agents from groups 1 and 2 who join the platform
- ϕ_1, ϕ_2: increasing demand functions mapping u_1, u_2 (utilities) to participation
- u_1, u_2: utilities offered to agents in groups 1 and 2

$$\pi = (p_1 - f_1)n_1 + (p_2 - f_2)n_2$$

Armstrong (2006): Monopoly Pricing

Profit Function

$$\pi(u_1, u_2) \;=\; \phi_1(u_1)\Big[\alpha_1\,\phi_2(u_2) - u_1 - f_1\Big] \;+\; \phi_2(u_2)\Big[\alpha_2\,\phi_1(u_1) - u_2 - f_2\Big] \quad (3)$$

- $\pi(u_1, u_2)$: platform's profit in terms of utilities
- f_1, f_2: per-agent costs for groups 1 and 2
- α_1, α_2: cross-group interaction benefits
- ϕ_1, ϕ_2: same demand functions as in the previous slide

$$p_1 = \alpha_1 n_2 - u_1, \quad p_2 = \alpha_2 n_1 - u_2$$

Armstrong (2006): Monopoly Pricing

Let the aggregate consumer surplus of group $i = 1, 2$ be $v_i(u_i)$, where $v_i(\cdot)$ satisfies the envelope condition $v_i'(u_i) \equiv \phi_i(u_i)$. Then welfare, as measured by the unweighted sum of profit and consumer surplus, is

$$w = \pi(u_1, u_2) \ + \ v_1(u_1) \ + \ v_2(u_2).$$

It is easily verified that the welfare-maximizing outcome has the utilities

$$u_1 = (\alpha_1 + \alpha_2)\, n_2 \ - \ f_1, \qquad u_2 = (\alpha_1 + \alpha_2)\, n_1 \ - \ f_2.$$

Armstrong (2006): Monopoly Pricing

From expression (1), the socially optimal prices satisfy

$$p_1 = f_1 \; - \; \alpha_2 \, n_2, \qquad p_2 = f_2 \; - \; \alpha_1 \, n_1.$$

From expression (2), the profit-maximizing prices satisfy:

$$p_1 \; = \; f_1 \; - \; \alpha_2 \, n_2 \; + \; \frac{\phi_1(u_1)}{\phi_1'(u_1)}, \quad p_2 \; = \; f_2 \; - \; \alpha_1 \, n_1 \; + \; \frac{\phi_2(u_2)}{\phi_2'(u_2)}.$$

Armstrong (2006): Monopoly Pricing

Proposition 1. Write

$$\eta_1(p_1 \mid n_2) = \frac{p_1\, \phi_1'(\alpha_1 n_2 - p_1)}{\phi_1(\alpha_1 n_2 - p_1)}, \quad \eta_2(p_2 \mid n_1) = \frac{p_2\, \phi_2'(\alpha_2 n_1 - p_2)}{\phi_2(\alpha_2 n_1 - p_2)}.$$

for a group's price elasticity of demand given the other group's participation level. Then the profit-maximizing pair of prices satisfy

$$\frac{p_1 - (f_1 - \alpha_2 n_2)}{p_1} = \frac{1}{\eta_1(p_1 \mid n_2)}, \quad \frac{p_2 - (f_2 - \alpha_1 n_1)}{p_2} = \frac{1}{\eta_2(p_2 \mid n_1)}.$$

Armstrong (2006): Price Distortions in Monopoly Pricing

Difference between profit-maximizing and efficient prices(Tan and Wright, 2018):

$$P_i^* - P_i^e = \underbrace{\eta_i\left(P_i^*; q_j^*\right)}_{\text{markup}} + \underbrace{q_j^e\left(\gamma_j(P_1^e, P_2^e) - \tilde{\gamma}_j(P_1^e, P_2^e)\right)}_{\text{Spence distortion}}$$
$$+ \underbrace{q_j^e\left(\tilde{\gamma}_j(P_1^e, P_2^e) - \tilde{\gamma}_j(P_1^*, P_2^*)\right)}_{\text{displacement distortion}} + \underbrace{\left(q_j^e - q_j^*\right)\left(\tilde{\gamma}_j(P_1^*, P_2^*) - \sigma\right)}_{\text{scale distortion}}$$

Combined effect may raise or lower prices compared to efficient levels on either side.

Armstrong (2006): Extensions of Monopoly Pricing

- **Chicken & Egg Problem** (Caillaud and Jullian, 2003): To overcome such pessimistic beliefs and successfully launch a platform, the platform must subsidize one side of the market through monetary payments or by offering additional services that raise the stand alone values and make them positive to attract users.
- **Vertical integration**: Offering content in addition to matching services allows the platform to raise the value that buyers derive from joining and secure their participation(Haigu and Spulber, 2013; Miao, 2009; Decorniere and Taylor, 2014; Wen and Zhu, 2019; Zhu and Liu, 2016)
- **Non-negative prices** (Amelio and Jullien, 2012): when the unconstrained monopoly price is negative, constraning the price to be non-negative has a detrimental effect not only on the platform but also on consumers

Armstrong (2006): Extensions of Monopoly Pricing

- **Distprtionary taxition and two-part tariffs**: taxation of usage has a distortionary effect and in this case the choice of transaction fees matters for efficiencies(Calvano, 2013)

- **Dynamic pricing**: The platform thus is treated as a durable good with two-sided network externalities and random obsolescence. Cabral (2019) finds that for a wide set of parameter values, platform size at time t follows a bimodal distribution: Either the platform achieves a “large” size or remains at very “low” size. Peitz et al. (2017) finds that provides an informational rationale for introductory pricing

Armstrong(2006): Competition for the Market

Basic Setting

1. Two groups of agents: 1, 2
2. Two platforms in the market: i, j
3. No fixed benefits from using the platform: $\xi_j^i = 0$
4. One group cares about the number of agents in the other group but not itself
5. Interaction benefits are different across groups but same across platforms: $\alpha_1^i = \alpha_2^i, \alpha_1^j = \alpha_2^j$
6. The platform can charge a per-agent fee to each group of agents
7. Each agent chooses to join a single platfom [single-homing]

Armstrong(2006): Competition for the Market

Basic model. There are two groups of agents, labeled 1 and 2, and there are two platforms, A and B, which enable these groups to interact. Groups 1 and 2 obtain the respective utilities $\left\{u_1^i,\, u_2^i\right\}$ if they join platform i. These utilities are determined similarly to the monopoly model in (1): if platform i attracts n_1^i and n_2^i members of the two groups, then the utilities on platform i are

$$u_1^i = \alpha_1 n_2^i - p_1^i, \qquad u_2^i = \alpha_2 n_1^i - p_2^i. \tag{5}$$

Armstrong(2006): Competition for the Market

When group 1 is offered a choice of utilities u_1^A and u_1^B from the two platforms, and group 2 is offered the choice u_2^A and u_2^B, suppose the number of each group who join platform i is given by the following Hotelling specification:

$$n_1^i = \tfrac{1}{2} + \frac{u_1^i - u_1^j}{2\,t_1}, \qquad n_2^i = \tfrac{1}{2} + \frac{u_2^i - u_2^j}{2\,t_2}, \tag{6}$$

Here, agents in each group are assumed to be uniformly located along a unit interval, with the two platforms at the endpoints. The parameters $t_1, t_2 > 0$ capture the extent of product differentiation (or "transport costs") for the two groups, describing how competitive the two-sided market is.

Armstrong(2006): Competition for the Market

Putting (6) together with (5), and using the fact that $n_1^j = 1 - n_1^i$, gives the following implicit expressions for market shares:

$$n_1^i \;=\; \tfrac{1}{2} + \frac{\alpha_1\left(2\,n_2^i \;-\; 1\right) \;-\; \left(p_1^i \;-\; p_1^j\right)}{2\,t_1}, \qquad n_2^i \;=\; \tfrac{1}{2} + \frac{\alpha_2\left(2\,n_1^i \;-\; 1\right) \;-\; \left(p_2^i \;-\; p_2^j\right)}{2\,t_2}$$

Keeping its group-2 price fixed, expression (7) shows that an extra group-1 agent on a platform attracts a further α_2/t_2 group-2 agents to that platform.

Armstrong(2006): Competition for the Market

Suppose platforms A and B offer the respective price pairs $\left(p_1^A,\, p_2^A\right)$ and $\left(p_1^B,\, p_2^B\right)$. Given these prices, solving the simultaneous equations (7) implies that the market shares are:

$$n_1^i \;=\; \tfrac{1}{2} + \tfrac{1}{2}\,\frac{\alpha_1\left(p_2^j - p_2^i\right) \;+\; t_2\left(p_1^j - p_1^i\right)}{t_1\, t_2 \;-\; \alpha_1\, \alpha_2}, \quad n_2^i \;=\; \tfrac{1}{2} + \tfrac{1}{2}\,\frac{\alpha_2\left(p_1^j - p_1^i\right) \;+\; t_1\left(p_2^j - p_2^i\right)}{t_1\, t_2 \;-\; \alpha_1\, \alpha_2}$$

Armstrong(2006): Competition for the Market

As with the monopoly model, suppose each platform has a per-agent cost f_1 for serving group 1 and f_2 for serving group 2. Then platform i's profit is:

$$(p_1^i - f_1) \times \left[\frac{1}{2} + \frac{1}{2}\,\frac{\alpha_1\,(p_2^j - p_2^i) + t_2\,(p_1^j - p_1^i)}{t_1\,t_2 - \alpha_1\,\alpha_2}\right] + (p_2^i - f_2) \times \left[\frac{1}{2} + \frac{1}{2}\,\frac{\alpha_2\,(p_1^j - p_1^i) + t_1\,(p_2^j - p_2^i)}{t_1\,t_2 - \alpha_1\,\alpha_2}\right]$$

If (8) is satisfied, one can show that no asymmetric equilibria exist, and in a *symmetric equilibrium* where each platform offers the same price pair (p_1, p_2), the first-order conditions yield:

$$p_1 = \underbrace{f_1}_{\text{cost}} + \underbrace{t_1}_{\text{market power}} - \underbrace{\left(\frac{\alpha_2}{t_2}\right)}_{\text{extra group-2 agents}} \times \underbrace{(\alpha_1 + p_2 - f_2)}_{\text{profit from an extra group-2 agent}} \tag{7}$$

Armstrong(2006): Competition for the Market

Alternative tariffs. Uniform prices.

Suppose $f_1 = f_2 = f$. It makes little sense to discuss price discrimination if the costs differ significantly across groups. Assume each platform cannot set different prices for groups 1 and 2, so platform i chooses a single uniform price p^i. (Perhaps sex discrimination laws prevent differential pricing by nightclubs.) Then platform i's profit is $\left(p^i - f\right)\left(n_1^i + n_2^i\right)$. From (9), total demand for platform i is

$$n_1^i + n_2^i = 1 + \frac{1}{2}\frac{t_1 + t_2 + \alpha_1 + \alpha_2}{t_1 t_2 - \alpha_1 \alpha_2}\left(p^j - p^i\right).$$

Solving the resulting first-order conditions implies the equilibrium uniform price is

$$p = f + 2\frac{t_1 t_2 - \alpha_1 \alpha_2}{t_1 + t_2 + \alpha_1 + \alpha_2} \tag{8}$$

Armstrong(2006): Competition for the Market

Two-Part Tariffs: a continuum of symmetric equilibria exist. Let $0 \leq \gamma_1 \leq 2\alpha_1$ and $0 \leq \gamma_2 \leq 2\alpha_2$ be the marginal prices charged to group 1 and group 2, respectively. An equilibrium exists in which both platforms offer the same pair of two-part tariffs $\left(T_1, T_2\right)$ defined by:

$$T_1 = p_1 + \gamma_1 n_2, \qquad T_2 = p_2 + \gamma_2 n_1,$$

where the *fixed fees* p_1, p_2 satisfy

$$p_1 = f_1 + t_1 - \alpha_2 + \tfrac{1}{2}\left(\gamma_2 - \gamma_1\right), \qquad p_2 = f_2 + t_2 - \alpha_1 + \tfrac{1}{2}\left(\gamma_1 - \gamma_2\right). \quad (17)$$

$$\pi = \frac{t_1 + t_2 - \alpha_1 - \alpha_2}{2} + \frac{\gamma_1 + \gamma_2}{4}.$$

Armstrong (2006): Main Conclusions of Competition for the Market

Key determinant: The nature of competition depends on the balance between:

- **Interaction benefits** across platforms, and
- **Horizontal differentiation** (measured by differences in agents' stand-alone values/ transportation costs)

Implications:

- **Small interaction benefits** $\Rightarrow$ Platforms can share the market
- **Large interaction benefits** $\Rightarrow$ One platform tends to dominate

Armstrong (2006): Extension of Competition for the Market

- **Divide-and-Conquer strategies**: Strategies that set prices below cost for some consumers compensated with prices above cost for other consumers (Caillaud and Jullien, 2001/2003; Jullien, 2011)

- **Congestion within sides**: Network externalities within sides may originate from congestion effects or competition between agents on the same sides (Belleflamme and Toulemonde, 2009), which may also prevent tipping (Halaburda et al., 2018)

- **Dynamic competition**: whether there is increasing dominance of the largest network eventually resulting in tipping? (Cabral, 2011; Halaburda et al., 2020; Biglaiser and Cremer, 2020))

Armstrong(2006): Competition on the Market

Basic Setting

1. Two groups of agents: 1, 2
2. Two platforms in the market: i, j
3. No fixed benefits from using the platform: $\xi_j^i = 0$
4. One group cares about the number of agents in the other group but not itself
5. Interaction benefits are different across groups but same across platforms: $\alpha_1^i = \alpha_2^i, \alpha_1^j = \alpha_2^j$
6. The platform can charge a per-agent fee to each group of agents
7. Each agent chooses to join a single platfom [single-homing]

Armstrong(2006): Competition on the Market

A general framework. Suppose there are two (possibly asymmetric) platforms that facilitate interaction between two groups of agents. Suppose that group-2 agents are heterogeneous: if there are n_1^i group-1 agents on platform i, the number of group-2 agents prepared to pay a fixed fee p_2^i to join that platform is

$$n_2^i = \phi^i(n_1^i, p_2^i)$$

where ϕ^i is decreasing in p_2^i and increasing in n_1^i. A group-2 agent's decision to join one platform does not depend on whether she also joins the rival platform.

Let $R^i(n_1^i, n_2^i)$ denote platform i's revenue from group 2 when it has n_1^i group-1 agents and sets its group-2 price so that n_2^i group-2 agents join. Formally,

$$R^i\Big(n_1^i, \phi^i(n_1^i, p_2^i)\Big) = p_2^i \phi^i(n_1^i, p_2^i)$$

$$u_1^i = U^i(n_2^i) - p_1^i.$$

Armstrong(2006): Competition on the Market

When a group-1 agent's utility with platform i is u_1^i, suppose the platform attracts

$$n_1^i = \Phi^i(u_1^i, u_1^j)$$

where Φ^i is increasing in its first argument and decreasing in its second. Let $C^i(n_1^i, n_2^i)$ denote the total cost to platform i of serving both groups. Then platform i's profit is

$$\pi^i = n_1^i p_1^i + R^i(n_1^i, n_2^i) - C^i(n_1^i, n_2^i)$$

where R^i is platform i's revenue from group 2 (as defined previously).

Armstrong(2006): Competition on the Market

Next, in equilibrium, the number of group-2 agents on each platform is derived from the equilibrium market shares for group 1. Suppose platform i sets a group-1 utility $\hat{u}_1^i$ and thus attracts $\hat{n}_1^i$ agents. By varying p_1^i and n_2^i so that $\hat{u}_1^i = U^i(n_2^i) - p_1^i$ remains constant, the platform's profit becomes

$$\pi^i = \hat{n}_1^i \left[U^i(n_2^i) - \hat{u}_1^i \right] + R^i(\hat{n}_1^i, n_2^i) - C^i(\hat{n}_1^i, n_2^i).$$

Given $\hat{n}_1^i$, platform i chooses $\hat{n}_2^i$ to maximize $\hat{n}_1^i U^i(\cdot) + R^i(\hat{n}_1^i, \cdot) - C^i(\hat{n}_1^i, \cdot)$, i.e.,

$$\hat{n}_2^i = \arg\max_{n_2} \left[\hat{n}_1^i U^i(n_2) + R^i(\hat{n}_1^i, n_2) - C^i(\hat{n}_1^i, n_2) \right]$$

The equilibrium price to group 2 is $\hat{p}_2^i$, where

$$\hat{n}_2^i = \phi^i(\hat{n}_1^i, \hat{p}_2^i)$$

Armstrong (2006): Main Conclusions of Competition on the Market

Proposition : In the competitive bottleneck model, in any equilibrium the number of group-2 agents on a platform is chosen to maximize the joint surplus of the platform and its group-1agents, and the interests of group 2 are ignored. Unless there are externalities within the set of group-2 agents, there are too few group-2 agents on each platform given the distribution of group-1agents on each platform

This highlights a welfare inefficiency due to platform-side optimization focused on group 1 without considering group 2

Armstrong (2006): Extension of Competition on the Market

- **Multihoming on both sides**: Doganoglu and Wright (2006) incremental pricing principle
- **Concentration and entry of platforms**: seesaw principle (Rochet and Tirole, 2006; Correia-da Silve et al., 2019; Anderson and Peitz, 2020)
- **Horizontal mergers**: Because of two-sidedness, a merger may not lead to higher prices and lower consumer surplus on both sides (Chandra and Collard Wexler, 2009; Tan and Zhou, 2020; Leonello, 2010)
- **Collusion**: Dewenter et al.(2001), Lefouili and Pinho(2020)
- **Exclusive dealing and Bundling**

Conclusion and Future Directions

Monopoly and Competition.

- Monopoly Platforms: Compared profit-maximizing versus welfare-maximizing outcomes, showing how network effects influence price levels.
- Platform Competition: Discussed how modeling "competition for the market" versus "competition on the market" must account for single-homing or multi-homing.

Gaps and Future Research.

- *Dynamic Aspects:* Literature remains largely static, with limited work on ignition or evolving platform competition.
- *Multi-Homing Nuances:* Different values for agents' interactions across sides require richer models of multi-homing behavior.
- *Discrimination and Design:* Need more flexible models covering discriminatory practices, bundling, feedback/recommendation systems, integration, and entry rules.
- *Industry-Specific Studies:* Areas like media, finance, housing, transportation, and health insurance offer promising directions for deeper analysis and policy implications.