Structural Analysis of Vertical Contracting

Empirical Framework

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Two-Stage Framework for Analysis

- Researchers often model vertical contracting using a two-stage framework.
- Stage 1: Supply
    - Firms negotiate contracts and take payoff-relevant actions (e.g., investment, pricing).
- Stage 2: Demand
    - Consumers purchase products/services provided by upstream and downstream firms.

Contracts

- Contracts represent agreements between upstream and downstream firms.
- Denoted as:

$$C_{ij} \in \mathbb{C}$$

- Where:
  - C_{ij}: Contract between upstream firm i and downstream firm j.
  - $\mathbb{C}$: Set of feasible contracts.
- Null contract (C_0) represents the disagreement (no contract) outcome.

Payoff-Relevant Actions

- Actions not explicitly specified in contracts but affect payoffs.
- Denoted by:

$$a = \{a_0, a_1(\mathbb{C}, a_0)\}$$

- Where:
  - a_0: Actions chosen simultaneously with contracts.
  - $a_1(\mathbb{C}, a_0)$: Actions chosen after contracts and initial actions.
- Examples:
  - Downstream pricing, product availability.
  - Effort provision or investment.

Payoff Representation

Detailed Representation:

- Payoffs for each upstream firm i and downstream firm j are represented as:

$$\Pi_{U_i}(\mathbb{C}, \boldsymbol{a}_0), \quad \Pi_{D_j}(\mathbb{C}, \boldsymbol{a}_0)$$

- These payoffs implicitly depend on:
    - Subsequent actions taken by firms ($\boldsymbol{a}_1(\cdot)$).
    - Consumer actions captured by demand functions.

Demand Representation:

- Upstream demand: $\bar{D}(\cdot)$.
- Downstream demand: $\underline{D}(\cdot)$.

Payoff Representation

Demand Representation:

- Example (successive monopoly with per-unit pricing):

$$\mathbb{C} = \{w\}, \quad a_0 = \emptyset, \quad a_1 = \{p\}$$
$$\bar{D}(w) = \underline{D}(w) = D\left(p^m\left(w + c_R\right)\right)$$

- {w}: upstream price;
- {p}: downstream price;
- p^m: monopoly price coefficient;
- c_R: retailer's marginal cost.
- In this case, upstream and downstream demand coincide because of a single retailer-manufacturer setup.

General Case:

- When there are multiple upstream firms or consumers do not always purchase upstream products, $\bar{D}(\cdot)$ and $\underline{D}(\cdot)$ will typically differ.

Example: Medical Devices (Grennan, 2013)

- Contracts $\mathbb{C}$ between stent manufacturers (i) and hospitals (j) specify linear prices $\boldsymbol{w} = \{w_{ij}\}$.
- The null contract $\mathbb{C}_0$ is represented by $w_{ij} = \infty$, meaning no trade occurs.

Example: Medical Devices (Grennan, 2013)

- The payoff for stent manufacturer i is:

$$\Pi_{U_i}(\mathbb{C}) = \sum_{j:\mathbb{C}_{ij} \neq \mathbb{C}_0} (w_{ij} - c_i)\, \bar{D}_{ij}\left(\{w_{kj}\}_{k \in \mathcal{I}}\right)$$

  - w_{ij} is the price in the contract between manufacturer i and hospital j.
  - c_i is the marginal cost of producing stent i.
  - $\bar{D}_{ij}(\cdot)$ is the quantity of stent i used at hospital j, which depends on:
    - Preferences of doctors and patients at hospital j.
    - Contracts $\{w_{kj}\}$ for all stents signed by hospital j.
- The payoff for hospital j is:

$$\Pi_{D_j}(\mathbb{C}) = W_j\left(\{w_{kj}\}_{k \in \mathcal{I}}\right)$$

  - $W_j(\cdot)$ is the welfare of hospital j, which depends on the prices $\{w_{kj}\}$ for all stents it uses.

Example: Health Insurers and Hospitals (Ho and Lee, 2017)

- Contracts between hospitals (upstream) and insurers (downstream) in the U.S. healthcare industry.
- Contracts $\mathbb{C}$ specify payments per hospital admission:

$$\boldsymbol{w} = \{w_{ij}\}$$

- Insurers also set premiums for households:

$$\boldsymbol{p} = \{p_j\}$$

- Demand terms:
  - $\underline{D}_j(\boldsymbol{p}, \boldsymbol{N})$: Households enrolled in insurer j.
  - $\bar{D}_{ij}(\boldsymbol{p}, \boldsymbol{N})$: Admissions from insurer j's enrollees to hospital i.
- $\boldsymbol{N} = \{ij : \mathbb{C}_{ij} \neq \mathbb{C}_0\}$ represents the network of contracts.

Example: Health Insurers and Hospitals (Ho and Lee, 2017)

- Hospital i's payoff:

$$\Pi_{U_i}(\mathbb{C}, \boldsymbol{p}) = \sum_{j:\mathbb{C}_{ij} \neq \mathbb{C}_0} (w_{ij} - c_i)\, \bar{D}_{ij}(\boldsymbol{p}, \boldsymbol{N})$$

  - w_{ij}: Payment per admission.
  - c_i: Per-admission cost.
- Insurer j's payoff:

$$\Pi_{D_j}(\mathbb{C}, \boldsymbol{p}) = (p_j - \eta_j)\, \underline{D}_j(\boldsymbol{p}, \boldsymbol{N}) - \sum_{i:\mathbb{C}_{ij} \neq \mathbb{C}_0} w_{ij} \bar{D}_{ij}(\boldsymbol{p}, \boldsymbol{N})$$

  - p_j: Insurer premium.
  - η_j: Non-hospital costs (e.g., physician or drug payments).
- Differences from the previous example:
  - Payoffs depend on premiums, an additional supply-side decision.
  - Insurers compete for households, so all firms' actions affect payoffs.

Example: Multichannel Television and Vertical Integration

- Crawford and Yurukoglu (2012) and Crawford et al. (2018) study negotiations between:
  - Upstream television channels (i).
  - Downstream multichannel video programming distributors (MVPDs, j), such as cable and satellite firms.
- Contracts $\mathbb{C}$ specify linear affiliate fees:

$$\boldsymbol{w} = \{w_{ij}\}$$

- w_{ij}: Amount distributor j pays channel i per subscriber.
- Distributors choose subscription prices:

$$\boldsymbol{p} = \{p_j\}$$

Timing and Demand in Multichannel Television

- Demand objects:
  - $\underline{D}_j(\boldsymbol{p}, \boldsymbol{N})$: Number of households subscribing to distributor j.
  - $\boldsymbol{N} = \{ij : \mathbb{C}_{ij} \neq \mathbb{C}_0\}$: Network of channel-distributor agreements.

Payoffs in Multichannel Television

- Channel i's payoff:

$$\Pi_{U_i}(\mathbb{C}, \boldsymbol{p}) = \sum_{j:\mathbb{C}_{ij} \neq \mathbb{C}_0} \left(w_{ij} \underline{D}_j(\boldsymbol{p}, \boldsymbol{N}) + ad_{ij}(\boldsymbol{p}, \boldsymbol{N}) \right)$$

  - $ad_{ij}(\cdot)$: Advertising revenue from distributor j's subscribers.
  - w_{ij}: channel fee received from distributor j.
- Distributor j's payoff:

$$\Pi_{D_j}(\mathbb{C}, \boldsymbol{p}) = \left(p_j - \sum_{i:\mathbb{C}_{ij} \neq \mathbb{C}_0} w_{ij} \right) \underline{D}_j(\boldsymbol{p}, \boldsymbol{N})$$

  - p_j: Subscription price set by distributor j.
  - $\underline{D}_j(\cdot)$: Number of households subscribing to distributor j.

Discussion

- Key difference from Example 11:
  - Here, upstream fees w_{ij} are paid for all subscribers ($\underline{D}_j$).
  - In Example 11, upstream fees were paid only for specific hospital admissions ($\bar{D}_{ij}$).

Sequential vs Simultaneous Timing

- Timing assumptions play a critical role in contracting models:
  - Simultaneous: Actions like pricing and contracting are decided together.
  - Sequential: Contracting concludes before other actions are taken (e.g., pricing).
- Example: Multichannel TV contracts often assume sequential timing for pricing decisions.

Modeling Contract Formation

- **Different Approaches to Modeling Contract Formation:**
  1. **Take-It-Or-Leave-It (TIOLI) Offers**
  2. **Nash-in-Nash Bargaining**

Take-It-Or-Leave-It (TIOLI) Offers

- In this framework, one side of the negotiation (e.g., the upstream firm, such as the manufacturer) **unilaterally proposes a contract offer** to the other side (e.g., the downstream firm, such as the retailer).
- The receiving party can either:
  - **Accept the offer**, in which case the contract terms are implemented as proposed.
  - **Reject the offer**, in which case no agreement is reached, and both sides receive their disagreement payoffs (e.g., profits they would earn without a deal).
- This approach assumes one party has the **power to dictate the terms of the contract**.
- Example: Villas-Boas (2007) uses a TIOLI framework to analyze manufacturer-retailer contracts, examining how manufacturers' offers affect the retailer's decision-making.
- Strengths:
  - Simple and easy to implement in theoretical and empirical models.
  - Useful when one party dominates the bargaining process.

Nash-in-Nash Bargaining

- In this framework, firms **bargain simultaneously** over contract terms.
- Each firm uses its **outside options and leverage** to negotiate favorable terms.
- Assumes mutual flexibility and the ability to reach efficient agreements.
- Useful for modeling industries where both upstream and downstream firms have significant bargaining power.
- Strengths:
  - Captures mutual influence in negotiation dynamics.
  - More flexible and realistic in industries with balanced power dynamics.
- Weaknesses:
  - Computational complexity in empirical applications.
  - Requires detailed data to estimate bargaining power and outside options.

Nash-in-Nash Bargaining Model

- Nash-in-Nash bargaining captures simultaneous negotiations between pairs of firms.
- Necessary condition:

$$\widehat{\mathbb{C}}_{ij} \in \arg \max_{\mathbb{C}_{ij} \in \mathcal{C}^{+}_{ij}(\widehat{\mathbb{C}}_{-ij})} \Bigg[\underbrace{\Big(\Pi_{Dj}(\{\mathbb{C}_{ij}, \widehat{\mathbb{C}}_{-ij}\}) - \Pi_{Dj}(\{\mathbb{C}_0, \widehat{\mathbb{C}}_{-ij}\})\Big)^{b_{ij}}}_{\Delta_{ij}\Pi_{Dj}(\{\mathbb{C}_{ij}, \widehat{\mathbb{C}}_{-ij}\})} \cdot \underbrace{\Big(\Pi_{Ui}(\{\mathbb{C}_{ij}, \widehat{\mathbb{C}}_{-ij}\}) - \Pi_{Ui}(\{\mathbb{C}_0, \widehat{\mathbb{C}}_{-ij}\})\Big)^{1-b_{ij}}}_{\Delta_{ij}\Pi_{Ui}(\{\mathbb{C}_{ij}, \widehat{\mathbb{C}}_{-ij}\})}$$

- Key terms:
  - $\Delta D_j(C)$: Gains from trade for downstream firm j.
  - $\Delta U_i(C)$: Gains from trade for upstream firm i.
  - b_{ij}: Bargaining parameter for downstream firm j.
- Assumes contracts of other pairs are held fixed during negotiations.

Health Insurer-Hospital Negotiations (Ho and Lee, 2017)

- Health insurers negotiate contracts $\mathbb{C}$ with hospitals, specifying per-admission payments w, while simultaneously negotiating premiums $\boldsymbol{p}$ with employers.
- Payoffs are based on Example 11 (Π_{U_i} for hospitals and Π_{D_j} for insurers).

Example: Health Insurers and Hospitals (Ho and Lee, 2017)

- Hospital i's payoff:

$$\Pi_{U_i}(\mathbb{C}, \boldsymbol{p}) = \sum_{j:\mathbb{C}_{ij} \neq \mathbb{C}_0} (w_{ij} - c_i)\, \bar{D}_{ij}(\boldsymbol{p}, \boldsymbol{N})$$

  - w_{ij}: Payment per admission.
  - c_i: Per-admission cost.
- Insurer j's payoff:

$$\Pi_{D_j}(\mathbb{C}, \boldsymbol{p}) = (p_j - \eta_j)\, \underline{D}_j(\boldsymbol{p}, \boldsymbol{N}) - \sum_{i:\mathbb{C}_{ij} \neq \mathbb{C}_0} w_{ij} \bar{D}_{ij}(\boldsymbol{p}, \boldsymbol{N})$$

  - p_j: Insurer premium.
  - η_j: Non-hospital costs (e.g., physician or drug payments).
  - Two demands are different: $\underline{D}_j(\boldsymbol{p}, \boldsymbol{N})$ and $\bar{D}_{ij}(\boldsymbol{p}, \boldsymbol{N})$.

Health Insurer-Hospital Negotiations (Ho and Lee, 2017)

- Nash-in-Nash bargaining conditions govern hospital payments w_{ij}:

$$\widehat{\mathbb{C}}_{ij} \in \arg \max_{\mathbb{C}_{ij} \in \mathcal{C}^{+}_{ij}(\widehat{\mathbb{C}}_{-ij})} \left[\Delta_{ij}\Pi_{Dj}(\{\mathbb{C}_{ij}, \widehat{\mathbb{C}}_{-ij}\})^{b_{ij}} \cdot \Delta_{ij}\Pi_{Ui}(\{\mathbb{C}_{ij}, \widehat{\mathbb{C}}_{-ij}\})^{1-b_{ij}} \right]$$

- Terms in the equation:
    - $\bar{D}_{ij}(\cdot)$: Number of insurer j's enrollees admitted to hospital i.
    - b_{ij}: Bargaining weight of hospital i in negotiations with insurer j.
    - $\Delta_{ij}\Pi_{D_j}$: Insurer j's gain from trade if the payment to hospital i is set to zero.
    - $\Delta_{ij}\Pi_{U_i}$: Hospital i's gain from trade under the same condition.

Modeling Contract Formation

Choosing a Model:

- The choice between TIOLI and Nash-in-Nash depends on:
  - The **industry structure**: Is one party dominant, or do both have leverage?
  - The **availability of data**: Nash-in-Nash requires more detailed data to estimate bargaining power and disagreement payoffs.
  - The **research objective**: TIOLI is simpler and easier for theoretical models, while Nash-in-Nash is better for realistic and flexible modeling.

What is Double Marginalization?

- Occurs in supply chains where a **manufacturer** and **retailer** independently maximize profits.
- Each adds a markup:
  - Manufacturer marks up the wholesale price (w).
  - Retailer marks up the retail price (p).
- Results in:
  - Higher retail price (p) than optimal.
  - Reduced consumer demand.
  - Lower joint profits for the supply chain.

Setup of the Problem

- A **manufacturer** and **retailer** negotiate the wholesale price (w) using **Nash bargaining**.
- The retailer simultaneously sets the retail price (p).
- Key assumptions:
  - Nash bargaining and retail pricing are **independent but simultaneous**.
  - The outcome depends on:
    - Bargaining power (b) of the retailer.
    - Marginal costs of the manufacturer (c_M) and retailer (c_R).
    - Consumer demand ($D(p)$).

Wholesale Price via Nash Bargaining

Nash bargaining condition:

$$\hat{w} = (1 - b)(\hat{p} - c_R) + bc_M$$

- b: Retailer's bargaining power ($0 \leq b \leq 1$).
- $\hat{p}$: Retail price (set by the retailer).
- c_R: Retailer's cost of selling the product.
- c_M: Manufacturer's marginal cost of production.

Key insights:

- If $b = 1$: $\hat{w} = c_M$ (retailer pays only the marginal cost).
- If $b < 1$: $\hat{w} > c_M$, leading to inefficiency (double marginalization).

Retail Price Setting

Key insight:

- Vertifical externality: when $\hat{w} > c_R$, the retailer's ignores the impact of its pricing on the manufacturer's profits.
- If R makes TIOLI offer, $\hat{w} = c_M$ and $\hat{p} = c_M + c_R$.

Supply Estimation and Identification

- Estimation involves recovering key parameters:
  - Marginal costs (c^U, c^D).
  - Bargaining parameters (b_{ij}).
  - Gains from trade ($\Delta D_j, \Delta U_i$).
- Ideal data scenario:
  - Observed wholesale prices (w).
  - Observed demand system ($D(p)$).
  - Marginal cost c_R and c_M.
- Missing data (e.g., marginal costs) requires additional assumptions or instruments.

Demand Estimation

- A key input for vertical contracting models is estimating consumer demand.
- Demand estimation helps predict:
  - Upstream and downstream quantities ($D(p, x, w)$).
  - Consumer responses to prices and product characteristics (x).
- Techniques:
  - Use exogenous variation in prices and characteristics to identify demand.
  - Estimate demand functions for both upstream and downstream firms.

Example: Estimating Demand in Healthcare

- Ho and Lee (2017): Model demand for health insurance plans and hospital services.
- Consumer utility for insurer j:

$$u_{cjm} = \beta v_{cjm} + x_{jm}\beta_x + \xi_{jm} + \epsilon_{cjm}$$

  - v_{cjm}: Willingness to pay (WTP) for insurer j's hospital network.
  - x_{jm}: Observed characteristics (e.g., premiums).
  - ξ_{jm}: Unobserved demand shocks.
  - ϵ_{cjm}: Idiosyncratic preferences.
- Model jointly estimates insurer and hospital demand.

Usage Models for Bundles

- When consumers purchase bundles, usage data can inform valuation.
- Example: Multichannel TV (Crawford et al., 2018)
  - Consumer utility for distributor j:

$$u_{cjm} = \beta v_{cjm}(C_j) + x_{jm}\beta_x + \xi_{jm} + \epsilon_{cjm}$$

  - $v_{cjm}(C_j)$: Viewership utility for channels in bundle C_j.
  - Viewership data helps estimate valuations for individual channels.
- Usage models reduce data requirements by linking upstream and downstream choices.

Upstream Choice-Only Models

- In some cases, only upstream demand is modeled.
- Example: Grennan (2013) - Medical devices
    - Focuses on hospitals' choice of medical devices.
    - Does not model patient flows across hospitals.
- Simplifies computation but ignores some competitive effects.

Consumer Selection in Demand Estimation

- Selection bias arises when observed consumption depends on unobserved preferences.
- Example: Multichannel TV (Crawford and Yurukoglu, 2012)
  - Consumers who purchase bundles may have higher valuations for included channels.
  - Ignoring selection leads to overestimating valuations.
- Solution:
  - Jointly estimate upstream and downstream demand.

Joint Estimation of Demand and Supply

- Demand and supply parameters can be jointly estimated for efficiency.
- Example: Crawford et al. (2018)
  - Wholesale prices (w_{ij}) used to infer demand valuations.
  - Assumes content with higher fees has higher consumer value.
- Benefits:
  - Increases precision of demand estimates.
  - Captures interactions between demand and supply.

Conclusion

- Vertical contracting models capture interactions between upstream and downstream firms.
- Estimation requires careful modeling of demand and supply.
- Techniques like Nash bargaining and demand estimation help recover key parameters.
- Joint estimation of demand and supply can improve precision and capture interactions.

Thank you!