Structural Analysis of Vertical Contracting

Empirical Framework

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Introduction

- Focus: theoretical insights into industry models to estimate firm behavior.
- Framework involves:
    1. Supply-side actions (Stage 1)
    2. Demand-side actions (Stage 2)
- Outcomes modeled over single or multiple time periods.

Two-Stage Framework for Analysis

- Researchers often model vertical contracting using a two-stage framework.
- Stage 1: Supply
  - Firms negotiate contracts and take payoff-relevant actions (e.g., investment, pricing).
- Stage 2: Demand
  - Consumers purchase products/services provided by upstream and downstream firms.

Contracts

- Contracts represent agreements between upstream and downstream firms.
- Denoted as:

$$C_{ij} \in \mathbb{C}$$

- Where:
  - C_{ij}: Contract between upstream firm i and downstream firm j.
  - $\mathbb{C}$: Set of feasible contracts.
- Null contract (C_0) represents the disagreement (no contract) outcome.

Payoff-Relevant Actions

- Actions not explicitly specified in contracts but affect payoffs.
- Denoted by:

$$a = \{a_0, a_1(\mathbb{C}, a_0)\}$$

- Where:
  - a_0: Actions chosen simultaneously with contracts.
  - $a_1(\mathbb{C}, a_0)$: Actions chosen after contracts and initial actions.
- Examples:
  - Downstream pricing, product availability.
  - Effort provision or investment.

Payoff Representation

Detailed Representation:

- Payoffs for each upstream firm i and downstream firm j are represented as:

$$\Pi_{U_i}(\mathbb{C}, \boldsymbol{a}_0), \quad \Pi_{D_j}(\mathbb{C}, \boldsymbol{a}_0)$$

- These payoffs implicitly depend on:
    - Subsequent actions taken by firms ($\boldsymbol{a}_1(\cdot)$).
    - Consumer actions captured by demand functions.

Demand Representation:

- Upstream demand: $\bar{D}(\cdot)$.
- Downstream demand: $\underline{D}(\cdot)$.

Payoff Representation

Demand Representation:

- Example (successive monopoly with per-unit pricing):

$$\mathbb{C} = \{w\}, \quad a_0 = \emptyset, \quad a_1 = \{p\}$$

$$\bar{D}(w) = \underline{D}(w) = D\left(p^m\left(w + c_R\right)\right)$$

- In this case, upstream and downstream demand coincide because of a single retailer-manufacturer setup.

General Case:

- When there are multiple upstream firms or consumers do not always purchase upstream products, $\bar{D}(\cdot)$ and $\underline{D}(\cdot)$ will typically differ.

Example: Medical Devices (Grennan, 2013)

- Contracts $\mathbb{C}$ between stent manufacturers (i) and hospitals (j) specify linear prices $\boldsymbol{w} = \{w_{ij}\}$.
- The null contract $\mathbb{C}_0$ is represented by $w_{ij} = \infty$, meaning no trade occurs.

Example: Medical Devices (Grennan, 2013)

- The payoff for stent manufacturer i is:

$$\Pi_{U_i}(\mathbb{C}) = \sum_{j:\mathbb{C}_{ij} \neq \mathbb{C}_0} (w_{ij} - c_i)\, \bar{D}_{ij}\left(\{w_{kj}\}_{k \in \mathcal{I}}\right)$$

  - w_{ij} is the price in the contract between manufacturer i and hospital j.
  - c_i is the marginal cost of producing stent i.
  - $\bar{D}_{ij}(\cdot)$ is the quantity of stent i used at hospital j, which depends on:
    - Preferences of doctors and patients at hospital j.
    - Contracts $\{w_{kj}\}$ for all stents signed by hospital j.
- The payoff for hospital j is:

$$\Pi_{D_j}(\mathbb{C}) = W_j\left(\{w_{kj}\}_{k \in \mathcal{I}}\right)$$

  - $W_j(\cdot)$ is the welfare of hospital j, which depends on the prices $\{w_{kj}\}$ for all stents it uses.

Example: Health Insurers and Hospitals (Ho and Lee, 2017)

- Contracts between hospitals (upstream) and insurers (downstream) in the U.S. healthcare industry.
- Contracts $\mathbb{C}$ specify payments per hospital admission:

$$\boldsymbol{w} = \{w_{ij}\}$$

- Insurers also set premiums for households:

$$\boldsymbol{p} = \{p_j\}$$

- Demand terms:
    - $\underline{D}_j(\boldsymbol{p}, \boldsymbol{N})$: Households enrolled in insurer j.
    - $\bar{D}_{ij}(\boldsymbol{p}, \boldsymbol{N})$: Admissions from insurer j's enrollees to hospital i.
- $\boldsymbol{N} = \{ij : \mathbb{C}_{ij} \neq \mathbb{C}_0\}$ represents the network of contracts.

Example: Health Insurers and Hospitals (Ho and Lee, 2017)

- Hospital i's payoff:

$$\Pi_{U_i}(\mathbb{C}, \boldsymbol{p}) = \sum_{j:\mathbb{C}_{ij} \neq \mathbb{C}_0} (w_{ij} - c_i)\, \bar{D}_{ij}(\boldsymbol{p}, \boldsymbol{N})$$

  - w_{ij}: Payment per admission.
  - c_i: Per-admission cost.
- Insurer j's payoff:

$$\Pi_{D_j}(\mathbb{C}, \boldsymbol{p}) = (p_j - \eta_j)\, \underline{D}_j(\boldsymbol{p}, \boldsymbol{N}) - \sum_{i:\mathbb{C}_{ij} \neq \mathbb{C}_0} w_{ij}\, \bar{D}_{ij}(\boldsymbol{p}, \boldsymbol{N})$$

  - p_j: Insurer premium.
  - η_j: Non-hospital costs (e.g., physician or drug payments).
- Differences from the previous example:
  - Payoffs depend on premiums, an additional supply-side decision.
  - Insurers compete for households, so all firms' actions affect payoffs.

Example: Multichannel Television and Vertical Integration

- Crawford and Yurukoglu (2012) and Crawford et al. (2018) study negotiations between:
  - Upstream television channels (i).
  - Downstream multichannel video programming distributors (MVPDs, j), such as cable and satellite firms.
- Contracts $\mathbb{C}$ specify linear affiliate fees:

$$\boldsymbol{w} = \{w_{ij}\}$$

- w_{ij}: Amount distributor j pays channel i per subscriber.
- Distributors choose subscription prices:

$$\boldsymbol{p} = \{p_j\}$$

Timing and Demand in Multichannel Television

- Timing assumptions:
  - Crawford and Yurukoglu (2012): Subscription prices are set after contracting.
  - Crawford et al. (2018): Subscription prices and contracts are set simultaneously.
- Demand objects:
  - $\underline{D}_j(\boldsymbol{p}, \boldsymbol{N})$: Number of households subscribing to distributor j.
  - $\boldsymbol{N} = \{ij : \mathbb{C}_{ij} \neq \mathbb{C}_0\}$: Network of channel-distributor agreements.

Payoffs in Multichannel Television

- Channel i's payoff:

$$\Pi_{U_i}(\mathbb{C}, \boldsymbol{p}) = \sum_{j:\mathbb{C}_{ij} \neq \mathbb{C}_0} \left(w_{ij} \underline{D}_j(\boldsymbol{p}, \boldsymbol{N}) + ad_{ij}(\boldsymbol{p}, \boldsymbol{N}) \right)$$

  - $ad_{ij}(\cdot)$: Advertising revenue from distributor j's subscribers.
  - w_{ij}: channel fee received from distributor j.
- Distributor j's payoff:

$$\Pi_{D_j}(\mathbb{C}, \boldsymbol{p}) = \left(p_j - \sum_{i:\mathbb{C}_{ij} \neq \mathbb{C}_0} w_{ij} \right) \underline{D}_j(\boldsymbol{p}, \boldsymbol{N})$$

  - p_j: Subscription price set by distributor j.
  - $\underline{D}_j(\cdot)$: Number of households subscribing to distributor j.

Discussion

- Key difference from Example 11:
  - Here, fees w_{ij} are paid for all subscribers ($\underline{D}_j$).
  - In Example 11, fees were paid only for specific hospital admissions ($\bar{D}_{ij}$).

Sequential vs Simultaneous Timing

- Timing assumptions play a critical role in contracting models:
    - Simultaneous: Actions like pricing and contracting are decided together.
    - Sequential: Contracting concludes before other actions are taken (e.g., pricing).
- Implications:
    - Simultaneous timing simplifies calculations but ignores downstream adjustments.
    - Sequential timing captures adjustment dynamics but increases computational complexity.
- Example: Multichannel TV contracts often assume sequential timing for pricing decisions.

Modeling Contract Formation

- Different approaches to modeling contract formation:
  1. Take-It-Or-Leave-It (TIOLI) Offers:
     - One side (e.g., upstream firms) makes offers to the other side.
     - Example: Villas-Boas (2007) for manufacturer-retailer contracts.
  2. Nash-in-Nash Bargaining:
     - Firms bargain simultaneously over terms.
     - Useful for capturing mutual leverage and flexibility in negotiations.
- Choice of model depends on the industry and available data.

Contracting Between Manufacturers and Retailers

- Villas-Boas (2007) models contracting and pricing between:
  - Manufacturers (yogurt producers, indexed by i).
  - Retailers (supermarkets, indexed by j).
- Two stages:
  - Stage 1: Manufacturers simultaneously offer contracts to retailers.
  - Stage 2: Retailers choose retail prices given contracts.
- Contracts may include:
  - Linear wholesale prices $\boldsymbol{w} = \{w_{ij}\}$.
  - Two-part tariffs (fixed fees and variable prices).
- Assumptions:
  - Each manufacturer produces one product.
  - Each retailer carries all products.

Retailer Profit and Pricing

- Retailer j's profit:

$$\Pi_{D_j}(\boldsymbol{w}, \boldsymbol{p}) = \sum_i (p_{ij} - w_{ij} - c_{Dj})\, D_{ij}(\boldsymbol{p})$$

  - p_{ij}: Retail price of product i at store j.
  - w_{ij}: Wholesale price offered by manufacturer i.
  - c_{Dj}: Retailer j's marginal cost.
  - $D_{ij}(\boldsymbol{p})$: Demand for product i at store j.
- Retail prices under Nash-Bertrand competition:

$$\boldsymbol{p} = \boldsymbol{w} + \boldsymbol{c}_D - (\boldsymbol{T} * \Lambda_D)^{-1}\, \boldsymbol{D}(\boldsymbol{p})$$

  - $\boldsymbol{T}(m, n)$: 1 if m, n share the same retailer, 0 otherwise.
  - $\Lambda_D(m, n) = \frac{\partial D_n}{\partial p_m}$: Demand sensitivity to price changes.
  - $*$: Element-wise multiplication.

Manufacturer Profit

- Manufacturer i's profit:

$$\Pi_{U_i}(\boldsymbol{w}) = \sum_j \left(w_{ij} - c_{Ui}\right) D_{ij}(\boldsymbol{p}(\boldsymbol{w}))$$

  - w_{ij}: Wholesale price offered to retailer j.
  - c_{Ui}: Manufacturer i's marginal cost.
  - $D_{ij}(\boldsymbol{p}(\boldsymbol{w}))$: Demand for product i at store j, given equilibrium prices.
- Equilibrium wholesale prices:

$$\boldsymbol{w} = \boldsymbol{c}_U - \left(I_{I\times J} * (\Lambda_P' \Lambda_D)\right)^{-1} \boldsymbol{D}(\boldsymbol{p}(\boldsymbol{w}))$$

  - $I_{I\times J}$: Identity matrix for all product-retailer combinations.
  - $\Lambda_P(m,n) = \frac{\partial p_n}{\partial w_m}$: Pass-through matrix.
- Villas-Boas uses this framework to analyze price-cost margins and conduct assumptions.

Nash-in-Nash Bargaining Model

- Nash-in-Nash bargaining captures simultaneous negotiations between pairs of firms.
- Necessary condition:

$$\widehat{\mathbb{C}}_{ij} \in \arg \max_{\mathbb{C}_{ij} \in \mathcal{C}^{+}_{ij}(\widehat{\mathbb{C}}_{-ij})} \Bigg[\underbrace{\Big(\Pi_{Dj}(\{\mathbb{C}_{ij}, \widehat{\mathbb{C}}_{-ij}\}) - \Pi_{Dj}(\{\mathbb{C}_0, \widehat{\mathbb{C}}_{-ij}\}) \Big)^{b_{ij}}}_{\Delta_{ij}\Pi_{Dj}(\{\mathbb{C}_{ij}, \widehat{\mathbb{C}}_{-ij}\})} \cdot \underbrace{\Big(\Pi_{Ui}(\{\mathbb{C}_{ij}, \widehat{\mathbb{C}}_{-ij}\}) - \Pi_{Ui}(\{\mathbb{C}_0, \widehat{\mathbb{C}}_{-ij}\}) \Big)^{1-b_{ij}}}_{\Delta_{ij}\Pi_{Ui}(\{\mathbb{C}_{ij}, \widehat{\mathbb{C}}_{-ij}\})}$$

- Key terms:
  - $\Delta D_j(C)$: Gains from trade for downstream firm j.
  - $\Delta U_i(C)$: Gains from trade for upstream firm i.
  - b_{ij}: Bargaining parameter for downstream firm j.
- Assumes contracts of other pairs are held fixed during negotiations.

Health Insurer-Hospital Negotiations (Ho and Lee, 2017)

- Health insurers negotiate contracts $\mathbb{C}$ with hospitals, specifying per-admission payments w, while simultaneously negotiating premiums $\boldsymbol{p}$ with employers.
- Payoffs are based on Example 11 (Π_{U_i} for hospitals and Π_{D_j} for insurers).

Example: Health Insurers and Hospitals (Ho and Lee, 2017)

- Hospital i's payoff:

$$\Pi_{U_i}(\mathbb{C}, \boldsymbol{p}) = \sum_{j:\mathbb{C}_{ij} \neq \mathbb{C}_0} (w_{ij} - c_i)\, \bar{D}_{ij}(\boldsymbol{p}, \boldsymbol{N})$$

  - w_{ij}: Payment per admission.
  - c_i: Per-admission cost.
- Insurer j's payoff:

$$\Pi_{D_j}(\mathbb{C}, \boldsymbol{p}) = (p_j - \eta_j)\, \underline{D}_j(\boldsymbol{p}, \boldsymbol{N}) - \sum_{i:\mathbb{C}_{ij} \neq \mathbb{C}_0} w_{ij}\, \bar{D}_{ij}(\boldsymbol{p}, \boldsymbol{N})$$

  - p_j: Insurer premium.
  - η_j: Non-hospital costs (e.g., physician or drug payments).
- Differences from the previous example:
  - Payoffs depend on premiums, an additional supply-side decision.
  - Insurers compete for households, so all firms' actions affect payoffs.

Health Insurer-Hospital Negotiations (Ho and Lee, 2017)

- Nash-in-Nash bargaining conditions govern hospital payments w_{ij}, incorporating premiums $\boldsymbol{p}$ as supply-side actions (first-order condition):

$$\widehat{w}_{ij} \times \bar{D}_{ij}(\cdot) = (1 - b_{ij}) \times \Delta_{ij} \Pi_{D_j} \left(\{w_{ij} = 0, \widehat{\mathbb{C}}_{-ij}\}, \widehat{\boldsymbol{p}} \right) - b_{ij} \times \Delta_{ij} \Pi_{U_i} \left(\{w_{ij} = 0, \widehat{\mathbb{C}}_{-ij}\}, \widehat{\boldsymbol{p}} \right)$$

- Terms in the equation:
  - $\bar{D}_{ij}(\cdot)$: Number of insurer j's enrollees admitted to hospital i.
  - b_{ij}: Bargaining weight of hospital i in negotiations with insurer j.
  - $\Delta_{ij} \Pi_{D_j}$: Insurer j's gain from trade if the payment to hospital i is set to zero.
  - $\Delta_{ij} \Pi_{U_i}$: Hospital i's gain from trade under the same condition.

Supply Estimation and Identification

- Estimation involves recovering key parameters:
  - Marginal costs (c^U, c^D).
  - Bargaining parameters (b_{ij}).
  - Gains from trade $(\Delta D_j, \Delta U_i)$.
- Ideal data scenario:
  - Observed wholesale prices (w).
  - Observed demand system $(D(p))$.
  - Marginal cost c_R and c_M.
- Missing data (e.g., marginal costs) requires additional assumptions or instruments.

Demand Estimation

- A key input for vertical contracting models is estimating consumer demand.
- Demand estimation helps predict:
  - Upstream and downstream quantities ($D(p, x, w)$).
  - Consumer responses to prices and product characteristics (x).
- Techniques:
  - Use exogenous variation in prices and characteristics to identify demand.
  - Estimate demand functions for both upstream and downstream firms.

Example: Estimating Demand in Healthcare

- Ho and Lee (2017): Model demand for health insurance plans and hospital services.
- Consumer utility for insurer j:

$$u_{cjm} = \beta v_{cjm} + x_{jm}\beta_x + \xi_{jm} + \epsilon_{cjm}$$

  - v_{cjm}: Willingness to pay (WTP) for insurer j's hospital network.
  - x_{jm}: Observed characteristics (e.g., premiums).
  - ξ_{jm}: Unobserved demand shocks.
  - ϵ_{cjm}: Idiosyncratic preferences.
- Model jointly estimates insurer and hospital demand.

Usage Models for Bundles

- When consumers purchase bundles, usage data can inform valuation.
- Example: Multichannel TV (Crawford et al., 2018)
  - Consumer utility for distributor j:

$$u_{cjm} = \beta v_{cjm}(C_j) + x_{jm}\beta_x + \xi_{jm} + \epsilon_{cjm}$$

  - $v_{cjm}(C_j)$: Viewership utility for channels in bundle C_j.
  - Viewership data helps estimate valuations for individual channels.
- Usage models reduce data requirements by linking upstream and downstream choices.

Upstream Choice-Only Models

- In some cases, only upstream demand is modeled.
- Example: Grennan (2013) - Medical devices
  - Focuses on hospitals' choice of medical devices.
  - Does not model patient flows across hospitals.
- Simplifies computation but ignores some competitive effects.

Consumer Selection in Demand Estimation

- Selection bias arises when observed consumption depends on unobserved preferences.
- Example: Multichannel TV (Crawford and Yurukoglu, 2012)
  - Consumers who purchase bundles may have higher valuations for included channels.
  - Ignoring selection leads to overestimating valuations.
- Solution:
  - Jointly estimate upstream and downstream demand.
  - Use techniques like Heckman correction or joint estimation of preferences.

Joint Estimation of Demand and Supply

- Demand and supply parameters can be jointly estimated for efficiency.
- Example: Crawford et al. (2018)
  - Wholesale prices (w_{ij}) used to infer demand valuations.
  - Assumes content with higher fees has higher consumer value.
- Benefits:
  - Increases precision of demand estimates.
  - Captures interactions between demand and supply.

Policy Applications: Horizontal Mergers in Vertical Markets

- Structural models are used to analyze merger impacts.
- Example: Gowrisankaran et al. (2015)
  - Study hospital mergers and their effects on insurer-hospital bargaining.
  - Simulate price increases under Nash-in-Nash bargaining.
- Policy implications:
  - Mergers can increase bargaining leverage but harm consumer welfare.
  - Quantifying competitive effects is crucial for antitrust decisions.

Policy Applications: Vertical Integration

- Vertical integration affects competition through:
    - Efficiencies: Reducing double marginalization.
    - Foreclosure: Denying rivals access to key inputs.
- Example: Crawford et al. (2018)
    - Study integration between cable distributors and sports channels.
    - Find positive average welfare effects but harm to rival distributors.
- Policy implications:
    - Effects of vertical integration vary across markets.
    - Requires case-specific analysis for mergers.