

Structural Analysis of Vertical Contracting

Empirical Framework

Siyuan (Bruce) Jin,
3rd Year PhD Candidate in Information Systems
siyuan-bruce.github.io

Hong Kong University of Science and Technology

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Two-Stage Framework for Analysis

- Researchers often model vertical contracting using a two-stage framework.
- Stage 1: Supply
 - Firms negotiate contracts and take payoff-relevant actions (e.g., investment, pricing).
- Stage 2: Demand
 - Consumers purchase products/services provided by upstream and downstream firms.

Contracts

- Contracts represent agreements between upstream and downstream firms.
- Denoted as:

$$C_{ij} \in \mathbb{C}$$

- Where:
 - C_{ij} : Contract between upstream firm i and downstream firm j .
 - \mathbb{C} : Set of feasible contracts.
- Null contract (C_0) represents the disagreement (no contract) outcome.

Payoff-Relevant Actions

- Actions not explicitly specified in contracts but affect payoffs.
- Denoted by:

$$a = \{a_0, a_1(\mathbb{C}, a_0)\}$$

- Where:
 - a_0 : Actions chosen simultaneously with contracts.
 - $a_1(\mathbb{C}, a_0)$: Actions chosen after contracts and initial actions.
- Examples:
 - Downstream pricing, product availability.
 - Effort provision or investment.

Payoff Representation

Detailed Representation:

- Payoffs for each upstream firm i and downstream firm j are represented as:

$$\Pi_{U_i}(\mathbb{C}, \mathbf{a}_0), \quad \Pi_{D_j}(\mathbb{C}, \mathbf{a}_0)$$

- These payoffs implicitly depend on:
 - Subsequent actions taken by firms ($\mathbf{a}_1(\cdot)$).
 - Consumer actions captured by demand functions.

Demand Representation:

- Upstream demand: $\bar{D}(\cdot)$.
- Downstream demand: $\underline{D}(\cdot)$.

Payoff Representation

Demand Representation:

- Example (successive monopoly with per-unit pricing):

$$\mathbb{C} = \{w\}, \quad a_0 = \emptyset, \quad a_1 = \{p\}$$

$$\bar{D}(w) = \underline{D}(w) = D(p^m(w + c_R))$$

- $\{w\}$: upstream price;
- $\{p\}$: downstream price;
- p^m : monopoly price coefficient;
- c_R : retailer's marginal cost.
- In this case, upstream and downstream demand coincide because of a single retailer-manufacturer setup.

General Case:

- When there are multiple upstream firms or consumers do not always purchase upstream products, $\bar{D}(\cdot)$ and $\underline{D}(\cdot)$ will typically differ.

Example: Medical Devices (Grennan, 2013)

- Contracts \mathbb{C} between stent manufacturers (i) and hospitals (j) specify linear prices $\mathbf{w} = \{w_{ij}\}$.
- The null contract \mathbb{C}_0 is represented by $w_{ij} = \infty$, meaning no trade occurs.

Example: Medical Devices (Grennan, 2013)

- The payoff for stent manufacturer i is:

$$\Pi_{U_i}(\mathbb{C}) = \sum_{j: \mathbb{C}_{ij} \neq \mathbb{C}_0} (w_{ij} - c_i) \bar{D}_{ij}(\{w_{kj}\}_{k \in \mathcal{I}})$$

- w_{ij} is the price in the contract between manufacturer i and hospital j .
- c_i is the marginal cost of producing stent i .
- $\bar{D}_{ij}(\cdot)$ is the quantity of stent i used at hospital j , which depends on:
 - ▶ Preferences of doctors and patients at hospital j .
 - ▶ Contracts $\{w_{kj}\}$ for all stents signed by hospital j .
- The payoff for hospital j is:

$$\Pi_{D_j}(\mathbb{C}) = W_j(\{w_{kj}\}_{k \in \mathcal{I}})$$

- $W_j(\cdot)$ is the welfare of hospital j , which depends on the prices $\{w_{kj}\}$ for all stents it uses.

Example: Health Insurers and Hospitals (Ho and Lee, 2017)

- Contracts between hospitals (upstream) and insurers (downstream) in the U.S. healthcare industry.
- Contracts \mathbb{C} specify payments per hospital admission:

$$\mathbf{w} = \{w_{ij}\}$$

- Insurers also set premiums for households:

$$\mathbf{p} = \{p_j\}$$

- Demand terms:
 - $\underline{D}_j(\mathbf{p}, \mathbf{N})$: Households enrolled in insurer j .
 - $\bar{D}_{ij}(\mathbf{p}, \mathbf{N})$: Admissions from insurer j 's enrollees to hospital i .
- $\mathbf{N} = \{ij : \mathbb{C}_{ij} \neq \mathbb{C}_0\}$ represents the network of contracts.

Example: Health Insurers and Hospitals (Ho and Lee, 2017)

- Hospital i 's payoff:

$$\Pi_{U_i}(\mathbb{C}, \mathbf{p}) = \sum_{j: \mathbb{C}_{ij} \neq \mathbb{C}_0} (w_{ij} - c_i) \bar{D}_{ij}(\mathbf{p}, \mathbf{N})$$

- w_{ij} : Payment per admission.
- c_i : Per-admission cost.
- Insurer j 's payoff:

$$\Pi_{D_j}(\mathbb{C}, \mathbf{p}) = (p_j - \eta_j) \underline{D}_j(\mathbf{p}, \mathbf{N}) - \sum_{i: \mathbb{C}_{ij} \neq \mathbb{C}_0} w_{ij} \bar{D}_{ij}(\mathbf{p}, \mathbf{N})$$

- p_j : Insurer premium.
- η_j : Non-hospital costs (e.g., physician or drug payments).
- Differences from the previous example:
 - Payoffs depend on premiums, an additional supply-side decision.
 - Insurers compete for households, so all firms' actions affect payoffs.

Example: Multichannel Television and Vertical Integration

- Crawford and Yurukoglu (2012) and Crawford et al. (2018) study negotiations between:
 - Upstream television channels (i).
 - Downstream multichannel video programming distributors (MVPDs, j), such as cable and satellite firms.
- Contracts \mathbb{C} specify linear affiliate fees:

$$\mathbf{w} = \{w_{ij}\}$$

- w_{ij} : Amount distributor j pays channel i per subscriber.
- Distributors choose subscription prices:

$$\mathbf{p} = \{p_j\}$$

Timing and Demand in Multichannel Television

- Demand objects:
 - $\underline{D}_j(\mathbf{p}, \mathbf{N})$: Number of households subscribing to distributor j .
 - $\mathbf{N} = \{ij : \mathbb{C}_{ij} \neq \mathbb{C}_0\}$: Network of channel-distributor agreements.

Payoffs in Multichannel Television

- Channel i 's payoff:

$$\Pi_{U_i}(\mathbb{C}, \mathbf{p}) = \sum_{j: \mathbb{C}_{ij} \neq \mathbb{C}_0} (w_{ij} \underline{D}_j(\mathbf{p}, \mathbf{N}) + ad_{ij}(\mathbf{p}, \mathbf{N}))$$

- $ad_{ij}(\cdot)$: Advertising revenue from distributor j 's subscribers.
- w_{ij} : channel fee received from distributor j .
- Distributor j 's payoff:

$$\Pi_{D_j}(\mathbb{C}, \mathbf{p}) = \left(p_j - \sum_{i: \mathbb{C}_{ij} \neq \mathbb{C}_0} w_{ij} \right) \underline{D}_j(\mathbf{p}, \mathbf{N})$$

- p_j : Subscription price set by distributor j .
- $\underline{D}_j(\cdot)$: Number of households subscribing to distributor j .

Discussion

- Key difference from Example 11:
 - Here, upstream fees w_{ij} are paid for all subscribers (\underline{D}_j).
 - In Example 11, upstream fees were paid only for specific hospital admissions (\bar{D}_{ij}).

Sequential vs Simultaneous Timing

- Timing assumptions play a critical role in contracting models:
 - Simultaneous: Actions like pricing and contracting are decided together.
 - Sequential: Contracting concludes before other actions are taken (e.g., pricing).
- Example: Multichannel TV contracts often assume sequential timing for pricing decisions.

Modeling Contract Formation

- **Different Approaches to Modeling Contract Formation:**
 1. **Take-It-Or-Leave-It (TIOLI) Offers**
 2. **Nash-in-Nash Bargaining**

Take-It-Or-Leave-It (TIOLI) Offers

- In this framework, one side of the negotiation (e.g., the upstream firm, such as the manufacturer) **unilaterally proposes a contract offer** to the other side (e.g., the downstream firm, such as the retailer).
- The receiving party can either:
 - **Accept the offer**, in which case the contract terms are implemented as proposed.
 - **Reject the offer**, in which case no agreement is reached, and both sides receive their disagreement payoffs (e.g., profits they would earn without a deal).
- This approach assumes one party has the **power to dictate the terms of the contract**.
- Example: Villas-Boas (2007) uses a TIOLI framework to analyze manufacturer-retailer contracts, examining how manufacturers' offers affect the retailer's decision-making.
- Strengths:
 - Simple and easy to implement in theoretical and empirical models.
 - Useful when one party dominates the bargaining process.

Nash-in-Nash Bargaining

- In this framework, firms **bargain simultaneously** over contract terms.
- Each firm uses its **outside options and leverage** to negotiate favorable terms.
- Assumes mutual flexibility and the ability to reach efficient agreements.
- Useful for modeling industries where both upstream and downstream firms have significant bargaining power.
- Strengths:
 - Captures mutual influence in negotiation dynamics.
 - More flexible and realistic in industries with balanced power dynamics.
- Weaknesses:
 - Computational complexity in empirical applications.
 - Requires detailed data to estimate bargaining power and outside options.

Nash-in-Nash Bargaining Model

- Nash-in-Nash bargaining captures simultaneous negotiations between pairs of firms.
- Necessary condition:

$$\hat{\mathbb{C}}_{ij} \in \arg \max_{\mathbb{C}_{ij} \in \mathcal{C}_{ij}^+(\hat{\mathbb{C}}_{-ij})} \left[\underbrace{\left(\Pi_{Dj}(\{\mathbb{C}_{ij}, \hat{\mathbb{C}}_{-ij}\}) - \Pi_{Dj}(\{\mathbb{C}_0, \hat{\mathbb{C}}_{-ij}\}) \right)^{b_{ij}}}_{\Delta_{ij} \Pi_{Dj}(\{\mathbb{C}_{ij}, \hat{\mathbb{C}}_{-ij}\})} \cdot \underbrace{\left(\Pi_{Ui}(\{\mathbb{C}_{ij}, \hat{\mathbb{C}}_{-ij}\}) - \Pi_{Ui}(\{\mathbb{C}_0, \hat{\mathbb{C}}_{-ij}\}) \right)^{1-b_{ij}}}_{\Delta_{ij} \Pi_{Ui}(\{\mathbb{C}_{ij}, \hat{\mathbb{C}}_{-ij}\})} \right]$$

- Key terms:
 - $\Delta D_j(C)$: Gains from trade for downstream firm j .
 - $\Delta U_i(C)$: Gains from trade for upstream firm i .
 - b_{ij} : Bargaining parameter for downstream firm j .
- Assumes contracts of other pairs are held fixed during negotiations.

Health Insurer-Hospital Negotiations (Ho and Lee, 2017)

- Health insurers negotiate contracts \mathbb{C} with hospitals, specifying per-admission payments w , while simultaneously negotiating premiums \mathbf{p} with employers.
- Payoffs are based on Example 11 (Π_{U_i} for hospitals and Π_{D_j} for insurers).

Example: Health Insurers and Hospitals (Ho and Lee, 2017)

- Hospital i 's payoff:

$$\Pi_{U_i}(\mathbb{C}, \mathbf{p}) = \sum_{j: \mathbb{C}_{ij} \neq \mathbb{C}_0} (w_{ij} - c_i) \bar{D}_{ij}(\mathbf{p}, \mathbf{N})$$

- w_{ij} : Payment per admission.
- c_i : Per-admission cost.
- Insurer j 's payoff:

$$\Pi_{D_j}(\mathbb{C}, \mathbf{p}) = (p_j - \eta_j) \underline{D}_j(\mathbf{p}, \mathbf{N}) - \sum_{i: \mathbb{C}_{ij} \neq \mathbb{C}_0} w_{ij} \bar{D}_{ij}(\mathbf{p}, \mathbf{N})$$

- p_j : Insurer premium.
- η_j : Non-hospital costs (e.g., physician or drug payments).
- Two demands are different: $\underline{D}_j(\mathbf{p}, \mathbf{N})$ and $\bar{D}_{ij}(\mathbf{p}, \mathbf{N})$.

Health Insurer-Hospital Negotiations (Ho and Lee, 2017)

- Nash-in-Nash bargaining conditions govern hospital payments w_{ij} :

$$\hat{\mathbb{C}}_{ij} \in \arg \max_{\mathbb{C}_{ij} \in \mathcal{C}_{ij}^+(\hat{\mathbb{C}}_{-ij})} \left[\Delta_{ij} \Pi_{D_j}(\{\mathbb{C}_{ij}, \hat{\mathbb{C}}_{-ij}\})^{b_{ij}} \cdot \Delta_{ij} \Pi_{U_i}(\{\mathbb{C}_{ij}, \hat{\mathbb{C}}_{-ij}\})^{1-b_{ij}} \right]$$

- Terms in the equation:
 - $\bar{D}_{ij}(\cdot)$: Number of insurer j 's enrollees admitted to hospital i .
 - b_{ij} : Bargaining weight of hospital i in negotiations with insurer j .
 - $\Delta_{ij} \Pi_{D_j}$: Insurer j 's gain from trade if the payment to hospital i is set to zero.
 - $\Delta_{ij} \Pi_{U_i}$: Hospital i 's gain from trade under the same condition.

Modeling Contract Formation

Choosing a Model:

- The choice between TIOLI and Nash-in-Nash depends on:
 - The **industry structure**: Is one party dominant, or do both have leverage?
 - The **availability of data**: Nash-in-Nash requires more detailed data to estimate bargaining power and disagreement payoffs.
 - The **research objective**: TIOLI is simpler and easier for theoretical models, while Nash-in-Nash is better for realistic and flexible modeling.

What is Double Marginalization?

- Occurs in supply chains where a **manufacturer** and **retailer** independently maximize profits.
- Each adds a markup:
 - Manufacturer marks up the wholesale price (w).
 - Retailer marks up the retail price (p).
- Results in:
 - Higher retail price (p) than optimal.
 - Reduced consumer demand.
 - Lower joint profits for the supply chain.

Setup of the Problem

- A **manufacturer** and **retailer** negotiate the wholesale price (w) using **Nash bargaining**.
- The retailer simultaneously sets the retail price (p).
- Key assumptions:
 - Nash bargaining and retail pricing are **independent but simultaneous**.
 - The outcome depends on:
 - ▶ Bargaining power (b) of the retailer.
 - ▶ Marginal costs of the manufacturer (c_M) and retailer (c_R).
 - ▶ Consumer demand ($D(p)$).

Example

Example Setup:

- Demand curve: $P = a - bQ$, where:
 - P : Price, Q : Quantity demanded.
 - a : Intercept, b : Slope of the demand curve.

Retailer's Total Revenue (TR):

$$TR = P \times Q = (a - bQ) \times Q = aQ - bQ^2$$

Retailer's Marginal Revenue (MR):

$$MR = \frac{d(TR)}{dQ} = a - 2bQ$$

Downstream Retailer Behavior

Profit Maximization:

- Retailer chooses Q such that $MR = MC$, where $MC = w$ (wholesale price).
- Solving:

$$Q_r = \frac{a - w}{2b}$$

- Retailer's Price:

$$P_r = a - bQ_r = \frac{a + w}{2}$$

Key Insight:

- Retailer's price P_r includes a markup over w .
- Final quantity Q_r is reduced compared to the optimal supply chain outcome.

Upstream Wholesaler Behavior & Double Marginalization

Wholesaler's Derived Demand Curve:

$$Q_r = \frac{a - w}{2b} \Rightarrow w = a - 2bQ$$

Wholesaler's Total Revenue (TR):

$$TR = w \times Q = (a - 2bQ) \times Q = aQ - 2bQ^2$$

Wholesaler's Marginal Revenue (MR):

$$MR = \frac{d(TR)}{dQ} = a - 4bQ$$

Profit Maximization

Profit Maximization:

- Wholesaler maximizes profit where $MR = MC$, where $MC = c$ (production cost).
- Solving:

$$Q_w = \frac{a - c}{4b}$$

- Wholesale Price:

$$w = a - 2bQ_w = \frac{a + c}{2}$$

Double Marginalization Problem:

- Retailer's markup on w means $Q_r < Q_w < Q_m$, where Q_m is the quantity a single monopolist would choose.
- Final price P_r is inflated, reducing consumer welfare and creating deadweight loss.

Wholesale Price via Nash Bargaining

Nash bargaining condition:

$$\hat{w} = (1 - b)(\hat{p} - c_R) + bc_M$$

- b : Retailer's bargaining power ($0 \leq b \leq 1$).
- \hat{p} : Retail price (set by the retailer).
- c_R : Retailer's cost of selling the product.
- c_M : Manufacturer's marginal cost of production.

Key insights:

- If $b = 1$: $\hat{w} = c_M$ (retailer pays only the marginal cost).
- If $b < 1$: $\hat{w} > c_M$, leading to inefficiency (double marginalization).

Retail Price Setting

Key insight:

- Vertical externality: when $\hat{w} > c_R$, the retailer's ignores the impact of its pricing on the manufacturer's profits.
- If R makes TIOLI offer, $\hat{w} = c_M$ and $\hat{p} = c_M + c_R$.

Supply Estimation and Identification

- Estimation involves recovering key parameters:
 - Marginal costs (c^U, c^D).
 - Bargaining parameters (b_{ij}).
 - Gains from trade ($\Delta D_j, \Delta U_i$).
- Ideal data scenario:
 - Observed wholesale prices (w).
 - Observed demand system ($D(p)$).
 - Marginal cost c_R and c_M .
- Missing data (e.g., marginal costs) requires additional assumptions or instruments.

Demand Estimation

- A key input for vertical contracting models is estimating consumer demand.
- Demand estimation helps predict:
 - Upstream and downstream quantities ($D(p, x, w)$).
 - Consumer responses to prices and product characteristics (x).
- Techniques:
 - Use exogenous variation in prices and characteristics to identify demand.
 - Estimate demand functions for both upstream and downstream firms.

Example: Estimating Demand in Healthcare

- Ho and Lee (2017): Model demand for health insurance plans and hospital services.
- Consumer utility for insurer j :

$$u_{cjm} = \beta v_{cjm} + x_{jm}\beta_x + \xi_{jm} + \epsilon_{cjm}$$

- v_{cjm} : Willingness to pay (WTP) for insurer j 's hospital network.
 - x_{jm} : Observed characteristics (e.g., premiums).
 - ξ_{jm} : Unobserved demand shocks.
 - ϵ_{cjm} : Idiosyncratic preferences.
- Model jointly estimates insurer and hospital demand.

Usage Models for Bundles

- When consumers purchase bundles, usage data can inform valuation.
- Example: Multichannel TV (Crawford et al., 2018)
 - Consumer utility for distributor j :

$$u_{cjm} = \beta v_{cjm}(C_j) + x_{jm}\beta_x + \xi_{jm} + \epsilon_{cjm}$$

- $v_{cjm}(C_j)$: Viewership utility for channels in bundle C_j .
 - Viewership data helps estimate valuations for individual channels.
- Usage models reduce data requirements by linking upstream and downstream choices.

Upstream Choice-Only Models

- In some cases, only upstream demand is modeled.
- Example: Grennan (2013) - Medical devices
 - Focuses on hospitals' choice of medical devices.
 - Does not model patient flows across hospitals.
- Simplifies computation but ignores some competitive effects.

Consumer Selection in Demand Estimation

- Selection bias arises when observed consumption depends on unobserved preferences.
- Example: Multichannel TV (Crawford and Yurukoglu, 2012)
 - Consumers who purchase bundles may have higher valuations for included channels.
 - Ignoring selection leads to overestimating valuations.
- Solution:
 - Jointly estimate upstream and downstream demand.

Joint Estimation of Demand and Supply

- Demand and supply parameters can be jointly estimated for efficiency.
- Example: Crawford et al. (2018)
 - Wholesale prices (w_{ij}) used to infer demand valuations.
 - Assumes content with higher fees has higher consumer value.
- Benefits:
 - Increases precision of demand estimates.
 - Captures interactions between demand and supply.

Conclusion

- Vertical contracting models capture interactions between upstream and downstream firms.
- Estimation requires careful modeling of demand and supply.
- Techniques like Nash bargaining help recover key parameters.
- Joint estimation of demand and supply can improve precision and capture interactions.

Thank you!