

1. Consider BPSK modulation with transmission power A^2 for both constellation points. The symbol duration is T and a rectangular pulse shaping function is used. Suppose that the a priori probability of transmitting 0 and 1 is not the same. Specifically, $p(s_n = 0) = 0.3$ and $p(s_n = 1) = 0.7$. The noise power spectral density is N_0 .

- a) (7 pts) Describe the optimal receiver that minimizes the BER in an AWGN channel. Derive the optimal BER. (hint: the optimal receiver is a MAP receiver).
- b) (6 pts) Consider a cellular system with circular cells. Suppose that the propagation follows a simplified path loss model with path loss exponent equal to 3. The average power measured at a reference point with $d_0 = 1m$ is 1W. The noise power spectrum density $N_0/2$ has $N_0 = 10^{-14} W/Hz$. The symbol duration is $10^{-2}ms$. In absence of log-normal shadowing and fast fading, what is the cell radius if the users can tolerate a BER of 10^{-4} ? (hint: $Q(x) \approx 0.5e^{-\frac{x^2}{2}}$)
- c) (7 pts) Now, suppose the propagation suffers flat Rayleigh fading. To achieve the same BER, what is the maximum cell radius? (hint: for $\alpha \geq 0, \beta > 0$, we have $\int_0^\infty e^{-\frac{\alpha}{4x} - \beta x} dx = \sqrt{\frac{\alpha}{\beta}} K_1(\alpha\beta)$, where $K_1(\cdot)$ is the Bessel function with parameter 1. In your derivations you would find that the $\alpha\beta$ above is a function of d ; please consider using approximation. You can express your result using $K_1(\cdot)$ without computing the numerical value.)

2. Consider a multipath channel with impulse response

$$h(\tau) = \alpha_0 \delta(\tau) + \alpha_1 \delta(\tau - \tau_1) + \alpha_2 \delta(\tau - \tau_2).$$

The α_i are Rayleigh fading coefficients, but their expected power varies such that

$$\begin{aligned} \Pr \{E[\alpha_0^2] = 5\} &= 0.5, & \Pr \{E[\alpha_0^2] = 10\} &= 0.5 \\ \Pr \{E[\alpha_1^2] = 0\} &= 0.5, & \Pr \{E[\alpha_1^2] = 20\} &= 0.5 \\ \Pr \{E[\alpha_2^2] = 5\} &= 0.75, & \Pr \{E[\alpha_2^2] = 10\} &= 0.25 \end{aligned}$$

The transmit power and noise power are such that a spread-spectrum receiver locked to the i^{th} multipath component will have an SNR of α_i^2 in the absence of other multipath components.

- a) (10 pts) What is the outage probability of BPSK modulation at an instantaneous BER of 10^{-3} for a single-branch receiver locked to the first path? (hint: $Q(x) \approx 0.5e^{-\frac{x^2}{2}}$)
- b) (10 pts) Find the outage probability of BPSK modulation at an instantaneous BER of 10^{-3} for a three-branch Rake receiver, where selection combining is used to combine the paths.

3. Consider a flat fading channel in which the channel gain (for channel h , the channel gain is $g = |h|^2$) is one of the four values: $g_1 = 0.5$, $g_2 = 0.4$, $g_3 = 0.2$, $g_4 = 0.1$ with probabilities 0.2, 0.3, 0.3, and 0.2, respectively. The received power without fading is 10dB over the noise power.
- (7 pts) Suppose that only the receiver knows the channel, what is the ergodic capacity per unit hertz for this channel?
 - (6 pts) Suppose that the channel is slow varying. Plot the outage capacity versus outage probability for $0 \leq P_{out} \leq 1$.
 - (7 pts) Suppose that both the transmitter and receiver know the channel. Find the optimal power adaptation for this channel, and the corresponding channel capacity per unit hertz.

4. Consider a MIMO system with channel matrix

$$\mathbf{H} = \begin{bmatrix} 0.3 & 0.5 \\ 0.7 & 0.2 \end{bmatrix} = \begin{bmatrix} -0.5946 & -0.8041 \\ -0.8041 & 0.5946 \end{bmatrix} \begin{bmatrix} 0.87 & 0 \\ 0 & 0.33 \end{bmatrix} \begin{bmatrix} -0.8507 & 0.5257 \\ -0.5257 & -0.8507 \end{bmatrix}$$

- (7 pts) Assume that \mathbf{H} is known at both transmitter and receiver. There is a total transmit power of 10mW, and noise power is 10^{-4} W across the communication bandwidth. Find the capacity of this channel.
 - (6 pts) Suppose now that the antennas are all used for diversity to maximize the SNR of the combiner output. Describe the optimal transmit and receive strategy that maximizes the SNR.
 - (7 pts) Repeat b) if the transmitter does not know the channel.
5. Consider a convolutional code with $k=1$, $n=3$, and $K=3$. The generator matrix is

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

- (6 pts) Draw the trellis diagram for this encoder
- (6 pts) Determine its transfer function $T(D, N, L)$.
- (3 pts) Determine the minimum Hamming distance of this code.
- (5 pts) Find the input sequence by decoding the received sequence $\mathbf{R}=[001010001]$.

Appendix: Q-function table.

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0.00	0.5	2.30	0.010724	4.55	2.6823×10^{-6}	6.80	5.231×10^{-12}
0.05	0.48006	2.35	0.0093867	4.60	2.1125×10^{-6}	6.85	3.6925×10^{-12}
0.10	0.46017	2.40	0.0081975	4.65	1.6597×10^{-6}	6.90	2.6001×10^{-12}
0.15	0.44038	2.45	0.0071428	4.70	1.3008×10^{-6}	6.95	1.8264×10^{-12}
0.20	0.42074	2.50	0.0062097	4.75	1.0171×10^{-6}	7.00	1.2798×10^{-12}
0.25	0.40129	2.55	0.0053861	4.80	7.9333×10^{-7}	7.05	8.9459×10^{-13}
0.30	0.38209	2.60	0.0046612	4.85	6.1731×10^{-7}	7.10	6.2378×10^{-13}
0.35	0.36317	2.65	0.0040246	4.90	4.7918×10^{-7}	7.15	4.3389×10^{-13}
0.40	0.34458	2.70	0.003467	4.95	3.7107×10^{-7}	7.20	3.0106×10^{-13}
0.45	0.32636	2.75	0.0029798	5.00	2.8665×10^{-7}	7.25	2.0839×10^{-13}
0.50	0.30854	2.80	0.0025551	5.05	2.2091×10^{-7}	7.30	1.4388×10^{-13}
0.55	0.29116	2.85	0.002186	5.10	1.6983×10^{-7}	7.35	9.9103×10^{-14}
0.60	0.27425	2.90	0.0018658	5.15	1.3024×10^{-7}	7.40	6.8092×10^{-14}
0.65	0.25785	2.95	0.0015889	5.20	9.9644×10^{-8}	7.45	4.667×10^{-14}
0.70	0.24196	3.00	0.0013499	5.25	7.605×10^{-8}	7.50	3.1909×10^{-14}
0.75	0.22663	3.05	0.0011442	5.30	5.7901×10^{-8}	7.55	2.1763×10^{-14}
0.80	0.21186	3.10	0.0009676	5.35	4.3977×10^{-8}	7.60	1.4807×10^{-14}
0.85	0.19766	3.15	0.00081635	5.40	3.332×10^{-8}	7.65	1.0049×10^{-14}
0.90	0.18406	3.20	0.00068714	5.45	2.5185×10^{-8}	7.70	6.8033×10^{-15}
0.95	0.17106	3.25	0.00057703	5.50	1.899×10^{-8}	7.75	4.5946×10^{-15}
1.00	0.15866	3.30	0.00048342	5.55	1.4283×10^{-8}	7.80	3.0954×10^{-15}
1.05	0.14686	3.35	0.00040406	5.60	1.0718×10^{-8}	7.85	2.0802×10^{-15}
1.10	0.13567	3.40	0.00033693	5.65	8.0224×10^{-9}	7.90	1.3945×10^{-15}
1.15	0.12507	3.45	0.00028029	5.70	5.9904×10^{-9}	7.95	9.3256×10^{-16}
1.20	0.11507	3.50	0.00023263	5.75	4.4622×10^{-9}	8.00	6.221×10^{-16}
1.25	0.10565	3.55	0.00019262	5.80	3.3157×10^{-9}	8.05	4.1397×10^{-16}
1.30	0.0968	3.60	0.00015911	5.85	2.4579×10^{-9}	8.10	2.748×10^{-16}
1.35	0.088508	3.65	0.00013112	5.90	1.8175×10^{-9}	8.15	1.8196×10^{-16}
1.40	0.080757	3.70	0.0001078	5.95	1.3407×10^{-9}	8.20	1.2019×10^{-16}
1.45	0.073529	3.75	8.8417×10^{-5}	6.00	9.8659×10^{-10}	8.25	7.9197×10^{-17}
1.50	0.066807	3.80	7.2348×10^{-5}	6.05	7.2423×10^{-10}	8.30	5.2056×10^{-17}
1.55	0.060571	3.85	5.9059×10^{-5}	6.10	5.3034×10^{-10}	8.35	3.4131×10^{-17}
1.60	0.054799	3.90	4.8096×10^{-5}	6.15	3.8741×10^{-10}	8.40	2.2324×10^{-17}
1.65	0.049471	3.95	3.9076×10^{-5}	6.20	2.8232×10^{-10}	8.45	1.4565×10^{-17}
1.70	0.044565	4.00	3.1671×10^{-5}	6.25	2.0523×10^{-10}	8.50	9.4795×10^{-18}
1.75	0.040059	4.05	2.5609×10^{-5}	6.30	1.4882×10^{-10}	8.55	6.1544×10^{-18}
1.80	0.03593	4.10	2.0658×10^{-5}	6.35	1.0766×10^{-10}	8.60	3.9858×10^{-18}
1.85	0.032157	4.15	1.6624×10^{-5}	6.40	7.7688×10^{-11}	8.65	2.575×10^{-18}
1.90	0.028717	4.20	1.3346×10^{-5}	6.45	5.5925×10^{-11}	8.70	1.6594×10^{-18}
1.95	0.025588	4.25	1.0689×10^{-5}	6.50	4.016×10^{-11}	8.75	1.0668×10^{-18}
2.00	0.02275	4.30	8.5399×10^{-6}	6.55	2.8769×10^{-11}	8.80	6.8408×10^{-19}
2.05	0.020182	4.35	6.8069×10^{-6}	6.60	2.0558×10^{-11}	8.85	4.376×10^{-19}
2.10	0.017864	4.40	5.4125×10^{-6}	6.65	1.4655×10^{-11}	8.90	2.7923×10^{-19}
2.15	0.015778	4.45	4.2935×10^{-6}	6.70	1.0421×10^{-11}	8.95	1.7774×10^{-19}
2.20	0.013903	4.50	3.3977×10^{-6}	6.75	7.3923×10^{-12}	9.00	1.1286×10^{-19}
2.25	0.012224						