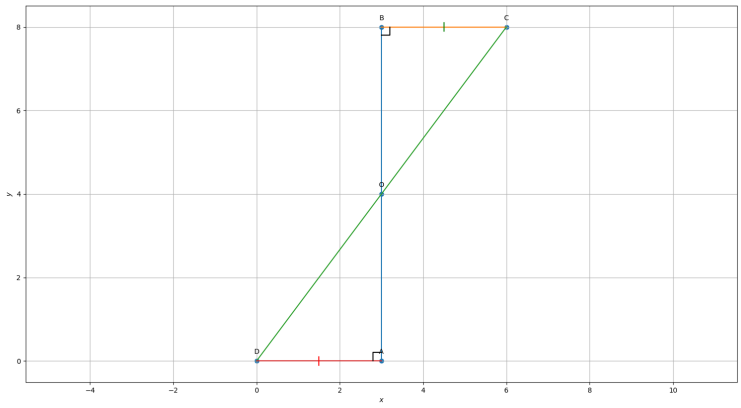
CHAPTER-7
TRIANGLES

1 Exercise 7.1

Q3. AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB.

Construction

The input parameters for construction are shown in 1:



Symbol	**Values**	**Description**
a	3	$AD = BC$
b	4	AB
$\mathbf{e}_1$	$\begin{pmatrix}1\\0\end{pmatrix}$	basis vector

Table 1: Parameters

$$\mathbf{A} = a\mathbf{e_1}, \mathbf{B} = \begin{pmatrix} a \\ b \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 2a \\ b \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

Solution: Given

$$\mathbf{D} - \mathbf{A} = \mathbf{B} - \mathbf{C} \tag{2}$$
$$\angle DAB = \angle CBA = 90^\circ \tag{3}$$

To Prove:

$$\mathbf{C} - \mathbf{O} = \mathbf{O} - \mathbf{D} \tag{4}$$

Proof:
Consider linesegment DC
Let $\mathbf{O}$ represent the Midpoint of DC

$$\mathbf{O} = \frac{1}{2}(\mathbf{C} + \mathbf{D}) \tag{5}$$
$$\implies = \frac{1}{2}\begin{pmatrix} 2a \\ b \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{6}$$
$$\implies = \frac{1}{2}\begin{pmatrix} 2a \\ b \end{pmatrix} \tag{7}$$
$$= \begin{pmatrix} a \\ \frac{b}{2} \end{pmatrix} \tag{8}$$
$$\tag{9}$$

$$\text{Since } AB \perp DA, \text{ } AB \text{ is parallel to } x = 0 \tag{10}$$
$$\text{Equation of } AB \text{ is defined as } x = a \tag{11}$$
$$\tag{12}$$
$$\tag{13}$$

from (8) and (12) $\mathbf{O}$ lies on linesegment CD and line DC intersects BA at its midpoint O.

$$\mathbf{C} - \mathbf{O} = \mathbf{O} - \mathbf{D} \tag{14}$$