Evaluating the Long-Run Validity of Purchasing Power Parity Between India and the United States

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Chapter 1

Introduction

1.1 Purpose of the Project

The primary purpose of this project is to evaluate the long run validity of the Purchasing Power Parity hypothesis between India and the United States using real exchange rate data. Purchasing Power Parity is a fundamental concept in international economics that suggests that, in the long run, identical goods should have the same price across countries when measured in a common currency. However, empirical evidence often shows persistent deviations from this theoretical condition, giving rise to what is known as the PPP puzzle.

This project aims to analyze whether the real exchange rate between India and the US exhibits mean reverting behavior, which would provide evidence in favor of the long run validity of PPP. By employing statistical and econometric techniques, including regression analysis of changes in the real exchange rate on its lagged value.

1.2 Overview of Purchasing Power Parity and the PPP Puzzle

Purchasing Power Parity (PPP) is an idea in economics that says the same goods should cost the same in different countries when prices are converted to a common currency. In simple terms, if a basket of goods costs $10 in the United States, and the same basket costs $Rs.$500 in India, the exchange rate should be \$1 $= Rs.50$ to make the prices equal.

Example:

1.Price of a Big Mac in the US = \$5

2.Price of a Big Mac in India = $Rs.250$

3.Nominal exchange rate is $Rs.50$ per \$1, the prices match perfectly so PPP holds. But if the exchange rate is $Rs.60$ per \$1, then the Big Mac is cheaper in India and PPP does not hold exactly.

Even though theory says the real exchange rate should eventually return to 1, in reality, it often deviates for a long time. This is called the PPP puzzle.

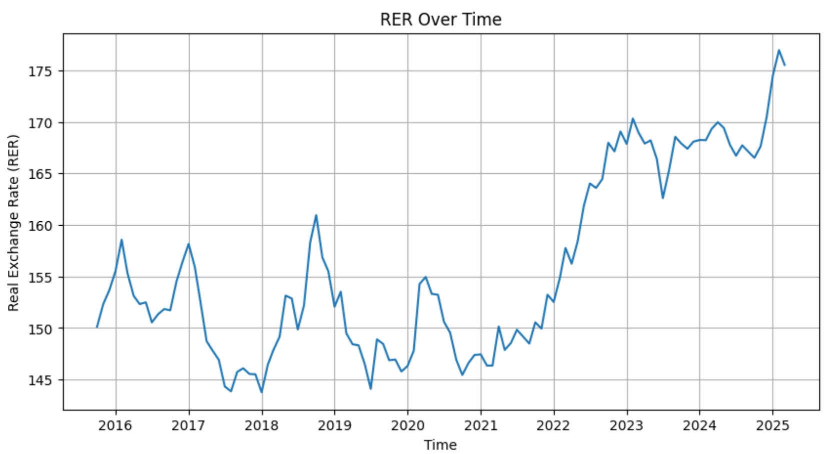

Figure 1.1: Real Exchange Rate (RER) between India and the US over Time

Chapter 2

Mathematical Framework

2.1 Definition of Real Exchange Rate

The Real Exchange Rate (RER) measures the relative price of goods between two countries, taking into account both the nominal exchange rate and the price levels in each country. It tells us how expensive domestic goods are compared to foreign goods in terms of a common currency.

Mathematically, it is defined as:

$$\text{RER} = S \times \frac{P^*}{P}$$

Where:

S = nominal exchange rate (domestic currency per unit of foreign currency)

P*= foreign price level (e.g., US CPI)

P = domestic price level (e.g., Indian CPI)

2.2 Theoretical Basis of PPP

When RER equals 1, it indicates that domestic and foreign goods cost the same in terms of purchasing power, meaning absolute Purchasing Power Parity (PPP) holds perfectly. If RER is greater than 1, it means that foreign goods are more expensive than domestic goods when converted into the domestic currency, implying that domestic goods are relatively cheaper and purchasing power is higher at home. Conversely, if RER is less than 1, domestic goods are more expensive than foreign goods in domestic currency terms, indicating that foreign goods are relatively cheaper. In this way, the RER provides a real measure of international price competitiveness and the deviations from parity reflect the extent to which absolute PPP does not hold.

For further analysis, we take the natural logarithm of both sides of the Real Exchange Rate equation.

$$\ln(RER) = \ln(S) + \ln(P^*) - \ln(P)$$

Where:

$$\begin{aligned}
\ln(RER) &= \text{log of the real exchange rate} \\
\ln(S) &= \text{log of the nominal exchange rate} \\
\ln(P^*) &= \text{log of the foreign price level} \\
\ln(P) &= \text{log of the domestic price level}
\end{aligned}$$

By working with, ln(RER), we can more conveniently test for deviations from PPP, analyze mean reversion, and interpret the results in terms of proportional changes

Chapter 3

Data Collection

3.1 Sources of Data

The data for this study has been collected from Federal Reserve Economic Data (FRED), a reliable source for historical economic time series. The nominal exchange rate of the Indian Rupee per US Dollar is obtained from FRED under the series code DEXINUS. This dataset is originally available on a daily frequency and has been converted to monthly data by taking the average of daily values for consistency in analysis. The Consumer Price Index (CPI) for India is taken from the series INDCPIALLMINMEI, while the CPI for the United States is taken from CPIAUCSL. All series cover the period from August 2015 to February 2025, providing the necessary data to compute the Real Exchange Rate and test the validity of Purchasing Power Parity (PPP) between India and the United States.

Here are some helpful links:

1. Nominal Exchange Rate (DEXINUS)
2. India CPI (INDCPIALLMINMEI)
3. US CPI (CPIAUCSL)

Using all available data, we take the natural logarithm of the Consumer Price Index (CPI) for the United States (CPIAUCSL), the CPI for India (INDCPIALLMINMEI), and the nominal exchange rate (DEXINUS). These transformations are used to compute the ln of the Real Exchange Rate as:

$$\ln(RER) = \ln(S) + \ln(P^*) - \ln(P)$$

Chapter 4

Empirical Methodology

4.1 Different tests for testing mean-reversion

To test the validity of the Purchasing Power Parity (PPP) hypothesis, economists commonly use the Real Exchange Rate (RER). The idea is to see whether the RER tends to revert to its long-run equilibrium over time.

4.1.1 The AR(1) Model

An autoregressive model of order 1, AR(1), is written as:

$$\ln(RER_t) = c + \rho \ln(RER_{t-1}) + u_t$$

First, estimate ρ from the AR(1) model using OLS:

$$\ln(RER_t) = c + \rho \ln(RER_{t-1}) + u_t$$

Then, check the estimated value of ρ :

$$\begin{cases} \rho < 1 & \Rightarrow \text{Stationary (mean-reverting)} \\ \rho \approx 1 & \Rightarrow \text{Non-stationary (unit root)} \\ \rho > 1 & \Rightarrow \text{Explosive (unstable)} \end{cases}$$

4.1.2 The Dickey–Fuller Test

The Dickey–Fuller (DF) Test and Model Transformation

The Dickey–Fuller (DF) test is a statistical test used to determine whether a time series is stationary or possesses a unit root (i.e., is non-stationary). It is based on estimating an autoregressive process of order one, AR(1), for the given series.

Consider the following AR(1) model for the logarithm of the real exchange rate:

$$\ln(RER_t) = c + \rho \ln(RER_{t-1}) + u_t$$

where c is a constant, ρ is the autoregressive coefficient, and u_t is a white noise error term.

The null hypothesis of the Dickey–Fuller test is that the series has a unit root, i.e. $\rho = 1$, which implies non-stationarity. To test this, we can transform the AR(1) equation by subtracting $\ln(RER_{t-1})$ from both sides:

$$\ln(RER_t) - \ln(RER_{t-1}) = c + \rho \ln(RER_{t-1}) - \ln(RER_{t-1}) + u_t$$

Simplifying the left-hand side gives:

$$\Delta \ln(RER_t) = c + (\rho - 1) \ln(RER_{t-1}) + u_t$$

We now define a new parameter $\beta = (\rho - 1)$, so that the equation can be rewritten as:

$$\Delta \ln(RER_t) = c + \beta \ln(RER_{t-1}) + u_t$$

This is the basic **Dickey–Fuller regression model**. The test of stationarity is then equivalent to testing whether $\beta = 0$ (i.e., $\rho = 1$).

Formally, the hypotheses are:

$$\begin{cases} H_0 : \beta = 0 & \text{(unit root, non-stationary)} \\ H_1 : \beta < 0 & \text{(stationary, mean-reverting)} \end{cases}$$

If the null hypothesis H_0 cannot be rejected, the time series $\ln(RER_t)$ is non-stationary. If H_0 is rejected in favor of H_1, it implies that $\ln(RER_t)$ is stationary and mean-reverting.

4.1.3 The Augmented Dickey–Fuller (ADF) Test

Overview. The Augmented Dickey–Fuller (ADF) test is an extension of the simple Dickey–Fuller (DF) test that accounts for higher-order serial correlation in the time series. It is used to determine whether a given time series possesses a unit root (i.e., is non-stationary) or is stationary (mean-reverting). The ADF test augments the basic DF regression by including lagged differences of the dependent variable to ensure white-noise residuals.

From the Dickey–Fuller Model to the ADF Model

We begin with the first-order autoregressive model (AR(1)) for the logarithm of the real exchange rate:

$$\ln(RER_t) = c + \rho \ln(RER_{t-1}) + u_t,$$

where c is a constant, ρ is the autoregressive coefficient, and u_t is a white noise error term.

Subtracting $\ln(RER_{t-1})$ from both sides yields:

$$\ln(RER_t) - \ln(RER_{t-1}) = c + (\rho - 1)\ln(RER_{t-1}) + u_t,$$

or equivalently,

$$\Delta \ln(RER_t) = c + (\rho - 1)\ln(RER_{t-1}) + u_t.$$

Defining $\beta = \rho - 1$, we obtain the basic Dickey–Fuller regression:

$$\Delta \ln(RER_t) = c + \beta \ln(RER_{t-1}) + u_t.$$

The null and alternative hypotheses are:

$$\begin{cases} H_0 : \beta = 0 & \text{(unit root, non-stationary)} \\ H_1 : \beta < 0 & \text{(stationary, mean-reverting)} \end{cases}$$

Motivation for the Augmentation

The simple DF regression assumes that the error term u_t is white noise. However, in many empirical time series, u_t may be serially correlated. To address this issue, the model is augmented by adding lagged values of the first difference of the dependent variable, ensuring that the residuals are uncorrelated.

Derivation from an AR(p) Process

Suppose the true data-generating process follows an AR(p) model:

$\ln(RER_t) = c + \phi_1 \ln(RER_{t-1}) + \phi_2 \ln(RER_{t-2}) + \cdots + \phi_p \ln(RER_{t-p}) + v_t.$

Subtracting $\ln(RER_{t-1})$ from both sides and rearranging terms gives:

$\Delta \ln(RER_t) = c + (\phi_1 - 1) \ln(RER_{t-1}) + \phi_2 \ln(RER_{t-2}) + \cdots + \phi_p \ln(RER_{t-p}) + v_t.$

By expressing lagged levels as lagged differences, we can reparameterize the model as:

$$\Delta \ln(RER_t) = c + \beta \ln(RER_{t-1}) + \sum_{i=1}^{p-1} \gamma_i \Delta \ln(RER_{t-i}) + \varepsilon_t,$$

where

$$\beta = \sum_{j=1}^{p} \phi_j - 1, \qquad \gamma_i = -\sum_{j=i+1}^{p} \phi_j,$$

and ε_t is a white-noise disturbance.

This is the general **Augmented Dickey–Fuller (ADF) regression model**. The inclusion of lagged differences $\Delta \ln(RER_{t-i})$ ensures that the residuals are not autocorrelated, which makes statistical inference valid.

ADF Regression Forms

In practice, the ADF test is estimated under three deterministic specifications, depending on the properties of the data:

$$\text{(1) No constant:} \quad \Delta \ln(RER_t) = \beta \ln(RER_{t-1}) + \sum_{i=1}^{p-1} \gamma_i \Delta \ln(RER_{t-i}) + \varepsilon_t,$$

$$\text{(2) Constant only:} \quad \Delta \ln(RER_t) = c + \beta \ln(RER_{t-1}) + \sum_{i=1}^{p-1} \gamma_i \Delta \ln(RER_{t-i}) + \varepsilon_t,$$

$$\text{(3) Constant and trend:} \quad \Delta \ln(RER_t) = c + \tau t + \beta \ln(RER_{t-1}) + \sum_{i=1}^{p-1} \gamma_i \Delta \ln(RER_{t-i}) + \varepsilon_t.$$

Choice among these depends on whether the series shows a deterministic mean or trend.

Hypotheses and Test Statistic

The ADF test examines:

$$\begin{cases} H_0 : \beta = 0 & \text{(unit root; non-stationary)} \\ H_1 : \beta < 0 & \text{(stationary; mean-reverting)} \end{cases}$$

After estimating the model by OLS, the t-statistic for β is computed as:

$$t_\beta = \frac{\hat{\beta}}{\mathrm{SE}(\hat{\beta})}.$$

Under the null hypothesis, the distribution of t_β is non-standard, and critical values are provided by Dickey and Fuller (1979). If t_β is more negative than the critical value, the null of a unit root is rejected, implying stationarity.

Lag Length Selection

The choice of lag length p is crucial:

- Use information criteria such as AIC or BIC.
- Ensure residuals are white noise (no serial correlation).
- Alternatively, use sequential testing to drop insignificant lags.

Chapter 5

Structural Break Analysis and ADF Test

5.1 Checking Stationarity for the Full Sample (up to 2025)

To begin the analysis, we apply the Augmented Dickey-Fuller (ADF) test on the full `ln_RER` time series to verify whether it is stationary. Stationarity is a crucial requirement for testing the long-run validity of the Purchasing Power Parity (PPP) hypothesis.

Hypotheses:

- **Null Hypothesis (H_0):** The series has a unit root (it is non-stationary), implying that shocks are permanent and PPP does not hold.
- **Alternative Hypothesis (H_1):** The series is stationary (mean-reverting), implying that PPP holds in the long run.

We use a significance level of $\alpha = 0.05$. The decision rule is:

- If the **p-value** ≤ 0.05, reject H_0 and conclude the series is stationary.
- If the **p-value** > 0.05, fail to reject H_0 and conclude the series is non-stationary.

The ADF test result for the full sample (up to 2025) is shown in Figure 5.1. The output reports:

- **ADF Statistic:** -0.6359
- **p-value:** 0.8627
- **Critical Values:** -3.490 (1%), -2.887 (5%), -2.581 (10%)

Since the p-value (0.8627) is much greater than the significance level (0.05), we fail to reject the null hypothesis (H_0). This indicates that the `ln_RER` series is **non-stationary** for the entire sample up to 2025.

This outcome suggests that the real exchange rate does not revert to a constant mean over time. Consequently, the PPP hypothesis does not appear to hold in the long run when considering the entire sample period. However, this non-stationarity may also indicate the presence of potential structural breaks in the series, which will be explored in subsequent sections.

5.2 Time Series Analysis of ln(**RER**)

To begin the analysis, it is useful to visually inspect the behavior of the real exchange rate in logarithmic form. Figure 5.2 shows the plot of ln(RER) over time.

The Augmented Dickey–Fuller (ADF) test was applied to monthly ln(RER) data around the suspected break in 2022. It yielded a p-value of **0.03096**, suggesting stationarity up to **2022-04-01**. However, including **2022-05-01** renders the series non-stationary, implying a possible structural break.

A Chow test further confirmed this, producing a p-value of $\mathbf{4.45 \times 10^{1}}$, providing strong statistical evidence of a structural break around **April 2022**. Hence, during **2015-08-01 to 2022-04-01**, the series appears stationary, indicating that **Purchasing Power Parity (PPP) holds** within this period.

To further examine the behavior of ln(RER) beyond the 2022 break, an Augmented Dickey–Fuller (ADF) test was conducted for the period **2022-05-01 to 2024-12-01**.

The test produced a p-value of **0.03113**, indicating that the series is **stationary** within this period. Hence, the **Purchasing Power Parity (PPP)** relationship continues to hold between **May 2022 and December 2024**.

To verify the stability of this relationship, a **Chow test** was performed around **December 2024**, yielding a p-value of **0.00111**. This provides

```python
from statsmodels.tsa.stattools import adfuller

# Perform the ADF test on the whole data
adf_test_full_data = adfuller(df['ln_RER'])

# Output the results
print('ADF Statistic:', adf_test_full_data[0])
print('p-value:', adf_test_full_data[1])
print('Critical Values:')
for key, value in adf_test_full_data[4].items():
    print('\t%s: %.3f' % (key, value))

# Determine if stationary
if adf_test_full_data[1] <= 0.05:
    print("The time series is likely stationary.")
else:
    print("The time series is likely non-stationary.")
```

```
ADF Statistic: -0.6358964881156401
p-value: 0.8626628772089528
Critical Values:
        1%: -3.490
        5%: -2.887
        10%: -2.581
The time series is likely non-stationary.
```

Figure 5.1: ADF Test Results for `ln_RER` (Full Sample up to 2025)

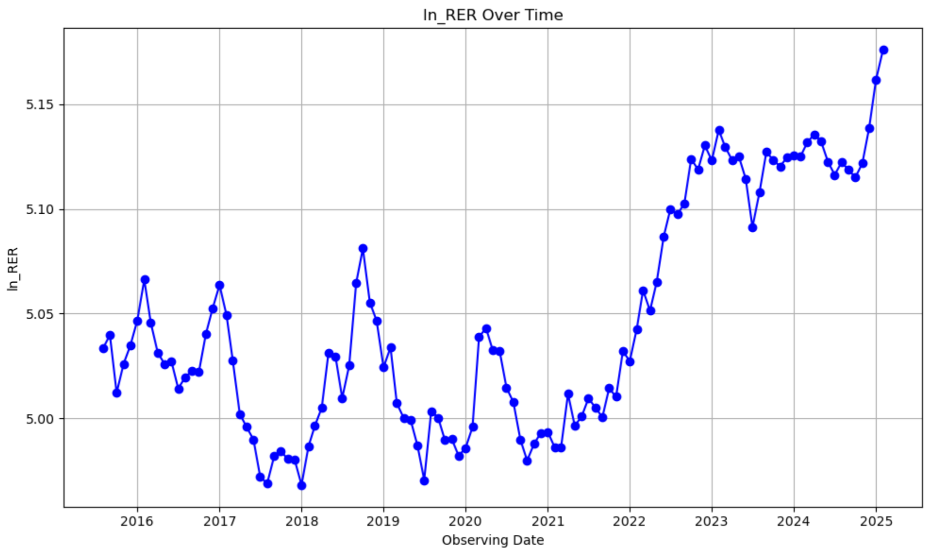

Figure 5.2: Plot of ln(RER) over time. Visual inspection indicates possible structural breaks around **2022** and near the boundary of **2024–2025**, which may reflect changes in economic dynamics or exchange rate policy. Subsequent analysis considers these potential breakpoints.

strong statistical evidence of a **structural break** near the end of 2024, suggesting a shift in the underlying exchange rate dynamics.

```python
from statsmodels.tsa.stattools import adfuller

# Filter the DataFrame up to 2022-04-01
df_adf = df[df['observation_date'] <= '2022-04-01'].copy()

# Perform the ADF test
adf_test = adfuller(df_adf['ln_RER'])

# Output the results
print('ADF Statistic:', adf_test[0])
print('p-value:', adf_test[1])
print('Critical Values:')
for key, value in adf_test[4].items():
    print('\t%s: %.3f' % (key, value))

# Determine if stationary
if adf_test[1] <= 0.05:
    print("The time series is likely stationary.")
else:
    print("The time series is likely non-stationary.")
```

```
ADF Statistic: -3.0442666417369026
p-value: 0.030962187108917728
Critical Values:
        1%: -3.516
        5%: -2.899
        10%: -2.587
The time series is likely stationary.
```

Figure 5.3: ADF and Chow test results confirming a structural break in April 2022.

```
ADF Statistic: -3.042238449223696
p-value: 0.03113374200709573
Critical Values:
        1%: -3.689
        5%: -2.972
        10%: -2.625
The time series is likely stationary.
```

Figure 5.4: ADF test result confirming stationarity of ln(RER) during May 2022–December 2024.

```
    print("Not enough observations in one or both
```

```
Chow Test Statistic: 11.195132862606602
p-value: 0.0011139293845184967
```

Figure 5.5: Chow test indicating a structural break in December 2024.

Chapter 6

Conclusion

6.1 Summary of Findings

Figure 6.1 summarizes the identified structural breaks in the ln(RER) series. Two significant breakpoints were detected — the first around **April 2022** and the second around **December 2024**.

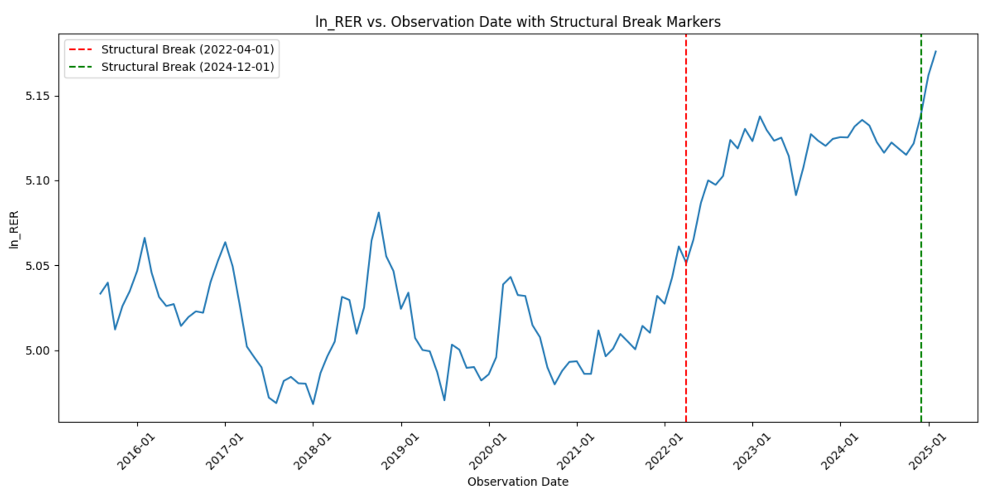

Figure 6.1: Structural breaks in ln(RER) identified in April 2022 and December 2024.

Based on the Augmented Dickey–Fuller and Chow test results, the analysis concludes that:

- From **2015-08-01 to 2022-04-01**, the ln(RER) series is **stationary**, indicating that the **Purchasing Power Parity (PPP)** hypothesis holds during this period.
- From **2022-05-01 to 2024-12-01**, the series remains **stationary** again, suggesting that **PPP continues to hold** in this sub-period as well.

Hence, it can be inferred that the real exchange rate follows mean-reverting behavior consistent with the PPP theory between **2015–2024**, with brief structural adjustments around the identified breakpoints.