GLASS-BRW

Precision-Gated Rule Ensemble with Abstention

1 Problem Setting

- $X \in \mathbb{R}^d$ denotes customer feature vectors.
- $Y \in \{0, 1\}$ denotes the target variable (STAY, CHURN).
- $S = \phi(X)$ denotes a deterministic segment assignment function.

$$S = \phi(X)$$

- The function $\phi(\cdot)$ maps each customer to a finite categorical lattice.
- Segment assignments are deterministic and fixed at inference time.
- GLASS-BRW does **not** learn a classifier $f : X \to Y$.
- Instead, it learns a partial decision function:

$$f_{\text{GLASS}} : X \to \{-1, 0, 1\}$$

- -1 denotes explicit **ABSTAIN**.
- Predictions are emitted only when symbolic segment rules meet strict precision guarantees.

2 Rule Representation

- A rule r is defined as an ordered pair:

$$r = (C_r, y_r)$$

- C_r is a conjunction of segment conditions.
- $y_r \in \{0, 1\}$ is the predicted class.

$$C_r = \{(f_1 = \ell_1), \ldots, (f_k = \ell_k)\}$$

- Each $(f_i = \ell_i)$ corresponds to a segment constraint.
- Rule complexity is bounded by $k \leq 3$.
- A customer x matches rule r if and only if:

$$\forall (f, \ell) \in C_r : \phi_f(x) = \ell$$

3 Candidate Rule Generation

Let:

- F denote the set of segment features
- L_f denote the discrete levels of feature f

Candidate rules are generated via bounded lattice enumeration:

$$\mathcal{R} = \bigcup_{k=1}^{3} \left\{ r \;\middle|\; C_r \subseteq \prod_{i=1}^{k} (f_i, \ell_i),\ |\mathrm{supp}(r)| \geq n_{\min} \right\}$$

where:

- $n_{\min}$ is a minimum support threshold (e.g. 50 samples)
- Separate hypotheses are generated for $y_r = 0$ and $y_r = 1$
- The resulting rule pool satisfies $|\mathcal{R}| \approx 200$–$500$

4 Rule Metrics (Validation-Based)

All metrics are computed on held-out validation data.

4.1 Precision (Primary Gate)

$$p_r = \frac{\mathrm{TP}_r}{\mathrm{TP}_r + \mathrm{FP}_r}$$

- Rules with $p_r < p_{\min}$ (typically 0.75) are ineligible for selection

4.2 Coverage

$$c_r = \frac{|\{x : x \models r\}|}{|X|}$$

- Measures the fraction of customers covered by rule r

4.3 Interpretability

$$i_r = \frac{1}{|C_r| + 1}$$

- Penalizes rule complexity
- Biases selection toward simpler symbolic explanations

4.4 Stability

$$\text{stability}_r = 1 - \frac{\text{std}(p_r^{(j)})}{\mathbb{E}[p_r^{(j)}]}$$

- $p_r^{(j)}$ denotes precision across cross-validation folds
- Low stability indicates non-stationarity or sample noise

4.5 Ensemble-Aware Metrics

Given a downstream EBM with output $\hat{p}_{\text{EBM}}(x)$:

Boundary Ambiguity

$$a_r = \mathbb{E}_{x \models r}\left[|\hat{p}_{\text{EBM}}(x) - 0.5|\right]$$

- Lower values indicate regions where EBM is uncertain

EBM Overlap

$$o_r = \mathbb{P}_{x \models r}\left(|\hat{p}_{\text{EBM}}(x) - 0.5| > 0.20\right)$$

- Higher values indicate redundancy with EBM's confident predictions

5 Rule Selection as Integer Optimization

5.1 Decision Variables

$$x_r \in \{0, 1\} \quad \forall r \in \mathcal{R}$$

5.2 Objective Function

$$\max \sum_{r \in \mathcal{R}} x_r \left(p_r^2 c_r i_r - \lambda_1 a_r - \lambda_2 o_r\right)$$

- Quadratic precision enforces superlinear preference for high-purity rules
- Penalties encode ensemble complementarity

5.3 Constraints

Cardinality

$$8 \leq \sum_r x_r \leq 10$$

Minimum Coverage

$$\sum_r x_r c_r \geq 0.60$$

Precision Gate

$$x_r = 0 \quad \text{if } p_r < p_{\min}$$

Class Balance

$$3 \leq \sum_r x_r \mathbb{I}(y_r = 1) \leq 5$$

$$4 \leq \sum_r x_r \mathbb{I}(y_r = 0) \leq 6$$

Feature Diversity

$$\sum_{r: f \in C_r} x_r \leq 3 \quad \forall f \in F$$

6 Execution Semantics (First-Match-Wins)

Selected rules $\mathcal{R}^*$ are ordered by descending p_r. Prediction for a customer x is defined as:

$$f_{\text{GLASS}}(x) = \begin{cases} y_r, & \exists r \in \mathcal{R}^* \text{ s.t. } x \models r \\ -1, & \text{otherwise} \end{cases}$$

- Mutual exclusivity
- Determinism
- Single-rule attribution

7 Probabilistic Output Encoding

$$P(Y = 1 \mid x) = \begin{cases} p_r, & f_{\text{GLASS}}(x) = 1 \\ 1 - p_r, & f_{\text{GLASS}}(x) = 0 \\ 0.5, & f_{\text{GLASS}}(x) = -1 \end{cases}$$

- Abstention propagates uncertainty to downstream models

8 Guarantees

Theorem 1 (Precision Lower Bound)

$$\mathbb{P}(Y = \hat{Y} \mid f_{\text{GLASS}}(x) \neq -1) \geq p_{\min}$$

- Each covered instance matches exactly one rule with precision $\geq p_{\min}$

Theorem 2 (Interpretability Bound)

- Every prediction is explained by at most three human-interpretable conditions

Theorem 3 (Ensemble Complementarity)

- For $\lambda_2 > 0$, the optimizer prefers rules covering regions of low EBM confidence