IEOR 8100 Network Project

Bail-ins and Bail-outs:

Extension to Multiple External Assets

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Contents

Summary

In the paper "Bail-ins and Bail-outs: Incentives, Connectivity, and Systemic Stability", we have assumed that each bank has outside investments e^i with a fixed fraction α recover rate when liquidated. But in reality, recovery rates are asset-specific, which may be related to the changes of asset prices and price elasticity of different assets. Besides, the outside investments of different banks are also not independent, which, in reality, are related by the asset prices. Therefore, it is natural to ask what if we extend this model into a more generalized one with multiple external assets with endogenous recovery ratio determined by supply-demand relationship of external assets in the market?

Moreover, when a bank suffers from financial shocks, liquidation action of external assets from other banks may have negative effect on asset prices, which will influence the interbank payments for all the banks in this system. Therefore, we may also wonder which liquidation strategy may have lower negative effect on social welfare. Here, we tested two liquidation strategies from the view of individual banks, the proportional liquidation strategy and liquidity-based strategy. From simulation results, we find that the liquidity-based liquidation strategy is nearly optimal in all settings, which is quite intuitive since the banks can liquidate less external assets to get the same amount of money.

As for the graph topology, the interbank payment network structure will have deterministic effect on the contagion of risks. We tested the ring and complete network as shown in the original paper, and find that the complete network will have lower welfare losses effect under small shocks due to its better ability of sharing risks among other banks in the system. However, when the shock size is large enough, complete network will become worse, since it will lead more banks to bankruptcy. Besides, we also tested on an asymmetric core-peripheral network, and find that we can actually find the relative importance of systemically important banks if they are suffering from shocks.

1. Model

Firstly, as in the paper "Price Contagion through Balance Sheet Linkages", we use the following formula for incremental demand for non-bank sector to capture the relationship between the amount of external assets that non-banking sector is willing to hold and the prices of these assets:

$$\Delta Q_k^{nb} = \frac{\gamma_k}{P_k + \Delta P_k} Q_k^{nb} (\Delta Z_k - \Delta P_k) \tag{1}$$

we set $\Delta Z_k = 0$ in the following analysis without loss of generality, which means that there's no asset-specific demand shocks in our model. And γ_k captures the price elasticity of asset k for the non-banking sector.

Indeed, we can also construct the incremental demand relationship for banking sector, but as our goal is not to track the leverage ratio as that shown in this paper, so we will not utilize these conditions here.

Secondly, we introduce the market clearing condition to create a link for holdings for different assets between non-banking sector and banking sector:

$$Q_k^{nb} + \sum_{i=1}^{N} Q_k^i = Q_{tot}^k \tag{2}$$

By taking derivatives, we can derive the relationship between the changes of Q_k^{nb} and Q_k^i:

$$\Delta Q_k^{nb} + \sum_{i=1}^{N} \Delta Q_k^i = 0 \tag{3}$$

From equation (1) and (3), we can derive the formula for percentage of price changes:

$$\frac{\Delta P_k}{P_k} = \frac{1}{\frac{\gamma_k Q_k^{nb}}{\sum_{i=1}^{N} \Delta Q_k^i} - 1} \tag{4}$$

Therefore, we can represent the value of external assets after clearing (which is equivalent to αe^i in the original paper) as:

$$\Delta Q_k^i (P_k + \Delta P_k) = \Delta Q_k^i P_k \left(1 + \frac{\Delta P_k}{P_k}\right) = \Delta Q_k^i P_k \frac{\gamma_k Q_k^{nb}}{\gamma_k Q_k^{nb} - \sum_{i=1}^{N} \Delta Q_k^i} \tag{5}$$

We define $\alpha(k, \Delta Q_k)$ as this percentage change for simplicity:

$$\alpha(k, \Delta Q_k) = \frac{\gamma_k Q_k^{nb}}{\gamma_k Q_k^{nb} - \sum_{i=1}^{N} \Delta Q_k^i} \tag{6}$$

which is a function of the asset class k and the clearing amount of asset k among all banks.

As can be seen, $\alpha(k, \Delta Q_k)$ is a decreasing function with $\sum_{i=1}^{N} \Delta Q_k^i$, which means that the percentage change of the price of asset k will be more severe if banks choose to liquidate a higher amount of this asset.

Next, we can follow the paper "Bail-ins and Bail-outs" to derive the main results:

The set of all fundamentally default banks will be:

$$\mathcal{F} = \left\{ i \middle| L^i > c^i + \sum_{k=1}^{K} Q_k^i P_k \alpha(k, \Delta Q_k) + (\pi L)^i \right\} \tag{7}$$

We use x to represent the clearing payment vector instead of p in the original paper to differentiate it from the asset price P_k here.

The set of default banks will become:

$$D(x) = \left\{ i \middle| x^i < L^i \right\} \tag{8}$$

Definition 1. The interbank payment vector is

$$x^i = \begin{cases} L^i & if\ c^i + \sum_{k=1}^{K} Q_k^i P_k \alpha(k, \Delta Q_k) + \sum_{j=1}^{n} \pi^{ij} x^j \geq L^i \\ \left(\beta \left(c_h^i + \sum_{k=1}^{K} Q_k^i P_k \alpha(k, \Delta Q_k) + \sum_{j=1}^{n} \pi^{ij} x^j \right) - C_f^i \right)^+ & else \end{cases} \tag{9}$$

Note that given $\alpha(k, \Delta Q_k)$, x can be determined from the fixed point theorem. This is because if we set the RHS of equation (9) to $H(x)$, then we have $H(0) \geq 0$, $H(L) \leq L$, and also $H(x)$ is an increasing function in x.

Besides, from the paper Rodrigo Cifuentes, Gianluigi Ferrucci (2005), we can derive that the uniqueness of x also holds under the following two trivial conditions:

1. The system is connected

2. At least 1 bank has positive equity value

Besides, in order to analyze the effect of liquidation strategies on the social welfare, we define ω_k^i as the proportion of assets k bank i has to liquidate in order to satisfy the clearing payment condition.

$$\sum_{k=1}^{K} \omega_k^i = 1, \quad for\ i = 1,2,\dots,N$$

For proportional liquidation strategy:

$$\omega_k^i = \frac{e_k^i}{\sum_{k=1}^{K} e_k^i} \tag{10}$$

For liquidity based liquidation strategy, one proxy for the liquidity state of asset k is the elasticity-weighted size of the nonbanking sector $\gamma_k e_k^{nb}$. So we can first rank each external assets buy this proxy, and then each bank will choose to liquidate the most liquid asset first, and the second most liquid asset next, and so on.

Following the equation (2.2) in "Price Contagion through Balance Sheet Linkages", we can derive a similar cash flow equation:

$$-\Delta Q_k^i (P_k + \Delta P_k) = \left(L^i - c^i - \sum_{j=1}^{n} \pi^{ij} x^j \right)^+ \omega_k^i \tag{11}$$

The RHS is the proportion of external asset k that bank i has to liquidate to meet the liability. And the LHS is the total amount of money bank i gains from liquidating asset k with amount $-\Delta Q_k^i$.

Therefore, from equation (6) and (11), we can derive the nominal liquidation amount of asset k for bank i as:

$$l_k^i(x) = min\left(\frac{\left(L^i - c^i - \sum_{j=1}^{n} \pi^{ij} x^j\right)^+ \omega_k^i}{\alpha(k, \Delta Q_k)}, e_k^i\right) \tag{12}$$

The value of bank $i's$ equity after liabilities are cleared with clearing payment vector x is equal to:

$$V^i(x) = \left(\pi x + c + \sum_{k=1}^{K} e_k^i - \sum_{k=1}^{K}\big(1-\alpha(k,\Delta Q_k)\big) l_k^i(x) - x\right)^i 1\{x^i = L^i\} \qquad (13)$$

This equation is similar to that in the original paper, except that we put the term $1 - \alpha$ into the summation since different assets will have different proportions of asset losses from liquidation.

If the payment x^i is positive, the senior creditors will be paid in full, and the remaining amount will be divided amount junior creditors of the bank. If $x^i = 0$, the junior creditors will receive nothing, and even the senior creditors will suffer a loss of:

$$\delta^i(x) = \left(c_f^i - \beta\left(c_h^i + \sum_{k=1}^{K} e_k^i\,\alpha(k,\Delta Q_k) + \sum_{j=1}^{n} \pi^{ij} x^j\right)\right)^+ \qquad (14)$$

And the welfare loss will become:

$$W_\lambda(x) = \sum_{i=1}^{n}\sum_{k=1}^{K}\big(1-\alpha(k,\Delta Q_k)\big) l_k^i(x)$$

$$+(1-\beta)\sum_{i\in D(x)}\left(c_h^i + \sum_{k=1}^{K} e_k^i\,\alpha(k,\Delta Q_k) + \sum_{j=1}^{n}\pi^{ij}x^j\right) + \lambda \sum_{i\in D(x)} \delta^i(x) \qquad (15)$$

The first part is the amount of loss due to inefficient recovery of liquidated assets. The second term captures the value loss due to default. The last term are losses by senior creditors. The weight λ captures the relative importance of senior creditor’ s losses compared with losses from asset liquidation and bankruptcy. We will expect that the regulator will behave differently with different values of λ.

1.1 No Intervention

Algorithm 1. Calculation of Clearing payment Vector with no intervention

Unlike the case in Definition 2.1 in the paper "Bail-ins and Bail-outs", the clearing payment x cannot be directly determined by a fixed point given a financial system (L, π, c_h, c_f, e).

However, we can use an iterative method to derive this clearing payment as follows:

Step 1: Set $\alpha(k, \Delta Q_k) = 1$ for all assets k. Set $\Delta Q_k^i = -Q_k^i$ for all bank i

Step 2: Find the set of fundamentally default banks $\mathcal{F}$:

$$\mathcal{F} = \left\{ i \middle| L^i > c^i + \sum_{k=1}^{K} -\Delta Q_k^i P_k \alpha(k, \Delta Q_k) + (\pi L)^i \right\}$$

Step 3: Calculate $\alpha(k, \Delta Q_k) = \frac{\gamma_k Q_k^{nb}}{\gamma_k Q_k^{nb} - \sum_{i=1}^{N} \Delta Q_k^i}$

Step 4: Determine the unique clearing payment vector x.

Step 5:

(a) If $L^i \leq c^i + (\pi x)^i$: Set $\Delta Q_k^i = 0 \quad \forall k = 1 \dots K,\ i = 1 \dots N$

(b) If $L^i \geq c^i + (\pi x)^i + \sum_{k=1}^{K} e_k^i \alpha(k, \Delta Q_k)$: Set $\Delta Q_k^i = -Q_k^i \quad \forall k = 1 \dots K,\ i = 1 \dots N$

(c) If $c^i + (\pi x)^i < L^i < c^i + (\pi x)^i + \sum_{k=1}^{K} e_k^i \alpha(k, \Delta Q_k)$:

Set $\Delta Q_k^i = -\frac{\left(L^i - c^i - \sum_{j=1}^{n} \pi^{ij} x^j\right)^+ \omega_k^i}{P_k \alpha(k, \Delta Q_k)} \quad \forall k = 1 \dots K,\ i = 1 \dots N$

Step 6: Go back to step 2. Continue until convergence.

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We note that since the absolute value of $\sum_{k=1}^{K} |\Delta Q_k|$ is monotonically non-decreasing in each iteration, so $\alpha(k, \Delta Q_k)$ is monotonically non-increasing, and the number of banks in the set $\mathcal{F}$ is also non-decreasing. Therefore, the clearing payment vector x is non-increasing in each iteration, so this vector will converge to a unique vector of values $\bar{x}$.

And we can calculate the welfare loss under no intervention by definition:

$$W_N = W_\lambda(\bar{x}(0,0,0)) \tag{16}$$

Lemma 1. The welfare-maximizing complete bailout S_P is given by;

$$S_P^i = \begin{cases} \left(L^i - c^i - (\pi L)^i\right)^+ & if \sum_{k=1}^{K} \omega_k^i \dfrac{1-\alpha(k,\Delta Q_k)}{\alpha(k,\Delta Q_k)} > \lambda \\ \left(L^i - c^i - \displaystyle\sum_{k=1}^{K} e_k^i \alpha(k,\Delta Q_k) - (\pi L)^i\right)^+ & else \end{cases} \tag{17}$$

Notice that the criterial for choosing S_P^i is determined by a weighted average of percentage changes of asset prices, which is reasonable from intuition.

Proof:

Since $\frac{\left(L^i - c^i - \sum_{j=1}^{n} \pi^{ij} x^j\right)^+ \omega_k^i}{\alpha(k,\Delta Q_k)} = -\Delta Q_k^i P_k$

So the welfare loss due to liquidation of assets before adding S_P is;

$$\sum_{k=1}^{K} \frac{\left(L^i - c^i - \sum_{j=1}^{n} \pi^{ij} x^j\right)^+ \omega_k^i}{\alpha(k,\Delta Q_k)} (1 - \alpha(k,\Delta Q_k))$$

And the welfare loss due to liquidation of assets after adding S_P is:

$$\sum_{k=1}^{K} \frac{\left(L^i - c^i - S_P^i - \sum_{j=1}^{n} \pi^{ij} x^j\right)^+ \omega_k^i}{\alpha(k,\Delta Q_k)} (1 - \alpha(k,\Delta Q_k))$$

Therefore, for those default banks, one unit of S_P^i will have an impact of saving liquidation loss of:

$$\sum_{k=1}^{K} \frac{\omega_k^i}{\alpha(k,\Delta Q_k)} (1 - \alpha(k,\Delta Q_k))$$

If this value is higher than the unit cost of using tax-payers money, then it will be preferable to provide more money for bail-out.

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1.2 Public Bail-outs

Algorithm 2. Calculation of Clearing payment Vector under Public Bail-outs

Step 1: Set $\alpha(k, \Delta Q_k) = 1$ for all assets k, Set $\Delta Q_k^i = -Q_k^i$ for all bank i

Step 2: Find the set of fundamentally default banks $\mathcal{F}$:

$$\mathcal{F} = \left\{ i \middle| L^i > c^i + \sum_{k=1}^{K} \Delta Q_k^i P_k \alpha(k, \Delta Q_k) + (\pi L)^i \right\}$$

Step 3: Set the unique clearing payment vector $x = L$. (Under the bail-outs, liabilities can be paid in full).

Step 4: Calculate S_P^i by definition (in the first iteration, $S_P^i = \left(L^i - c^i - \sum_{k=1}^{K} e_k^i - (\pi L)^i\right)^+$) since $\lambda > 0$.

$$S_P^i = \begin{cases} \left(L^i - c^i - (\pi L)^i\right)^+ & if \sum_{k=1}^{K} \omega_k^i \dfrac{1 - \alpha(k, \Delta Q_k)}{\alpha(k, \Delta Q_k)} > \lambda \\ \left(L^i - c^i - \displaystyle\sum_{k=1}^{K} e_k^i \alpha(k, \Delta Q_k) - (\pi L)^i\right)^+ & else \end{cases}$$

Step 5:

(a) If $L^i \le c^i + (\pi x)^i + S_P^i$: Set $\Delta Q_k^i = 0 \quad \forall k = 1 \dots K,\ i = 1 \dots N$

(b) If $L^i \ge c^i + (\pi x)^i + S_P^i$:

$$\text{Set } \Delta Q_k^i = -\frac{\left(L^i - c^i - \sum_{j=1}^{n} \pi^{ij} x^j - S_P^i\right)^+ \omega_k^i}{P_k \alpha(k, \Delta Q_k)} \quad \forall k = 1 \dots K,\ i = 1 \dots N$$

Step 6: Calculate $\alpha(k, \Delta Q_k) = \frac{\gamma_k Q_k^{nb}}{\gamma_k Q_k^{nb} - \sum_{i=1}^{N} \Delta Q_k^i}$

Step 7: Go back to step 2.

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We note that $\alpha(k, \Delta Q_k)$, $\sum_{i=1}^{N} \Delta Q_k^i$, and S_P^i do not satisfy monotonicity during the iteration process in Algorithm 2. So we cannot guarantee that this method will converge to the true value of S_P^i in any case.

In the implementation of this procedure, this algorithm can converge in less than 10 iterations for our parameter settings, so whether this algorithm can convergence is not a severe problem in our case. But we cannot guarantee that it will work well for other parameters.

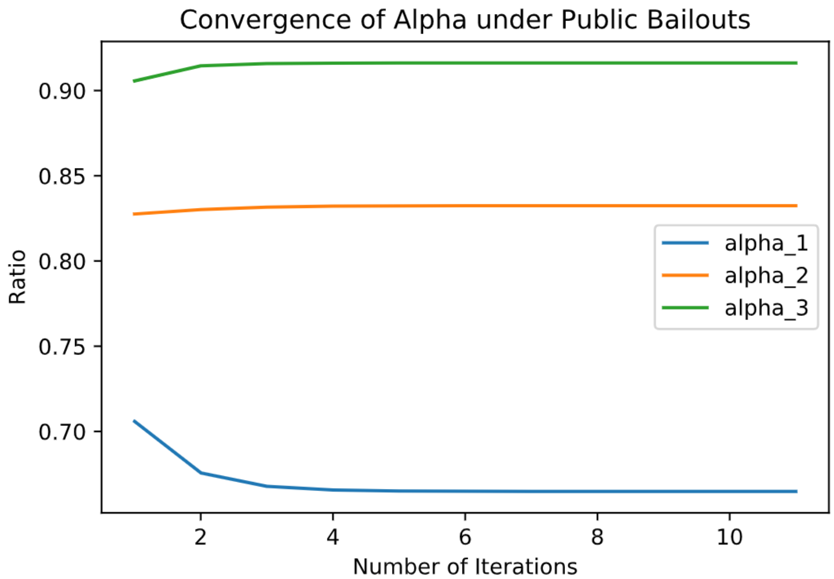

Figure 1: Number of iterations needed for converge in public bailouts

And we can calculate the welfare loss under public bail-outs as follows:

$$W_P = W_\lambda(0, S_P, 1) = W_\lambda(\bar{x}(0, S_P, 1)) + \lambda \sum_{i=1}^{N} S_P^i \tag{18}$$

1.3 Subsidized Bail-ins

The nominal amount that any bank i has to liquidate to retrieve its net contribution $[b^i 1\{a^i = 1\} - s^i]$ is:

$$l_c^{k,i}\left(b^i 1\{a^i = 1\} - s^i\right) = min\left(\frac{\left(L^i + [b^i 1\{a^i = 1\} - s^i] - c^i - \sum_{j=1}^{n} \pi^{ij} x^j\right)^+ \omega_k^i}{\alpha(k, \Delta Q_k)}, e_k^i\right) \quad (19)$$

This formula can be seen as adding $[b^i 1\{a^i = 1\} - s^i]$ to L^i in equation (12).

Bank $i's$ contribution to the bail-in reduces welfare losses by:

$$f^i(b, s, a) = \lambda b^i 1\{a^i = 1\} - \sum_{k=1}^{K} \left(1 - \alpha(k, \Delta Q_k)\right)\left(l_c^{k,i}\left(b^i 1\{a^i = 1\} - s^i\right) - l_c^{k,i}(-s^i)\right) \quad (20)$$

The first part is the saving of losses from tax-payers, which will be paid by bank i instead. But the bank may liquidate some external assets to make such a contribution. The second term captures the extra losses from liquidating more external assets to meet this contribution.

Theorem 1. Let $i_1, i_2, \dots, i_{|F^c|}$ be a non-increasing ordering of banks according to

$$v^i = \lambda \eta^i - \sum_{k=1}^{K} \left(1 - \alpha(k, \Delta Q_k)\right)\left(l_c^{k,i}\left(\eta^i - S_P^i\right) - l_c^{k,i}\left(-S_P^i\right)\right) \quad (21)$$

where

$$\eta^i = \begin{cases} \min\left(\sum_{j=1}^{n} \pi^{ij}\left(L^j - P_N^j\right), \left(c^i + (\pi L)^i + \sum_{k=1}^{K} e_k^i \alpha\left(k, \Delta Q_k\right) - L^i\right)\right) & if \sum_{k=1}^{K} \omega_k^i \frac{1 - \alpha\left(k, \Delta Q_k\right)}{\alpha\left(k, \Delta Q_k\right)} \le \lambda \\ \min\left(\sum_{j=1}^{n} \pi^{ij}\left(L^j - P_N^j\right), \left(c^i + (\pi L)^i - L^i\right)\right) & if \sum_{k=1}^{K} \omega_k^i \frac{1 - \alpha\left(k, \Delta Q_k\right)}{\alpha\left(k, \Delta Q_k\right)} > \lambda \end{cases} \quad (22)$$

Let $m = \min\left(k \middle| W_P - \sum_{m=1}^{M} v^{i_m} < W_N\right)$ (23)

Following the paper, this theorem can be derived from the extensions we illustrated before. The inequality in the if condition is similar with that in Lemma 1.

Following the paper "Bail-ins and Bail-outs", we can use the same condition to calculate welfare losses.

Welfare Losses

If $W_P < W_N$, then the no intervention threat will not be credible. The unique equilibrium is the public bailout S_P by the regulator, and the welfare loss will be the same as W_P.

If $W_P \geq W_N$, then the welfare losses in any weakly renegotiation proof equilibrium are equal to $w_* = \min\left(W_P - \sum_{m=1}^{M} v^{i_m}, W_N - v^{i_{m+1}}\right)$

Algorithm 3. Calculation of Clearing Payment Vector under Bail-ins

Step 1:

If $W_P < W_N$: choose public bail-out. $b_i^* = 0$, $s_i^* = S_P$. Return b_i^*, s_i^*.

Else, go to step 2.

Step 2: Set $\alpha(k, \Delta Q_k) = 1$ for all assets k, Set $\Delta Q_k^i = -Q_k^i$ for all bank i

Step 3: Set the unique clearing payment vector $x = L$. (Under the bail-ins, liabilities can be paid in full).

Step 4: set η^i, v^i and m by definition in Theorem 1.

$$\eta^i = \begin{cases} \min\left(\sum_{j=1}^{n} \pi^{ij}\left(L^j - P_N^j\right), \left(c^i + (\pi L)^i + \sum_{k=1}^{K} e_k^i \alpha\left(k, \Delta Q_k\right) - L^i\right)^+\right) & if \sum_{k=1}^{K} \omega_k^i \frac{1 - \alpha\left(k, \Delta Q_k\right)}{\alpha\left(k, \Delta Q_k\right)} \leq \lambda \\ \min\left(\sum_{j=1}^{n} \pi^{ij}\left(L^j - P_N^j\right), \left(c^i + (\pi L)^i - L^i\right)^+\right) & if \sum_{k=1}^{K} \omega_k^i \frac{1 - \alpha\left(k, \Delta Q_k\right)}{\alpha\left(k, \Delta Q_k\right)} > \lambda \end{cases}$$

Let $m = \min\left(k \middle| W_P - \sum_{m=1}^{k} v^{i_m} < W_N\right)$

$$set\ v^i = \lambda \eta^i - \sum_{k=1}^{K} \left(1 - \alpha(k, \Delta Q_k)\right) \left(l_c^{k,i}\left(\eta^i - S_P^i\right) - l_c^{k,i}\left(-S_P^i\right)\right)$$

Step 5 :

If $W_P - \sum_{j=1}^{m} v^{i_j} < W_N - v^{i_{m+1}}$:

set $b_i^* = \eta^i$ for $i = 1,2, \ldots, m$, set $s_i^* = S_P$.

Else if $W_P - \sum_{j=1}^{m} v^{i_j} \geq W_N - v^{i_{m+1}}$, set b_i^* and s_i^* satisfying the bail-in proposal as that in the original paper.

Step 6:

(a) If $b^i + L^i \le c^i + (\pi x)^i + s^i$: Set $\Delta Q_k^i = 0 \quad \forall k = 1 \dots K,\ i = 1 \dots N$

(b) If $b^i + L^i \ge c^i + (\pi x)^i + s^i$:

Set $\Delta Q_k^i = -\frac{\left(b^i + L^i - c^i - \sum_{j=1}^{n} \pi^{ij} x^j - s^i\right)^+ \omega_k^i}{P_k \alpha(k, \Delta Q_k)} \quad \forall k = 1 \dots K,\ i = 1 \dots N$

Step 7: Calculate $\alpha(k, \Delta Q_k) = \frac{\gamma_k Q_k^{nb}}{\gamma_k Q_k^{nb} - \sum_{i=1}^{N} \Delta Q_k^i}$

Step 8: Go back to step 3.

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Same as Algorithm 2, the convergence of algorithm 3 is not sure. Our code implementation can converge in less than 10 iterations.

Amplification of losses

The portion of the shortfall that is amplified through the system:

$$\chi_0 = \sum_{i=1}^{n} \left(L^i - c^i - (\pi L)^i - \sum_{k=1}^{K} e_k^i \alpha\left(k, \Delta Q_k\right) \right)^+ - \sum_{i \in D(x)} \delta^i (P_N) \tag{24}$$

The losses accruing to the financial system after the shock has spread through the network:

$$\chi_N = \sum_{i \in \mathcal{F}} \sum_{k=1}^{K} \left(1 - \alpha\left(k, \Delta Q_k\right)\right) e_k^i + \sum_{i \notin \mathcal{F}} \left(c^i + \sum_{k=1}^{K} e_k^i + (\pi L)^i - L^i - V^i (P_N) \right) \tag{25}$$

Since the formula is quite complex, the monotonicity relationship between $\chi_N - \chi_0$ and λ is ambiguous here.

2. Network Structure

2.1 Core-periphery Model

In the implementation later, we will test our model on ring and complete networks. However, interbank markets are tiered rather than flat. Most banks do not lend to each other directly but through money center banks as intermediaries. Therefore, in this section, we developed a core-periphery model to capture this structure of interbank lending.

The notion "tiered" is in contrast to a flat structure. Top-tier banks tend to lend to each other, borrow and lead to lower-tier banks. And the lower-tier banks do not lend to each other directly.

In the adjacency matrix, we can put all the core banks in the left top corner and all the peripheral banks in the right bottom corner. If the binary value in i^{th} row and j^{th} column is 1, then it means that there's lending activity from bank i to bank j. In this way, we can divide this matrix into four parts:

$$A = \begin{bmatrix} CC & CP \\ PC & PP \end{bmatrix} = \begin{bmatrix} 1 & CP \\ PC & 0 \end{bmatrix} \tag{26}$$

Where CC represents core-to-core lending, PC represents peripheral-to-core lending, and so on. We set all elements in the block CC to 1 by assuming that all core banks will have interbank lending activities with each other. And we also assume that peripheral banks have to be involved into the interbank market through these money center banks.

3. Simulation Results

3.1 Ring and Complete Networks

In this part, we will test our result by using similar parameters settings as that shown in the original paper.

Firstly, we assume that there' re three external assets with different liquidity conditions in the market, with the first asset having lowest liquidity, and the third asset having the highest liquidity. And we assume that there' s a shock for bank 1, and the level of interbank liabilities, net cash position and external assets positions are as follows:

Table 1: Simulation parameters for ring and complete network

Bank	1	2	3	4	5	6
$\boldsymbol{L}$	1	1	1	1	1	1
$\boldsymbol{c}$	-1	0.05	0.05	0.05	0.05	0.05
$\boldsymbol{e_1}$	0.17	0.17	0.13	0.23	0.07	0.03
$\boldsymbol{e_2}$	0.17	0.17	0.13	0.23	0.07	0.03
$\boldsymbol{e_3}$	0.17	0.17	0.13	0.23	0.07	0.03

$\boldsymbol{\gamma_1}$	$\boldsymbol{\gamma_2}$	$\boldsymbol{\gamma_3}$	$\boldsymbol{\beta}$	$\boldsymbol{Q_{nb}^1}$	$\boldsymbol{Q_{nb}^2}$	$\boldsymbol{Q_{nb}^3}$
0.5	1	2	0.85	0.8	0.8	0.8

Table 2: Simulation result for proportional and liquidity-based liquidation strategy

Proportional				
	$\hat{p}$	$\|D\|$	w_N	w^*
Complete	$(0.22,1,1,1,1,0.87)$	2	0.56	0.38
Ring	$(0.23,0.57,0.82,1,1,1)$	3	0.73	0.61
Liquidity-based				

	$\hat{p}$	$\|D\|$	w_N	w^*
Complete	$(0.24,1,1,1,1,0.88)$	2	0.50	0.32
Ring	$(0.23,0.58,0.83,1,1,1)$	3	0.70	0.54

As can be seen, the interbank payment vector is higher under liquidity-based liquidation strategy, and the welfare losses are also lower. This result is intuitive since liquidity-based strategy will generate lower price impact on external assets, which can reduce the gap between nominal asset liquidation values and their realized liquidation values.

Table 3: External asset liquidation ratios

Bank	**1**	**2**	**3**	**4**	**5**	**6**
Proportional						
e_1	100%	42%	53%	30%	100%	100%
e_2	100%	33%	42%	24%	83%	100%
e_3	100%	31%	39%	23%	77%	100%
Liquidity-based						
e_1	100%	0%	0%	0%	27%	100%
e_2	100%	0%	21%	0%	100%	100%
e_3	100%	99%	100%	71%	100%	100%

The table above shows the percentage of assets liquidated for each bank. Since the third asset is most liquid, so the liquidity based strategy will always liquidate it first. And as for the proportional liquidation strategy, banks also need to liquidate the first and second assets more so as to have the same realized liquidation value as the third asset.

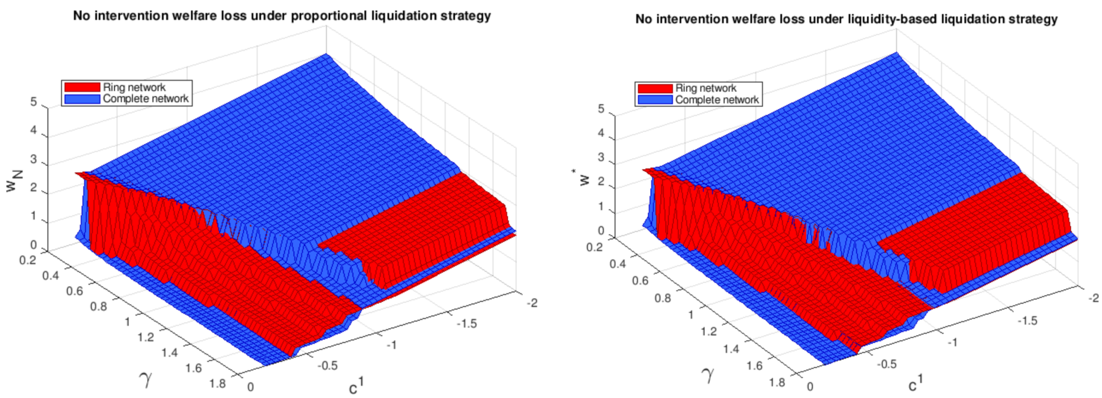


Figure 2: Comparison of no intervension welfare under different c_1 and γ

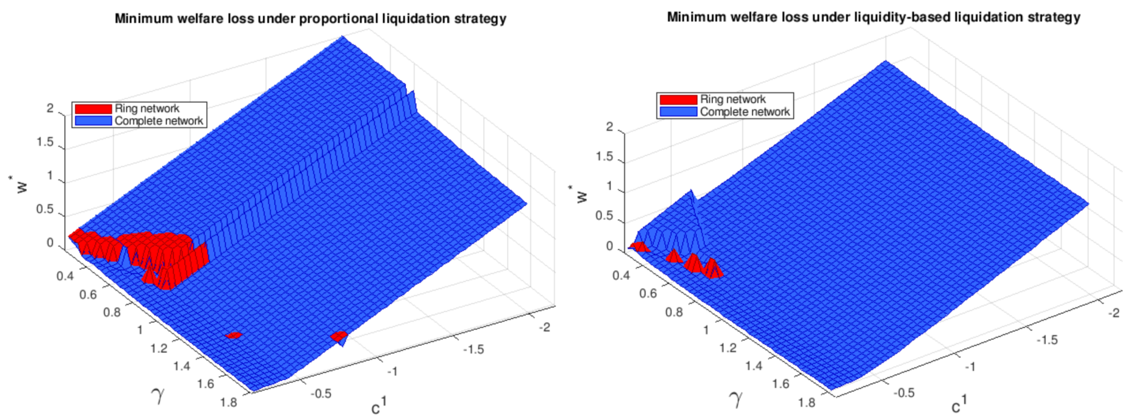


Figure 3: Comparison of minimum welfare under different c_1 and γ

Figure 2 and Figure 3 show us the conditions under which ring network will be better than the complete one. Figure 2 compares w_N under proportional and liquidity-based strategy on ring and complete networks. To make it more conclusive, we summarize the result in the top row with the following table:

Table 4: Welfare losses under different settings in ring and complete network

	Small γ	Large γ
Small shock	Complete < Ring	Complete < Ring
Large shock	Ring < Complete	Complete < Ring

As can be seen, under small shocks, the complete network performs better than the ring network. This is because all other banks can be involved to liquidate some external assets to avoid default of the bank suffering from shock. However, when the shock size is large enough, then the complete network will become worse due to its higher contagion effect than the ring network.

Figure 3 shows the minimum welfare losses between w_N and w_P on two networks. We can see that the minimum welfare of the ring network is lower than that of the complete network nearly in all situations. On reason for this is that given a certain size of shock c_1, the no-intervention threat under ring network is always more credible than that under complete network, which means that $w_P < w_N$ in the ring network is more credible. Therefore, we would expect the red area in Figure 3 to be smaller than that in Figure 2. Figure 4 below confirms this explanation.

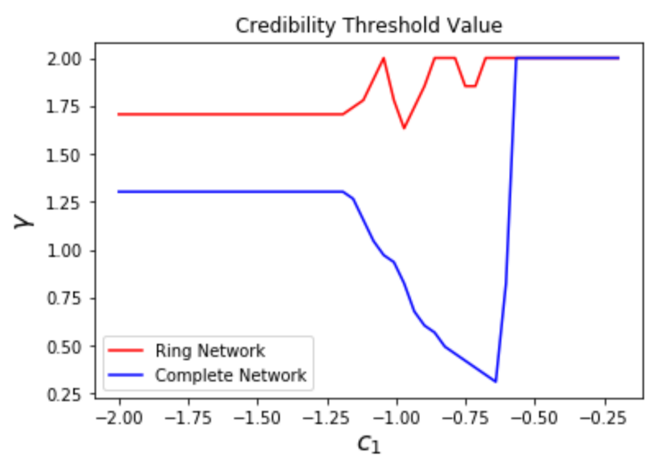


Figure 4: Credibility threshold of no intervention under different shock sizes

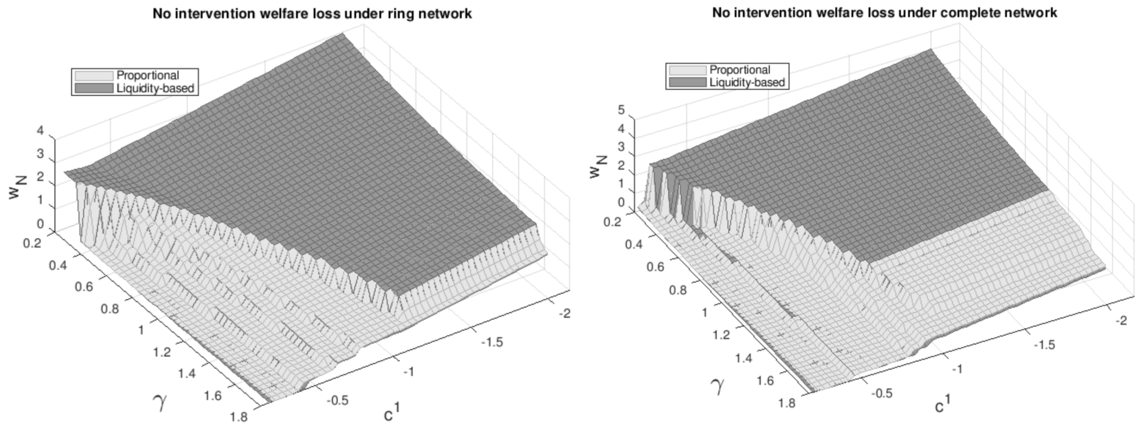


Figure 5: Comparison between proportional and liquidity-based liquidation strategies

Figure 5 shows the no intervention welfare under two liquidation strategies. The liquidity based strategy seems to dominate the proportional liquidation strategy except in the case when the shock size is large. But actually, the different between welfare losses saving by using proportional liquidation strategy when there' s large shock is very tiny (lower than 0.01), so we can conclude that liquidity-based liquidation strategy is a better choice.

3.2 Core-peripheral Networks

In section 2, we have introduced core-peripheral model. The following is an example:

$$\begin{bmatrix} 1 & CP \\ PC & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

And we can visualize this adjacency matrix into graphs as follows. We also listed two other situations that the definition in equation (26) is violated.

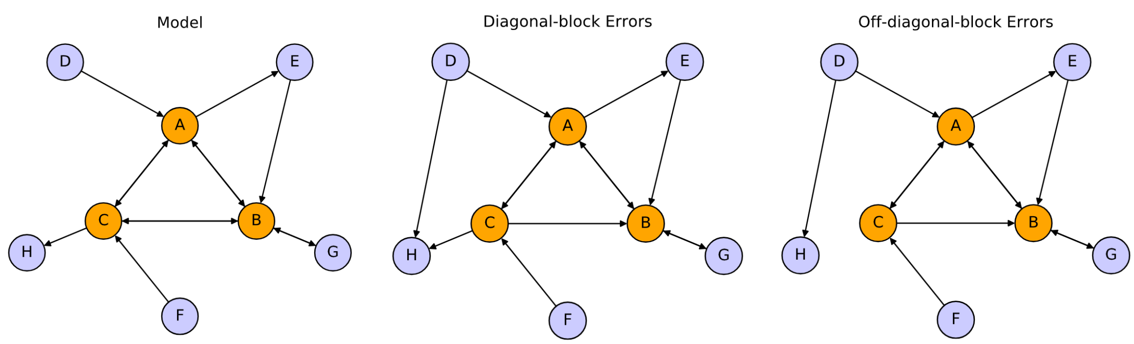


Figure 6: Core-peripheral model for interbank market

From the statistics of Ben Craig, Goetz von Peter (2014), it shows the number of links between core and peripheral blocks, as well as the credit exposures of each block. Therefore, the exposures ratios per link can be calculated. We summarize it as follows:

Table 5: Relative exposures per link in core-peripheral model

Links per block	
6.6%	26.1%
58.5%	8.8%

Credit exposures	
34.6%	47.6%
16.0%	1.8%

Exposures ratio per link	
25.63	8.92
1.34	1.00

Based on the exposure ratio per link, we can assume that the relative size of money centered banks is about 25 times the size of peripheral banks.

Now, as can be seen in the first standard model, bank D lends to A, without borrowing any money from money-centered banks. Therefore, we will expect that if there’ s shock in one of the money centered banks A, B or C, then this bank D will never default. Similarly, the default of bank F will also be impossible.

Besides, bank E and G both borrow and lend money to money centered banks. From the exposure relationship above, the ratio in PC block is 1.34, which is lower than the ratio in CP block. This indicates that the peripheral banks tend to borrow more money from the centered banks than lending to them.

Take the first standard structure that conforms to the definition as an example, we can set the parameters for simulation from this observation as follows:

Table 6: Simulation parameters for core-peripheral network

Bank	1	2	3	4	5	6	7	8
$\boldsymbol{L}$	0.9753	1	0.9753	0	0.1711	0.1711	0.1711	0.1711
$\boldsymbol{c}$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
$\boldsymbol{e_1}$	0.5	0.5	0.5	0.1	0.1	0.1	0.1	0.1
$\boldsymbol{e_2}$	0.5	0.5	0.5	0.1	0.1	0.1	0.1	0.1
$\boldsymbol{e_3}$	0.5	0.5	0.5	0.1	0.1	0.1	0.1	0.1

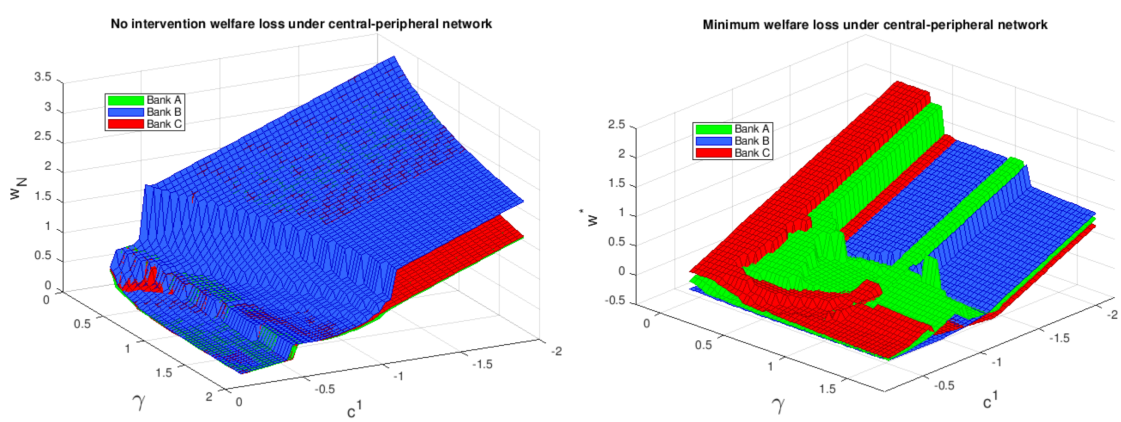


Figure 7: Comparison between losses for core banks under first graph topology

In the left graph above, we set shock with size c_1 on different systemic important banks A, B and C. Bank A has lowest welfare loss than the other two core banks. If we compare the position of bank A with bank C, we will find that they both borrow from one peripheral banks (bank D for A, bank F for C). However, since bank E, which is the bank that bank A lends money to, also lends money to bank B, so E is more resistant to shock than H. Therefore, A will be more insensitive to shocks than C.

As for bank B and C, since bank C has less connections with peripheral banks, so the welfare loss from contagion of shocks starting from bank C will be lower than bank B.

In the right graph of Figure 7, we compared the minimum welfare losses for three banks, but the result is kind of ambiguous here. It' s hard for us to interpret it without future investigation.

In the following graph, we set $\gamma = (0.5,1,2)$, and let c_1 varies. We can confirm our analysis above by looking at the number of default banks as we increase shock sizes in the left figure.

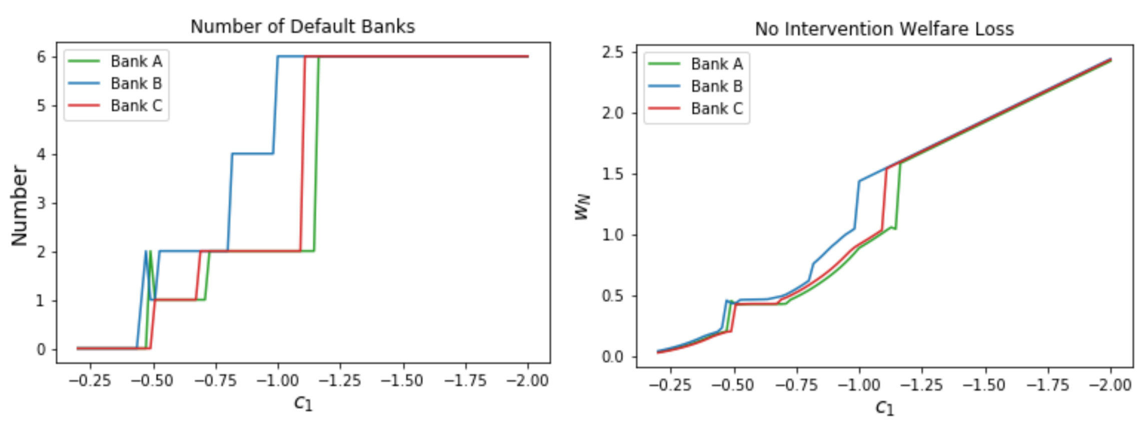
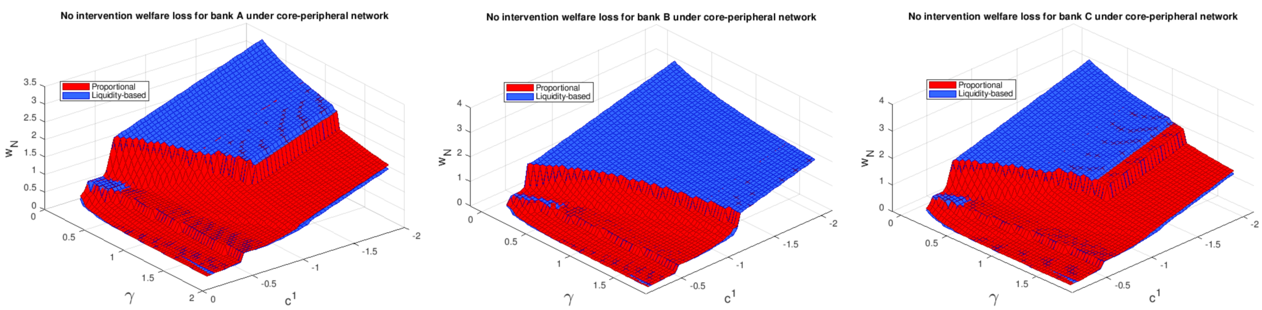
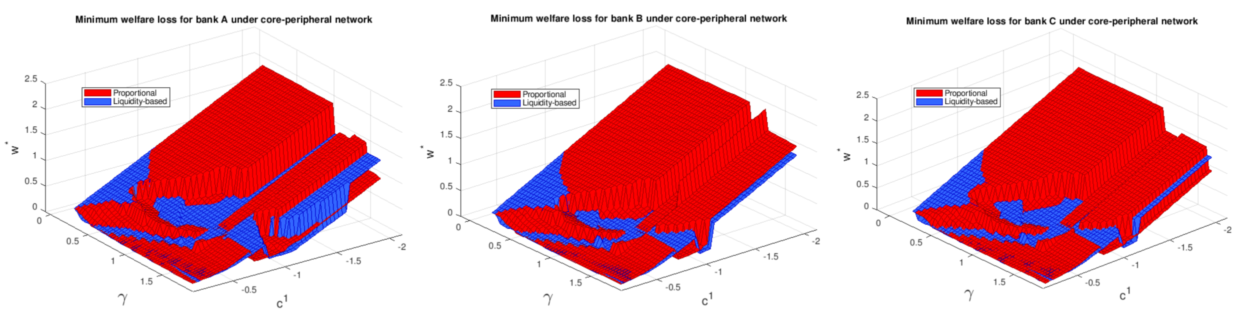


Figure 8: Comparison between core banks under certain price elasticity

As for the two liquidation strategies, we can see the result compared for no intervention welfare in the following figures. The liquidity-based liquidation strategy is nearly optimal in all situations. Note that in the blue area, the welfare losses of these two strategies will be very close, just as what we have plotted in the right panel of Figure 8 when shock size c_1 is large. As for the minimum welfare losses, the liquidity-based strategy seems to perform better under most situations, but it' s hard to interpret the result where the shock size is low.



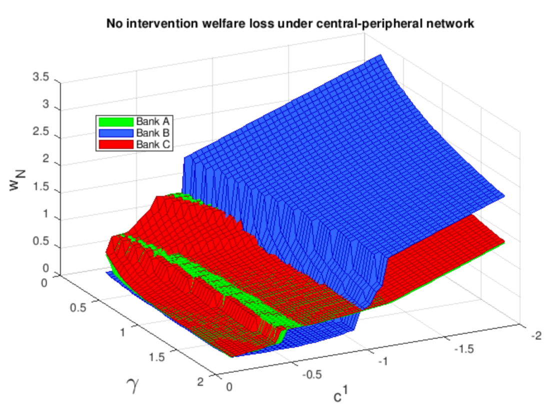
(a) No intervention welfare losses



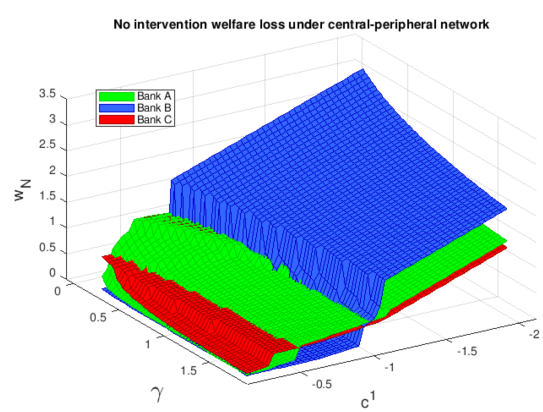
(b) Minimum welfare losses

Figure 9: Comparison of liquidation strategies for three graph topology

We can also test our result in other two structures of core-peripheral networks:



(a) Diagonal block errors



(b) Off-diagonal block errors

Figure 10: No intervention welfare losses for other network structures

As we can see, when we set shock to bank B with size larger than 1, the welfare loss will have a sharp increase. This is because the interbank lending from bank C to bank B constitutes the highest part of the interbank assets for bank C. When bank B is unable to pay enough back to C, the risk will contage to bank C pretty fast. Moreover, we can find that if the shock size is large enough, then welfare loss with starting shock in bank A will be higher in the off-diagonal block error structure than the diagonal block error structure. This is because firstly, bank D will be affected if there’ s shock in bank A, and then bank H will be affected. But bank H can also get some extra interbank subsidy from C in the second structure, so it will be less vulnerable than the third structure.

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