MATHEMATICAL FORMULATION FOR INTELLIGENT RESOURCE ALLOCATION IN WIRELESS COMMUNICATIONS SYSTEMS

We consider a multi-channel cellular system which allows direct communication among users, i.e., device-to-device (D2D) communication. We assume that the number of channels is K, and for each channel, one cellular user equipment (CUE) transmits data to the base station (BS), i.e., uplink transmission. The number of D2D user equipments (DUEs) is set to N where each DUE consists of one transmitter and one receiver, and $\mathbb{I}$ denotes the set of D2D transmitters. Moreover, the channel gain between the i-th transmitter and the j-th receiver for channel k, which comprises both the distance-related channel gain (i.e., path loss) and multipath fading, is denoted as $h_{i,j}^k$, where the index 0 is assigned to the CUE and BS, e.g., $h_{0,j}^k$ is the channel gain between the CUE and j-th DUE receiver for channel k.

The transmit power of the DUEs allocated to each channel is denoted as P_i^k where k and i are the index of the channel and the DUE, respectively. Moreover, we assume that the maximum transmit power of DUEs is P_T such that $\sum\limits_{k \le K} P_i^k \le P_\text{T}$ for all i. Furthermore, we assume that the transmit power of CUE is P_M. In addition, we denote W and N_0 as the bandwidth and the noise spectral density, respectively. Accordingly, in our system model, the spectral efficiency (SE) of DUE i, SE_i, can be written as

$$\text{SE}_i = \sum_{k \le K} \log_2 \left(1 + \frac{h_{i,i}^k P_i^k}{N_0 W + \sum_{l \in \mathbb{I} \setminus \{i\}} h_{l,i}^k P_l^k + h_{0,i}^k P_\text{M}} \right). \tag{1}$$

Moreover, the energy efficiency (EE) of DUE i, EE_i, can be written as

$$\text{EE}_i = \frac{\text{SE}_i}{\sum\limits_{k \le K} P_i^k + P_\text{C}}, \tag{2}$$

where P_C is the circuit power of the DUE.

In our work, we consider the optimization with three different objectives which are 1) maximization of SE, 2) maximization of EE, and 3) minimization of the total transmit power. Moreover, we take three constraints into account. First, the transmit power allocated to each channel should be non-negative and the sum of the transmit power for a single DUE must not exceed the maximum transmit power, P_T (transmit power constraint). Second, the amount of interference caused to the cellular transmission must be less than the threshold I_T (interference constraint). Third, each D2D transmission should satisfy the minimum QoS requirement, i.e., the SE achieved by each DUE must be greater than the threshold, R_T (QoS constraint).

Accordingly, the optimization problem to maximize the SE of DUEs can be formulated as follows:

$$
\begin{aligned}
\underset{P_i^k}{\text{maximize}} \quad & \sum_{i\in\mathbb{I}} \mathrm{SE}_i \\
\text{s.t.} \quad & \sum_{k\le K} P_i^k \le P_{\mathrm{T}} && \forall i \in \mathbb{I} \\
& 0 \le P_i^k && \forall i \in \mathbb{I},\ \forall k \le K \\
& \textstyle\sum_{l\in\mathbb{I}} h_{l,0}^k P_l^k \le I_{\mathrm{T}} && \forall k \le K \\
& \mathrm{SE}_i \ge R_{\mathrm{T}} && \forall i \in \mathbb{I}.
\end{aligned}
\tag{3}
$$

Similarly, the optimization problem to maximize the EE of DUEs can be formulated as follows:

$$
\begin{aligned}
\underset{P_i^k}{\text{maximize}} \quad & \sum_{i\in\mathbb{I}} \mathrm{EE}_i \\
\text{s.t.} \quad & \sum_{k\le K} P_i^k \le P_{\mathrm{T}} && \forall i \in \mathbb{I} \\
& 0 \le P_i^k && \forall i \in \mathbb{I},\ \forall k \le K \\
& \textstyle\sum_{l\in\mathbb{I}} h_{l,0}^k P_l^k \le I_{\mathrm{T}} && \forall k \le K \\
& \mathrm{SE}_i \ge R_{\mathrm{T}} && \forall i \in \mathbb{I}.
\end{aligned}
\tag{4}
$$

Finally, the optimization problem to minimize the total transmit power of DUEs can be formulated as follows:

$$
\begin{aligned}
\underset{P_i^k}{\text{minimize}} \quad & \sum_{i\in\mathbb{I}} \sum_{k\le K} P_i^k \\
\text{s.t.} \quad & \sum_{k\le K} P_i^k \le P_{\mathrm{T}} && \forall i \in \mathbb{I} \\
& 0 \le P_i^k && \forall i \in \mathbb{I},\ \forall k \le K \\
& \textstyle\sum_{l\in\mathbb{I}} h_{l,0}^k P_l^k \le I_{\mathrm{T}} && \forall k \le K \\
& \mathrm{SE}_i \ge R_{\mathrm{T}} && \forall i \in \mathbb{I}.
\end{aligned}
\tag{5}
$$

The optimization problems formulated in (3) - (5) are nonconvex such that they are extremely hard to be solved analytically with low computational complexity. Moreover, given that the optimal solutions depend on the channel gain ($h_{i,j}^k$), the transmit power has to be re-derived whenever the channel gain changes. Accordingly, in our work, we have proposed a DNN-based resource allocation (RA) to find the optimal transmit power which can solve the aforementioned problems with low computation time.

Mathematical Formulation for Loss Functions

For the training, in order to achieve the goal while satisfying the constraints, the weighted sum of the objective function and the functions of the constraints are taken into account. Accordingly, in order to train the DNN model to maximize the SE, the loss function, L_{SE}, can be set as follows:

$$L_{\text{SE}} \qquad -\sum \text{SE}_i + \lambda_1 \sum \tanh\left([I_{\text{CUE}}^k - I_{\text{T}}]^+\right) + \lambda_2 \sum \tanh\left([\text{SE}_i - R_{\text{T}}]^+\right), \tag{6}$$

where λ_1 and λ_2 are the controlling parameters, and I_{CUE}^k is the interference caused to the CUE at channel k.

As shown by the formulation, the loss function increases as the SE of the DUEs decreases. It also increases when the interference or QoS constraints are not satisfied. Given that the DNN is trained to reduce the loss, through training, the SE of the DUEs will be increased and the violation caused by the interference and QoS constraints will be reduced. Note that λ_1 and λ_2 determine the penalty for the violation of the constraints. When the value of λ_1 is small, for example, the interference constraint is barely considered in the loss function, such that the transmit power is learned without considering the interference caused to the CUE.

Similarly, the loss function to maximize EE, L_{EE}, can be written as follows:

$$L_{\text{EE}} \qquad -\sum \text{EE}_i + \lambda_1 \sum \tanh\left([I_{\text{CUE}}^k - I_{\text{T}}]^+\right) + \lambda_2 \sum \tanh\left([\text{SE}_i - R_{\text{T}}]^+\right), \tag{7}$$

where the EE of DUE i, i.e., EE_i, can be written as $\frac{\text{SE}_i}{\sum P_i^k + P_{\text{C}}}$ and P_{C} is the circuit power of the DUE.

Finally, the loss function to minimize the total transmit power, which we denote as L_{PW}, can be written as follows:

$$L_{\text{PW}} \qquad \sum P_i^k + \lambda_1 \sum \tanh\left([I_{\text{CUE}}^k - I_{\text{T}}]^+\right) + \lambda_2 \sum \tanh\left([\text{SE}_i - R_{\text{T}}]^+\right). \tag{8}$$

Computational Complexity Analysis

Herein, we formulate the computational complexity of the proposed scheme and that of exhaustive search (ES). In our proposed scheme, the channel gain has to be pre-processed before being fed into DNN. Note that in our complexity analysis, we only consider the computational complexity incurred during inference phase (i.e., forward path) and the complexity during training (i.e., back propagation path) is not taken into account because the training can be executed beforehand its actual usage [1].

The computational complexity incurred for pre-processing will be $\mathcal{O}((N+1)^2K)$, where N and K are the number of DUEs and channels, respectively, and $\mathcal{O}(\cdot)$ is the big-$\mathcal{O}$ notation. Then, the pre-processed channel gain is fed into two independent networks, which are total transmit power network (Tnet) and power allocation network (Pnet). Given that the computation of DNN can be considered as a matrix computation, the computational complexity of Tnet becomes $\mathcal{O}((N+1)^2KZ_T) + \mathcal{O}(Z_T^2(L_T-1)) + \mathcal{O}(Z_TN)$, where Z_T and L_T are the number of hidden nodes and layers in Tnet, respectively [2]. Similarly, the computational complexity of Pnet can be calculated as $\mathcal{O}((N+1)^2KZ_P) + \mathcal{O}(Z_P^2(L_P-1)) + \mathcal{O}(Z_PN)$, where Z_P and L_P are the number of hidden nodes and layers in Pnet, respectively. Finally, the outputs from Tnet and Pnet are multiplied whose computational complexity becomes $\mathcal{O}(NK)$. In summary, the total computational complexity of our proposed scheme can be formulated as $\mathcal{O}(N^2K(Z_T+Z_P) + Z_T^2L_T + Z_P^2L_P)$. When the size of DNN structure is fixed, the computational complexity can be simplified to $\mathcal{O}(N^2K)$, such that the our proposed scheme has polynomial complexity regarding the number of users and channels. The computational complexity of DNN-based RA using SL [3] can be derived similarly.

On the other hand, the optimal performance can be found through ES where the transmit power level of DUE on each channel is quantized with Q equally-spaced values and all combinations of quantized values are examined. In this case, the computational complexity becomes $\mathcal{O}\big(Q^{NK}\big)$ such that it grows exponentially with N and K. Accordingly, as N and K increase, the computation time of ES will increase more rapidly compared to that of our proposed scheme and the gap between the computation time for both schemes will increase significantly, which reveals the benefit of our proposed scheme, especially for a large system configuration.

ADDITIONAL SIMULATION RESULTS

In this section, we have provided simulation results which were not added to the manuscript due to the limitation of the IEEE Communications Magazine on the total length of paper and the total number of figures.

First, in Figs. 1-3 of this response letter, we show the performance of our proposed scheme as a function of the number of DUEs, N, when K = 10 and the size of area, D, is 30 m. In the simulation, we were unable to evaluate the performance of ES due to the excessive computation time. For example, the required computation time for ES is longer than 30 seconds for one channel realization when $N = K = 4$, such that obtaining one simulation point requires approximately 7 days in our current simulation setting. Consequently, we were also unable to show the performance of DNN-based RA using SL because the label data for training cannot be obtained due to the unavailability of ES.

First of all, we can find that our proposed DNN-based RA achieves desired goals. Specifically, the DNN-based RA for SE maximization shows the highest SE, the DNN-based RA for EE maximization shows the highest EE, and the DNN-based RA for transmit power minimization shows the lowest total transmit power. Especially, we can observe that our proposed scheme can achieve sufficiently higher performance compared to the random scheme which justifies the usefulness of our proposed DNN-based RA.

Moreover, as can be seen from the simulation results, the average SE and EE decrease as the number of DUEs increases because more interference is incurred among DUEs. Moreover, we can find that the total transmit power of DNN with the maximization of SE (Max. SE) and the maximization of EE (Max. EE) decreases as the number of DUEs increases in order to meet the constraint on the interference regarding the cellular transmission, i.e., $\sum_{l \in \mathbb{I}} h_{l,0}^{k} P_l^k \leq I_{\mathrm{T}}$. However, for the DNN with the minimization of transmit power (Min. PW), the quality-of-service (QoS) constraint regarding DUEs, $\mathrm{SE}_i \geq R_{\mathrm{T}}$, is more critical and harder to satisfy such that the total transmit power increases as N increases.

Next, we compare the performance of our proposed scheme with the weighted minimum mean square error (WMMSE)-based RA. To this end, we have modified our original system model and performed additional simulations. Specifically, we have removed the interference and QoS constraints and also taken into account single channel system same as [3]. Accordingly, the

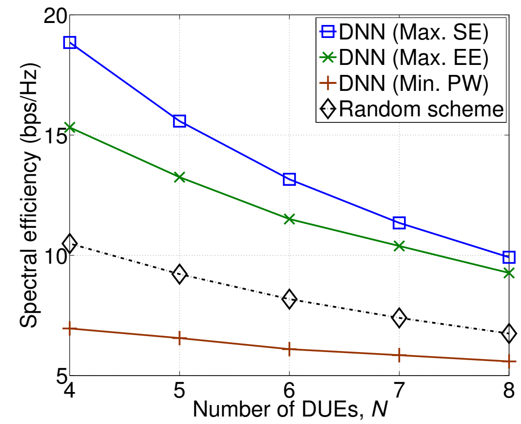

Fig. 1. Spectral efficiency vs. N when D = 30 m and K = 10.

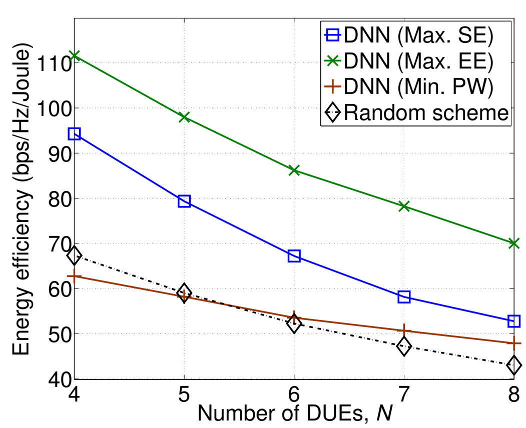

Fig. 2. Energy efficiency vs. N when D = 30 m and K = 10.

modified optimization problem is written as follows:

$$\begin{aligned} \underset{P_i}{\text{maximize}} \quad & \sum_{i \in \mathbb{I}} \log_2 \left(1 + \frac{h_{i,i} P_i}{N_0 W + \sum_{l \in \mathbb{I} \setminus \{i\}} h_{l,i} P_l + h_{0,i} P_{\mathrm{M}}} \right) \\ \text{s.t.} \quad & 0 \leq P_i \leq P_{\mathrm{T}} \qquad \forall i \in \mathbb{I}, \end{aligned} \tag{9}$$

where the superscript k is dropped since we considered the single channel system.

The optimization problem in (9) can be solved by the WMMSE-based RA proposed in [3]. In Figs. 4 and 5, we show the average SE of DUEs for various values of the number of DUEs when D = 30 and 60 m, respectively. Moreover, we also show the measured computation time in Fig. 6. Furthermore, in the performance evaluation, we have also considered DNN-based RA using supervised learning (SL) proposed in [3].

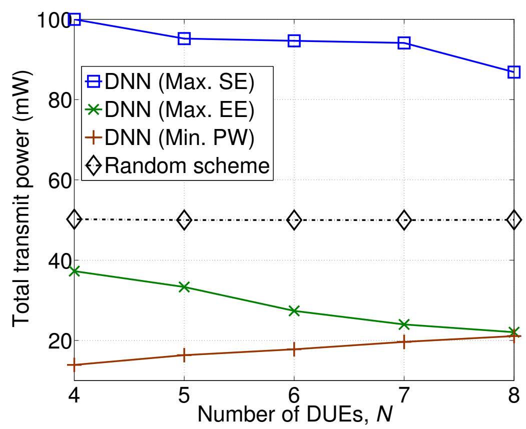

Fig. 3. Total transmit power vs. N when D = 30 m and K = 10.

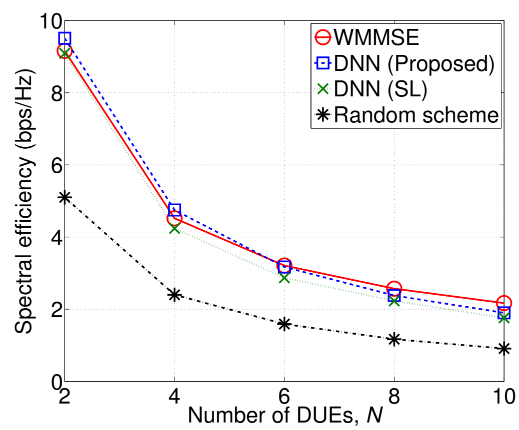

Fig. 4. Spectral efficiency vs. N when D = 30 m.

As can be seen from the simulation results, our proposed scheme and DNN with SL can achieve similar SE with the WMMSE-based scheme and they show higher SE compared to the random scheme, which again confirms the necessity of a proper RA. Especially, we can find that the SE of our proposed scheme is slightly higher than that of WMMSE-based scheme when N is small while the SE of DNN with SL is always smaller than that of WMMSE-based scheme. However, as N increases, the SE of our proposed scheme becomes smaller than that of WMMSE-based scheme mainly due to the lack of computational capability of DNN structure since we have considered small DNN structure. Moreover, we can find that the SE decreases as the number of DUEs increases, due to the increased interference among DUEs. For the same reason, the SE of considered schemes when D = 30 m is lower than that of SE when D = 60 m.

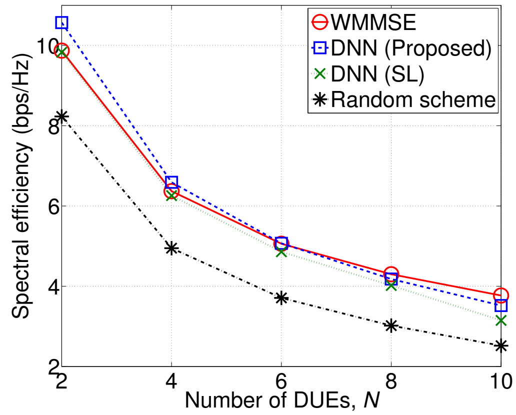

Fig. 5. Spectral efficiency vs. N when D = 60 m.

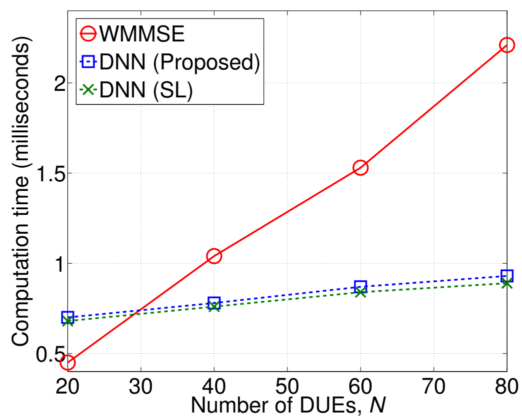

Fig. 6. Computation time vs. N.

Furthermore, we can find that the computation time of WMMSE-based scheme can be smaller than that of DNN-based scheme for small N, however, as N increases, the computation time of the DNN-based scheme becomes smaller than that of WMMSE-based scheme. From these results, we can confirm that our proposed scheme is beneficial in terms of computation time when the number of users is large, e.g., ultra-dense network (UDN).

References

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