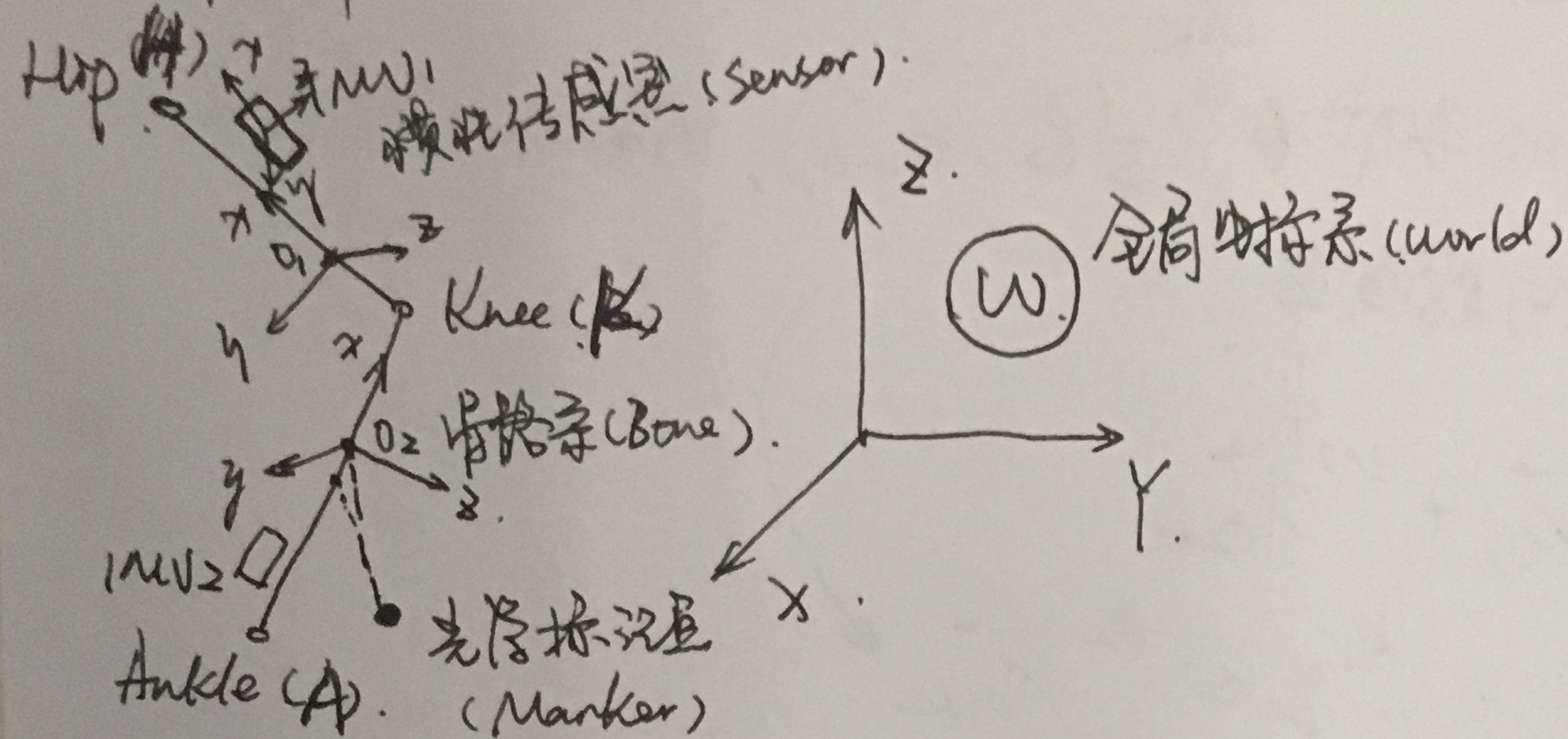
运动链四元数运动学表示方法:



问题描述:

给定IMU在全局系的测量姿态 $\left\{ {}^{S_1}_{W}q,\ {}^{S_2}_{W}q \right.$，光点在全局系的测量位置 ${}^{W}\vec{x}_M$

及校准后的安装参数:

IMU在骨骼系中的相对安装姿态 $\{ {}^{S_1}_{B_1}q,\ {}^{S_2}_{B_2}q \}$，光点在小腿骨骼系中的相对安装位置 ${}^{B_2}\vec{x}_M$ (认为小腿骨骼系原点O2在小腿骨中点).

以及骨骼尺寸: 大腿长 l_1，小腿长 l_2.

求: 髋关节H点在全局系的位置 ${}^{W}\vec{x}_H$.

解: 1. 求骨骼姿态(相对于全局系).

$$\begin{cases} {}^{W}_{B_1}q = {}^{S_1}_{B_1}q \otimes {}^{W}_{S_1}q = {}^{S_1}_{B_1}q \otimes {}^{S_1}_{W}q^{-1} \\ {}^{W}_{B_2}q = {}^{S_2}_{B_2}q \otimes {}^{W}_{S_2}q = {}^{S_2}_{B_2}q \otimes {}^{S_2}_{W}q^{-1} \end{cases}$$

2. 求小腿中点(骨骼系原点)在全局系位置.

$${}^{W}\vec{x}_{O_2} = {}^{W}\vec{x}_M + {}^{W}\vec{v}_{MO_2}.$$

其中 ${}^{W}\vec{v}_{MO_2} = {}^{B_2}_{W}q \otimes {}^{B_2}\vec{v}_{MO_2} \otimes {}^{B_2}_{W}q^{-1}$ (通常简写: ${}^{B_2}_{W}q \otimes {}^{B_2}\vec{v}_{MO_2}$).

$$= {}^{B_2}_{W}q \otimes {}^{B_2}\vec{v}_{MO_2} = {}^{W}_{B_2}q^{-1} \otimes (-{}^{B_2}\vec{v}_{O_2M})$$

$$= {}^{W}_{B_2}q^{-1} \otimes (-{}^{B_2}\vec{x}_M) = -{}^{W}_{B_2}q^{-1} \otimes {}^{B_2}\vec{x}_M.$$

3. 求膝关节点位置。

膝关节在小腿系中的坐标为 ${}^{B_2}\vec{x}_K = (\frac{1}{2}l_2, 0, 0)$，因为小腿骨骼系 x轴由踝关节指向膝关节。

所以
$$
\begin{aligned}
{}^{W}\vec{x}_K &= {}^{W}\vec{x}_{O_2} + {}^{W}\vec{x}_{O_2K} \\
&= {}^{W}\vec{x}_{O_2} + {}^{B_2}_{W}q \otimes {}^{B_2}\vec{x}_{O_2K} \\
&= {}^{W}\vec{x}_{O_2} + {}^{B_2}_{W}q \otimes {}^{B_2}\vec{x}_K .
\end{aligned}
$$

4. 求髋关节点 H 在全局系位置。

髋关节在大腿系中坐标为 ${}^{B_1}\vec{x}_H = (\frac{1}{2}l_1, 0, 0)$，而从膝到髋的向量在大腿系中表示为 ${}^{B_1}\vec{v}_{KH} = (l_1, 0, 0)$。

则
$$
\begin{aligned}
{}^{W}\vec{x}_H &= {}^{W}\vec{x}_K + {}^{W}\vec{v}_{KH} \\
&= {}^{W}\vec{x}_K + {}^{B_1}_{W}q \otimes {}^{B_1}\vec{v}_{KH} .
\end{aligned}
$$

综上：

$$
\begin{aligned}
{}^{W}\vec{x}_H &= {}^{W}\vec{x}_K + {}^{B_1}_{W}q \otimes {}^{B_1}\vec{v}_{KH} . \\
&= ({}^{W}\vec{x}_{O_2} + {}^{B_2}_{W}q \otimes {}^{B_2}\vec{x}_K) + {}^{B_1}_{W}q \otimes {}^{B_1}\vec{v}_{KH} \\
&= {}^{W}\vec{x}_m + (-{}^{W}_{B_2}q^{-1} \otimes {}^{B_2}\vec{x}_m) + {}^{W}_{B_2}q^{-1} \otimes {}^{B_2}\vec{x}_K + {}^{W}_{B_1}q^{-1} \otimes {}^{B_1}\vec{v}_{KH} . \\
&= {}^{W}\vec{x}_m - ({}^{S_2}_{B_2}q \otimes {}^{S_2}_{W}q^{-1})^{-1} \otimes {}^{B_2}\vec{x}_m + ({}^{S_2}_{B_2}q \otimes {}^{S_2}_{W}q^{-1})^{-1} \otimes {}^{B_2}\vec{x}_K \\
&\quad + ({}^{S_1}_{B_1}q \otimes {}^{S_1}_{W}q^{-1})^{-1} \otimes {}^{B_1}\vec{v}_{KH} \\
&= \underbrace{{}^{W}\vec{x}_m}_{\text{测量}} + \underbrace{{}^{S_2}_{W}q}_{\text{测量}} \otimes \underline{{}^{S_2}_{B_2}q^{-1}} \otimes (\underline{{}^{B_2}\vec{x}_K} - \underline{{}^{B_2}\vec{x}_m}) + \underbrace{{}^{S_1}_{W}q}_{\text{测量}} \otimes \underline{{}^{S_1}_{B_1}q^{-1}} \otimes \underline{{}^{B_1}\vec{v}_{KH}}
\end{aligned}
$$

测量。　　校准后模型参数。