Weekly Report

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Review of NUFFT

Background

In 1995, Dutt *et al.* [1] combined previous classical fast Fourier transform and proposed a multipole method to do Fast Fourier Transforms for Nonequispaced Data with complexity - $O(N \cdot \log N + N \cdot \log 1/\epsilon)$, where ϵ is precision and N is number of elements. In order to save the processing time, Greengard *et al.* [2] accelerated NUFFT by removing precomputation and storage of the interpolation weights in 2004. And they implemented a single-threaded library by Fortran [3]. What is more, Keiner *et al.* [4] implemented a multi-threads software library with various gridding kernels, however, large amount of RAM are required.
Furthermore, some GPU versions are implemented to reduce the time and achieve high-efficiency. A general Gaussian kernel based GPU NUFFT was proposed in 2012 with a fast speed and fine accuracy [5]. And some implementations are for MRI [6] or OCT [7], limitations like accuracy and dimensionality are existed due to the specified applications.

$$\phi_{KB,\beta}(z) = \begin{cases} I_0(\beta\sqrt{1-z^2})/I_0(\beta), & |z| \leq 1 \\ 0, & otherwise \end{cases} \tag{1}$$

There are two commonly used kernels for convolution gridding - Gaussian kernel [1, 2] and Kaiser-Bessel kernel [6, 8, 9] (and their derivatives). In KB kernel see Euq. 1, I_0 is the regular modified Bessel function of order zero. Recently, a new kernel was presented by Barnett *et al.* [10] - "exponential of semicircle (ES) kernel (see Equ. 2) which is simpler and faster to evaluate. And the process is paralleled by OpenMP. And based on their work, cuFINUFFT -a GPU version of general NUFFT- was implemented by Shih *et al.* [11] in 2021.

$$\phi_{\beta}(z) = \begin{cases} e^{\beta(\sqrt{1-z^2}-1)}, & |z| \leq 1 \\ 0, & otherwise \end{cases} \tag{2}$$

Kernel analysis

Therefore, three kernels - Gaussian, KB, ES - are implemented in CUDA. And Chan *et al.* [12] found that simpler Gaussian function offers a better accuracy-speed tradeoff. With pre-computed kernel, the processing speed of KB kernel, however, is independent on the kernel function type. For the error convergence rate, ES kernel is close to that of KB and twice faster than that of Gaussian kernel. What is more, ES kernel is fast to evaluate and does not need precomputation stage [10].
The figure 1 shows the performance of three GPU versions with different kernel and one CPU version (the figure is from [11]).

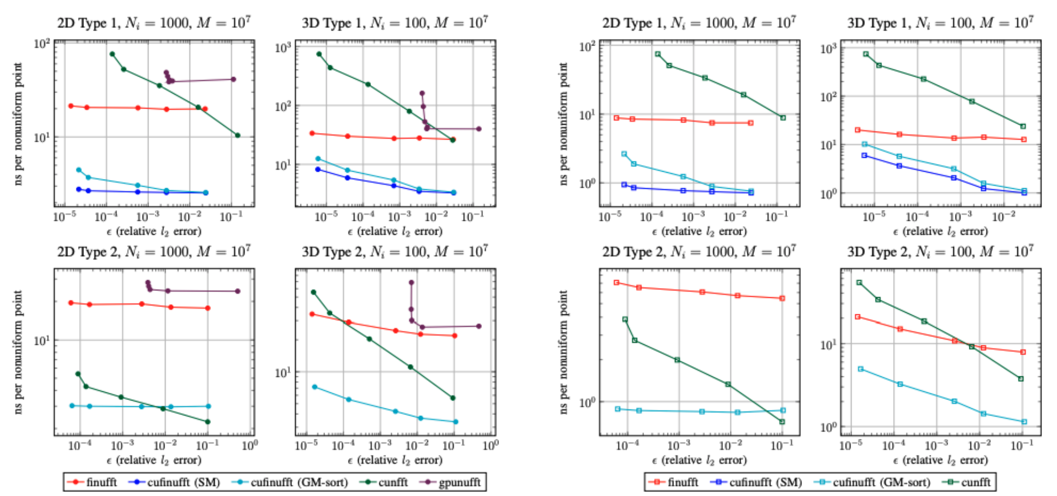

Fig. 4: Single precision NUFFT comparisons in 2D (left) and 3D (right), for type 1 (upper) and 2 (lower). "total+mem" ("total" for FINUFFT) time per nonuniform point vs accuracy is shown, for the named libraries, for the distribution "rand".

Fig. 5: Single precision comparisons in 2D and 3D. "exec" time per nonuniform point vs accuracy is shown for the tested libraries, except for gpuNUFFT. For explanation see caption of Fig. 4.

Figure 1: Comparison

Conclusion

Overall, there is not general NUFFT using GPU based on KB kernel or specified GPU version NUFFT for radio astronomy. The Gaussian kernel GPU version [5] which presented in 2012 may be improved.

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