1. $A = \begin{bmatrix} 4 & 3 & 3 & 2 \\ 3 & 6 & 3 & 2 \\ 3 & 3 & 4 & 2 \\ 2 & 2 & 2 & 2 \end{bmatrix}$

a) $x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $q_0 = \frac{x_0}{\|x_0\|} = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$, $\lambda_0^{max} = q_0^T A q_0 = 11,5$

$x_1 = Aq_0 = \begin{bmatrix} 6 \\ 7 \\ 6 \\ 4 \end{bmatrix}$, $q_0 = \frac{x_1}{\|x_1\|} = \frac{x_1}{11,7047} = \begin{bmatrix} 0,5128 \\ 0,5983 \\ 0,5128 \\ 0,3419 \end{bmatrix}$,

$\lambda_1^{max} = q_1^T A q_1 = 11,9658$

$x_2 = Aq_1 = \begin{bmatrix} 6,0684 \\ 7,3504 \\ 6,0684 \\ 3,9316 \end{bmatrix}$, $q_2 = \frac{x_2}{\|x_2\|} = \frac{x_2}{11,964} = \begin{bmatrix} 0,5072 \\ 0,6144 \\ 0,5072 \\ 0,3286 \end{bmatrix}$

$\lambda_2^{max} = q_2^T A q_2 = 11,9632$

$x_3 = Aq_2 = \begin{bmatrix} 6,0509 \\ 7,3868 \\ 6,0509 \\ 3,9149 \end{bmatrix}$, $q_3 = \begin{bmatrix} 0,5058 \\ 0,6175 \\ 0,5058 \\ 0,3272 \end{bmatrix}$, $\lambda_3^{max} = 11,9634$

$x_4 = Aq_3 = \begin{bmatrix} 6,0474 \\ 7,394 \\ 6,0474 \\ 3,9126 \end{bmatrix}$, $q_3 = \begin{bmatrix} 0,5055 \\ 0,6181 \\ 0,5055 \\ 0,327 \end{bmatrix}$, $\lambda_4^{max} = 11,9634$

c)

$$B = A - \lambda_4^{max} I = \begin{bmatrix} -7,9634 & 3 & 3 & 2 \\ 3 & -5,9634 & 3 & 2 \\ 3 & 3 & -7,9634 & 2 \\ 2 & 2 & 2 & -9,9634 \end{bmatrix}$$

$$x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad y_0 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \quad \nabla_0^{max} = y_0^T A y_0 = 11,5$$

$$x_1 = B y_0 = \begin{bmatrix} -0,4817 \\ 0,5183 \\ -0,4817 \\ 0,5092 \end{bmatrix}, \quad y_1 = \frac{x_1}{\|x_1\|} = \begin{bmatrix} -0,4836 \\ 0,5204 \\ -0,4836 \\ 0,5112 \end{bmatrix}, \quad \nabla_1^{max} = -10,4751$$

$$x_2 = B y_1 = \begin{bmatrix} 4,9842 \\ -4,9828 \\ 4,9842 \\ -5,9872 \end{bmatrix}, \quad y_2 = \begin{bmatrix} 0,4744 \\ -0,4743 \\ 0,4744 \\ -0,5699 \end{bmatrix}, \quad \nabla_2^{max} = -10,5948$$

$$x_3 = \begin{bmatrix} -4,9177 \\ 4,5354 \\ -4,9177 \\ 6,6276 \end{bmatrix}, \quad y_3 = \begin{bmatrix} -0,4629 \\ 0,4269 \\ -0,4629 \\ 0,6239 \end{bmatrix}, \quad \nabla_3^{max} = -10,7079$$

$$x_4 = \begin{bmatrix} 4,8260 \\ -4,0756 \\ 4,8260 \\ -7,2134 \end{bmatrix}, \quad y_4 = \begin{bmatrix} 0,4496 \\ -0,3797 \\ 0,4496 \\ -0,6720 \end{bmatrix}, \quad \nabla_4^{max} = -10,8103$$

$$\nabla_4^{max} \approx \lambda^{min} - \lambda_4^{max} \Rightarrow \lambda^{min} \approx \nabla_4^{max} + \lambda_4^{max} = 1,1531$$

2.

a) $K(A) \approx 10,3748$

b) $\omega_{opt} = \frac{2}{\lambda^{min} + \lambda_4^{max}} = 0,1525$

$$\rho_{opt} = \frac{K(A)-1}{K(A)+1} = 0,8242, \quad k \geq \frac{\log 10^{-2}}{\log 0,8242} = 23,8188$$