6.1 Convergence and limit laws

Yuchen Liu

November 20, 2023

In last chapter, we defined real number by formal limits: LIM. Then, we'll define lim and define what actually real number is via lim.

We start from restating ϵ-near **real** sequences' main structure.

Definition 1. (distance between two real numbers) Given by 2 real number x and y, we defined the distance between them $d(x, y)$ as $d(x, y) := |x - y|$.

Definition 2. (ϵ-near real numbers) Suppose $\epsilon > 0$ is a real number, we state 2 real number x and y is ϵ-near, if and only if $d(x, y) \leqslant \epsilon$.

Now suppose $(a_n)_{n=m}^{\infty}$ is a sequence of real number. Then redefine Cauchy sequences the same way as before.

Definition 3. (Cauchy sequences of real numbers) Suppose $\epsilon > 0$ is a real number, a sequence $(a_n)_{n=N}^{\infty}$ started from an integer N can be stated **ϵ-stable** if and only if to arbitrary $j, k \geq N$, a_j and a_k is ϵ-near.

...