

AE/ME 330 Spring 2013, Homework VII, Due Monday April 29

1. Using composite Trapezoidal and Simpson's 1/3 rule, approximate

$$\int_0^{\pi} x \sin(x) dx.$$

Obtain your results for $n = 2, 4, 6, \dots, 40$ panels. For each case, calculate the error in the approximation using the exact value of the integral. For each n (number of panels), tabulate h (mesh size), result obtained with the trapezoidal method, error of the trapezoidal method, result obtained with Simpson's method, and the error of the Simpson's method. On the same log-log plot, show the change of the error with the mesh size for each method. Comment on your results.

2. Use 3 and 4 point Gaussian Quadrature to approximate the integral given in question 1.
3. To numerically evaluate

$$\int_0^{0.5} \int_0^{0.5} xy e^{(y-x)} dy dx$$

- (a) Use Simpson's 1/3 method with two panels in each direction
- (b) Use 3 point Gaussian quadrature for each direction

Show your calculations clearly and evaluate the error for each method using the exact value of the integral.

4. Following integral arises in the study of dihedral angle for assuring the lateral stability of an aircraft:

$$\int_0^1 \sqrt{(1-\eta^2)} \eta^2 d\eta$$

- (a) Numerically integrate the above integral using: **(i)** Simpson's method with two panels, **(ii)** three-point Gaussian Quadrature. Compare your results with the exact value of the integral.

- (b) Another three-point quadrature rule for this type of integrals can be written as

$$\int_0^1 \sqrt{1-\eta^2} f(\eta) d\eta \approx \omega_0 f(0) + \omega_1 f(0.5) + \omega_2 f(1)$$

which is exact for quadratic polynomials (*i.e.*, above formula will be exact for $f(\eta) = 1$, $f(\eta) = \eta$, and $f(\eta) = \eta^2$). Using this fact, determine the weight coefficients ω_0 , ω_1 , and ω_2 .