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Theory and Methodology

Commonality in product design: Cost saving, valuation change and cannibalization

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Abstract

Offering a variety of products is important for a firm to attract different consumer segments. However, high product variety increases production and distribution costs. Modular product design and parts commonality are approaches used to counter this trend in cost and still offer a variety of products. This paper develops a model to examine when modular products should be introduced and how much modularity to offer. The model looks at a market consisting of a high segment and a low segment. Customers choose the product that maximizes their surplus, which is defined as the product's utility minus its price. Presence of commonality affects the utility of a product. Greater commonality decreases production cost but makes the products more indistinguishable from one another. This makes the product more desirable for the low segment but less desirable for the high segment. The firm's objective is to design the products and set the prices so as to maximize its profit.

Keywords: Marketing; Modelling; Product design

1. Introduction

It is generally accepted that a proliferation of products results in deterioration in manufacturing/logistics performance. Higher product variety leads to higher forecast errors, excessive inventory for some products and shortages for others, higher overhead and administrative costs, and higher manufacturing costs due to more specialized processes, materials, changeovers, and quality assurance methods (Lee and Billington, 1994). Stalk and Webber (1993) trace many problems faced by several Japanese manufacturers to excessive product proliferation. Many companies have started to address this problem. Pollack (1992) gives several examples of Japanese Manufacturers and goes on to say that "Nissan expected to save 3 percent of its costs by reducing the number of parts it uses by 30 percent in next three years". Martin et al. (1998) give examples of four manufacturers (Boeing, Denso, Hewlet-Packard and

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Seagate) concerned about product variety. Child et al. (1991) discuss the operational problems related to the complexity of product variety, and recognize that 80% of manufacturing costs, 50% of quality, 50% of order leadtime, and 80% of business complexity can be influenced by product and process design.

The use of modularity/commonality in the design of a product family or generations of products provides the firm with a chance to meet diverse customer needs with less cost due to economies-of-scale in procurement, production and distribution. Hence the concept of commonality in product design is widely used in many industries ranging from automobile to household appliance (Sanchez and Mahoney, 1995). Honda manufactures different versions of Accord targeting local markets around the world based on a single design platform (Business Week, 1997). Toyota Camry and Lexus ES300, the entry-level luxury sedan, are built on the same design platform (Automotive Industries, 1996). Answering machines in different price categories share common features such as 'memo' and 'remote access' while they are differentiated through the length of recording time and the number of outgoing messages.

The issue of part commonality on cost saving in inventory is first addressed and studied by Rutenberg (1969) and Rutenberg and Shaftel (1971). They simultaneously consider the design of modules and the decision of common modules to be used for different applications and markets. In their model, the problem of commonality is posed as a problem of economic balance between economies-of-scale and the disutility of refusing to provide each customer segment with an item fitting its exact requirements. On the supply side of the firm, research on modularity/commonality has evolved and now includes studies on understanding the impact of component commonality on stocking level (Gerchak et al., 1988; Baker et al., 1986), designing of product family and generations (Sanderson and Uzumeri, 1997; Smith and Reinertsen, 1991), and designing of products for postponement (Lee and Billington, 1994). In many cases, it is possible to achieve tangible cost saving through modular product design. The reasons for cost savings are several and include the economies-of-scale in production by using a common product module for multiple products (Rutenberg, 1969), reduction of inventory holding costs due to the pooling effect against demand uncertainty (Baker et al., 1986), and reduction in investment in production equipment (Cooper, 1994; Pisano and Rossi, 1994).

The use of commonality is not free. Excessive commonality may make products very similar (Ulrich and Tung, 1991). This may affect the customers' valuation of products and can negatively affect the firm's profits if a product does not appeal to the customers for whom it is designed. This will result in product *cannibalization*. Note that this type of cannibalization, which is due to product similarity, differs from the cannibalization due to the substitution effect. Cannibalization due to the substitution effect does not assume physical or functional similarity among products, but it takes place when different products exist and satisfy the same customers' needs in different ways. Microwaves will cannibalize the market for ovens not because of physical or functional similarity to the oven but because both can meet the same customers' needs in different ways. On the other hand, cannibalization due to product similarity occurs when different products satisfy the needs in a similar way. When high- and low-end microwaves are in the market and they share many common parts, two things can happen. Either high-end customers may find the low-end product is also appealing, or low-end customers may also find the high-end product is appealing if it is affordable, or both cases can happen simultaneously.

In this paper, we are interested in the effects of commonality on customers' valuation of products when it is used to design products in different classes. Common features increase similarity between products and influence customers' choice of products (Chernev, 1997; Tversky, 1977). Consider the example of 1997 Toyota Camry and 1997 Lexus ES300. Because of the common design platform, buyers may perceive Camry similar to Lexus ES300 and associate the quality of Camry with that of Lexus. The buyers of Camry may also be willing to pay a higher price for the product, knowing that an important part of the design is the same as the high-end product, Lexus. This type of valuation premium is similar to the 'Veblen effect' in a theory of conspicuous consumption in economics. The Veblen effect, defined to be 'willingness to pay a higher price for a functionally equivalent good' (Bagwell and Berheim, 1996), arises from the desire to signal wealth. In our model of product design for product classes, valuation premium arises when a customer is likely to choose one product over the other. If this type of valuation premium prevails, product similarity due to common design will result in increased revenue from the customers of low-end product. On the other hand, the similarity may lower the customers' willingness to pay for high-end product and even lead high-end customers to switch to the low-end product.

We adopt the economic modeling framework developed by Moorthy and Png (1992) and extend it to incorporate the issues of commonality and similarity. We address the two types of cannibalization problems associated with the modular product design for two product classes. The contribution of this paper is in providing a framework to answer these questions: (1) Under what condition does commonality help the firm to increase revenue? When is the common design strategy optimal compared to non-common product design strategies? (2) How does the product similarity interact with cannibalization due to the substitution effect? and (3) How does the product design with commonality differ from the one without commonality?

This paper is organized as follows. Section 2 introduces the model. Section 3 presents various product design strategies the firm can consider. We provide a sensitivity analysis of each strategy in Section 4. Section 5 presents the conditions where each of strategies is optimal, Section 6 presents model implications, and the conclusion is in Section 7. All proofs are in Appendix A.

2. Model

In this paper, we assume that the overall distinguishing features of a product can be represented in a single dimension which we call 'quality' and denote its value by q. Thus, if there are multiple products in the market, they are differentiated through overall levels of quality. The model considers a monopolistic firm who serves a market consisting of a high segment, h, and a low segment, l. Customers' utility linearly increases with quality so that a quality q provides a utility of $v_{\mathrm{h}} q$ to the high segment and a utility of $v_{\mathrm{l}} q$ to the low segment. The two segments differ in that the high segment values the quality higher than the low segment so that $v_{\mathrm{h}} > v_{\mathrm{l}}$. Cost of providing a unit of quality increases at an increasing rate as the level of quality increases and is given by cq^2 where c is a constant. The size of each segment, n_{h} and n_{l}, is given and all consumers in each segment choose the product which maximizes their surplus which is defined as product's utility minus its price.

To operationalize the notion of commonality, we assume that the overall quality, q, is provided to customers through a modular design, q_{m}, and a custom design specific to the types of customer, $q - q_{\mathrm{m}}$. Although the quality provided through the common design will yield the same utility as the quality provided through the custom design, there is a valuation change due to the product similarity if the common design is used. This valuation change factor, $0 \leqslant \beta \leqslant 1$, affects the perceived quality of products. We model the valuation change such that the common design used in the low-end product will provide a *valuation premium*, β_{p}, for its buyers while the high-end product will undergo a *valuation discount*, β_{d}. Thus the perceived quality of the high-end product reduces to $q_{\mathrm{h}} - \beta_{\mathrm{d}} q_{\mathrm{m}}$ and the perceived quality of the low-end product increases to $q_{\mathrm{l}} + \beta_{\mathrm{p}} q_{\mathrm{m}}$. Note that this is regardless of the segment type who values the products. Valuation discount and premium take place because of the existence of two products with common parts, not because of the existence of customer segments with different quality valuation. Therefore, if the firm offers a single product, then there wouldn't be any valuation change. On the supply side, cost saving of $c\alpha f(q_{\mathrm{m}})$ will occur if a common modular design is used for the design of multiple products so that the total cost function is $c(q^2 - \alpha f(q_{\mathrm{m}}))$ where $0 \leqslant \alpha < 1$ is the cost saving parameter and $f(q_{\mathrm{m}})$ is the cost saving function that is non-decreasing in q_{m}.

The firm wants to design two products with quality q_{h} and q_{l} for the two segments with a common module, q_m. (In Section 3, we expand the options of the firm by considering single product strategies.) The model for the case when the firm introduces *two products* can be stated as follows:

$$\max_{P_l,P_h,q_l,q_h,q_m} \quad n_l\left(P_l - cq_l^2 + \alpha cf(q_m)\right) + n_h\left(P_h - cq_h^2 + \alpha cf(q_m)\right)$$

s.t.

$$v_l\left(q_l + \beta_p q_m\right) - P_l \geqslant v_l(q_h - \beta_d q_m) - P_h, \tag{1}$$

$$v_h(q_h - \beta_d q_m) - P_h \geqslant v_h\left(q_l + \beta_p q_m\right) - P_l, \tag{2}$$

$$v_l\left(q_l + \beta_p q_m\right) \geqslant P_l, \tag{3}$$

$$v_h(q_h - \beta_d q_m) \geqslant P_h, \tag{4}$$

$$q_h \geqslant 0, \quad q_l \geqslant 0, \quad q_h \geqslant q_m, \quad q_l \geqslant q_m, \quad q_m \geqslant 0.$$

In Eq. (1), $v_l(q_l + \beta_p q_m)$ $(v_l(q_h - \beta_d q_m))$ is segment l's utility of the low (high) product and left (right) hand side is the surplus derived from the low (high) product. Eq. (2) is similarly for the high segment. Eqs. (1) and (2) are the self-selection constraints which are to ensure that each segment will voluntarily buy the product designed for that segment. Eqs. (3) and (4) are the participation constraints that ensure that each segment gets non-negative surplus from the purchase of the product. Product price and quality are decided in a manner so that they do not violate the consumer self-selection and participation constraints, and the firm's goal is to design and price the products so as to maximize its profit.

Now we analyze which constraints will be binding. If the firm tries to extract all the surplus of customers in the high segment by setting $P_h = v_h(q_h - \beta_d q_m)$ (level a in Fig. 1), then these customers will switch to the low-end product to get a positive surplus since

$$v_h(q_l + \beta_p q_m) > v_l(q_l + \beta_p q_m) \geqslant P_l.$$

However, nothing prevents the firm from extracting all the surplus from the customers in the low segment, so that constraint (3) will be binding (level b in Fig. 1) and becomes,

$$P_l = v_l\left(q_l + \beta_p q_m\right). \tag{5}$$

Substituting this in (1) and (2) and rearranging the terms give us

$$v_h(q_h - \beta_d q_m) - (v_h - v_l)\left(q_l + \beta_p q_m\right) \geqslant P_h \geqslant v_l(q_h - \beta_d q_m). \tag{6}$$

Given P_l, the firm charges the highest possible price for the high-end product which is given by the left-hand side term of (6). Note that at this price the high segment is indifferent in buying either of the products as it gets the same surplus from both products. Thus constraint (2) is binding (level c in Fig. 1) and can be rewritten as

$$P_h = v_h q_h - q_l(v_h - v_l) - q_m\left\{v_h\beta_d + \beta_p(v_h - v_l)\right\}. \tag{7}$$

By considering the lower and upper bounds on P_h in Eq. (6) we get

$$q_h - q_l - \left(\beta_p + \beta_d\right)q_m \geqslant 0. \tag{8}$$

An interpretation of Eq. (8) is that the firm must ensure that the perceived quality of the high-end product is greater than or equal to that of the low-end product, i.e., $q_h - \beta_d q_m \geqslant q_l + \beta_p q_m$. We assume

$$q_h - \beta_d q_m > q_l + \beta_p q_m \tag{9}$$

so that the perceived quality of the high-end product is still higher than that of the low-end product.

3. Firm's product design strategies

Substituting the values of P_l and P_h in the objective function with (5) and (7), we have the following objective function:

$$\max_{q_h,q_l,q_m} \quad n_h v_h q_h - n_h c q_h^2 + (n_l + n_h)v_l(1 - T)q_l - n_l c q_l^2 + (n_l + n_h)v_l\left\{\beta_p - T\left(\beta_p + \beta_d\right)\right\}q_m + (n_l + n_h)\alpha cf(q_m), \tag{10}$$

where $T = wv_l^h$, $w = n_h/(n_h + n_l)$ and $v_l^h = v_h/v_l$.

The objective function in (10) is concave in q_h and q_l. The changes in revenue and the total cost savings due to commonality are represented by

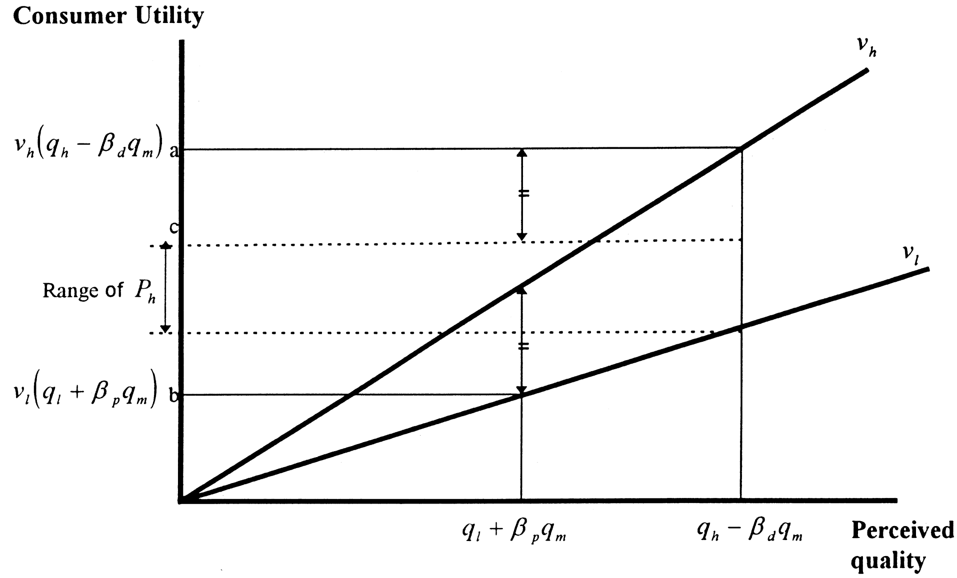

Fig. 1. Price-quality schedule.

$$(n_\mathrm{l} + n_\mathrm{h})v_\mathrm{l}\{\beta_\mathrm{p} - T(\beta_\mathrm{p} + \beta_\mathrm{d})\}q_\mathrm{m} \text{ and } (n_\mathrm{l} + n_\mathrm{h})\alpha c f(q_\mathrm{m}),$$

respectively. The changes in revenue consist of two components, one from the low segment and the other from the high segment, and given by

$$(v_\mathrm{l}\beta_\mathrm{p}n_\mathrm{l})q_\mathrm{m} \text{ and } -\{(v_\mathrm{h} - v_\mathrm{l})\beta_\mathrm{p} + v_\mathrm{h}\beta_\mathrm{d}\}n_\mathrm{h}q_\mathrm{m},$$

respectively. We refer to the increase in revenue from the low segment as the premium effect whereas the decrease in revenue from the high segment as cannibalization due to similarity. We first note that when $T \leqslant \beta_\mathrm{p}/(\beta_\mathrm{p} + \beta_\mathrm{d})$, commonality always increases the profit for any form of the cost saving, $f(q_\mathrm{m})$, which is non-decreasing in q_m so that the optimal design of commonality is $q_\mathrm{m} = q_\mathrm{l}$, which we term as completely modular design (we use the term 'partially modular design' when $0 < q_m < q_\mathrm{l}$). The value of T represents the degree of cannibalization. The cannibalization is severe when relative size of the high segment, w, is large and the valuation difference, v_l^h, is large. Above observations are summarized in Propositions 1 and 2.

Proposition 1. *The existence of valuation change factors increases revenue when the degree of cannibalization, T, is less than $\beta_\mathrm{p}/(\beta_\mathrm{p} + \beta_\mathrm{d})$, while it decreases revenue when T is greater than $\beta_\mathrm{p}/(\beta_\mathrm{p} + \beta_\mathrm{d})$.*

Proposition 2. *Completely modular design is the optimal design of commonality when the degree of cannibalization, T, is less than $\beta_\mathrm{p}/(\beta_\mathrm{p} + \beta_\mathrm{d})$.*

When $T > \beta_\mathrm{p}/(\beta_\mathrm{p} + \beta_\mathrm{d})$, the commonality results in a loss of revenue so that the optimal design of commonality will be decided from the balance between the loss of revenue and cost saving due to commonality. When the cost saving is any convex function in q_m, then for any value of q_l, the optimal solution for q_m lies at the extreme point, $q_\mathrm{m} = 0$ or $q_\mathrm{m} = q_\mathrm{l}$. Therefore the optimal design of commonality, if used, is again completely modular design. For other forms of cost saving functions partially modular design can be the optimal design (see Appendix A). To obtain more insights on different product design strategies, we now assume a particular form of cost saving, $f(q_\mathrm{m}) = q_\mathrm{m}^2$. Bagchi and Gutierrez (1992) studied the impact of number of common components on inventory costs. They found not only that the introduction of more common components results in larger cost reduction but also that the cost reduction increases at an increasing rate. However, we also note that there are situations where concave cost saving is more reasonable.

With the convex cost saving assumption, the optimal design of commonality is either $q_m = 0$ or $q_m = q_l$. When $q_m = 0$, we have the first strategy (S1), two products without modular design; when $q_m = q_l$, we have the second strategy (S2), two products with modular design. Two additional product design strategies available to the firm are, a single product for both segments with modular design (S3), and a single product for only the high segment (S4) (do not serve the low segment). Note that the strategy of a single product for only the low segment is not possible since a single product designed for the low segment would also be attractive to the high segment. We now derive the firm's product design and consequent profit under each strategy.

3.1. S1: Two unique products

In the two unique products strategy, since no modular design is used, there is no valuation discount and premium and the problem reduces to the one considered by Moorthy and Png (1992). With $\beta_p = \beta_d = 0$ we note that the objective function (10) is concave in q_h and q_l. Using the first order condition we obtain the following equations:

$$q_h^* = \frac{v_h}{2c}, \quad q_l^* = \frac{v_l(1-T)}{2c(1-w)}.$$

These products will be priced at $P_l = v_l q_l^*$, $P_h = v_h q_h^* - (v_h - v_l) q_l^*$ and will result in a profit of

$$\pi_1 = \frac{v_l^2(n_l + n_h)(1-T)^2}{4c(1-w)} + \frac{n_h v_h^2}{4c}.$$

To ensure that q_l^* is positive, we need $T < 1$.

Product quality for the high segment is set at its efficient level which is the quality that gives the firm maximum possible profit from the high segment. Product quality for the low segment decreases as the value of T increases to mitigate cannibalization.

3.2. S2: Two modular products

In the second strategy, we set $q_m = q_l$, and solve it using the first order condition to obtain the following equations:

$$q_h^* = \frac{v_h}{2c},$$
$$q_l^* = q_m^* = \frac{v_l\{(1+\beta_p) - T(1+\beta_p+\beta_d)\}}{2c\{1-w-\alpha\}}.$$

The resulting profit will be

$$\pi_2 = \frac{v_l^2(n_l + n_h)\{(1+\beta_p) - T(1+\beta_p+\beta_d)\}^2}{4c\{1-w-\alpha\}} + \frac{n_h v_h^2}{4c}.$$

The constraint $T < (1+\beta_p)/(1+\beta_p+\beta_d)$ is necessary to have two products with positive quality, and

$$1 - \alpha > (1+\beta_p)(1+\beta_p+\beta_d)\frac{v_l}{v_h} + w\left\{1 - (1+\beta_p+\beta_d)^2\right\} \tag{11}$$

results from constraint (9). Note that when (11) holds and $T < (1+\beta_p)/(1+\beta_p+\beta_d)$, we will have $1-\alpha > w$ and the profit will be positive. While π_2 can be positive even when $1-\alpha \leqslant w$ but S4 will dominate S2.

3.3. S3: Single product for both segments

In this case, $q_h = q_l = q$. As there is only one product in the market, there is no valuation change since a valuation change occurs due to the existence of an alternate product with common design. However, there is cost saving due to economies-of-scale as q is provided for both segments and consequently, the cost function becomes $c(1-\alpha)q^2$. In this strategy, the maximum price the firm can charge is $v_l q$; otherwise the low segment would get negative surplus. The high segment would get the positive surplus since

$v_h q > v_l q = P$ and both segments will buy the product at this price. The optimal product design, price and profit are as follows:

$$q = \frac{v_l}{2c(1-\alpha)}, \quad P = \frac{v_l^2}{2c(1-\alpha)},$$
$$\pi_3 = \frac{(n_l + n_h)v_l^2}{4c(1-\alpha)}.$$

3.4. S4: Single product for high segment only

This strategy may make sense when the cannibalization problem is so severe that the firm cannot afford to have two products in the market. There is no cost saving since the product is provided only for the high segment. The product design, price and the resulting profit are

$$q = \frac{v_h}{2c}, \quad P = \frac{v_h^2}{2c}, \quad \pi_4 = \frac{n_h v_h^2}{4c}.$$

4. Sensitivity analysis

In this section, we explore how the profit and product design for each strategy changes with different parameters of the model. Since the central question concerns two unique products versus two modular products, we also compare strategies S1 and S2.

4.1. S1: Two unique products

Product quality for the high segment is increasing in v_h, and product quality for the low segment is increasing in v_l, but is decreasing in w and v_h. Recall that w is the proportion of high-end customers. As v_h increases, for given v_l and w, the cannibalization becomes so severe that the firm has to reduce the product quality for the low segment. However, optimal profit, π_1, is increasing at an increasing rate in v_h and v_l, respectively (see Proposition 3). When $T \geqslant 1$, the two unique products strategy does not exist since $q_l \leqslant 0$ when $T \geqslant 1$, and S1 essentially reduces to S4. Note that the profit approaches the efficient profit as v_l^h approaches 1.

4.2. S2: Two modular products

The product quality for the high segment is again linearly increasing in v_h. The product quality for the low segment is increasing in v_l, but is decreasing in v_h up to

$$v_l^h = \frac{(1+\beta_p)}{(1+\beta_p+\beta_d)}\left(\frac{1}{w}\right).$$

Beyond this value of v_l^h, it is not feasible to provide two modular products.

As α increases, more cost saving is possible so that q_m increases. Increased q_m increases the valuation premium and this allows the firm to charge a higher price for the low-end product so that P_l increases. On the other hand, increased q_m makes the low-end product more attractive to the high segment so, to prevent the high segment from switching to the low-end product, P_h should be lower.

As β_p increases, the firm can increase both q_l and q_m (with $q_l = q_m$) and charge a higher price for the low-end product by exploiting the larger amount of valuation premium from the low segment. On the other hand, the high segment has an incentive to switch to the low-end product and enjoy the valuation premium. To prevent this the firm needs to compensate the high segment with lower P_h.

As β_d increases, q_m goes down. This reduces the total amount of valuation discount of the high-end product, and prevents the high segment from switching to the low-end product. As q_m reduces, P_l also reduces. On the other hand, increased β_d may increase or decrease P_h depending on other parameter values. To understand this, consider Eq. (7) for P_h, with $q_l = q_m$.

$$P_h = v_h(q_h - \beta_d q_m) - (v_h - v_l)q_m(1+\beta_p). \qquad (12)$$

Eq. (12) shows that the price of the high-end product is given by the difference between the high segment's utility from the high-end product (the first term in the right-hand side) and the high segment's surplus from the low-end product (the second term in the right-hand side). Substituting for q_h and q_m in Eq. (12) and taking a derivative with respect to β_d yield

$$\frac{\partial P_h}{\partial \beta_d} = -\frac{v_h v_l}{2c(1-w-\alpha)}\left\{(1+\beta_p) - T(1+\beta_p+2\beta_d)\right\} + \frac{(v_h - v_l)(1+\beta_p)v_l T}{2c(1-w-\alpha)}. \tag{13}$$

When

$$v_l^h < \frac{(1+\beta_p)(1+w)}{(1+\beta_p+\beta_d)2w},$$

the derivative is less than zero, and P_h will decrease with an increase in β_d. When

$$v_l^h > \frac{(1+\beta_p)(1+w)}{(1+\beta_p+\beta_d)2w},$$

an increase in β_d would make P_h higher. We note that the high segment's surplus from the low-end product is always decreasing in β_d since increased β_d reduces q_m and makes the low-end product less appealing to the high segment. Less satisfied with the low-end product, the high segment is more likely to purchase the high-end product so the increased β_d provides an incentive for the firm to increase P_h. While the sign of the second term in the right-hand side of (13) is always positive, the sign of $\partial P_h/\partial \beta_d$ also depends on the changes in the high segment's utility from the high-end product (the first term in the right-hand side of (13)), which is essentially the changes in the total amount of valuation discount associated with the high-end product, $\beta_d q_m$. When

$$v_l^h > \frac{(1+\beta_p)}{(1+\beta_p+2\beta_d)w},$$

the sign of the first term is positive, meaning that the high segment's utility from the high-end product increases in β_d so that P_h increases in β_d. When

$$v_l^h < \frac{(1+\beta_p)}{(1+\beta_p+2\beta_d)w} \quad \text{and} \quad \frac{(1+\beta_p)(1+w)}{(1+\beta_p+\beta_d)2w} < v_l^h,$$

the sign of the first term is negative, but $\partial P_h/\partial \beta_d > 0$; the decrease in utility from the high-end product is less than the decrease in surplus from the low-end product so that P_h increases in β_d. When

$$v_l^h < \frac{(1+\beta_p)}{(1+\beta_p+2\beta_d)w} \quad \text{and}$$
$$v_l^h < \frac{(1+\beta_p)(1+w)}{(1+\beta_p+\beta_d)2w},$$

both sign of the first term in the right-hand side of (13) and $\partial P_h/\partial \beta_d$ are negative; the decrease in utility from the high-end product is more than the decrease in surplus from the low-end product so that the high segment is more likely to switch to the low-end product. To prevent the switching, P_h should be lower with an increase in β_d.

Higher values of β_p and β_d increase the high segment's chance of switching to the low-end product. Therefore, both β_p and β_d are sources of cannibalization due to product similarity. On the other hand, higher β_p helps the firm to improve its revenue from the low segment.

For a fixed value of w, both P_h and q_h linearly increase in v_h. However, π_2 increases at an increasing rate in v_h and v_l, respectively. The changes of π_1 and π_2 in v_h and v_l are summarized in Proposition 3.

Proposition 3. *In their respective feasible regions,* π_1 *and* π_2 *increase at an increasing rate in* v_h *and* v_l.

Note that

$$\frac{\partial q_m}{\partial v_h} = \frac{-w(1+\beta_p+\beta_d)}{2c(1-w-\alpha)} < 0.$$

Thus, to mitigate cannibalization, q_m linearly decreases in v_h. Table 1 provides aditional sensitivity analyses.

If the firm serves the low segment in isolation, the efficient quality for the low segment from the firm's perspective is $v_l/2c$. Without the valuation change, in the first strategy, the optimal q_l is always lower than $v_l/2c$ because of cannibalization whereas $q_h^* = v_h/2c$. In the second strategy, q_l increases as β_p increases. The following Proposition

Table 1
Sensitivity analysis of α, β_p, β_d, v_h on price, quality and π_2

$v_l = 1$	P_h	P_l	q_h	$q_l = q_m$	π_2
$\alpha \uparrow$	$\downarrow$	$\uparrow$	–	$\uparrow$	$\uparrow$
$\beta_p \uparrow$	$\downarrow$	$\uparrow$	–	$\uparrow$	$\uparrow$
$\beta_d \uparrow$	$\uparrow$ if $v_l^h > \dfrac{(1+\beta_p)(1+w)}{(1+\beta_p+\beta_d)2w}$	$\downarrow$	–	$\downarrow$	$\downarrow$
	$\downarrow$ otherwise				
$v_h \uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$

shows the existence of the region where the optimal q_l is above $v_l/2c$ and the profit is larger than $n_l v_l^2/4c + n_h v_h^2/4c$ which is the profit from the efficient quality for each segment.

Proposition 4. *With the presence of valuation change, there exists a region under strategy $S2$, where the optimal q_l is higher than $v_l/2c$ and the firm's profit is larger than $n_l v_l^2/(4c) + n_h v_h^2/(4c)$.*

Proposition 4 shows that there is a region where the firm can completely restore the loss of profit due to cannibalization through the premium effect from the low segment. The existence of such a region depends on the relative size of the low segment; a large proportion of the low segment justifies the loss of profits from the small proportion of high segment.

4.3. Comparison of strategy 1 and strategy 2

Since the first and the second product design strategies are the central part of this paper, we compare revenues, profits and the product designs under each of these two strategies. In this discussion we denote product quality for the low segment under the first and second strategies by q_l^1 and q_l^2, respectively. Note that while we only have the cannibalization problem due to the substitution effect in the first strategy, in the second strategy we have the cannibalization due to similarity and premium effect on top of cannibalization due to substitution.

With $q_l^1 = q_l^2$, the changes in revenue and cost in going from strategy 1 to strategy 2 are

$$\Delta R = \left\{1 - T\left(\frac{\beta_p+\beta_d}{\beta_p}\right)\right\} v_l(n_l+n_h)\beta_p q_l^1,$$

$$\Delta C = c\alpha(n_l+n_h)\left(q_l^1\right)^2,$$

respectively. The valuation change factors affect revenue in strategy 2 in two different ways depending on the value of v_l^h. When

$$v_l^h < \frac{\beta_p}{\beta_p+\beta_d}\left(\frac{1}{w}\right),$$

it is easy to see that net effect of valuation change factors is the increased revenue for strategy 2 as compared to strategy 1; the premium effect dominates the cannibalization due to similarity. Thus when valuation change exists it is more effective for the firm to use modular design.

Recall that a feasibility condition for the second strategy is

$$v_l^h < \frac{1+\beta_p}{1+\beta_p+\beta_d}\left(\frac{1}{w}\right).$$

The valuation change decreases revenue when

$$\frac{\beta_p}{\beta_p+\beta_d}\left(\frac{1}{w}\right) < v_l^h < \frac{1+\beta_p}{1+\beta_p+\beta_d}\left(\frac{1}{w}\right).$$

In this region, the valuation change intensifies the cannibalization problem as the low-end product is more appealing to the high segment, which results in a loss of revenue.

Note that for the valuation change to be a revenue increasing factor, $w < \beta_p/(\beta_p+\beta_d)$ should hold since $v_l^h > 1$. This condition implies that the relative size of the low segment should be large enough to compensate the valuation discount from the high segment. The following observations result from the profit comparisons.

Observation 1. *With no valuation discount* ($\beta_d = 0$), *the two modular products strategy dominates the two unique products strategy* (*i.e.*, $\pi_2 \geqslant \pi_1$).

Observation 2. *When the degree of cannibalization is greater than* $\beta_p/(\beta_p + \beta_d)$ *and there is no cost saving, then the two unique products strategy dominates the two modular products strategy.*

In comparing product designs, q_h^* is the same under both strategies whereas q_l^{1*} and q_l^{2*} may not be the same. Looking at the relative values,

$$q_l^{1*} < q_l^{2*} \text{ if } v_l^h < \left(\frac{1}{w}\right)\left(\frac{\alpha + (1-w)\beta_p}{\alpha + (1-w)(\beta_p + \beta_d)}\right)$$

and $q_l^{1*} \geqslant q_l^{2*}$ otherwise. We now compare the profits under the two strategies for different values of v_l^h. Consider the following four cases and note that $\Delta C \geqslant 0$:

Case 1: $$v_l^h < \left(\frac{1}{w}\right)\left(\frac{\beta_p}{\beta_p + \beta_d}\right) < \left(\frac{1}{w}\right)\left(\frac{\alpha + 2(1-w)\beta_p}{\alpha + 2(1-w)(\beta_p + \beta_d)}\right) < \left(\frac{1}{w}\right)\left(\frac{\alpha + (1-w)\beta_p}{\alpha + (1-w)(\beta_p + \beta_d)}\right).$$

In this case $\Delta R > 0$. As in Proposition 1, the degree of cannibalization is low so that the increase in revenue due to the premium effect is always larger than the decrease in revenue caused by the cannibalization due to similarity from the high segment. In this case, the firm has an incentive to make $q_l^2 > q_l^1$. The profit will increase even without accounting for cost saving due to the modular design.

Case 2: $$\left(\frac{1}{w}\right)\left(\frac{\beta_p}{\beta_p + \beta_d}\right) \leqslant v_l^h < \left(\frac{1}{w}\right)\left(\frac{\alpha + 2(1-w)\beta_p}{\alpha + 2(1-w)(\beta_p + \beta_d)}\right).$$

In this case, $\Delta R < 0$ since the decrease in revenue from the high segment is larger than the increase in revenue from the low segment in the second strategy compared to the first strategy. Nevertheless profit in the second strategy will be higher as long as $q_l^2 > q_l^{1*}$ (the optimal low-segment quality found under the strategy 1).

Case 3: $$\left(\frac{1}{w}\right)\left(\frac{\alpha + 2(1-w)\beta_p}{\alpha + 2(1-w)(\beta_p + \beta_d)}\right) \leqslant v_l^h < \left(\frac{1}{w}\right)\left(\frac{\alpha + (1-w)\beta_p}{\alpha + (1-w)(\beta_p + \beta_d)}\right).$$

As in Case 2, we have $\Delta R \leqslant 0$ and $\Delta C \geqslant 0$ but, unlike Case 2, $\Delta R + \Delta C \leqslant 0$. The optimal low-segment quality in strategy 2 will be higher than the optimal low-segment quality in strategy 1 to make strategy 2 attractive.

Case 4: $$\left(\frac{1}{w}\right)\left(\frac{\alpha + (1-w)\beta_p}{\alpha + (1-w)(\beta_p + \beta_d)}\right) \leqslant v_l^h.$$

As in Case 3, $\Delta R \leqslant 0$, $\Delta C \geqslant 0$ and $\Delta R + \Delta C \leqslant 0$. However, unlike other cases, the cannibalization is so severe that the firm should lower the quality in the second strategy even if there is cost saving associated with it so $q_l^{2*} \leqslant q_l^{1*}$ (where q_l^{2*} is the optimal quality for the low-segment in S2). In other words, the cost saving should not result in an increase of quality for the low segment since it only worsens the cannibalization in the second strategy. In terms of overall profits, strategy 1 dominates strategy 2.

4.4. S3: Single product for both segments

Given w, quality and profit depend only on v_l and α. As the cost saving increases, quality and profit increase at an increasing rate. As v_h increases, this strategy becomes relatively unattractive because in this strategy the firm has to charge the price based on v_l.

4.5. S4: Single product for high segment only

When the cannibalization is severe, the firm may serve the high market only. This is true when the cost saving is not significant and the gap in quality valuation is large. In this strategy, the firm will increase the level of quality and increase the profit as v_h increases.

5. Optimal product design strategies

Given profits from the four different strategies, we make all possible pair-wise comparisons of strategies to obtain regions where one strategy dominates the others.

$$\pi_1 \geqslant \pi_2 \quad \Longleftrightarrow \quad 1-\alpha \geqslant w + (1-w)\frac{\left\{(1+\beta_p) - wv_l^h(1+\beta_p+\beta_d)\right\}^2}{(1-wv_l^h)^2}, \tag{14}$$

$$\pi_1 \geqslant \pi_3 \quad \Longleftrightarrow \quad 1-\alpha \geqslant \frac{1-w}{(1-wv_l^h)^2 + wv_l^{h^2}(1-w)}, \tag{15}$$

$$\pi_2 \geqslant \pi_3 \quad \Longleftrightarrow \quad 1-\alpha \geqslant \{(1-w-\alpha)\}/ \{\{(1+\beta_p) - wv_l^h(1+\beta_p+\beta_d)\}^2 + wv_l^{h^2}(1-w-\alpha)\}, \tag{16}$$

$$\pi_4 \geqslant \pi_3 \quad \Longleftrightarrow \quad 1-\alpha \geqslant \frac{1}{wv_l^{h^2}}, \tag{17}$$

$$\pi_4 \geqslant \pi_1 \quad \Longleftrightarrow \quad v_l^h \geqslant \frac{1}{w}, \tag{18}$$

$$\pi_2 \geqslant \pi_4 \quad \Longleftrightarrow \quad v_l^h \leqslant \frac{1+\beta_p}{1+\beta_p+\beta_d}. \tag{19}$$

By changing the above inequalities into equalities, we get iso-profit lines. For brevity, we will use the same equation numbers (14) through (19) to refer to these lines. Using these iso-profit lines and the dominating regions, we plot the optimum product design strategy with $1-\alpha$ on the y-axis and v_h on the x-axis. To obtain insights we draw these lines using specific parameter values. Figs. 2–5 provide pair-wise comparisons of four different strategies with $\beta_p = \beta_d = \beta = 0.1,\ v_l = 1,\ c = 0.1$ and $w = 1/3$. For example, the region marked π_1 in Fig. 2 shows the range of values of α and v_h where strategy 1 is better than strategy 2, and π_2 marks the range where strategy 2 is better than strategy 1.

The iso-profit line (14), marked (a) in Fig. 2, is decreasing in v_h at a decreasing rate. While increasing α makes the modular product strategy more desirable, increasing v_h makes the cannibalization more intense due to the existence of valuation change in the second strategy. The line (b) in Fig. 2 is from Eq. (11) which is the feasibility condition of the second strategy. The feasible region for the second strategy increases as v_h increases.

To compare π_2 and π_3, we need to consider the relevant region where both these strategies are feasible. Recall that in strategy S2, Eq. (11) has to hold, which is the region above the line marked (b) in Fig. 3. The iso-profit line between π_2 and π_3 (marked as (a) in Fig. 3) intersects line (b) at

$$v_l^{h^*} = \frac{(1+2\beta)(2+\beta)}{4(1+\beta)^2 w}$$

which satisfies

$$\frac{1}{2w} \leqslant v_l^{h^*} \leqslant \frac{(1+\beta)}{(1+2\beta)w}.$$

For the lower values of $v_l^h < v_l^{h^*}$, the line of constraint lies above the iso-profit line and it is below when $v_l^h > v_l^{h^*}$. For $v_l^h < v_l^{h^*}$, if $1-\alpha$ decreases beyond line (b), S2 becomes infeasible since the perceived quality of the low product is higher than that of the high product so that S3 is optimal. For

$$v_l^{h^*} < v_l^h < \frac{(1+\beta)}{(1+2\beta)w},$$

as $1-\alpha$ decreases, it is better for the firm to offer the single product for both segments rather than to keep a two-product strategy since significant cost saving allows the firm to offer the single product with high quality and to enjoy higher profits.

Fig. 4 is the comparison of S2 and S4. As noted previously, S2 is bounded by its feasibility constraints and dominates S4 whenever it is feasible.

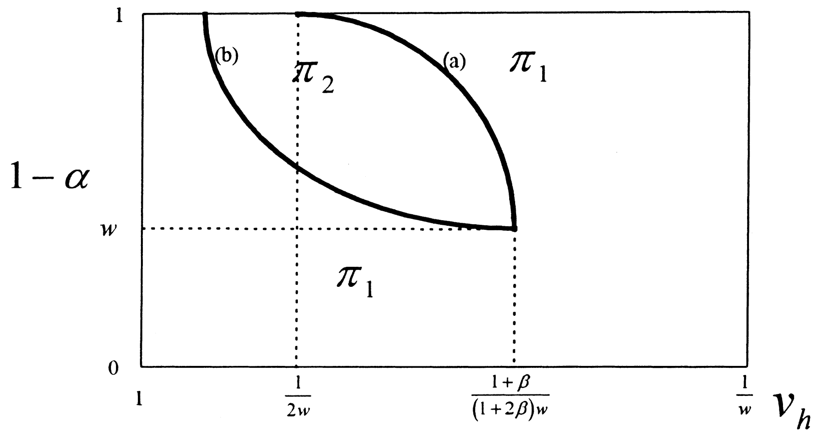

Fig. 2. π_1 vs. π_2. $v_l = 1,\ c = 0.1,\ \beta_p = \beta_d = \beta = 0.1,\ w = 1/3$.

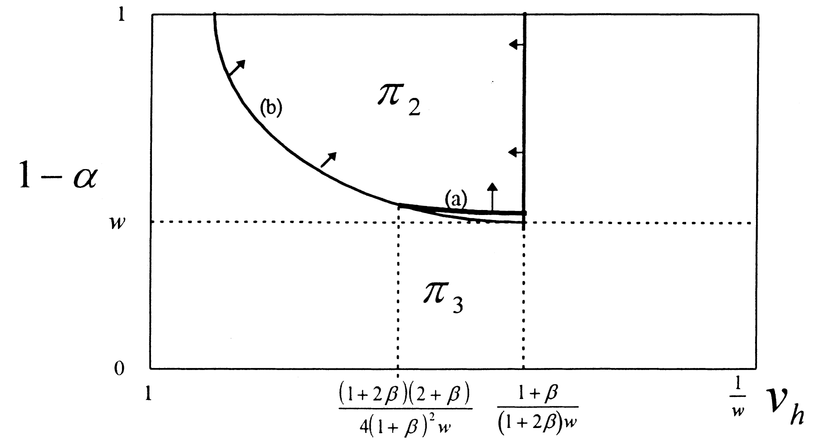

Fig. 3. π_2 vs. π_3.

The iso-profit line between π_1 and π_3 first decreases at a decreasing rate and then decreases at an increasing rate when v_l^h is greater than $1 + \sqrt{(1-w)/(4-w)}$. The iso-profit line between π_3 and π_4 decreases at an increasing rate in v_l^h. As v_l^h increases, it requires more cost saving for the single product for the both segments strategy to be an optimal strategy (Fig. 5).

When $\alpha = 0$ and v_l^h is small ($< 1/2w$), the completely modular product solution is optimal since, given the small difference of quality valuations, the firm should take advantage of the premium effect (Proposition 1). However, as v_l^h increases beyond $1/2w$, valuation change intensifies cannibalization so that valuation change adversely affects the firm's profit. In this case, the two unique products strategy is better since a more modular design lessens profit (Observation 2). When $\alpha = 0$ it can be shown that the single product for both segments solution is never optimal. This result, which is also observed in Moorthy and Png (1992), is given below.

Proposition 5. *When $\alpha = 0$, the single product for both segments strategy is never optimal.*

We now compare all four strategies to find the optimum product strategy. Overlaying the

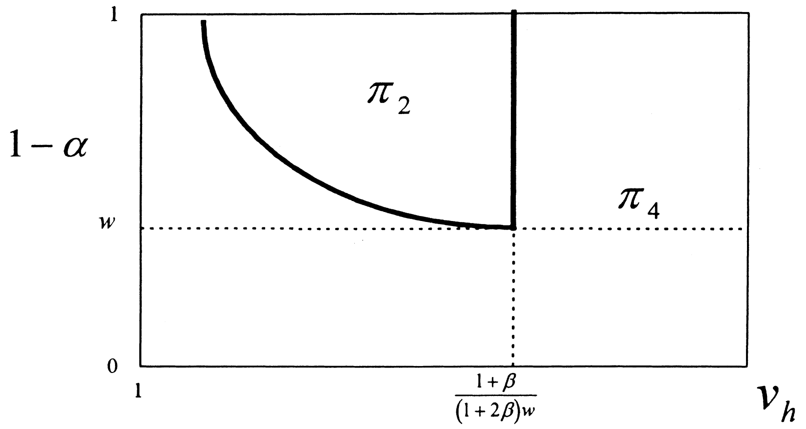

Fig. 4. π_2 vs. π_4.

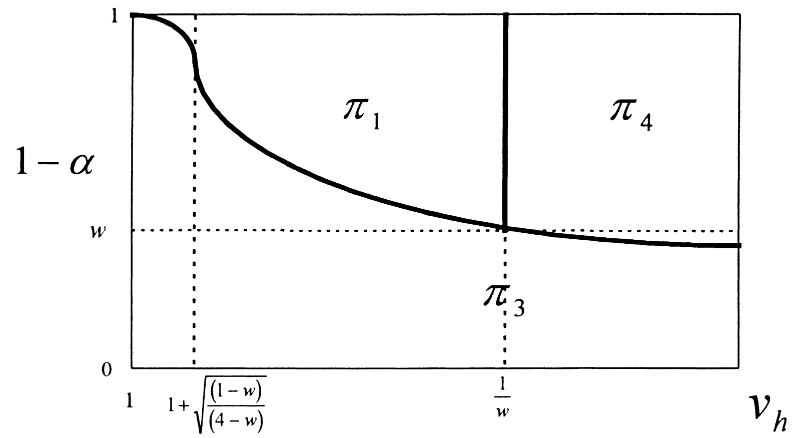

Fig. 5. π_1 vs. π_3 vs. π_4.

pair-wise comparisons discussed above, Fig. 6 is obtained. In drawing this figure, as before, we make α and v_h changing variables and set $\beta_p = \beta_d = \beta = 0.1$, $v_l = 1$, $c = 0.1$ and $w = 1/3$. In Fig. 6, first consider the case where the difference in the quality valuation is large. In this region, upper part of line (a) in Fig. 6, cannibalization is so severe that the best strategy for the firm is to offer a single product for the high segment only. A single product for both segments is the optimal strategy when the cost saving is significant. At these extreme values of v_h, a two-product solution is not feasible.

For mid range values of v_h, π_2 increases with cost saving parameter α (see line (b) in Fig. 6). The increased economies-of-scale allows the firm to design the same modular product with less costs so as to increase the profit (as α increases q_m also increases). However, as α further increases, the value of q_m approaches q_h so that two products are less and less distinguishable from each other, which eventually leads to the adoption of single product for both segments strategy. The two unique products strategy becomes an optimal strategy in the small region at the northwest corner of Fig. 6 (see Proposition 6).

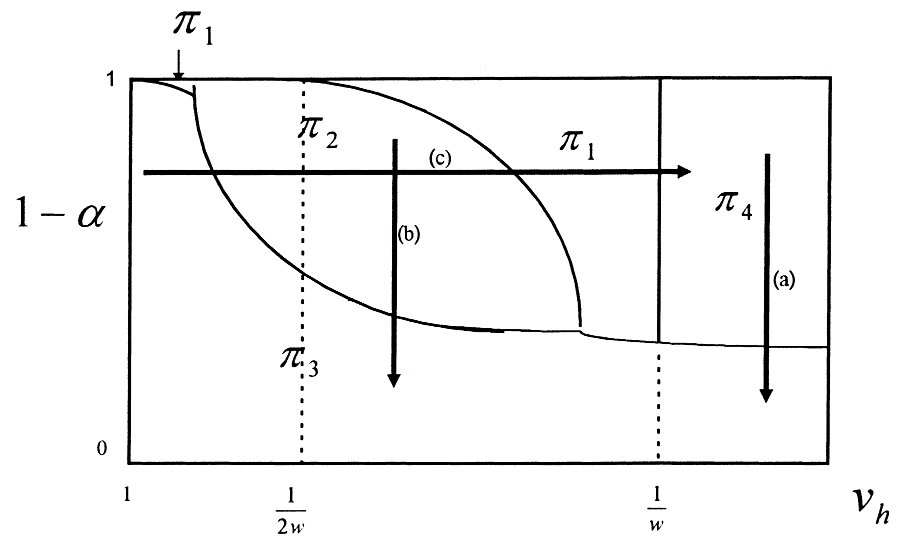

Fig. 6. Optimal product design strategies: $v_l = 1,\ c = 0.1,\ \beta_p = \beta_d = 0.1,\ w = 1/3$.

Given an appropriate value of α, for low values of v_h (see line (c) in Fig. 6) at first the firm can take advantage of cost saving and low valuation difference by adopting S3. As v_h increases, the firm can improve the profit by differentiating products using the modular design strategy while enjoying cost savings. As v_h further increases, the firm is better off by adopting the two unique products strategy not to suffer from the cannibalization due to product similarity. As v_h increases, the quality for the low segment decreases in S1. When v_h increases beyond $1/w$, the two unique products strategy is not feasible since q_l^* is reduced to zero.

An interesting observation from the profit comparison is that there has to be some degree of valuation difference between the segments for the firm to use the modular design strategy. This is because the firm must design high and low end products so that the perceived quality of products should be in the correct order after the valuation change takes place. When there is a cost saving and the relative size of the low segment is large, the optimal q_l^* is higher ($\partial q_l^*/\partial\alpha > 0$, $\partial q_l^*/\partial w < 0$ if Eq. (11) holds), which in turn increases the perceived quality of the low end product, given $\beta > 0$. The following proposition derived from Eq. (11) indicates the necessary amount of valuation difference for the firm to use the modular design.

Proposition 6. *To adopt the modular design strategy, a minimum degree of valuation difference is required, which is equal to*

$$v^{h}_{l,\min} = \frac{(1+\beta_p)(1+\beta_p+\beta_d)}{(1-\alpha) - w\left\{1-(1+\beta_p+\beta_d)^2\right\}}.$$

To study the effect of the relative size of the two segments, Figs. 7 and 8 are drawn with $w = 0.5$ and $w = 0.75$, respectively, keeping other parameters the same as in Fig. 6. We observe that the region where S2 is the optimal strategy decreases with w. For the modular product design strategy to be viable we have the constraint that

$$v_l^h < \frac{(1+\beta_p)}{(1+\beta_p+\beta_d)w}.$$

Since the minimum value of v_l^h is 1, this implies that

$$w < \frac{(1+\beta_p)}{(1+\beta_p+\beta_d)}$$

should hold for the modular product design strategy to be feasible. If the relative size of the high segment is larger than this value, then we wouldn't have the modular product design

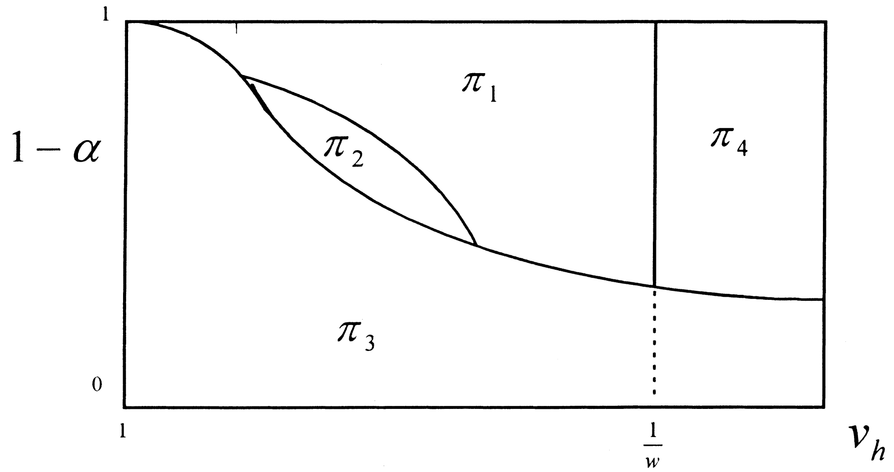

Fig. 7. Optimal product design strategies: $v_l = 1,\ c = 0.1,\ \beta_p = \beta_d = 0.1,\ w = 0.5$.

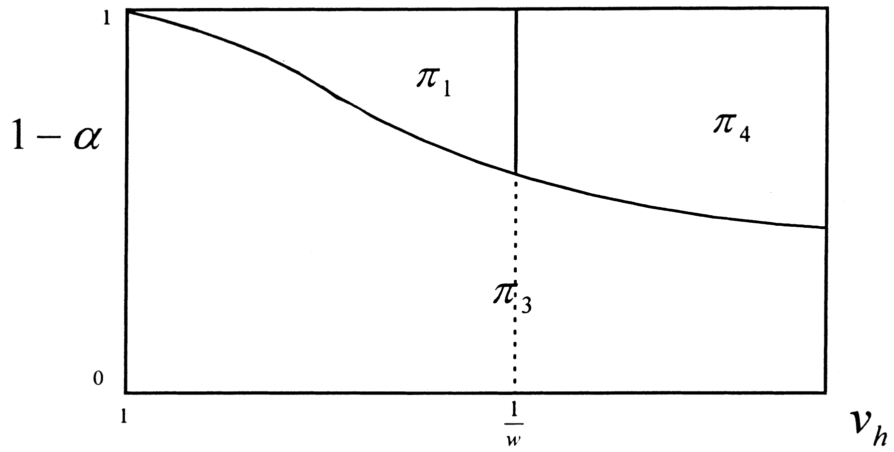

Fig. 8. Optimal product design strategies: $v_l = 1,\ c = 0.1,\ \beta_p = \beta_d = 0.1,\ w = 0.75$.

strategy, which is intuitive since the larger the size of the high segment, the worse is the cannibalization problem (see Figs. 7 and 8).

When comparing S1 and S2, the firm has an incentive to adopt S2 even if there is no cost saving (Proposition 1). However, the existence of the region depends on the relative size of each segment. Proposition 7 provides a condition for this.

Proposition 7. *When there is no cost saving, the modular product design strategy can be optimal only if*

$$w < \frac{\beta_p}{(1+\beta_d)(\beta_p+\beta_d)}.$$

6. Implications

With cost saving and valuation change, our analysis shows that the use of commonality in the products in different classes is desirable when target segments' quality valuations are not too different from each other. For example, some commonality between Camry and Lexus ES300 might be good one, whereas one between Toyota Tercel and Lexus ES300 might not be. A large difference in quality valuation of the two segments increases the amount of quality discount with the high-end product, whereas the premium effect associated with the low-end product is relatively small due to the low segment's low willingness to pay for quality. Buyers of the higher-end products seek quality that cannot be found in the lower-end products. However, commonality reduces the perceived quality difference, and despite of its cost saving, it will further adversely affect the profit when there is a higher expectation for quality difference. In this regard, commonality would only be used for a group of products in similar price ranges, and the products in vastly different price ranges should be distinguished by adopting a design that has unique attributes. Recent use of memory chips in answering machines is a case in point. The memory chips are used for message recording in machines with higher prices whereas cassette tapes are used in lower-price machines. If the chips were used in the low-end products as well it would have blurred merits of the machines with higher prices.

Although we haven't considered multiple attributes in this paper, the notion of valuation change carries some implications for the selection of common attributes when products possess multiple attributes. If the cost saving is similar across the attributes, a common attribute should be such that either it does not influence the valuation of products, or it is viewed favorably by the low segment but less influential in the judgment of the high-end product (Observation 1). For example, if there is an attribute that no longer characterizes the high-end product, then incorporating the attribute into the low-end products would not be critical in the evaluation of high-end product while low segments might benefit from this feature addition. The indexed gear shifting mechanism that was used only in expensive bicycles now can be found in less expensive bicycles as well. The high-segment now considers the attribute less unique so that the manufacturers offer the attribute in lower-end bicycles to enhance the product value while enjoying the cost saving through economies-of-scale.

In using commonality, the firm should take advantage of the premium effect ($v_l \beta_p q_m n_l$) associated with the low-end product, and the premium effect is greater when the size of the low-segment is larger. In the case of bicycles discussed before, if there isn't a sizable low-segment who might benefit from the addition of common features, the firm should deter from it. This is because the premium effect and total cost saving that depend on the size of the low-segment wouldn't justify the loss of a product attribute that can be used for product differentiation, and the subsequent loss of profit from the high-segment.

As cost saving parameter (α) increases, the amount of common quality increases when other parameter values are fixed (Table 1). Higher cost saving certainly motivates the firm to adopt more commonality between products. In the product development stage, Nissan Motor Company identifies ways to increase the commonality of parts across variations and models to achieve the desired cost level (Cooper, 1994). Commonality with higher-end product helps the firm in increasing the price of the low-end products. However, the increased similarity may weaken the justification for any price difference between products so that the firm has to reduce the price of the high-end product. For instance, incorporating the indexed gear shifting system into lower-end bicycle may make the high-end bicycle less attractive due to valuation discount so that the price of the high-end bicycle has to be lowered.

Our analysis has shown that commonality in design strategy can be optimal even if there is no cost savings; the valuation change due to commonality itself can be profitable to the firm. This happens when the target segments' quality valuations are such that the premium effect dominates the valuation discount with high-end products (Proposition 1), or when the valuation discount doesn't exist so that there is no negative effect of commonality (Observation 1). In addition to judicious selection of target segments and of common attributes as discussed before, firms can be proactive in avoiding the negative effects of commonality. In practice, this can be done in several ways including enhancing unique attributes on high-end products to deviate buyers' attention from common attributes (Chernev, 1997), and displaying products in separate places (e.g., Camry and Lexus are displayed and sold at separate dealer shops).

7. Concluding remarks

We have assumed in the model that cost saving due to the modular design increases at an increasing rate. This convex cost saving assumption makes the optimal value of q_m either 0 or q_l and allows us to compare two main strategies. This reduces mathematical complexity without obscuring, in our view, the main idea of the paper. Different cost saving functions, of course, will yield different design strategies (e.g., $0 < q_m < q_l$) as shown in Appendix A. In case of concave cost saving function, when the condition in Proposition 2 is not met, we conjecture that as the cost saving increases q_m will approach q_l, and further increase in cost saving will make $q_m = q_l = q_h$. However, qualitative insights obtained here will still hold since the effect of valuation change depends on the existence of common quality regardless of the form of cost saving which leads to different designs of common quality.

Our model assumes a 0–1 market share with known size of each segment, n_h and n_l, so cost savings due to volume is known for sure when a customer segment favors a particular product. If the customer choice process is probabilistic then the cost savings will reduce and its computation may become more involved. However, examining other types of market share model, such as a logit model, will be a useful extension. In the logit model, proportional market share of a product is given by a ratio between utility from the product and the sum of the all utility from products in the market. As commonality increases the perceived similarity between products, it will lead to convergence of utility and thereby market shares of the products.

In this model, we exogenously assume the existence of β_p and β_d. Examining the existence of β_p and β_d, and factors influencing the magnitude of them are interesting subjects of our on-going research. Furthermore as we assumed that commonality linearly influences the perceived quality of products, and customers' utility functions are linear in perceived quality, it is useful to understand the pattern of changes in perceived quality and utility with the magnitude of commonality. Incorporating multiple attributes in the model will also be a useful extension for the theory of modularity and commonality in product design. A multiple-attribute model of product line design may help in answering questions such as: Which attributes should be made common? Which set of products/customers must get common attributes?

Appendix A

The purpose of this appendix is to show through numerical examples that the partially modular design can be the optimal design of commonality when the cost saving is concave in q_m. We consider a specific form of cost saving, $\alpha c\sqrt{q_m}$, thus the total cost function is $c(q^2 - \alpha\sqrt{q_m})$. The product design problem, (10), becomes as follows:

$$\begin{aligned}\max_{q_h, q_l, q_m}\ & n_h v_h q_h - n_h c q_h^2 + (n_l + n_h) v_l (1 - T) q_l \\ & - n_l c q_l^2 + (n_l + n_h) v_l \{\beta_p - T(\beta_p + \beta_d)\} q_m \\ & + (n_l + n_h)\alpha c\sqrt{q_m}\end{aligned}$$

s.t.

$$q_h - \beta_d q_m \geqslant q_l + \beta_p q_m,$$

$$q_h \geqslant q_m, \quad q_l \geqslant q_m, \quad q_h \geqslant 0, \quad q_l \geqslant 0, \quad q_m \geqslant 0.$$

Table 2 is obtained using GAMS, a standard optimization software, with $n_h = 1$, $n_l = 2$, $v_l = 1$, $c = 0.1$, $\beta_p = \beta_d = 0.1$ and $\alpha = 0.05$. It shows how the values of q_h, q_l and q_m change as v_h varies. First note that above problem formulation covers strategies 1, 2 and 4, but not strategy 3. By letting $\beta_p = \beta_d = 0$ and $q_h = q_l = q_m = q$, we obtain a formulation for the single product for both

Table 2
Numerical examples

v_h	1.4	1.7	2.0	2.3	2.6	2.9
q_h^*	7.0	8.5	10.0	11.5	13.0	14.5
q_l^*	4.06	3.25	2.50	1.75	1.00	0.25
q_m^*	4.06	0.035	0.006	0.002	0.001	0.0007
Profit	8.21	9.34	11.25	13.87	17.10	21.04

segments strategy (S3), which yields the optimal solutions of $q = 5.006$ and profit of 7.5336 regardless of the values of v_h. Note that the profit of S3 is dominated by the profit obtained in Table 2.

Note that

$$\left(\frac{\beta_p}{\beta_p + \beta_d}\right)\left(\frac{1}{w}\right) = 1.5.$$

Table 2 shows that, when $v_h = 1.4 < 1.5$, the completely modular design is the optimal design of commonality (Proposition 2), and that as v_h increases beyond 1.5, the partially modular design is the optimal design of commonality.

Proof of Proposition 3. We make a proof only for v_h and omit a proof for v_l as it is similar to one for v_h. We take derivatives of π_1 and π_2 w.r.t. v_h to obtain the following:

$$\frac{\partial \pi_1}{\partial v_h} = \frac{2(v_h - v_l)n_h(n_h + n_l)}{n_l 4c} > 0 \quad \text{and}$$

$$\frac{\partial^2 \pi_1}{\partial v_h^2} = \frac{2n_h(n_h + n_l)}{n_l 4c} > 0.$$

$$\frac{\partial \pi_2}{\partial v_h} = \frac{2n_h v_h}{4c\{1 - w - \alpha\}}\left[1 - \alpha - (1 + \beta_p)(1 + \beta_p + \beta_d)\frac{v_l}{v_h} - w\left\{1 - (1 + \beta_p + \beta_d)^2\right\}\right] > 0$$

since

$$1 - \alpha \geqslant (1 + \beta_p)(1 + \beta_p + \beta_d)\frac{v_l}{v_h} + w\left\{1 - (1 + \beta_p + \beta_d)^2\right\}$$

is a feasibility condition in strategy 2.

$$\frac{\partial^2 \pi_2}{\partial v_h^2} = \frac{2n_h}{4c\{1 - w - \alpha\}}\left[1 - \alpha - w\left\{1 - (1 + \beta_p + \beta_d)^2\right\}\right] > 0. \qquad \square$$

Proof of Proposition 4. Let

$$q_l^* = \frac{v_l\{(1 + \beta_p) - T(1 + \beta_p + \beta_d)\}}{2c\{1 - w - \alpha\}} > \frac{v_l}{2c}.$$

Using this and the feasibility condition (11), we get the RHS and LHS, respectively, of the following inequalities:

$$\frac{(1 + \beta_p)(1 + \beta_p + \beta_d)}{(1 - w - \alpha) + w(1 + \beta_p + \beta_d)^2} < v_l^h < \frac{\beta_p + w + \alpha}{(1 + \beta_p + \beta_d)w}. \tag{A.1}$$

To show that there is positive amount of region for v_l^h in (A.1), let's assume $\alpha = 0$. We need to show (1) RHS of (A.1) is greater than 1 since $v_l^h > 1$, and (2) RHS > LHS in (A.1). First, RHS of (A.1) is greater than 1 when

$$w < \frac{\beta_p}{(\beta_p + \beta_d)}. \tag{A.2}$$

We now show (2), which can be reduced to

$$w < \frac{\beta_p}{(\beta_p + \beta_d)(\beta_p + \beta_d + 2)}. \tag{A.3}$$

Note that when (A.3) holds, (A.2) also holds. Thus when (A.3) holds there is a positive amount of region for v_l^h in (A.1) where $q_l^* > v_l/2c$.

Next, to show that the profit is larger than $(n_l v_l^2 + n_h v_h^2)/4c$ in this region, we need to show

$$(1 - w - \alpha)(1 - w) < \left\{(1 + \beta_p) - T(1 + \beta_p + \beta_d)\right\}^2. \tag{A.4}$$

Again assuming $\alpha = 0$, (A.4) holds when

$$v_l^h < \frac{\beta_p + w}{(1 + \beta_p + \beta_d)w},$$

which is the RHS of (A.1). Therefore when (A.3) holds, the proposition is true. $\square$

Numerical example. Let $c = 0.1$, $v_l = 1$, $v_h = 1.3$, $\beta_p = \beta_d = 0.1$, $n_l = 50$, $n_h = 10$ and $\alpha = 0$. Then $w = 1/6 < 1/4.4$ so (A.3) is satisfied. Furthermore $q_l^* = 5.04 > v_l/2c = 5$ and the resulting profit is 169.26 which is higher than 167.25, the profit from the efficient quality.

Proof of Observation 1. Using the optimal profit expressions derived in Sections 3.1 and 3.2 (with $\beta_d = 0$) we get

$$\pi_2 - \pi_1 = \frac{v_l^2(n_l+n_h)\{(1-T)(1+\beta_p)\}^2}{4c(1-w-\alpha)} - \frac{v_l^2(n_l+n_h)(1-T)^2}{4c(1-w)} = \frac{v_l^2(n_l+n_h)(1-T)^2}{4c}\left\{\frac{(1+\beta_p)^2}{(1-w-\alpha)} - \frac{1}{1-w}\right\} > 0.$$

Note that $1-w-\alpha > 0$ for S2 to be feasible. □

Proof of Observation 2. With $\alpha = 0$,

$$\pi_1 - \pi_2 = \frac{v_l^2(n_l+n_h)}{4c(1-w)}\left\{(1-T)^2 - \{(1-T)+\beta_p - T(\beta_p+\beta_d)\}^2\right\}.$$

Now suppose $T > \beta_p/(\beta_p+\beta_d)$. Substituting $\beta_p/(\beta_p+\beta_d)$ for T in the last term and simplifying terms we get

$$\pi_1 - \pi_2 > \frac{v_l^2(n_l+n_h)}{4c(1-w)}\left\{(1-T)^2 - \{(1-T)+\beta_p-\beta_p\}^2\right\} = 0. \quad \square$$

Proof of Proposition 5. When $\alpha = 0$,

$$\pi_1 - \pi_3 = \frac{(n_l+n_h)^2 v_l^2}{4cn_l}\left[(1-T)^2 + T\frac{v_h}{v_l}(1-w) - (1-w)\right] = \frac{(n_l+n_h)^2 v_l^2}{4cn_l}\left\{w\left(\frac{v_h}{v_l}-1\right)^2\right\} > 0$$

but π_1 is valid only when $T < 1$. When $T \geqslant 1$ with $\alpha = 0$, from (17) we get $\pi_4 \geqslant \pi_3 \Longleftrightarrow v_l^h \geqslant 1/T$. Therefore when $\alpha = 0$, S3 is not a dominant strategy. □

Proof of Proposition 7. If $T < \beta_p/(\beta_p+\beta_d)$, then valuation change becomes the revenue increasing factor in S2 (see Proposition 1) but $v_l^h > 1$ thus $w < \beta_p/(\beta_p+\beta_d)$ should hold. Substituting $\alpha = 0$ and $v_l^h = (\beta_p/(\beta_p+\beta_d))(1/w)$ in Eq. (11) we obtain $w < \beta_p/(1+\beta_d)(\beta_p+\beta_d)$. Both $w < \beta_p/(\beta_p+\beta_d)$ and $w < \beta_p/(1+\beta_d)(\beta_p+\beta_d)$ yield $w < \beta_p/(1+\beta_d)(\beta_p+\beta_d)$. □

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