Particle Swarm Optimization: Introduction and Applications to Nonlinear Bifurcation Diagrams

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Outline

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- PSO with Speciation
  - Theory
  - Example: Himmelblau Function
- Applications to Bifurcation Diagrams of Nonlinear Systems
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Particle Swarm Optimization

- Inspired by collective behavior of social animals.
- Does not require any derivative calculations.
- Uses concept of swarm intelligence.

Theory

- "Swarm" of points randomly spread out in search domain.
- Position of individual point updated with best personal position and global best of measured fitness value.
- Swarm converges towards points with better fitness values.
- Stochastic values applied to position to simulate random component of individual movement.

Theory

- Position of individual point updated by velocity vector which indicates direction to move towards.

$$\bar{x}_{k+1} = \bar{x}_k + \bar{v}_{k+1}$$

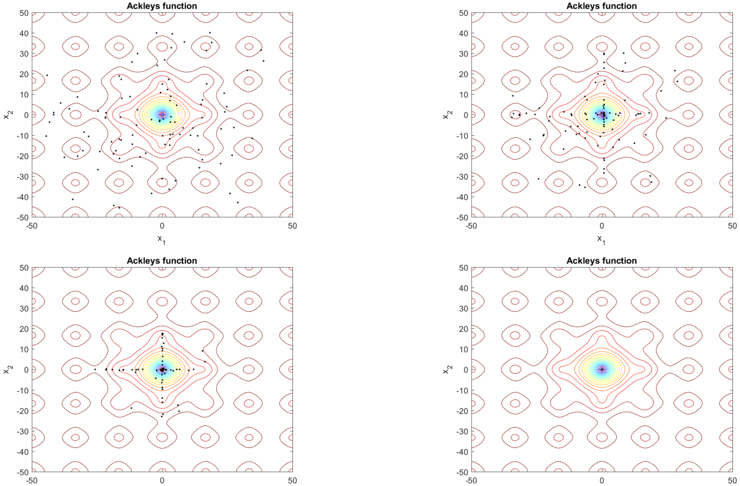
- Velocity vector found by:

$$\bar{v}_{k+1} = \omega \bar{v}_k + c_1 r_1 \bar{p}_{best} + c_2 r_2 \bar{g}_{best}$$

- ω is a damping factor to prevent velocity "explosion" which might cause points to overshoot out of search domain.
- Random numbers, $r_1, r_2 \in (0, 1]$, simulate random component of movement.
- $\bar{p}_{best}$ indicates point's personal best position and $\bar{g}_{best}$ best global position.
- c_1, c_2 are scaling factors. Usually set to equal 2.

Example: Ackley's Function

$$f(x_1, x_2) = -20e^{-0.2\sqrt{0.5\sum_{i=1}^{2} x_i^2}} - e^{\sqrt{0.5\sum_{i=1}^{2} cos(2\pi x_i)}} + 20 + e$$



Drawbacks

- Fine tuning ω parameter difficult.
- There's still chance of being trapped in local minima.
- Performs poorly for high dimensional problems with high accuracy requirements.
- Not optimal for problems with multiple global minima.

PSO with Speciation

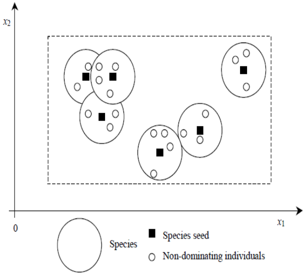
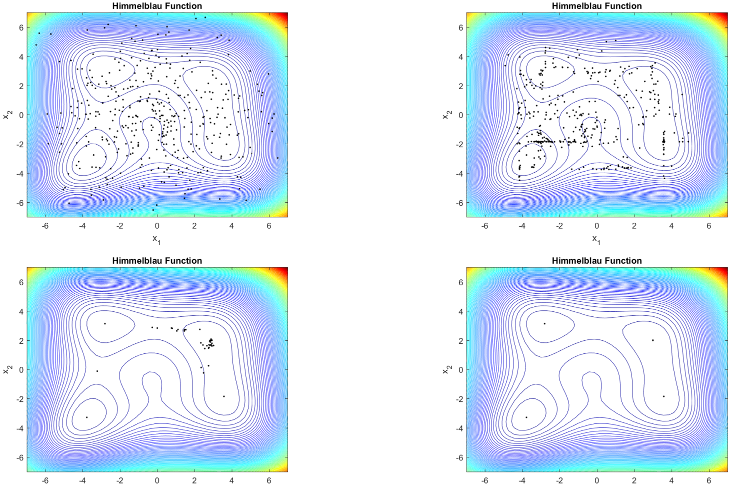


Fig. 1 A sample distribution of species in a two-dimensional domain.

- Splits up points into several species centered around species seed.
- Species formed of actual individuals and occupies region of feasibility domain.
- Performs PSO algorithm for each individual species with species seed as species global best.

Example: Himmelblau's Function

$$f(x_1, x_2) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$



Nonlinear System Bifurcation Diagrams

- Solve:

$$G(\bar{x}, \alpha_1) = 0$$

- Historical method is to solve nonlinear system using Newton's Method (or Quasi-Newton) at α_k and use result as initial condition for solving at α_{k+1}. For example,

$$G_x(x_k, \alpha_1)\Delta x_k = -G(x_k, \alpha_1)$$

$$x_{k+1} = x_k + \Delta x_k$$

- Ineffective for bifurcation diagrams with limit points.
- Pseudoarclength methods such as Gauss-Newton method with parametrized α help resolve problem.

Bifurcation Diagrams Using SPSO

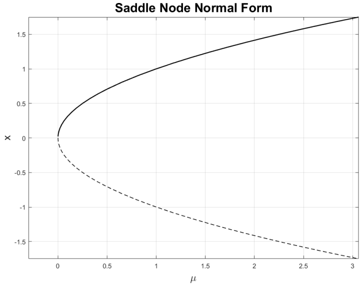
- Solving for fixed points at varying parameter by applying PSO to $L_2 - norm$ of the system. AKA solve:

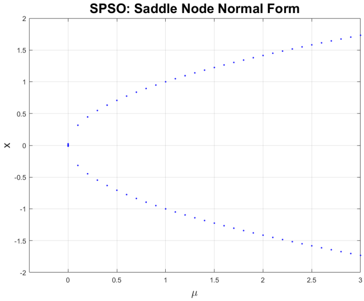
$$||G(x, \alpha_k)||_2^2 = 0$$

- Considering that bifurcations such as saddle node and pitchfork bifurcations have more than one fixed point values of α, Particle Swarm Optimization with Speciation was used.
- Used normal forms of different bifurcations to test SPSO.

Saddle Node Bifurcation

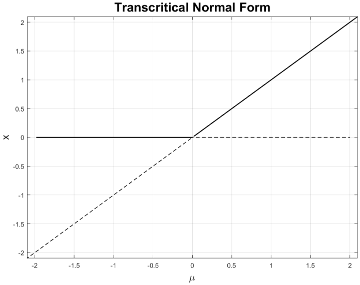
- Solved for normal form: $\dot{x} = \alpha - x^2$

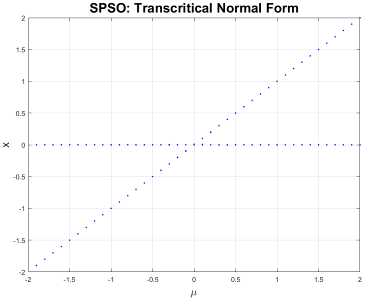




Transcritical Bifurcation

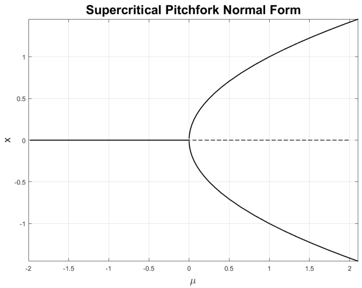
- Solved for normal form: $\dot{x} = (\alpha - x)x$

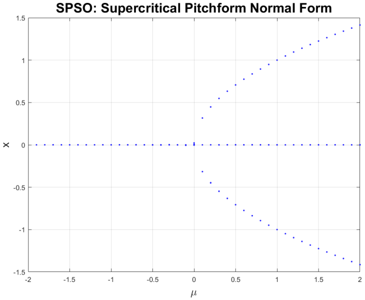




Supercritical Pitchfork Bifurcation

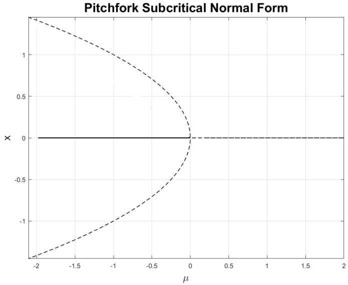
- Solved for normal form: $\dot{x} = (\alpha - x^2)x$

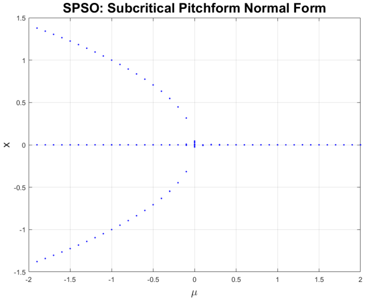




Subcritical Pitchfork Bifurcation

- Solved for normal form: $\dot{x} = (\alpha + x^2)x$



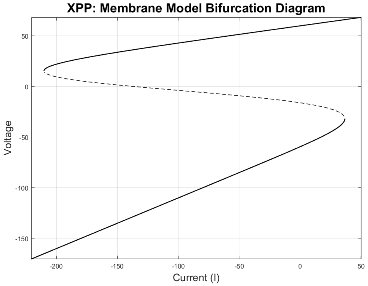


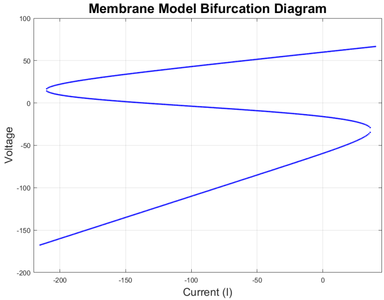
Cell Membrane: AUTO Tutorial

- Solved for system:

$$C\frac{dV}{dt} = I + g_l(V_L - V) + g_{Ca}m_{\infty}(V)(V_{Ca} - V)$$

$$m_{\infty} = .5(1 + tanh((V - V_1)/V_2))$$





Discussion

Pros:

- Good, robust tool to use in drawing bifurcation diagrams without need calculating nor approximating Jacobian.
- Better, fine-tuned variants of Particle Swarm optimization exist.

Cons:

- Might require fine tuning of parameters for large dimensional problems that require more accuracy.
- Unless cleverly coded (AKA parallel computing), significantly slower than numerical continuation methods.

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