In[186]:=

```
(*Title:Muon Pair Production Calculation (e⁺e⁻→μ⁺μ⁻)
  Author:[Hirokaxzu Maruyama]
  Date:January 2025 Version:1.0 Description:This code calculates
      the cross section for muon pair production in e⁺e⁻ collisions:
     1. Standard calculation using 4x4 gamma matrices 2. Extended formalism
      using 256x256 matrices 3. Implementation in both Minkowski and curved
      spacetime Physical Significance:-Basic QED process for lepton pair production-
        Important for particle accelerator physics-Test of lepton universality*)

(*Key Variables:s-total center-of-mass energy squared t,
u-Mandelstam variables me-electron mass mμ-
 muon mass α-fine structure constant θ-scattering angle Energy
  Scales:√s-center-of-mass energy (GeV) σ-cross section (nb)*)

(*Step 1:Define matrix elements and spinor states*)
(*Step 2:Calculate scattering amplitude*)
(*Step 3:Compute differential cross section Note:
 Integration over angular variables gives total cross section*)
(*Step 4:Plot cross section vs.center-of-mass energy Note:
 Shows characteristic 1/s behavior at high energies*)




Print[Style["Example of calculation of muon pair generation", Blue]];




Print[
  "****************************************************************************
    ******************"];
Print[Style["1.Calculation of muon generation
     using 4 γ matrices (4*4) (conventional calculation)", Blue]];

y1 =.;
y2 =.;
y3 =.;
y4 =.;
T1 =.;
s =.;
t =.;
u =.;
(*γ matrix(4×4)*)
```

```
gu[0] = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}};
gu[1] = {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}};
gu[2] = {{0, 0, 0, -I}, {0, 0, I, 0}, {0, I, 0, 0}, {-I, 0, 0, 0}};
gu[3] = {{0, 0, 1, 0}, {0, 0, 0, -1}, {-1, 0, 0, 0}, {0, 1, 0, 0}};
e4 = IdentityMatrix[4];

gd[0] = 1 * gu[0];
gd[1] = -gu[1];
gd[2] = -gu[2];
gd[3] = -gu[3];


sl[q] = gu[0] * q0 + gu[1] * (-q1) + gu[2] * (-q2) + gu[3] * (-q3) + m * e4;
sl[p] = gu[0] * p0 + gu[1] * (-p1) + gu[2] * (-p2) + gu[3] * (-p3) + m * e4;
sl[k] = gu[0] * k0 + gu[1] * (-k1) + gu[2] * (-k2) + gu[3] * (-k3);
sl[j] = gu[0] * j0 + gu[1] * (-j1) + gu[2] * (-j2) + gu[3] * (-j3);

s1 = 0;
s2 = 0;
y1 = 0;
me = m1 * e4;
mμ = m * e4;
m =.;
m1 = 0;

For[x = 0, x ≤ 3, x++,
  For[y = 0, y ≤ 3, y++,
   s1 = Tr[(sl[q] - me).gu[x].(sl[p] + me).gu[y]];
   s2 = Tr[(sl[k] + mμ).gd[x].(sl[j] - mμ).gd[y]];
   y1 = y1 + s1 * s2;
  ]];


(*Conversion using Mandelstam variables*)

m = 0;
T1 = Simplify[y1 //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0,
     k3 → -p3, q0 → p0, q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3,
     j1 → -p3 * Sqrt[1 - z^2], j2 → 0, j3 → -p3 * z, p0 → (u + m^2) / (2 Sqrt[u]),
     p3 → (u - m^2) / (2 Sqrt[u]), z → 1 + s / (2 p3^2), s → 2 m^2 - u - t}]; (*f(ut,u)*)


Print["f(s,u);", T1];

keisuu = Coefficient[T1, t, 2];

y2 = α^2 / (2 * s^3) * T1 / keisuu;

t = -1 / 2 * s * (1 - Cos[θ]);
u = -1 / 2 * s * (1 + Cos[θ]);
```

```
Print["Scattering cross section of laboratory system (conventional calculation);",
  Expand[y2]];

ya2 = 32 / 9 * Integrate[y2, {θ, 0, Pi}];

Print["Total cross section integrated with respect to Θ;", Expand[ya2]];


y3 = 32 / 9 * Integrate[y2, {θ, 0, Pi}] //. {s → sw^2};

xt =.;
yt =.;
xt[min_, max_, lbl_] :=
  Table[If[PossibleZeroQ@Mod[i, 10], {i, If[lbl, i, Null], {0.04, 0}},
    {i, Null, {0.02, 0}}], {i, Ceiling[min], Floor[max], 2}];
yt[min_, max_, lbl_] := Join @@ Table[Join[{{10^i, If[lbl, 10^i, Null], {0.04, 0}}},
     If[10^i < max, Table[{j 10^i, Null, {0.02, 0}}, {j, 2, 9}], {}]],
    {i, Log10[min], Log10[max]}];
xt1[min_, max_] := xt[min, max, True];
xt0[min_, max_] := xt[min, max, False];
yt1[min_, max_] := yt[min, max, True];
yt0[min_, max_] := yt[min, max, False];

α = 1 / 137;
y4 = y3 * (10^6 / 2.57);
LogPlot[y4, {sw, 4, 40},
 PlotRange → {{0, 40}, {1 / 100, 10}}, PlotStyle → Red, Frame → True,
 FrameLabel → {Labeled[Sqrt[s], "(GeV)", Right], Labeled[σ, "(nb)", Right]},
 FrameTicks → {{yt1, yt0}, {xt1, xt0}}, AspectRatio → 1]



Print[
  "*****************************************************************************
    ******************"];
Print[Style["2.Muon pair generation calculation using γ matrix (256*256)", Blue]];

(*γ行列(256×256)*)

y10 =.;
y20 =.;
y30 =.;
y40 =.;
T10 =.;
s =.;
t =.;
u =.;

(*Find 16 combinations of γ matrices (256*256) that satisfy the anti-
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 commutative relationship*)
demoteRank4to2[y_] := Flatten[Map[Flatten, Transpose[y, {1, 3, 2, 4}], {2}], 1];
pauli8times[g1_, g2_, g3_, g4_, g5_, g6_, g7_, g8_] :=
  demoteRank4to2[Outer[Times, demoteRank4to2[Outer[Times,
      demoteRank4to2[Outer[Times, g1, g2]], demoteRank4to2[Outer[Times, g3, g4]]]],
    demoteRank4to2[Outer[Times, demoteRank4to2[Outer[Times, g5, g6]],
      demoteRank4to2[Outer[Times, g7, g8]]]]]];

g[1] = {{1, 0}, {0, -1}};
g[2] = {{0, -I}, {I, 0}};
g[3] = {{0, 1}, {1, 0}};
g[0] = {{1, 0}, {0, 1}};

e256 = IdentityMatrix[256];


γuv[0] = pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[0], g[3]];
γuv[1] = I * pauli8times[g[0], g[0], g[0], g[0], g[3], g[2], g[2], g[2]];
γuv[2] = I * pauli8times[g[0], g[0], g[0], g[1], g[2], g[2], g[2], g[2]];
γuv[3] = I * pauli8times[g[0], g[0], g[3], g[2], g[2], g[2], g[2], g[2]];

γuv[4] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[0], g[1]];
γuv[5] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[3], g[2]];
γuv[6] = I * pauli8times[g[1], g[2], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[7] = I * pauli8times[g[0], g[0], g[1], g[2], g[2], g[2], g[2], g[2]];

γuv[8] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[3], g[2], g[2]];
γuv[9] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[1], g[2]];
γuv[10] = I * pauli8times[g[3], g[2], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[11] = I * pauli8times[g[0], g[0], g[0], g[0], g[1], g[2], g[2], g[2]];

γuv[12] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[1], g[2], g[2]];
γuv[13] = I * pauli8times[g[0], g[1], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[14] = I * pauli8times[g[0], g[3], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[15] = I * pauli8times[g[0], g[0], g[0], g[3], g[2], g[2], g[2], g[2]];


num =
  115 792 089 237 316 195 423 570 985 008 687 907 853 269 984 665 640 564 039 457 584 007 913 129 639 \
   936;(*Confirm determinant*)


(*16 γ matrices (256×256) Calculation to confirm
 that the anticommutative relationship is satisfied*)
yg = 0;
For[kh = 0, kh ≤ 15, kh++,
  For[ks1 = 0, ks1 ≤ 15, ks1++,
   yf = Det[γuv[kh].γuv[ks1] + γuv[ks1].γuv[kh]];
   yg = yf + yg;
   If[kh =!= ks1 && yf == num * 16, Print["No.", km, ",x=", kh, ",y=", ks1]];
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  ]];

If[kh == 16 && ks1 == 16 && yg / num == 16, Print[""],
  Print["γ matrix (256*256) 16 pieces Anti-commutation relation confirmation NG"]];


(*Set metric tensor*)


Print[Style[
   "2.1.Calculation using 4 γ matrices (256*256) under Minkowski spacetime", Blue]];

gd[0] = 1;
gd[1] = 1;
gd[2] = 1;
gd[3] = 1;
gd[4] = 0;
gd[5] = 0;
gd[6] = 0;
gd[7] = 0;
gd[8] = 0;
gd[9] = 0;
gd[10] = 0;
gd[11] = 0;
gd[12] = 0;
gd[13] = 0;
gd[14] = 0;
gd[15] = 0;

m256 = 1 * m;


(*γ matrix multiplied by metric*)

For[km1 = 0, km1 ≤ 15, km1++,
  γu[km1] = -gd[km1] * γuv[km1];
 ];


For[km2 = 0, km2 ≤ 15, km2++,
  γd[km2] = 1 * γu[km2];
 ];
γd[0] = -1 * γu[0];


metric = {{-gd[0], gd[10], gd[12], gd[14]}, {gd[11], gd[1], gd[4], gd[6]},
    {gd[13], gd[5], gd[2], gd[8]}, {gd[15], gd[7], gd[9], gd[3]}} / gd[0];

Print["Calculate the metric tensor as ", MatrixForm[metric]];
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Print["det (determinant of the metric tensor)=", Det[metric]];

sl[q] = γu[0] * q0 + γu[1] * -q1 + γu[2] * -q2 + γu[3] * -q3 + m256 * e256;
sl[p] = γu[0] * p0 + γu[1] * -p1 + γu[2] * -p2 + γu[3] * -p3 + m256 * e256;
sl[k] = γu[0] * k0 + γu[1] * -k1 + γu[2] * -k2 + γu[3] * -k3;
sl[j] = γu[0] * j0 + γu[1] * -j1 + γu[2] * -j2 + γu[3] * -j3;


s10 = 0;
s20 = 0;
y10 = 0;
me = m1 * e256;
mμ = m * e256;
m =.;
m1 = 0;

For[x = 0, x ≤ 3, x++,
  For[y = 0, y ≤ 3, y++,
   s10 = Tr[(sl[q] - me).γu[x].(sl[p] + me).γu[y]];
   s20 = Tr[(sl[k] + mμ).γd[x].(sl[j] - mμ).γd[y]];
   y10 = y10 + s10 * s20;
  ]];


(*Conversion using Mandelstam variables*)

m = 0;
T10 =
  Simplify[y10 / gd[0]^8 //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
     q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
     j3 → -p3 * z, p0 → (u + m^2) / (2 Sqrt[u]), p3 → (u - m^2) / (2 Sqrt[u]),
     z → 1 + s / (2 p3^2), s → 2 m^2 - u - t}];(*f(ut,u)*)


Print["f(s,u);", T10];

keisuu = Coefficient[T10, t, 2];

y20 = α^2 / (2 * s^3) * T10 / keisuu;

t = -1 / 2 * s * (1 - Cos[θ]);
u = -1 / 2 * s * (1 + Cos[θ]);

Print["Scattering cross section of laboratory system
    (consistent with conventional calculation results);", Expand[y20]];


y30 = 32 / 9 * Integrate[y20, {θ, 0, Pi}] //. {s → sw^2};

xt =.;
```

```
yt =.;
xt[min_, max_, lbl_] :=
  Table[If[PossibleZeroQ@Mod[i, 10], {i, If[lbl, i, Null], {0.04, 0}},
    {i, Null, {0.02, 0}}], {i, Ceiling[min], Floor[max], 2}];
yt[min_, max_, lbl_] := Join @@ Table[Join[{{10^i, If[lbl, 10^i, Null], {0.04, 0}}},
     If[10^i < max, Table[{j 10^i, Null, {0.02, 0}}, {j, 2, 9}], {}]],
    {i, Log10[min], Log10[max]}];
xt1[min_, max_] := xt[min, max, True];
xt0[min_, max_] := xt[min, max, False];
yt1[min_, max_] := yt[min, max, True];
yt0[min_, max_] := yt[min, max, False];
```

```
Print[
  "*****************************************************************************
    *****************"];

Print[Style["2.2.Trial calculation example
     using 16 γ matrices (256*256) in curved space-time", Blue]];

y100 =.;
y200 =.;
y300 =.;
y400 =.;
T100 =.;
s =.;
t =.;
u =.;
α =.;


gd[0] = 9 / 10;
gd[1] = 1;
gd[2] = 1;
gd[3] = 1;
gd[4] = 1 / 10;
gd[5] = 1 / 10;
gd[6] = 1 / 10;
gd[7] = 1 / 10;
gd[8] = 1 / 10;
gd[9] = 1 / 10;
gd[10] = 1 / 10;
gd[11] = 1 / 10;
gd[12] = 1 / 10;
gd[13] = 1 / 10;
gd[14] = 1 / 10;
gd[15] = 1 / 10;
```

```
m256 = 1 * m;
```

```
(*γ matrix multiplied by metric*)

For[km1 = 0, km1 ≤ 15, km1++,
  γu[km1] = -gd[km1] * γuv[km1];
 ];


For[km2 = 0, km2 ≤ 15, km2++,
  γd[km2] = 1 * γu[km2];
 ];
γd[0] = -1 * γu[0];


metric = {{-gd[0], gd[10], gd[12], gd[14]}, {gd[11], gd[1], gd[4], gd[6]},
    {gd[13], gd[5], gd[2], gd[8]}, {gd[15], gd[7], gd[9], gd[3]}} / gd[0];

Print["Calculate the metric tensor as ", MatrixForm[metric]];

Print["det(determinant of metric tensor)=", Det[metric]];

sl[q] = γu[0] * q0 + γu[1] * -q1 + γu[2] * -q2 + γu[3] * -q3 + m256 * e256;
sl[p] = γu[0] * p0 + γu[1] * -p1 + γu[2] * -p2 + γu[3] * -p3 + m256 * e256;
sl[k] = γu[0] * k0 + γu[1] * -k1 + γu[2] * -k2 + γu[3] * -k3;
sl[j] = γu[0] * j0 + γu[1] * -j1 + γu[2] * -j2 + γu[3] * -j3;


s100 = 0;
s200 = 0;
y100 = 0;
me = m1 * e256;
mμ = m * e256;
m =.;
m1 = 0;

For[x = 0, x ≤ 15, x++,
  For[y = 0, y ≤ 15, y++,
   s100 = Tr[(sl[q] - me).γu[x].(sl[p] + me).γu[y]];
   s200 = Tr[(sl[k] + mμ).γd[x].(sl[j] - mμ).γd[y]];
   y100 = y100 + s100 * s200;
  ]];
```

```
(*Conversion using Mandelstam variables*)

m = 0;
T100 =
  Simplify[y100 / gd[0]^8 //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
     q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
     j3 → -p3 * z, p0 → (u + m^2) / (2 Sqrt[u]), p3 → (u - m^2) / (2 Sqrt[u]),
     z → 1 + s / (2 p3^2), s → 2 m^2 - u - t}, TimeConstraint → 5000]; (*f(ut,u)*)


Print["f(s,u);", T100];

keisuu = Coefficient[T100, t, 2];

y200 = α^2 / (2 * s^3) * T100 / keisuu;

t = -1 / 2 * s * (1 - Cos[θ]);
u = -1 / 2 * s * (1 + Cos[θ]);

Print[
  "Scattering cross section in a laboratory system (trial calculation example using
    16 γ matrices (256*256) for curved space);", Expand[y200]];



ya20 = 32 / 9 * Integrate[y200, {θ, 0, Pi}];

Print["Total cross section integrated with respect to Θ;", Expand[ya20]];


y300 = 32 / 9 * Integrate[y200, {θ, 0, Pi}] //. {s → sw^2};

xt =.;
yt =.;
xt[min_, max_, lbl_] :=
  Table[If[PossibleZeroQ@Mod[i, 10], {i, If[lbl, i, Null], {0.04, 0}},
    {i, Null, {0.02, 0}}], {i, Ceiling[min], Floor[max], 2}];
yt[min_, max_, lbl_] := Join @@ Table[Join[{{10^i, If[lbl, 10^i, Null], {0.04, 0}}},
     If[10^i < max, Table[{j 10^i, Null, {0.02, 0}}, {j, 2, 9}], {}]],
    {i, Log10[min], Log10[max]}];
xt1[min_, max_] := xt[min, max, True];
xt0[min_, max_] := xt[min, max, False];
yt1[min_, max_] := yt[min, max, True];
yt0[min_, max_] := yt[min, max, False];



Print[
  "****************************************************************************
```

```
    ******************"];

α = 1 / 137;
y40 = y30 * (10^6 / 2.57);
y400 = y300 * (10^6 / 2.57);

Print[Style["3.When using four γ matrices under Minkowski spacetime (conventional
     calculation) and γ matrix (256*256)\ under curved spacetime.
Comparison of trial calculation examples using 16 (trial
     calculation examples in this paper)", Blue]];
(*LogPlot[{y40,y400},{sw,4,40},PlotRange→{{0,40},{1/100,10}},
 PlotStyle→{{Red,Dashed},Blue},
 PlotLegend→{"Before","After"},LegendPosition→{0.2,0.3},
 LegendSize→{0.6,0.4},LegendShadow→{.03,-.03},
 Frame→True,FrameLabel→{Labeled[Sqrt[s],"(GeV)",Right],Labeled[σ,"(nb)",Right]},
 FrameTicks→{{yt1,yt0},{xt1,xt0}},AspectRatio→1]*)

LogPlot[{y40, y400}, {sw, 4, 40}, PlotRange → {{0, 40}, {1 / 100, 10}},
 PlotStyle → {{Red, Dashed}, Blue}, Frame → True, FrameTicks → {{yt1, yt0}, {xt1, xt0}},
 FrameLabel → {Labeled[Sqrt[s], "(GeV)", Right], Labeled[σ, "(nb)", Right]},
 PlotLegends → Placed[LineLegend[Automatic, {"Conventional calculation",
     "Example of calculation for this paper"}, LabelStyle → 8,
    LegendFunction → "Frame", LegendLayout → "Column"], {{0.975, 0.925}, {1, 0.9}}],
 AspectRatio → 1]
```

Example of calculation of muon pair generation

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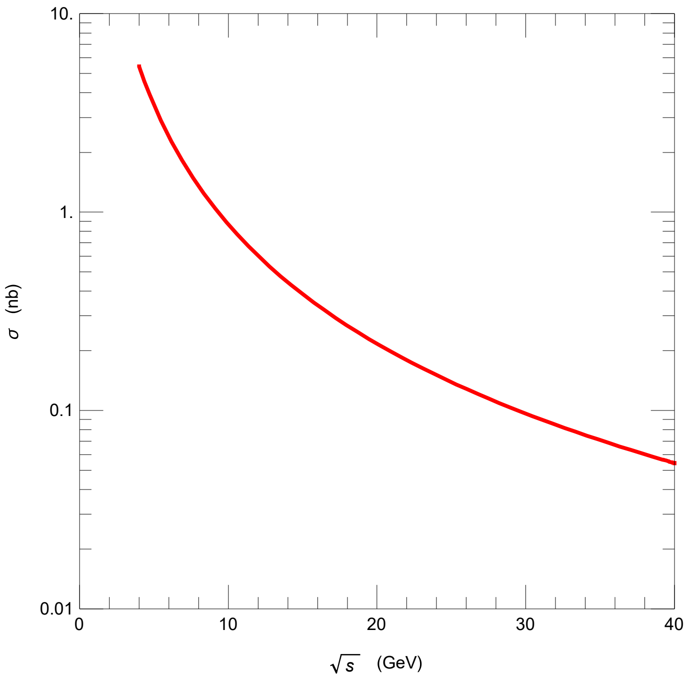
1.Calculation of muon generation using 4 γ matrices (4*4) (conventional calculation)

f(s,u);8 $\left(t^2+u^2\right)$

Scattering cross section of laboratory system (conventional calculation); $\frac{\alpha^2}{4\,s}+\frac{\alpha^2\,\mathrm{Cos}[\Theta]^2}{4\,s}$

Total cross section integrated with respect to Θ; $\frac{4\,\pi\,\alpha^2}{3\,s}$

Out[239]=



****

2.Muon pair generation calculation using γ matrix (256*256)

2.1.Calculation using 4 γ matrices (256*256) under Minkowski spacetime

Calculate the metric tensor as $\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

det (determinant of the metric tensor) = -1

f(s,u) ; $32768\left(t^2+u^2\right)$

Scattering cross section of laboratory system

(consistent with conventional calculation results) ; $\frac{1}{75076\,s}+\frac{\text{Cos}[\theta]^2}{75076\,s}$

****

2.2.Trial calculation example using 16 γ matrices (256*256) in curved space-time

Calculate the metric tensor as $\begin{pmatrix} -1 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{10}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{10}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{10}{9} \end{pmatrix}$

det (determinant of metric tensor) = $-\frac{37}{27}$

f(s,u) ; $\frac{4096\left(662920000\,t^2+249852400\,t\,u+661591653\,u^2\right)}{43046721}$

Scattering cross section in a laboratory system (trial calculation example using 16 γ matrices (256*256) for curved space) ;

$$\frac{1574364053\,\alpha^2}{5303360000\,s}-\frac{1328347\,\alpha^2\,\text{Cos}[\theta]}{2651680000\,s}+\frac{1074659253\,\alpha^2\,\text{Cos}[\theta]^2}{5303360000\,s}$$

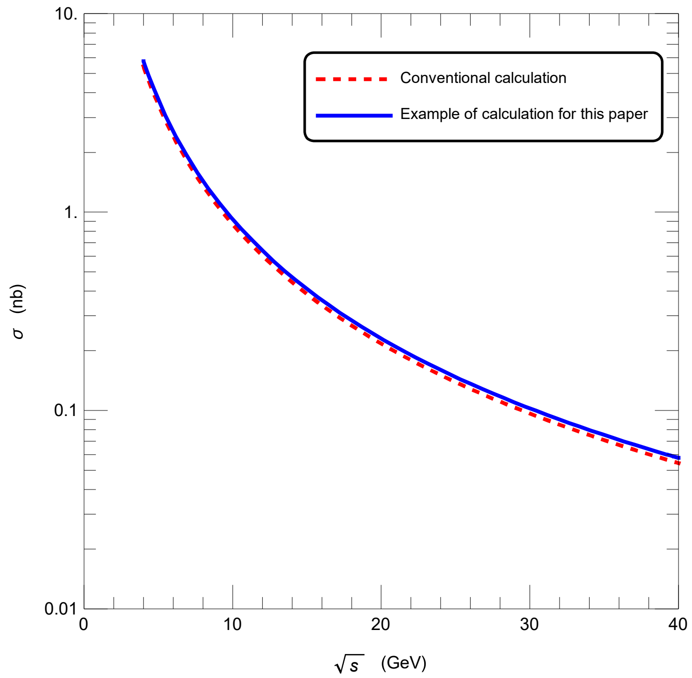
Total cross section integrated with respect to Θ; $\frac{4223387359\,\pi\,\alpha^2}{2983140000\,s}$

****

3.When using four γ matrices under Minkowski spacetime (conventional calculation) and γ matrix (256*256) under curved spacetime.

Comparison of trial calculation examples using 16 (trial calculation examples in this paper)

Out[400]=


10.
1.
0.1
0.01
σ (nb)
0
10
20
30
40
√s (GeV)
Conventional calculation
Example of calculation for this paper