In[1]:=

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(*Title:Muon Decay Rate Calculation (μ⁻→e⁻νₑνμ̄)
  Author:[Hirokaxzu Maruyama] Date:January 2025 Version:
   1.0 Description:This code calculates the muon decay rate using:
     1. Standard V-A theory with 4x4 gamma matrices 2. Extended formalism with 256
       x256 matrices 3. Comparison between Minkowski and curved spacetime Physical
       Significance:-Fundamental weak interaction process-Test of lepton flavor
          conservation-Important for determination of Fermi constant GF*)

(*Key Variables:mμ-muon mass me-electron mass G-Fermi coupling constant ω-
   electron energy θ-angle between e⁻ and νₑ Matrix Definitions:
  γ[μ]-gamma matrices ω[μ]-omega matrices (1-γ₅)-left-handed projection operator*)


(*Step 1:Define matrix elements and spinor states*)
(*Step 2:Calculate transition amplitude Note:Include V-A interaction structure*)
(*Step 3:Integrate over phase space Note:Account for three-body final state*)
(*Step 4:Compare results between different formalisms Note:Standard result gives Γ=
 G²mμ⁵/(192π³)*)



Print[Style["Example of calculation of muon decay", Blue]];




Print[
  "****************************************************************************
    ******************"];
Print[Style[
   "1.Muon decay calculation using 4 γ matrices (4*4) (conventional calculation)γ",
   Blue]];

(*γ matrix(4×4)*)

m1 =.;
m =.;
me =.;
mμ =.;
gu[0] = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}};
gu[1] = {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}};
gu[2] = {{0, 0, 0, -I}, {0, 0, I, 0}, {0, I, 0, 0}, {-I, 0, 0, 0}};
gu[3] = {{0, 0, 1, 0}, {0, 0, 0, -1}, {-1, 0, 0, 0}, {0, 1, 0, 0}};


e4 = IdentityMatrix[4];

gd[0] = 1 * gu[0];
gd[1] = -gu[1];
gd[2] = -gu[2];
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gd[3] = -gu[3];
gd[5] = I * gu[0].gu[1].gu[2].gu[3];

sl[q] = gu[0] * q0 + gu[1] * (-q1) + gu[2] * (-q2) + gu[3] * (-q3) + m * e4;
sl[p] = gu[0] * p0 + gu[1] * (-p1) + gu[2] * (-p2) + gu[3] * (-p3) + m * e4;
sl[k] = gu[0] * k0 + gu[1] * (-k1) + gu[2] * (-k2) + gu[3] * (-k3);
sl[j] = gu[0] * j0 + gu[1] * (-j1) + gu[2] * (-j2) + gu[3] * (-j3);
m1 = 0;
m =.;
me = m1 * e4;
mμ = m * e4;


s1 = 0;
s2 = 0;
y1 = 0;

For[x = 0, x ≤ 3, x++,
  For[y = 0, y ≤ 3, y++,
   s1 = Tr[(sl[k] + me).gu[x].(e4 - gd[5]).(sl[p] - me).gu[y].(e4 - gd[5])];
   s2 = Tr[(sl[j] + mμ).gd[x].(e4 - gd[5]).(sl[q] + mμ).gd[y].(e4 - gd[5])];
   y1 = y1 + s1 * s2;
  ]];


f1 = Simplify[y1 //. {p0 → m, p1 → 0, p2 → 0, p3 → 0, k0 → ω, k1 → 0, k2 → ω, k3 → 0,
     q0 → m / 2, q1 → 0, q2 → 0, q3 → -m / 2, j0 → m / 2 - ω, j1 → 0, j2 → 0, j3 → -m / 2 - ω}];


keisuu1 = Coefficient[f1, m^3 * ω, 1];
f2 = m * G^2 / (2 * Pi^3) * f1 / keisuu1;
f3 = Integrate[f2 / m^2, {ω, 1 / 2 m, 1 / 2 m - s}];
f4 = Integrate[f3, {s, 1 / 2 m, 0}];

Print["Muon decay rate (conventional calculation);", f4];
Print[Style["2.Muon decay calculation using γ matrix (256*256)", Blue]];


(*γ matrix(256×256)*)
(*Find 16 combinations of γ matrices (256×256)
 that satisfy the anticommutative relationship*)
demoteRank4to2[y_] := Flatten[Map[Flatten, Transpose[y, {1, 3, 2, 4}], {2}], 1];
pauli8times[g1_, g2_, g3_, g4_, g5_, g6_, g7_, g8_] :=
  demoteRank4to2[Outer[Times, demoteRank4to2[Outer[Times,
      demoteRank4to2[Outer[Times, g1, g2]], demoteRank4to2[Outer[Times, g3, g4]]]],
    demoteRank4to2[Outer[Times, demoteRank4to2[Outer[Times, g5, g6]],
      demoteRank4to2[Outer[Times, g7, g8]]]]]];


g[1] = {{1, 0}, {0, -1}};
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g[2] = {{0, -I}, {I, 0}};
g[3] = {{0, 1}, {1, 0}};
g[0] = {{1, 0}, {0, 1}};

e256 = IdentityMatrix[256];


γuv[0] = pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[0], g[3]];
γuv[1] = I * pauli8times[g[0], g[0], g[0], g[0], g[3], g[2], g[2], g[2]];
γuv[2] = I * pauli8times[g[0], g[0], g[0], g[1], g[2], g[2], g[2], g[2]];
γuv[3] = I * pauli8times[g[0], g[0], g[3], g[2], g[2], g[2], g[2], g[2]];

γuv[4] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[0], g[1]];
γuv[5] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[3], g[2]];
γuv[6] = I * pauli8times[g[1], g[2], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[7] = I * pauli8times[g[0], g[0], g[1], g[2], g[2], g[2], g[2], g[2]];

γuv[8] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[3], g[2], g[2]];
γuv[9] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[1], g[2]];
γuv[10] = I * pauli8times[g[3], g[2], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[11] = I * pauli8times[g[0], g[0], g[0], g[0], g[1], g[2], g[2], g[2]];

γuv[12] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[1], g[2], g[2]];
γuv[13] = I * pauli8times[g[0], g[1], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[14] = I * pauli8times[g[0], g[3], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[15] = I * pauli8times[g[0], g[0], g[0], g[3], g[2], g[2], g[2], g[2]];

γd[500] = I * γuv[0].γuv[1].γuv[2].γuv[3].γuv[4].γuv[5].γuv[6].γuv[7].
    γuv[8].γuv[9].γuv[10].γuv[11].γuv[12].γuv[13].γuv[14].γuv[15];

num =
  115 792 089 237 316 195 423 570 985 008 687 907 853 269 984 665 640 564 039 457 584 007 913 129 639 \
   936;
(*Confirm determinant*)

(*16 γ matrices (256×256) Calculation to confirm
 that the anticommutative relationship is satisfied*)
yt = 0;
For[kh = 0, kh ≤ 15, kh++,
  For[ks1 = 0, ks1 ≤ 15, ks1++,
   yf = Det[γuv[kh].γuv[ks1] + γuv[ks1].γuv[kh]];
   yt = yf + yt;
   If[kh =!= ks1 && yf == num * 16, Print["No.", km, ",x=", kh, ",y=", ks1]];
  ]];

If[kh == 16 && ks1 == 16 && yt / num == 16, Print[""],
  Print["γ matrix (256*256) 16 pieces Anti-commutation relation confirmation NG"]];

(**Set metric tensor*)
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Print[Style[
   "2.1. Calculation using 4 γ matrices (256*256) under Minkowski spacetime", Blue]];

m1 =.;
m =.;
me =.;
mμ =.;

gd[0] = 1;
gd[1] = 1;
gd[2] = 1;
gd[3] = 1;
gd[4] = 0;
gd[5] = 0;
gd[6] = 0;
gd[7] = 0;
gd[8] = 0;
gd[9] = 0;
gd[10] = 0;
gd[11] = 0;
gd[12] = 0;
gd[13] = 0;
gd[14] = 0;
gd[15] = 0;

m256 = 1 * m;


(*γ matrix multiplied by metric*)

For[km1 = 0, km1 ≤ 15, km1++,
  γu[km1] = -gd[km1] * γuv[km1];
 ];


For[km2 = 0, km2 ≤ 15, km2++,
  γd[km2] = 1 * γu[km2];
 ];
γd[0] = -1 * γu[0];


metric = {{-gd[0], gd[10], gd[12], gd[14]}, {gd[11], gd[1], gd[4], gd[6]},
    {gd[13], gd[5], gd[2], gd[8]}, {gd[15], gd[7], gd[9], gd[3]}} / gd[0];

Print["Calculate the metric tensor as", MatrixForm[metric]];

Print["det(Determinant of the metric tensor)=", Det[metric]];
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```
sl[q] = (γu[0] * q0 + γu[1] * - q1 + γu[2] * - q2 + γu[3] * - q3 + m256 * e256);
sl[p] = (γu[0] * p0 + γu[1] * - p1 + γu[2] * - p2 + γu[3] * - p3 + m256 * e256);
sl[k] = (γu[0] * k0 + γu[1] * - k1 + γu[2] * - k2 + γu[3] * - k3);
sl[j] = (γu[0] * j0 + γu[1] * - j1 + γu[2] * - j2 + γu[3] * - j3);
```

```
m =.;
me = m1 * e256;
mμ = m * e256;
m1 = 0;

s10 = 0;
s20 = 0;
y10 = 0;
```

```
For[x = 0, x ≤ 3, x++,
  For[y = 0, y ≤ 3, y++,
   s10 = Tr[(sl[k] + me).γu[x].(e256 - γd[500]).(sl[p] - me).γu[y].(e256 - γd[500])];
   s20 = Tr[(sl[j] + mμ).γd[x].(e256 - γd[500]).(sl[q] + mμ).γd[y].(e256 - γd[500])];
   y10 = y10 + s10 * s20;
  ]];
```

```
f10 = Simplify[
   y10 / ((m256 / m)^8) //. {p0 → m, p1 → 0, p2 → 0, p3 → 0, k0 → ω, k1 → 0, k2 → ω, k3 → 0,
     q0 → m / 2, q1 → 0, q2 → 0, q3 → -m / 2, j0 → m / 2 - ω, j1 → 0, j2 → 0, j3 → -m / 2 - ω}];
```

```
keisuu10 = Coefficient[f10, m^3 * ω, 1];
f20 = m * G^2 / (2 * Pi^3) * f10 / keisuu10;
f30 = Integrate[f20 / m^2, {ω, 1 / 2 m, 1 / 2 m - s}];
f40 = Integrate[f30, {s, 1 / 2 m, 0}];
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Print["Muon decay rate (consistent with conventional calculation results);", f40];
```

```
Print[
  "***************************************************************************
    *****************"];
```

```
Print[Style["2.2.Trial calculation example
     using 16 γ matrices (256*256) in curved space-time", Blue]];
```

```
m1 =.;
m =.;
me =.;
mμ =.;

gd[0] = 9 / 10;
gd[1] = 1;
gd[2] = 1;
gd[3] = 1;
gd[4] = 1 / 10;
gd[5] = 1 / 10;
gd[6] = 1 / 10;
gd[7] = 1 / 10;
gd[8] = 1 / 10;
gd[9] = 1 / 10;
gd[10] = 1 / 10;
gd[11] = 1 / 10;
gd[12] = 1 / 10;
gd[13] = 1 / 10;
gd[14] = 1 / 10;
gd[15] = 1 / 10;

m256 = 1 * m;
```

```
(*γ matrix multiplied by metric*)

For[km1 = 0, km1 ≤ 15, km1++,
  γu[km1] = -gd[km1] * γuv[km1];
 ];


For[km2 = 0, km2 ≤ 15, km2++,
  γd[km2] = 1 * γu[km2];
 ];
γd[0] = -1 * γu[0];


metric = {{-gd[0], gd[10], gd[12], gd[14]}, {gd[11], gd[1], gd[4], gd[6]},
    {gd[13], gd[5], gd[2], gd[8]}, {gd[15], gd[7], gd[9], gd[3]}} / gd[0];

Print["Calculate the metric tensor as ", MatrixForm[metric]];

Print["det(Determinant of the metric tensor)=", Det[metric]];
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```
sl[q] = (γu[0] * q0 + γu[1] * - q1 + γu[2] * - q2 + γu[3] * - q3 + m256 * e256);
sl[p] = (γu[0] * p0 + γu[1] * - p1 + γu[2] * - p2 + γu[3] * - p3 + m256 * e256);
sl[k] = (γu[0] * k0 + γu[1] * - k1 + γu[2] * - k2 + γu[3] * - k3);
sl[j] = (γu[0] * j0 + γu[1] * - j1 + γu[2] * - j2 + γu[3] * - j3);


m1 = 0;
m =.;
me = m1 * e256;
mμ = m * e256;

s100 = 0;
s200 = 0;
y100 = 0;


For[x = 0, x ≤ 15, x++,
  For[y = 0, y ≤ 15, y++,
   s100 = Tr[(sl[k] + me).γu[x].(e256 - γd[500]).(sl[p] - me).γu[y].(e256 - γd[500])];
   s200 = Tr[(sl[j] + mμ).γd[x].(e256 - γd[500]).(sl[q] + mμ).γd[y].(e256 - γd[500])];
   y100 = y100 + s100 * s200;
  ]];


f100 = Simplify[
   y100 / ((m256 / m)^8) //. {p0 → m, p1 → 0, p2 → 0, p3 → 0, k0 → ω, k1 → 0, k2 → ω, k3 → 0,
     q0 → m / 2, q1 → 0, q2 → 0, q3 → -m / 2, j0 → m / 2 - ω, j1 → 0, j2 → 0, j3 → -m / 2 - ω}];


keisuu100 = Coefficient[f100, m^3 * ω, 1];
f200 = m * G^2 / (2 * Pi^3) * f100 / keisuu100;
f300 = Integrate[f200 / m^2, {ω, 1 / 2 m, 1 / 2 m - s}];
f400 = Integrate[f300, {s, 1 / 2 m, 0}];


Print["Muon decay rate (calculation example using
    16 γ matrices (256*256) in case of curved space);", f400];
```

Example of calculation of muon decay

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1.Muon decay calculation using 4 γ matrices (4*4) (conventional calculation)γ

Muon decay rate (conventional calculation); $\frac{G^2 m^5}{192 \pi^3}$

2.Muon decay calculation using γ matrix (256*256)

2.1. Calculation using 4 γ matrices (256*256) under Minkowski spacetime

Calculate the metric tensor as $\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

det(Determinant of the metric tensor)=-1

Muon decay rate (consistent with conventional calculation results); $\frac{G^2 m^5}{192 \pi^3}$

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2.2.Trial calculation example using 16 γ matrices (256*256) in curved space-time

Calculate the metric tensor as $\begin{pmatrix} -1 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{10}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{10}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{10}{9} \end{pmatrix}$

det(Determinant of the metric tensor)=$-\frac{37}{27}$

Muon decay rate (calculation example using 16 γ matrices (256*256) in case of curved space);
$\frac{546\,713\,G^2 m^5}{502\,524\,096 \pi^3}$