In[1]:=

```
(*Title:Møller Scattering Calculation (e⁻e⁻→e⁻e⁻)
  Author:[Hirokazu Maruyama] Date:January 2025 Version:
   1.0 Description:This code computes the Møller scattering cross section using:
     1. Conventional approach with 4x4 gamma matrices 2. Extended formalism with 256
      x256 gamma matrices 3. Implementation in both Minkowski and curved spacetime
      Physical Background:-Electron-electron scattering involves exchange diagrams-
        Features identical particle effects in final state-
        Shows characteristic angular distribution*)


(*Key Variables and Parameters:m-electron mass s,t,
u-Mandelstam variables α-fine structure constant (1/137) θ-
 scattering angle E-electron energy in lab frame Matrix
  Definitions:gu[μ]-gamma matrices with upper index gd[μ]-
    gamma matrices with lower index sl[p]-Dirac slash notation for momentum p*)

(*Step 1:Define gamma matrices and spinor states*)
(*Step 2:Calculate scattering amplitudes Note:
 Include both direct and exchange terms due to identical particles*)
(*Step 3:Convert to center of mass frame coordinates*)
(*Step 4:Compute differential cross section Note:
 Include 1/2 factor for identical particles in final state*)
(*Step 5:Calculate angular distributions and compare between formalisms*)


(*Results
 Interpretation:-Angular distribution shows characteristic 1/sin⁴(θ/2) behavior-
   Cross section includes identical particle interference effects-
   Curved space calculation demonstrates[specific effects]*)


(*Plot Description:-X-axis:
  cos(θ) from-1 to 1-Y-axis:differential cross section (nb/GeV²/sr)-Red dashed line:
    conventional calculation result-Blue solid line:result in curved spacetime-Note:
      Asymmetry in angular distribution due to identical particle effects*)


Print[Style[
   "Example of calculation of electron and electron scattering (Möller scattering)",
   Blue]];


Print[
  "***************************************************************************
    ******************"];
```

```
Print[
  Style["1.Scattering of electrons and positrons using four γ matrices (4*4) (Meller
     scattering) (conventional calculation)", Blue]];


m =.;
s =.;
t =.;
u =.;
dt =.;
pu =.;
ε =.;
θ =.;
re =.;
α =.;

(*γ matrix(4×4)*)
γu[0] = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}};
γu[1] = {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}};
γu[2] = {{0, 0, 0, -I}, {0, 0, I, 0}, {0, I, 0, 0}, {-I, 0, 0, 0}};
γu[3] = {{0, 0, 1, 0}, {0, 0, 0, -1}, {-1, 0, 0, 0}, {0, 1, 0, 0}};
e4 = IdentityMatrix[4];
ms = m * e4;

γd[0] = 1 * γu[0];
γd[1] = -γu[1];
γd[2] = -γu[2];
γd[3] = -γu[3];

sl[p] = γu[0] * p0 + 1 * (γu[1] * (-p1) + γu[2] * (-p2) + γu[3] * (-p3) + ms);
sl[q] = γu[0] * q0 + 1 * (γu[1] * (-q1) + γu[2] * (-q2) + γu[3] * (-q3) + ms);
sl[k] = γu[0] * k0 + 1 * (γu[1] * (-k1) + γu[2] * (-k2) + γu[3] * (-k3) + ms);
sl[j] = γu[0] * j0 + 1 * (γu[1] * (-j1) + γu[2] * (-j2) + γu[3] * (-j3) + ms);

ftu1 = 0;
gtu1 = 0;
fut1 = 0;
gut1 = 0;

y1 = 0;
y2 = 0;
y3 = 0;
y4 = 0;

For[x = 0, x ≤ 3, x++,
  For[y = 0, y ≤ 3, y++,
   ftu1 = 1 / (16 * t^2) * Tr[sl[j].γu[x].sl[k].γu[y]] * Tr[sl[q].γd[x].sl[p].γd[y]];
   gtu1 = -1 / (16 * t * u) * Tr[sl[j].γu[x].sl[k].γu[y].sl[q].γd[x].sl[p].γd[y]];
   gut1 = -1 / (16 * t * u) * Tr[sl[k].γu[x].sl[j].γu[y].sl[p].γd[x].sl[q].γd[y]];
```

```
    y1 = y1 + FullSimplify[ExpandAll[ftu1]];
    y3 = y3 + FullSimplify[ExpandAll[gtu1]];
    y4 = y4 + FullSimplify[ExpandAll[gut1]];

   ]];


(*Conversion using Mandelstam variables*)

M = m;
T1 = Simplify[y1 //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
      q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
      j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
      z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];

T2 = Simplify[T1 //. {u → t}];

T3 = Simplify[y3 //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
      q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
      j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
      z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];
T4 = Simplify[y4 //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
      q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
      j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
      z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];

y5 = T1 + T2 + T3 + T4;

(*Center of gravity system*)

pu =.;
m =.;
ε =.;
θ =.;
t = 4 m^2 - s - u;
s = 4 * (pu^2 + m^2);
t = -4 * pu^2 * Sin[θ / 2]^2;
u = -4 * pu^2 * Cos[θ / 2]^2;

ε = Sqrt[pu^2 + m^2];
jy = Pi * re^2 * 4 * m1^2 * dt / (s (s - 4 m^2));
re = e1^2 / m1;
dt = pu^2 * do / Pi;

y6 = Simplify[jy * y5];
Print["Center-of-mass scattering cross section (conventional calculation);", y6];

(*Cross section in ultra-relativistic case (conventional calculation)*)
y7 = Simplify[y6 //. {pu → ε, m → 0, do → 1, e1 → 1}];
yf1 = 4 * ε^2 * α^2 * y7;
```

```
b = Extract[yf1, First@Position[yf1, α^2] - 1] * 64;
yf2 = yf1 / b;
Print["Scattering cross section in the
    ultra-relativistic case (conventional calculation);", yf2];
x = Cos[θ];
α = 1 / 137;
ListLogPlot[Table[{-x, yf2 * (10^6 / 2.57)}, {θ, Pi / 36, Pi, Pi / 36}], PlotStyle → Red,
 AspectRatio → 1.5, Joined → True, Frame → {{True, None}, {True, None}},
 FrameLabel → {"cos θ", Labeled[Subscript["×(dσ/dΩ)", CoM], {Superscript[4 E, 2],
     Labeled[nb Superscript[GeV, 2] Superscript[sterad, -1], {"(", ")"},
      {Left, Right}, Spacings → 0.1]}, {Left, Right}, Spacings → 0.1]}, FrameTicks →
  {{{{10, Superscript[10, 1]}, {100, Superscript[10, 2]}, {1000, Superscript[10, 3]},
     {10 000, Superscript[10, 4]}, {100 000, Superscript[10, 5]}}, None},
   {{{-1, "1.0"}, {-0.5, 0.5}, {0, 0}, {0.5, -0.5}, {1, "-1.0"}}, None}},
 PlotRange → {{-1, 1}, {1*^0, 2*^5}}, Axes -> None]



Print[
  "****************************************************************************
    ******************"];


Print[Style["2.Scattering of electrons and positrons
     using γ matrix (256*256) (Möller scattering)", Blue]];

m =.;
s =.;
t =.;
u =.;
dt =.;
pu =.;
ε =.;
θ =.;
k =.;
α =.;
re =.;


(*Find 16 combinations of gamma matrix (256 rows and 256 columns)
 that satisfy the anticommutation relationship*)
demoteRank4to2[y_] := Flatten[Map[Flatten, Transpose[y, {1, 3, 2, 4}], {2}], 1];
pauli8times[g1_, g2_, g3_, g4_, g5_, g6_, g7_, g8_] :=
  demoteRank4to2[Outer[Times, demoteRank4to2[Outer[Times,
      demoteRank4to2[Outer[Times, g1, g2]], demoteRank4to2[Outer[Times, g3, g4]]]],
    demoteRank4to2[Outer[Times, demoteRank4to2[Outer[Times, g5, g6]],
      demoteRank4to2[Outer[Times, g7, g8]]]]]];

g[1] = {{0, 1}, {1, 0}};
g[2] = {{0, -I}, {I, 0}};
```

```
g[3] = {{1, 0}, {0, -1}};
g[0] = {{1, 0}, {0, 1}};

e256 = IdentityMatrix[256];


γuv[0] = pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[0], g[3]];
γuv[1] = I * pauli8times[g[0], g[0], g[0], g[0], g[3], g[2], g[2], g[2]];
γuv[2] = I * pauli8times[g[0], g[0], g[0], g[1], g[2], g[2], g[2], g[2]];
γuv[3] = I * pauli8times[g[0], g[0], g[3], g[2], g[2], g[2], g[2], g[2]];

γuv[4] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[0], g[1]];
γuv[5] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[3], g[2]];
γuv[6] = I * pauli8times[g[1], g[2], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[7] = I * pauli8times[g[0], g[0], g[1], g[2], g[2], g[2], g[2], g[2]];

γuv[8] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[3], g[2], g[2]];
γuv[9] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[1], g[2]];
γuv[10] = I * pauli8times[g[3], g[2], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[11] = I * pauli8times[g[0], g[0], g[0], g[0], g[1], g[2], g[2], g[2]];

γuv[12] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[1], g[2], g[2]];
γuv[13] = I * pauli8times[g[0], g[1], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[14] = I * pauli8times[g[0], g[3], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[15] = I * pauli8times[g[0], g[0], g[0], g[3], g[2], g[2], g[2], g[2]];


num =
  115792089237316195423570985008687907853269984665640564039457584007913129639\
   936;(*Determinant confirmation*)



(*16 γ matrices (256×256) Calculation to confirm
 that the anticommutative relationship is satisfied*)
yt = 0;
For[kh = 0, kh ≤ 15, kh++,
  For[ks1 = 0, ks1 ≤ 15, ks1++,
   yf = Det[γuv[kh].γuv[ks1] + γuv[ks1].γuv[kh]];
   yt = yf + yt;
   If[kh =!= ks1 && yf == num * 16, Print["No.", km, ",x=", kh, ",y=", ks1]];
  ]];

If[kh == 16 && ks1 == 16 && yt / num == 16, Print[""],
  Print["γ matrix (256*256) 16 pieces Anti-commutation relation confirmation NG"]];

Print[Style[
   "2.1.Calculation using 4 γ matrices (256*256) under Minkowski spacetime", Blue]];

gd1[0] = 100;
```

```
gd1[1] = 100;
gd1[2] = 100;
gd1[3] = 100;
gd1[4] = 0;
gd1[5] = 0;
gd1[6] = 0;
gd1[7] = 0;
gd1[8] = 0;
gd1[9] = 0;
gd1[10] = 0;
gd1[11] = 0;
gd1[12] = 0;
gd1[13] = 0;
gd1[14] = 0;
gd1[15] = 0;

m256 = 100 * m;


(*γ matrix multiplied by metric*)

For[km1 = 0, km1 ≤ 15, km1++,
  γu[km1] = gd1[km1] * γuv[km1];
 ];

For[km2 = 0, km2 ≤ 15, km2++,
  γd[km2] = -1 * γu[km2];
 ];
γd[0] = 1 * γu[0];

metric = {{-gd1[0], gd1[10], gd1[12], gd1[14]}, {gd1[11], gd1[1], gd1[4], gd1[6]},
    {gd1[13], gd1[5], gd1[2], gd1[8]}, {gd1[15], gd1[7], gd1[9], gd1[3]}} / gd1[0];

Print["Calculate the metric tensor as ", MatrixForm[metric]];

Print["det(Determinant of the metric tensor)=", Det[metric]];



sl[q] = γu[0] * q0 + γu[1] * -q1 + γu[2] * -q2 + γu[3] * -q3 + m256 * e256;
sl[p] = γu[0] * p0 + γu[1] * -p1 + γu[2] * -p2 + γu[3] * -p3 + m256 * e256;
sl[k] = γu[0] * k0 + γu[1] * -k1 + γu[2] * -k2 + γu[3] * -k3 + m256 * e256;
sl[j] = γu[0] * j0 + γu[1] * -j1 + γu[2] * -j2 + γu[3] * -j3 + m256 * e256;



ftu10 = 0;
gtu10 = 0;
fut10 = 0;
gut10 = 0;
```

```
y10 = 0;
y20 = 0;
y30 = 0;
y40 = 0;


For[x = 0, x ≤ 3, x++,
  For[y = 0, y ≤ 3, y++,
   ftu10 = 1 / (64 * t^2) * Tr[sl[j].γu[x].sl[k].γu[y]] * Tr[sl[q].γd[x].sl[p].γd[y]];
   gtu10 = -1 / (t * u) * Tr[sl[j].γu[x].sl[k].γu[y].sl[q].γd[x].sl[p].γd[y]];
   gut10 = -1 / (t * u) * Tr[sl[k].γu[x].sl[j].γu[y].sl[p].γd[x].sl[q].γd[y]];

   y10 = y10 + FullSimplify[ExpandAll[ftu10]];
   y30 = y30 + FullSimplify[ExpandAll[gtu10]];
   y40 = y40 + FullSimplify[ExpandAll[gut10]];

  ]];



(*Conversion using Mandelstam variables*)

m256 = m;
M = m;
T10 = Simplify[
   y10 / ((m256 / m)^8) //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
     q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
     j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
     z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];

T20 = Simplify[T10 //. {u → t}];

T30 = Simplify[
   y30 / ((m256 / m)^8) //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
     q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
     j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
     z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];
T40 = Simplify[y40 / ((m256 / m)^8) //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0,
     k2 → 0, k3 → -p3, q0 → p0, q1 → p3 * Sqrt[1 - z^2], q2 → 0,
     q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0, j3 → -p3 * z,
     p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
     z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];

y50 = T10 + T20 + T30 + T40;



(*Center of gravity system*)
pu =.;
```

```
m =.;
ε =.;
θ =.;
t = 4 m^2 - s - u;
s = 4 * (pu^2 + m^2);
t = -4 * pu^2 * Sin[θ / 2]^2;
u = -4 * pu^2 * Cos[θ / 2]^2;

ε = Sqrt[pu^2 + m^2];
jy = Pi * re^2 * 4 * m1^2 * dt / (s (s - 4 m^2));
re = e1^2 / m1;
dt = pu^2 * do / Pi;
bf100 = 16 * 1024 * gd1[0]^8;
y60 = Simplify[jy * y50];
yf61 = y60 / bf100;
Print[yf61];

Print["Center-of-mass scattering cross section (conventional calculation);", yf61];


(*Cross section in ultra-relativistic case (conventional calculation)*)
y70 = Simplify[y60 //. {pu → ε, m → 0, do → 1, e1 → 1}];
yf10 = 4 * ε^2 * α^2 * y70;
bf10 = 16 * 1024 * gd1[0]^8;
yf20 = yf10 / bf10;
Print["Scattering cross section in the
    ultra-relativistic case (conventional calculation);", yf20];



Print[
  "***************************************************************************
    *****************"];

Print[Style["2.2.Trial calculation example using
     16 γ matrices (256*256) under curved space-time", Blue]];

m =.;
s =.;
t =.;
u =.;
dt =.;
pu =.;
ε =.;
θ =.;
k =.;
α =.;


(*Set metric tensor*)
```

```
gd2[0] = 9 / 10;
gd2[1] = 1;
gd2[2] = 1;
gd2[3] = 1;
gd2[4] = 1 / 10;
gd2[5] = 1 / 10;
gd2[6] = 1 / 10;
gd2[7] = 1 / 10;
gd2[8] = 1 / 10;
gd2[9] = 1 / 10;
gd2[10] = 1 / 10;
gd2[11] = 1 / 10;
gd2[12] = 1 / 10;
gd2[13] = 1 / 10;
gd2[14] = 1 / 10;
gd2[15] = 1 / 10;

m256 = 1 * m;



(*γ matrix multiplied by metric*)

For[km1 = 0, km1 ≤ 15, km1++,
  γu[km1] = gd2[km1] * γuv[km1];
 ];


For[km2 = 0, km2 ≤ 15, km2++,
  γd[km2] = -1 * γu[km2];
 ];
γd[0] = 1 * γu[0];

metric = {{-gd2[0], gd2[10], gd2[12], gd2[14]}, {gd2[11], gd2[1], gd2[4], gd2[6]},
    {gd2[13], gd2[5], gd2[2], gd2[8]}, {gd2[15], gd2[7], gd2[9], gd2[3]}} / gd2[0];


Print["Calculate the metric tensor as ", MatrixForm[metric]];

Print["det(Determinant of the metric tensor)=", Det[metric]];

sl[q] = γu[0] * q0 + γu[1] * -q1 + γu[2] * -q2 + γu[3] * -q3 + m256 * e256;
sl[p] = γu[0] * p0 + γu[1] * -p1 + γu[2] * -p2 + γu[3] * -p3 + m256 * e256;
sl[k] = γu[0] * k0 + γu[1] * -k1 + γu[2] * -k2 + γu[3] * -k3 + m256 * e256;
sl[j] = γu[0] * j0 + γu[1] * -j1 + γu[2] * -j2 + γu[3] * -j3 + m256 * e256;

ftu100 = 0;
gtu100 = 0;
```

```
fut100 = 0;
gut100 = 0;

y100 = 0;
y200 = 0;
y300 = 0;
y400 = 0;



For[x = 0, x ≤ 15, x++,
  For[y = 0, y ≤ 15, y++,
   ftu100 = 1 / (64 * t^2) * Tr[sl[j].γu[x].sl[k].γu[y]] * Tr[sl[q].γd[x].sl[p].γd[y]];
   gtu100 = -1 / (t * u) * Tr[sl[j].γu[x].sl[k].γu[y].sl[q].γd[x].sl[p].γd[y]];
   gut100 = -1 / (t * u) * Tr[sl[k].γu[x].sl[j].γu[y].sl[p].γd[x].sl[q].γd[y]];

   y100 = y100 + FullSimplify[ExpandAll[ftu100]];
   y300 = y300 + FullSimplify[ExpandAll[gtu100]];
   y400 = y400 + FullSimplify[ExpandAll[gut100]];

  ]];


m256 = m;

(*Conversion using Mandelstam variables*)

M = m;
T100 = Simplify[
   y100 / ((m256 / m)^8) //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
     q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
     j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
     z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];

T200 = Simplify[T100 //. {u → t}];

T300 = Simplify[
   y300 / ((m256 / m)^8) //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
     q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
     j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
     z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];
T400 = Simplify[y400 / ((m256 / m)^8) //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0,
     k2 → 0, k3 → -p3, q0 → p0, q1 → p3 * Sqrt[1 - z^2], q2 → 0,
     q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0, j3 → -p3 * z,
     p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
     z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];

y500 = T100 + T200 + T300 + T400;
```

```
(*Center of gravity system*)

t = 4 m^2 - s - u;
s = 4 * (pu^2 + m^2);
t = -4 * pu^2 * Sin[θ / 2]^2;
u = -4 * pu^2 * Cos[θ / 2]^2;
ε = Sqrt[pu^2 + m^2];
jt = Pi * re^2 * 4 * m1^2 * dt / (s (s - 4 m^2));
re = e1^2 / m1;
dt = pu^2 * do / Pi;

bf100 = 16 * 1024 * gd2[0]^8;
y600 = Simplify[jt * y500];
yf601 = y600 / bf100;


Print["Center-of-mass scattering cross section (conventional calculation);", yf601];

(*Cross section in ultra-relativistic case*)
y700 = Simplify[y600 //. {pu → ε, m → 0, do → 1, e1 → 1}];
yf100 = 4 * ε^2 * α^2 * y700;

yf200 = yf100 / bf100;

Print["Scattering cross section in the ultra-relativistic case (calculation example
    using 16 γ matrices (256*256) in the case of curved space);", yf200];


Print[
  "***************************************************************************
    *****************"];

x = Cos[θ];
α = 1 / 137;
data1 = Table[{-x, yf20 * (10^6 / 2.57)}, {θ, Pi / 37, Pi, Pi / 37}];
data2 = Table[{-x, yf200 * (10^6 / 2.57)}, {θ, Pi / 37, Pi, Pi / 37}];


Print[Style[
   "3.When using 4 γ matrices under Minkowski spacetime (conventional calculation)
     and γ matrix (256*256) under curved spacetime.
Comparison of trial calculation examples using 16 (trial
     calculation examples in this paper)", Blue]];

ListLogPlot[{data1, data2}, PlotStyle → {{Red, Dashed}, Blue},
 AspectRatio → 1.5, Joined → True, Frame → {{True, None}, {True, None}},
 PlotLegends → Placed[LineLegend[Automatic, {"Conventional calculation",
     "Example of calculation for this paper"}, LabelStyle → 8,
```

```
    LegendFunction → "Frame", LegendLayout → "Column"], {{1, 0.15}, {1.15, 0.075}}],
 FrameLabel → {"cos θ", Labeled[Subscript["×(dσ/dΩ)", CoM], {Superscript[4 E, 2],
     Labeled[nb Superscript[GeV, 2] Superscript[sterad, -1], {"(", ")"}, {Left, Right},
      Spacings → 0.1]}, {Left, Right}, Spacings → 0.1]}, FrameTicks →
  {{{{10, Superscript[10, 1]}, {100, Superscript[10, 2]}, {1000, Superscript[10, 3]},
     {10000, Superscript[10, 4]}, {100000, Superscript[10, 5]}}, None},
   {{{-1, "1.0"}, {-0.5, 0.5}, {0, 0}, {0.5, -0.5}, {1, "-1.0"}}, None}},
 PlotRange → {{-1, 1}, {1*^0, 2*^5}}, Axes -> None]
```

Example of calculation of electron and electron scattering (Möller scattering)

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****

1.Scattering of electrons and positrons using four
γ matrices (4*4) (Meller scattering) (conventional calculation)

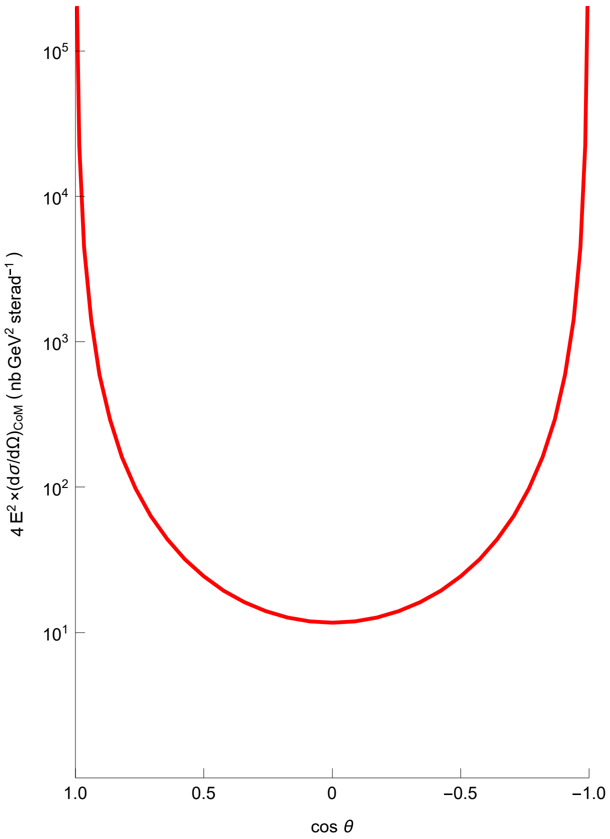
Center-of-mass scattering cross section (conventional calculation);

$$\left(\left(\text{do e1}^4 \left(20\,m^4 + 80\,m^2\,pu^2 + 99\,pu^4 + 4\left(3\,m^4 + 12\,m^2\,pu^2 + 7\,pu^4\right)\text{Cos}[2\,\theta] + pu^4\,\text{Cos}[4\,\theta]\right)\text{Csc}[\theta]^4\right)\Big/\left(32\,pu^4\left(m^2 + pu^2\right)\right)\right)$$

Scattering cross section in the ultra-relativistic case (conventional calculation);

$$\frac{1}{64}\alpha^2\,(7 + \text{Cos}[2\,\theta])^2\,\text{Csc}[\theta]^4$$

Out[64]=



****

2.Scattering of electrons and positrons using γ matrix (256*256) (Möller scattering)

2.1.Calculation using 4 γ matrices (256*256) under Minkowski spacetime

Calculate the metric tensor as $\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

det(Determinant of the metric tensor)=-1

$$\left(\text{do}\ \text{e1}^4 \left(20\, m^4 + 80\, m^2\, \text{pu}^2 + 99\, \text{pu}^4 + 4 \left(3\, m^4 + 12\, m^2\, \text{pu}^2 + 7\, \text{pu}^4\right) \text{Cos}[2\,\theta] + \text{pu}^4\, \text{Cos}[4\,\theta]\right) \text{Csc}[\theta]^4\right) \Big/ \left(512\, \text{pu}^4 \left(m^2 + \text{pu}^2\right)\right)$$

Center-of-mass scattering cross section (conventional calculation);

$$\left(\left(\text{do}\ \text{e1}^4 \left(20\, m^4 + 80\, m^2\, \text{pu}^2 + 99\, \text{pu}^4 + 4 \left(3\, m^4 + 12\, m^2\, \text{pu}^2 + 7\, \text{pu}^4\right) \text{Cos}[2\,\theta] + \text{pu}^4\, \text{Cos}[4\,\theta]\right) \text{Csc}[\theta]^4\right) \Big/ \left(512\, \text{pu}^4 \left(m^2 + \text{pu}^2\right)\right)\right)$$

Scattering cross section in the ultra-relativistic case (conventional calculation);

$$\frac{1}{64}\,\alpha^2\,(7 + \text{Cos}[2\,\theta])^2\,\text{Csc}[\theta]^4$$

****

2.2.Trial calculation example using 16 γ matrices (256*256) under curved space-time

Calculate the metric tensor as $\begin{pmatrix} -1 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{10}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{10}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{10}{9} \end{pmatrix}$

det(Determinant of the metric tensor)=$-\frac{37}{27}$

Center-of-mass scattering cross section (conventional calculation);

$$\frac{1}{705\,277\,476\,864\, \text{pu}^4 \left(m^2 + \text{pu}^2\right)^3} \text{do}\ \text{e1}^4 \Big(33\,291\,591\,971\, m^8 + 212\,950\,936\,912\, m^6\, \text{pu}^2 + 533\,960\,529\,672\, m^4\, \text{pu}^4 + 573\,123\,425\,216\, m^2\, \text{pu}^6 + 219\,126\,989\,104\, \text{pu}^8 - 13\,968\,800\, m^4\, \text{pu}^2 \left(m^2 + \text{pu}^2\right) \text{Cos}[\theta] + \left(21\,105\,753\,261\, m^8 + 138\,302\,914\,992\, m^6\, \text{pu}^2 + 266\,595\,951\,352\, m^4\, \text{pu}^4 + 202\,967\,489\,856\, m^2\, \text{pu}^6 + 53\,662\,339\,664\, \text{pu}^8\right) \text{Cos}[2\,\theta] + 13\,968\,800\, m^6\, \text{pu}^2\, \text{Cos}[3\,\theta] + 13\,968\,800\, m^4\, \text{pu}^4\, \text{Cos}[3\,\theta] + 2\,323\,440\,000\, m^4\, \text{pu}^4\, \text{Cos}[4\,\theta] + 4\,646\,880\,000\, m^2\, \text{pu}^6\, \text{Cos}[4\,\theta] + 2\,323\,440\,000\, \text{pu}^8\, \text{Cos}[4\,\theta]\Big) \text{Csc}[\theta]^4$$

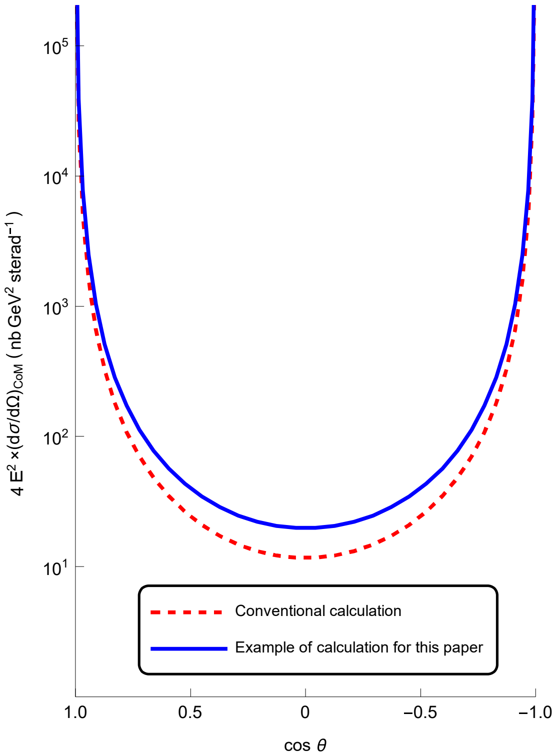
Scattering cross section in the ultra-relativistic case (calculation example using 16 γ matrices (256*256) in the case of curved space);

$$\frac{1}{11\,019\,960\,576}\alpha^2\,(13\,695\,436\,819 + 3\,353\,896\,229\,\text{Cos}[2\,\theta] + 145\,215\,000\,\text{Cos}[4\,\theta])\,\text{Csc}[\theta]^4$$

****

3.When using 4 γ matrices under Minkowski spacetime (conventional calculation) and γ matrix (256*256) under curved spacetime. Comparison of trial calculation examples using 16 (trial calculation examples in this paper)

Out[248]=



10^5
10^4
10^3
10^2
10^1
$4\,E^2 \times (d\sigma/d\Omega)_{CoM}$ (nb GeV^2 sterad^{-1})
Conventional calculation
Example of calculation for this paper
1.0
0.5
0
−0.5
−1.0
cos θ