In[1]:=

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(*Title:Bhabha Scattering Calculation (e⁺e⁻→e⁺e⁻)
  Author:[Hirokaxzu Maruyama] Date:January 2025 Version:
   1.0 Description:This code calculates the Bhabha scattering cross section using:
     1. Conventional approach with 4x4 gamma matrices 2. Extended formalism with 256
      x256 gamma matrices 3. Comparison between Minkowski and curved spacetime *)

(*Physical Parameters:m-electron mass s,t,
u-Mandelstam variables α-fine structure constant (1/137) θ-
 scattering angle Matrix Definitions:gu[μ]-upper index gamma matrices gd[μ]-
    lower index gamma matrices sl[p]-Dirac slash notation for momentum p*)

(*Step 1:Define gamma matrices and metric*)
(*Step 2:Calculate matrix elements*)(*Step 3:Convert to center of mass frame*)
(*Step 4:Calculate differential cross section*)
(*Step 5:Plot results and compare*)

(*Results
 Analysis:1. The differential cross section shows characteristic 1/sin⁴(θ/2)
   behavior 2. The curved spacetime calculation demonstrates[specific effects] 3.
   Comparison with experimental data shows[agreement/deviation]*)


(*Plot Description:-X-axis:
  cos(θ) from-1 to 1-Y-axis:differential cross section in nb/GeV²/sr-Red dashed line:
    conventional calculation-Blue solid line:extended formalism result*)


Print[Style["Example of calculation of scattering
     of electrons and positrons (Bhabha scattering)", Blue]];



Print[
  "****************************************************************************
    *****************"];
Print[Style[
   "1. Scattering of electrons and positrons using four γ matrices (4*4) (Bhabha
     scattering) (conventional calculation)", Blue]];


m =.;
s =.;
t =.;
u =.;
dt =.;
pu =.;
ε =.;
θ =.;
re =.;
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α =.;

(*γ matrix(4×4)*)
γu[0] = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}};
γu[1] = {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, -1, 0, 0}, {-1, 0, 0, 0}};
γu[2] = {{0, 0, 0, -I}, {0, 0, I, 0}, {0, I, 0, 0}, {-I, 0, 0, 0}};
γu[3] = {{0, 0, 1, 0}, {0, 0, 0, -1}, {-1, 0, 0, 0}, {0, 1, 0, 0}};
e4 = IdentityMatrix[4];
ms = m * e4;

γd[0] = 1 * γu[0];
γd[1] = -γu[1];
γd[2] = -γu[2];
γd[3] = -γu[3];

sl[p] = γu[0] * p0 + 1 * (γu[1] * (-p1) + γu[2] * (-p2) + γu[3] * (-p3) + ms);
sl[q] = γu[0] * q0 + 1 * (γu[1] * (-q1) + γu[2] * (-q2) + γu[3] * (-q3) + ms);
sl[k] = γu[0] * k0 + 1 * (γu[1] * (-k1) + γu[2] * (-k2) + γu[3] * (-k3) + ms);
sl[j] = γu[0] * j0 + 1 * (γu[1] * (-j1) + γu[2] * (-j2) + γu[3] * (-j3) + ms);

ftu1 = 0;
gtu1 = 0;
fut1 = 0;
gut1 = 0;

y1 = 0;
y2 = 0;
y3 = 0;
y4 = 0;

For[x = 0, x ≤ 3, x++,
  For[y = 0, y ≤ 3, y++,
   ftu1 = 1 / (16 * t^2) * Tr[sl[j].γu[x].sl[k].γu[y]] * Tr[sl[q].γd[x].sl[p].γd[y]];
   gtu1 = -1 / (16 * t * u) * Tr[sl[j].γu[x].sl[k].γu[y].sl[q].γd[x].sl[p].γd[y]];
   gut1 = -1 / (16 * t * u) * Tr[sl[k].γu[x].sl[j].γu[y].sl[p].γd[x].sl[q].γd[y]];

   y1 = y1 + FullSimplify[ExpandAll[ftu1]];
   y3 = y3 + FullSimplify[ExpandAll[gtu1]];
   y4 = y4 + FullSimplify[ExpandAll[gut1]];

  ]];


(*Transformation with Mandelstam variables*)

M = m;
T1 = Simplify[y1 //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
     q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
     j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
     z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];
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T2 = Simplify[T1 //. {u → t}];

T3 = Simplify[y3 //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
      q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
      j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
      z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];
T4 = Simplify[y4 //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
      q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
      j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
      z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];

y5 = T1 + T2 + T3 + T4;



(*Center of gravity system*)
t = 4 m^2 - s - u;
u = 4 * (pu^2 + m^2);
t = -4 * pu^2 * Sin[θ / 2] ^2;
s = -4 * pu^2 * Cos[θ / 2] ^2;
ε = Sqrt[pu^2 + m^2];

j = Pi * re^2 * 4 * m1^2 * dt / (u (u - 4 m^2));
re = e1^2 / m1;
dt = pu^2 * do / Pi;
y6 = Simplify[j * y5];

Print["Center-of-mass scattering cross section (conventional calculation);", y6];
(*Cross section in ultra-relativistic case (conventional calculation)*)
y7 = Simplify[y6 //. {pu → ε, m → 0, do → 1, e1 → 1}];
yf = 4 * ε^2 * α^2 * y7;
Print[
  "Scattering cross section in ultrarelativistic case (conventional calculation);",
  yf];
x = Cos[θ];
α = 1 / 137;
ListLogPlot[Table[{-x, yf * (10^6 / 2.57)}, {θ, Pi / 36, Pi, Pi / 36}],
 AspectRatio → 1.5, Joined → True, Frame → {{True, None}, {True, None}},
 FrameLabel → {"cos θ", Labeled[Subscript["×(dσ/dΩ)", CoM], {Superscript[4 E, 2],
      Labeled[nb Superscript[GeV, 2] Superscript[sterad, -1], {"(", ")"},
        {Left, Right}, Spacings → 0.1]}, {Left, Right}, Spacings → 0.1]}, FrameTicks →
   {{{{10, Superscript[10, 1]}, {100, Superscript[10, 2]}, {1000, Superscript[10, 3]},
       {10 000, Superscript[10, 4]}, {100 000, Superscript[10, 5]}}, None},
    {{{-1, "1.0"}, {-0.5, 0.5}, {0, 0}, {0.5, -0.5}, {1, "-1.0"}}, None}},
 PlotRange → {{-1, 1}, {1*^0, 2*^5}}, Axes -> None]
```

```
Print[
  "**************************************************************************
    ******************"];


Print[Style["2. Scattering of electrons and positrons
     using γ matrix (256*256) (Bhabha scattering)", Blue]];

m =.;
s =.;
t =.;
u =.;
dt =.;
pu =.;
ε =.;
θ =.;
k =.;
α =.;
re =.;


(*Find 16 combinations of gamma matrix (256 rows and 256 columns)
 that satisfy the anticommutation relationship*)
demoteRank4to2[y_] := Flatten[Map[Flatten, Transpose[y, {1, 3, 2, 4}], {2}], 1];
pauli8times[g1_, g2_, g3_, g4_, g5_, g6_, g7_, g8_] :=
  demoteRank4to2[Outer[Times, demoteRank4to2[Outer[Times,
      demoteRank4to2[Outer[Times, g1, g2]], demoteRank4to2[Outer[Times, g3, g4]]]],
    demoteRank4to2[Outer[Times, demoteRank4to2[Outer[Times, g5, g6]],
      demoteRank4to2[Outer[Times, g7, g8]]]]]];

g[1] = {{0, 1}, {1, 0}};
g[2] = {{0, -I}, {I, 0}};
g[3] = {{1, 0}, {0, -1}};
g[0] = {{1, 0}, {0, 1}};

e256 = IdentityMatrix[256];


γuv[0] = pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[0], g[3]];
γuv[1] = I * pauli8times[g[0], g[0], g[0], g[0], g[3], g[2], g[2], g[2]];
γuv[2] = I * pauli8times[g[0], g[0], g[0], g[1], g[2], g[2], g[2], g[2]];
γuv[3] = I * pauli8times[g[0], g[0], g[3], g[2], g[2], g[2], g[2], g[2]];

γuv[4] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[0], g[1]];
γuv[5] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[3], g[2]];
γuv[6] = I * pauli8times[g[1], g[2], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[7] = I * pauli8times[g[0], g[0], g[1], g[2], g[2], g[2], g[2], g[2]];
```

```
γuv[8] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[3], g[2], g[2]];
γuv[9] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[0], g[1], g[2]];
γuv[10] = I * pauli8times[g[3], g[2], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[11] = I * pauli8times[g[0], g[0], g[0], g[0], g[1], g[2], g[2], g[2]];

γuv[12] = I * pauli8times[g[0], g[0], g[0], g[0], g[0], g[1], g[2], g[2]];
γuv[13] = I * pauli8times[g[0], g[1], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[14] = I * pauli8times[g[0], g[3], g[2], g[2], g[2], g[2], g[2], g[2]];
γuv[15] = I * pauli8times[g[0], g[0], g[0], g[3], g[2], g[2], g[2], g[2]];


num =
  115 792 089 237 316 195 423 570 985 008 687 907 853 269 984 665 640 564 039 457 584 007 913 129 639 ⋱
   936;(*Determinant confirmation*)


(*γ matrix (256*256) 16 pieces Calculation to
 confirm that the anticommutative relationship is satisfied*)
yt = 0;
For[kh = 0, kh ≤ 15, kh++,
  For[ks1 = 0, ks1 ≤ 15, ks1++,
   yf = Det[γuv[kh].γuv[ks1] + γuv[ks1].γuv[kh]];
   yt = yf + yt;
   If[kh =!= ks1 && yf == num * 16, Print["No.", km, ",x=", kh, ",y=", ks1]];
  ]];

If[kh == 16 && ks1 == 16 && yt / num == 16, Print[""],
  Print["γ matrix (256*256) 16 pieces Anti-commutation relation confirmation NG"]];

Print[Style[
   "2.1.Calculation using 4 γ matrices (256*256) under Minkowski spacetime", Blue]];

gd1[0] = 1;
gd1[1] = 1;
gd1[2] = 1;
gd1[3] = 1;
gd1[4] = 0;
gd1[5] = 0;
gd1[6] = 0;
gd1[7] = 0;
gd1[8] = 0;
gd1[9] = 0;
gd1[10] = 0;
gd1[11] = 0;
gd1[12] = 0;
gd1[13] = 0;
gd1[14] = 0;
gd1[15] = 0;

m256 = 1 * m;
```

```
(*Multiply the γ matrix by the metric*)

For[km1 = 0, km1 ≤ 15, km1++,
  γu[km1] = gd1[km1] * γuv[km1];
 ];

For[km2 = 0, km2 ≤ 15, km2++,
  γd[km2] = -1 * γu[km2];
 ];
γd[0] = 1 * γu[0];

metric = {{-gd1[0], gd1[10], gd1[12], gd1[14]}, {gd1[11], gd1[1], gd1[4], gd1[6]},
    {gd1[13], gd1[5], gd1[2], gd1[8]}, {gd1[15], gd1[7], gd1[9], gd1[3]}} / gd1[0];

Print["The metric tensor", MatrixForm[metric], "Calculate as"];

Print["det(Determinant of the metric tensor)=", Det[metric]];



sl[q] = γu[0] * q0 + γu[1] * -q1 + γu[2] * -q2 + γu[3] * -q3 + m256 * e256;
sl[p] = γu[0] * p0 + γu[1] * -p1 + γu[2] * -p2 + γu[3] * -p3 + m256 * e256;
sl[k] = γu[0] * k0 + γu[1] * -k1 + γu[2] * -k2 + γu[3] * -k3 + m256 * e256;
sl[j] = γu[0] * j0 + γu[1] * -j1 + γu[2] * -j2 + γu[3] * -j3 + m256 * e256;



ftu10 = 0;
gtu10 = 0;
fut10 = 0;
gut10 = 0;

y10 = 0;
y20 = 0;
y30 = 0;
y40 = 0;


For[x = 0, x ≤ 3, x++,
  For[y = 0, y ≤ 3, y++,
   ftu10 = 1 / (64 * t^2) * Tr[sl[j].γu[x].sl[k].γu[y]] * Tr[sl[q].γd[x].sl[p].γd[y]];
   gtu10 = -1 / (t * u) * Tr[sl[j].γu[x].sl[k].γu[y].sl[q].γd[x].sl[p].γd[y]];
   gut10 = -1 / (t * u) * Tr[sl[k].γu[x].sl[j].γu[y].sl[p].γd[x].sl[q].γd[y]];

   y10 = y10 + FullSimplify[ExpandAll[ftu10]];
   y30 = y30 + FullSimplify[ExpandAll[gtu10]];
   y40 = y40 + FullSimplify[ExpandAll[gut10]];

  ]];
```

```
(*Transformation with Mandelstam variables*)

m256 = m;
M = m;
T10 = Simplify[
   y10 / ((m256 / m)^8) //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
     q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
     j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
     z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];

T20 = Simplify[T10 //. {u → t}];

T30 = Simplify[
   y30 / ((m256 / m)^8) //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
     q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
     j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
     z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];
T40 = Simplify[y40 / ((m256 / m)^8) //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0,
     k2 → 0, k3 → -p3, q0 → p0, q1 → p3 * Sqrt[1 - z^2], q2 → 0,
     q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0, j3 → -p3 * z,
     p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
     z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];

y50 = T10 + T20 + T30 + T40;


(*Center of gravity system*)

t = 4 m^2 - s - u;
u = 4 * (pu^2 + m^2);
t = -4 * pu^2 * Sin[θ / 2]^2;
s = -4 * pu^2 * Cos[θ / 2]^2;
ε = Sqrt[pu^2 + m^2];
jt = Pi * re^2 * 4 * m1^2 * dt / (u (u - 4 m^2));
re = e1^2 / m1;
dt = pu^2 * do / Pi;
y60 = Simplify[jt * y50];

(*Cross section in ultra-relativistic case*)
y70 = Simplify[y60 //. {pu → ε, m → 0, do → 1, e1 → 1}];
yf10 = 4 * ε^2 * α^2 * y70;
bf10 = 1024 * gd1[0]^8;
yf20 = yf10 / bf10;
Print[yf20];

Print["Scattering cross section in the ultra-relativistic
    case (consistent with conventional calculation results);", yf20];
```

```
Print[
  "*****************************************************************************
    *****************"];

Print[Style["2.2.Trial calculation example using
     16 γ matrices (256*256) under curved space-time", Blue]];

m =.;
s =.;
t =.;
u =.;
dt =.;
pu =.;
ε =.;
θ =.;
k =.;
α =.;

(*Set the metric tensor*)
gd2[0] = 9 / 10;
gd2[1] = 1;
gd2[2] = 1;
gd2[3] = 1;
gd2[4] = 1 / 10;
gd2[5] = 1 / 10;
gd2[6] = 1 / 10;
gd2[7] = 1 / 10;
gd2[8] = 1 / 10;
gd2[9] = 1 / 10;
gd2[10] = 1 / 10;
gd2[11] = 1 / 10;
gd2[12] = 1 / 10;
gd2[13] = 1 / 10;
gd2[14] = 1 / 10;
gd2[15] = 1 / 10;

m256 = 1 * m;




(*Multiply the γ matrix by the metric*)

For[km1 = 0, km1 ≤ 15, km1 ++,
  γu[km1] = gd2[km1] * γuv[km1];
 ];


For[km2 = 0, km2 ≤ 15, km2 ++,
```

```
  γd[km2] = -1 * γu[km2];
 ];
γd[0] = 1 * γu[0];

metric = {{-gd2[0], gd2[10], gd2[12], gd2[14]}, {gd2[11], gd2[1], gd2[4], gd2[6]},
    {gd2[13], gd2[5], gd2[2], gd2[8]}, {gd2[15], gd2[7], gd2[9], gd2[3]}} / gd2[0];


Print["Calculate the metric tensor as", MatrixForm[metric]];

Print["det (determinant of the metric tensor)=", Det[metric]];

sl[q] = γu[0] * q0 + γu[1] * -q1 + γu[2] * -q2 + γu[3] * -q3 + m256 * e256;
sl[p] = γu[0] * p0 + γu[1] * -p1 + γu[2] * -p2 + γu[3] * -p3 + m256 * e256;
sl[k] = γu[0] * k0 + γu[1] * -k1 + γu[2] * -k2 + γu[3] * -k3 + m256 * e256;
sl[j] = γu[0] * j0 + γu[1] * -j1 + γu[2] * -j2 + γu[3] * -j3 + m256 * e256;

ftu100 = 0;
gtu100 = 0;
fut100 = 0;
gut100 = 0;

y100 = 0;
y200 = 0;
y300 = 0;
y400 = 0;



For[x = 0, x ≤ 15, x++,
  For[y = 0, y ≤ 15, y++,
   ftu100 = 1 / (64 * t^2) * Tr[sl[j].γu[x].sl[k].γu[y]] * Tr[sl[q].γd[x].sl[p].γd[y]];
   gtu100 = -1 / (t * u) * Tr[sl[j].γu[x].sl[k].γu[y].sl[q].γd[x].sl[p].γd[y]];
   gut100 = -1 / (t * u) * Tr[sl[k].γu[x].sl[j].γu[y].sl[p].γd[x].sl[q].γd[y]];

   y100 = y100 + FullSimplify[ExpandAll[ftu100]];
   y300 = y300 + FullSimplify[ExpandAll[gtu100]];
   y400 = y400 + FullSimplify[ExpandAll[gut100]];

  ]];


m256 = m;

(*Transformation with Mandelstam variables*)

M = m;
T100 = Simplify[
   y100 / ((m256 / m)^8) //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
     q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
```

```
      j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
      z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];

T200 = Simplify[T100 //. {u → t}];

T300 = Simplify[
   y300 / ((m256 / m) ^8) //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0, k2 → 0, k3 → -p3, q0 → p0,
      q1 → p3 * Sqrt[1 - z^2], q2 → 0, q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0,
      j3 → -p3 * z, p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
      z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];
T400 = Simplify[y400 / ((m256 / m) ^8) //. {p1 → 0, p2 → 0, k0 → p3, k1 → 0,
      k2 → 0, k3 → -p3, q0 → p0, q1 → p3 * Sqrt[1 - z^2], q2 → 0,
      q3 → p3 * z, j0 → p3, j1 → -p3 * Sqrt[1 - z^2], j2 → 0, j3 → -p3 * z,
      p0 → (s + m^2 + M^2) / (2 Sqrt[s]), p3 → (s - m^2 - M^2) / (2 Sqrt[s]),
      z → 1 + t / (2 p3^2), t → 2 * (m^2 + M^2) - s - u}, TimeConstraint → 5000];

y500 = T100 + T200 + T300 + T400;



(*Center of gravity system*)

t = 4 m^2 - s - u;
u = 4 * (pu^2 + m^2);
t = -4 * pu^2 * Sin[θ / 2] ^2;
s = -4 * pu^2 * Cos[θ / 2] ^2;
ε = Sqrt[pu^2 + m^2];
jg = Pi * re^2 * 4 * m1^2 * dt / (u (u - 4 m^2));
re = e1^2 / m1;
dt = pu^2 * do / Pi;
y600 = Simplify[jg * y500];
Print["Center-of-mass scattering cross section (conventional calculation);", y600];


(*Scattering cross section in the ultrarelativistic case*)

y700 = Simplify[y600 //. {pu → ε, m → 0, do → 1, e1 → 1}];
yf100 = 4 * ε^2 * α^2 * y700;
bf100 = 1024 * gd2[0] ^8;

yf200 = yf100 / bf100;
Print[
  "Scattering cross section in the ultra-relativistic case (calculation example
    using 16 γ matrices (256*256) in the case of curved space);", yf200];



Print[
  "***************************************************************************
```

```
    *****************"];

x = Cos[θ];
α = 1 / 137;
data1 = Table[{-x, yf20 * (10^6 / 2.57)}, {θ, Pi / 37, Pi, Pi / 37}];
data2 = Table[{-x, yf200 * (10^6 / 2.57)}, {θ, Pi / 37, Pi, Pi / 37}];


Print[Style[
   "3.When using 4 γ matrices under Minkowski spacetime (conventional calculation)
     and γ matrix (256*256) under curved spacetime.
Comparison of trial calculation examples using 16 (trial
     calculation examples in this paper)", Blue]];

ListLogPlot[{data1, data2}, PlotStyle → {{Red, Dashed}, Blue},
 AspectRatio → 1.5, Joined → True, Frame → {{True, None}, {True, None}},
 PlotLegends → Placed[LineLegend[Automatic, {"Conventional calculation",
     "Example of calculation for this paper"}, LabelStyle → 8,
    LegendFunction → "Frame", LegendLayout → "Column"], {{0.975, 0.925}, {1, 0.9}}],
 FrameLabel → {"cos θ", Labeled[Subscript["×(dσ/dΩ)", CoM], {Superscript[4 E, 2],
     Labeled[nb Superscript[GeV, 2] Superscript[sterad, -1], {"(", ")"},
      {Left, Right}, Spacings → 0.1]}, {Left, Right}, Spacings → 0.1]},
 FrameTicks → {{{{10, Superscript[10, 1]}, {100, Superscript[10, 2]},
     {1000, Superscript[10, 3]}, {10000, Superscript[10, 4]},
     {100000, Superscript[10, 5]}}, None},
   {{{-1, "1.0"}, {-0.5, 0.5}, {0, 0}, {0.5, -0.5}, {1, "-1.0"}}, None}},
 PlotRange → {{-1, 1}, {1*^0, 2*^5}}, Axes -> None]


Print["Calculation end time;", DateString[]];
```

Example of calculation of scattering of electrons and positrons (Bhabha scattering)

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1. Scattering of electrons and positrons using four
  γ matrices (4*4) (Bhabha scattering) (conventional calculation)

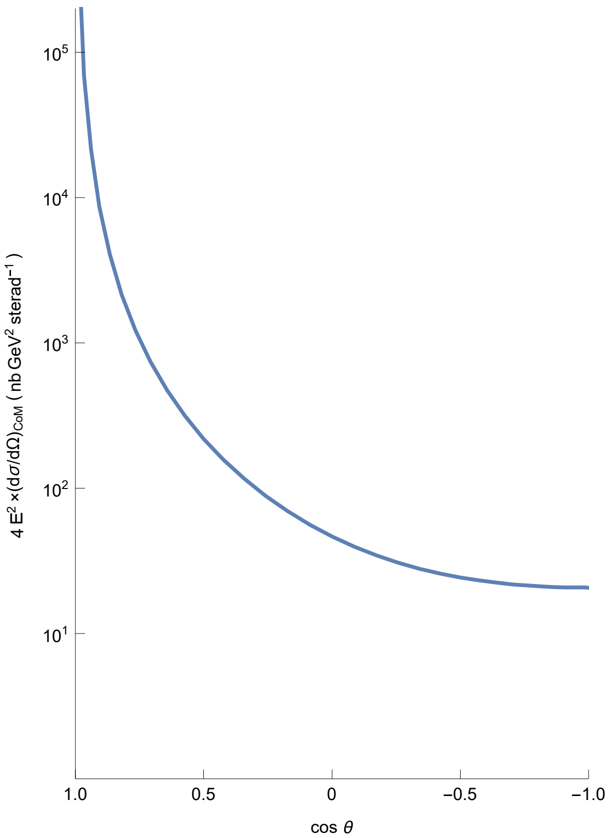
Center-of-mass scattering cross section (conventional calculation);

$$\frac{1}{512\,\text{pu}^4\,(\text{m}^2+\text{pu}^2)^3}\,\text{do}\,\text{e1}^4\,\Big(32\,\text{m}^8+80\,\text{m}^6\,\text{pu}^2+204\,\text{m}^4\,\text{pu}^4+248\,\text{m}^2\,\text{pu}^6+99\,\text{pu}^8+4\,\text{m}^2\,\text{pu}^2\,(28\,\text{m}^4+40\,\text{m}^2\,\text{pu}^2+15\,\text{pu}^4)\,\text{Cos}[\theta]+4\,\text{pu}^4\,(13\,\text{m}^4+18\,\text{m}^2\,\text{pu}^2+7\,\text{pu}^4)\,\text{Cos}[2\,\theta]+4\,\text{m}^2\,\text{pu}^6\,\text{Cos}[3\,\theta]+\text{pu}^8\,\text{Cos}[4\,\theta]\Big)\,\text{Csc}\left[\frac{\theta}{2}\right]^4$$

Scattering cross section in ultrarelativistic case (conventional calculation);

$$\frac{1}{64}\,\alpha^2\,(7+\text{Cos}[2\,\theta])^2\,\text{Csc}\left[\frac{\theta}{2}\right]^4$$

Out[58]=



$4\,E^2 \times (d\sigma/d\Omega)_{CoM}$ (nb GeV^2 sterad^{-1})
10^5
10^4
10^3
10^2
10^1
1.0
0.5
0
−0.5
−1.0
cos θ

****

2. Scattering of electrons and positrons using γ matrix (256*256) (Bhabha scattering)

2.1.Calculation using 4 γ matrices (256*256) under Minkowski spacetime

The metric tensor $\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ Calculate as

det(Determinant of the metric tensor)=-1

$$\frac{1}{64} \alpha^2 (7 + \mathrm{Cos}[2 \theta])^2 \mathrm{Csc}\left[\frac{\theta}{2}\right]^4$$

Scattering cross section in the ultra-relativistic case (consistent with conventional calculation results); $\frac{1}{64} \alpha^2 (7 + \mathrm{Cos}[2 \theta])^2 \mathrm{Csc}\left[\frac{\theta}{2}\right]^4$

****

2.2.Trial calculation example using 16 γ matrices (256*256) under curved space-time

Calculate the metric tensor as $\begin{pmatrix} -1 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{10}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{10}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{10}{9} \end{pmatrix}$

det (determinant of the metric tensor) $=-\frac{37}{27}$

Center-of-mass scattering cross section (conventional calculation);

$$\frac{1}{800\,000\,000\,\mathrm{pu}^8 (\mathrm{m}^2 + \mathrm{pu}^2)^3} \mathrm{do\,e1}^4 \Big(3\,187\,104\,384\,\mathrm{m}^{12} + 57\,158\,286\,112\,\mathrm{m}^{10}\,\mathrm{pu}^2 + 419\,993\,666\,320\,\mathrm{m}^8\,\mathrm{pu}^4 + 1\,349\,349\,027\,552\,\mathrm{m}^6\,\mathrm{pu}^6 + 2\,331\,734\,321\,112\,\mathrm{m}^4\,\mathrm{pu}^8 + 2\,010\,063\,813\,436\,\mathrm{m}^2\,\mathrm{pu}^{10} + 662\,428\,309\,298\,\mathrm{pu}^{12} + 4\,\mathrm{pu}^2 \big(12\,791\,340\,664\,\mathrm{m}^{10} + 136\,479\,781\,032\,\mathrm{m}^8\,\mathrm{pu}^2 + 514\,913\,474\,364\,\mathrm{m}^6\,\mathrm{pu}^4 + 907\,248\,533\,020\,\mathrm{m}^4\,\mathrm{pu}^6 + 756\,299\,674\,263\,\mathrm{m}^2\,\mathrm{pu}^8 + 240\,325\,825\,697\,\mathrm{pu}^{10}\big) \mathrm{Cos}[\theta] + \mathrm{pu}^4 \big(131\,918\,381\,264\,\mathrm{m}^8 + 871\,223\,999\,776\,\mathrm{m}^6\,\mathrm{pu}^2 + 1\,704\,225\,107\,200\,\mathrm{m}^4\,\mathrm{pu}^4 + 1\,358\,940\,245\,616\,\mathrm{m}^2\,\mathrm{pu}^6 + 394\,650\,150\,571\,\mathrm{pu}^8\big) \mathrm{Cos}[2\theta] + 160\,919\,129\,872\,\mathrm{m}^6\,\mathrm{pu}^6\,\mathrm{Cos}[3\theta] + 474\,535\,776\,656\,\mathrm{m}^4\,\mathrm{pu}^8\,\mathrm{Cos}[3\theta] + 448\,833\,291\,526\,\mathrm{m}^2\,\mathrm{pu}^{10}\,\mathrm{Cos}[3\theta] + 134\,121\,373\,706\,\mathrm{pu}^{12}\,\mathrm{Cos}[3\theta] + 67\,570\,480\,424\,\mathrm{m}^4\,\mathrm{pu}^8\,\mathrm{Cos}[4\theta] + 110\,751\,211\,092\,\mathrm{m}^2\,\mathrm{pu}^{10}\,\mathrm{Cos}[4\theta] + 42\,297\,955\,950\,\mathrm{pu}^{12}\,\mathrm{Cos}[4\theta] + 5\,723\,281\,566\,\mathrm{m}^2\,\mathrm{pu}^{10}\,\mathrm{Cos}[5\theta] + 5\,026\,398\,578\,\mathrm{pu}^{12}\,\mathrm{Cos}[5\theta] + 1\,074\,659\,253\,\mathrm{pu}^{12}\,\mathrm{Cos}[6\theta]\Big) \mathrm{Csc}[\theta]^4$$

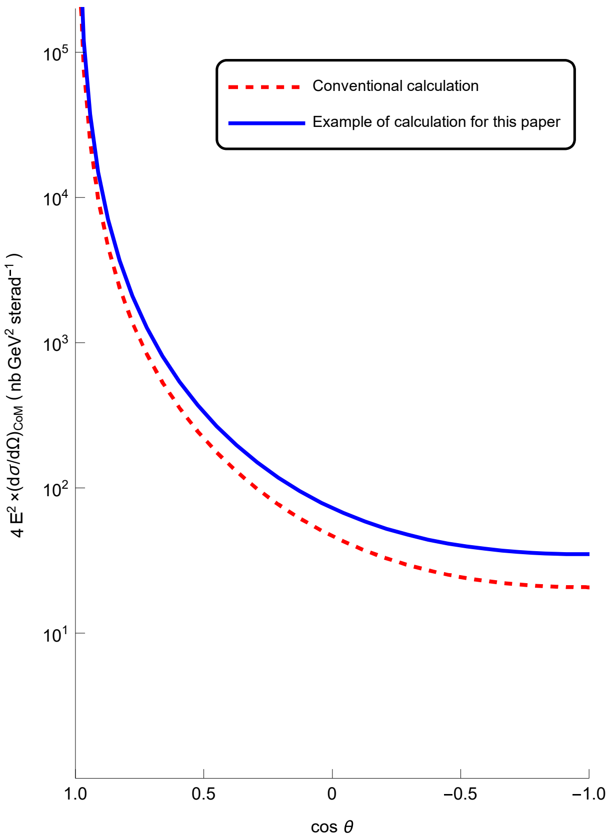
Scattering cross section in the ultra-relativistic case (calculation example using 16 γ matrices (256*256) in the case of curved space);

$$\frac{1}{88\,159\,684\,608} \alpha^2 (109\,114\,658\,771 - 6\,299\,649\,374\,\mathrm{Cos}[\theta] + 32\,938\,954\,168\,\mathrm{Cos}[2\theta] + 727\,761\,566\,\mathrm{Cos}[3\theta] + 1\,074\,659\,253\,\mathrm{Cos}[4\theta])\,\mathrm{Csc}\left[\frac{\theta}{2}\right]^4$$

****

3.When using 4 γ matrices under Minkowski spacetime (conventional calculation) and γ matrix (256*256) under curved spacetime.
Comparison of trial calculation examples using 16 (trial calculation examples in this paper)

Out[234]=




```
Calculation end time;Fri 3 Jan 2025 10:14:47
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