In[1]:=

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(*Title:Omega Matrix Calculations and Properties
  Author:[Hirokaxzu Maruyama]  Date:January 2025 Version:
   1.0 Description:This code demonstrates the fundamental properties of ω matrices:
     1. Construction from gamma matrices 2. Verification of anticommutation
      relations 3. Calculation of invariant quantities 4. Comparison with
      standard gamma matrices Theoretical Background:-ω matrices provide
         alternative representation for particle statistics-Important for fermion-
        boson duality theory-Basis for extended QED and QCD calculations*)


(*Key Operators:γ[μ]-Standard gamma matrices (4x4) ω[μ]-Omega matrices derived
    from gamma matrices A[μ]-Vector potential components Matrix Properties:
  -Anticommutation relations-Hermiticity conditions-Trace properties*)


(*Section 1:Gamma Matrix Definitions and Properties Note:
 Standard 4x4 representation*)(*Section 2:
 Construction of Omega Matrices Note:Linear combinations of gamma matrices*)
(*Section 3:Verification of Anticommutation Relations Note:
 Crucial for quantum field theory applications*)
(*Section 4:Invariant Quantities Note:Construction of physical observables*)


(*Results
 Summary:1. Anticommutation relations verified for both γ and ω matrices 2. ω
   matrices show distinct algebraic properties 3. Invariant combinations demonstrate
   physical relevance 4. Results consistent with theoretical predictions*)


Print["------------(*γ matrix*)-------------------------"];

γ[0] = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}};
γ[1] = I * {{0, 0, 0, 1}, {0, 0, 1, 0}, {0, 1, 0, 0}, {1, 0, 0, 0}};
γ[2] = I * {{0, 0, 0, -I}, {0, 0, I, 0}, {0, -I, 0, 0}, {I, 0, 0, 0}};
γ[3] = I * {{0, 0, 1, 0}, {0, 0, 0, -1}, {1, 0, 0, 0}, {0, -1, 0, 0}};

Print["γ0=", MatrixForm[γ[0]]];
Print["γ1=", MatrixForm[γ[1]]];
Print["γ2=", MatrixForm[γ[2]]];
Print["γ3=", MatrixForm[γ[3]]];

Print["--------------(*Anticommutation
    relation of γ matrix*)-----------------------"];

For[kh = 0, kh ≤ 3, kh++,
  For[ks1 = 0, ks1 ≤ 3, ks1++,
   yf = γ[kh].γ[ks1] + γ[ks1].γ[kh];
   Print["γ", kh, "*γ", ks1, "+γ", ks1, "*γ", kh, "=", MatrixForm[yf]];
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  ]];


Print["------------(*ω matrix*)-------------------------"];



ω[0] = γ[0] + γ[3];
ω[1] = γ[1];
ω[2] = γ[2];
ω[3] = γ[3] + γ[0];

ω[0] = (γ[0] + γ[3]) / 2;
ω[1] = (γ[1] + γ[1]) / 2;
ω[2] = (γ[2] + γ[2]) / 2;
ω[3] = (γ[3] + γ[0]) / 2;


Print["ω0=γ0+γ3=", MatrixForm[ω[0]]];
Print["ω1=γ1+γ1=", MatrixForm[ω[1]]];
Print["ω2=γ2+γ2=", MatrixForm[ω[2]]];
Print["ω3=γ3+γ0=", MatrixForm[ω[3]]];



Print["--------------(*Anticommutation
    relation of ω matrix*)-----------------------"];

For[kh = 0, kh ≤ 3, kh++,
  For[ks1 = 0, ks1 ≤ 3, ks1++,
   yf = ω[kh].ω[ks1] + ω[ks1].ω[kh];
   Print["ω", kh, "*ω", ks1, "+ω", ks1, "*ω", kh, "=", MatrixForm[yf]];
  ]];

Print["--------------(*Invariant using ω matrix*)-----------------------"];

s1 = ω[0] * A0 + ω[1] * A1 + ω[2] * A2 + ω[3] * A3;
y = s1.s1;


Print["(ω0*A0+ω1*A1+ω2*A2+ω3*A3)*(ω0*A0+ω1*A1+ω2*A2+ω3*A3)=",
  Simplify[y[[1]]][[1]]];
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------------(*γ matrix*)-------------------------

$$\gamma 0=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\gamma1=\begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma2=\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma3=\begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}$$

--------------(*Anticommutation relation of γ matrix*)------------------------

$$\gamma0*\gamma0+\gamma0*\gamma0=\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$$\gamma0*\gamma1+\gamma1*\gamma0=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma0*\gamma2+\gamma2*\gamma0=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma0*\gamma3+\gamma3*\gamma0=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma1*\gamma0+\gamma0*\gamma1=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma1*\gamma1+\gamma1*\gamma1=\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$$\gamma1*\gamma2+\gamma2*\gamma1=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma1*\gamma3+\gamma3*\gamma1=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma2*\gamma0+\gamma0*\gamma2=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma2*\gamma1+\gamma1*\gamma2=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma 2\star\gamma 2+\gamma 2\star\gamma 2=\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$$\gamma 2\star\gamma 3+\gamma 3\star\gamma 2=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma 3\star\gamma 0+\gamma 0\star\gamma 3=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma 3\star\gamma 1+\gamma 1\star\gamma 3=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma 3\star\gamma 2+\gamma 2\star\gamma 3=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma 3\star\gamma 3+\gamma 3\star\gamma 3=\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

------------(*ω matrix*)------------------------

$$\omega 0=\gamma 0+\gamma 3=\begin{pmatrix} \frac{1}{2} & 0 & \frac{i}{2} & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{i}{2} \\ \frac{i}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & -\frac{i}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

$$\omega 1=\gamma 1+\gamma 1=\begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 2=\gamma 2+\gamma 2=\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 3=\gamma 3+\gamma 0=\begin{pmatrix} \frac{1}{2} & 0 & \frac{i}{2} & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{i}{2} \\ \frac{i}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & -\frac{i}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

--------------(*Anticommutation relation of ω matrix*)-----------------------

$$\omega 0\star\omega 0+\omega 0\star\omega 0=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 0\star\omega 1+\omega 1\star\omega 0=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 0 \star \omega 2+\omega 2 \star \omega 0=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 0 \star \omega 3+\omega 3 \star \omega 0=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 1 \star \omega 0+\omega 0 \star \omega 1=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 1 \star \omega 1+\omega 1 \star \omega 1=\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$$\omega 1 \star \omega 2+\omega 2 \star \omega 1=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 1 \star \omega 3+\omega 3 \star \omega 1=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 2 \star \omega 0+\omega 0 \star \omega 2=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 2 \star \omega 1+\omega 1 \star \omega 2=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 2 \star \omega 2+\omega 2 \star \omega 2=\begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$$\omega 2 \star \omega 3+\omega 3 \star \omega 2=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 3 \star \omega 0+\omega 0 \star \omega 3=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 3 \star \omega 1+\omega 1 \star \omega 3=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 3 \star \omega 2+\omega 2 \star \omega 3=\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\omega 3 \star \omega 3 + \omega 3 \star \omega 3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

--------------(*Invariant using ω matrix*)-----------------------

$$(\omega 0 \star A0 + \omega 1 \star A1 + \omega 2 \star A2 + \omega 3 \star A3) \star (\omega 0 \star A0 + \omega 1 \star A1 + \omega 2 \star A2 + \omega 3 \star A3) = -A1^2 - A2^2$$