

So, the AC dir during the first or second half cycles are equal but opposite in sign i.e. they are alternatively positive and negative so that the avg over one cycle is zero.

**RMS/effective/virtual value:** The effective value of an AC is that steady current which, when passed through a given resistance for a certain time will develop the same amount of heat as the actual AC shall develop when passed for the same time.

The rate of heat generation in a current carrying resistance is given by  $I^2 R$ .

If we represent the effective value as  $I_{eff}$ , then,

$$I_{eff}^2 = \bar{I}^2$$

$$\text{or, } I_{eff} = \sqrt{\bar{I}^2} = I_{rms}$$

$$\bar{I}^2 = \text{Mean value}$$

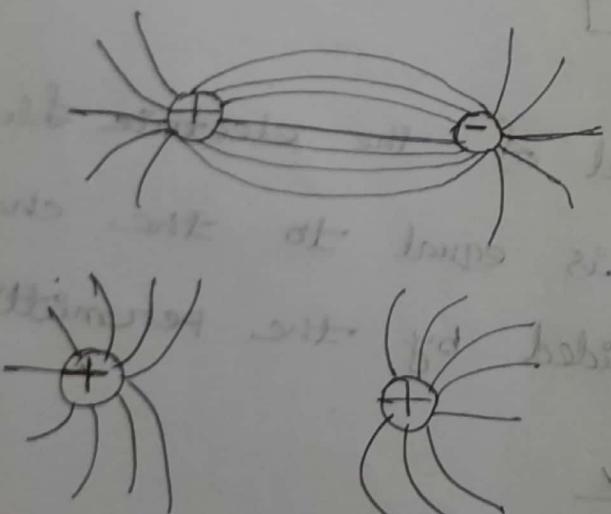
The eff. value of an AC current is the square root of the mean of the square of the instantaneous values over one complete cycle.

Coulomb's law: The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\therefore F = k \frac{|q_1 q_2|}{r^2}$$

Magnetic field: The portion of space near a magnet body or a current-carrying body in which the magnetic field forces due to the body or current can be detected is known as magnetic field.



Attraction

$$\frac{V}{\theta} = \Phi$$

Repulsion

$$V = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r}$$

The magnetic force on a moving charge:

M. Force  $\vec{F}$  does not have the same direction as the magnetic field  $\vec{B}$ , but instead is always perpendicular to both  $\vec{B}$  and the velocity  $\vec{v}$ .

The magnitude  $F$  of the force is found to be proportional to the component of  $\vec{v}$  perpendicular to the field field. When the component is zero, the force is zero.

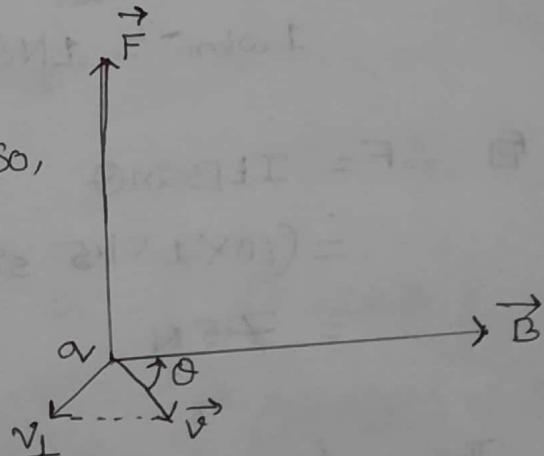
The direction of  $\vec{F}$  is always perpendicular to the plane containing  $\vec{v}$  and  $\vec{B}$ . Its magnitude is given by,  $F = |av| v_L B$

$$= |av| v B \sin \theta$$

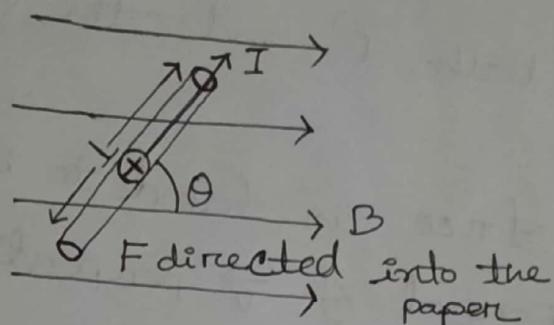
As  $\vec{F}$  is perpendicular to the plane  $\vec{v}$  and  $\vec{B}$ . So,

$$\theta = 90^\circ$$

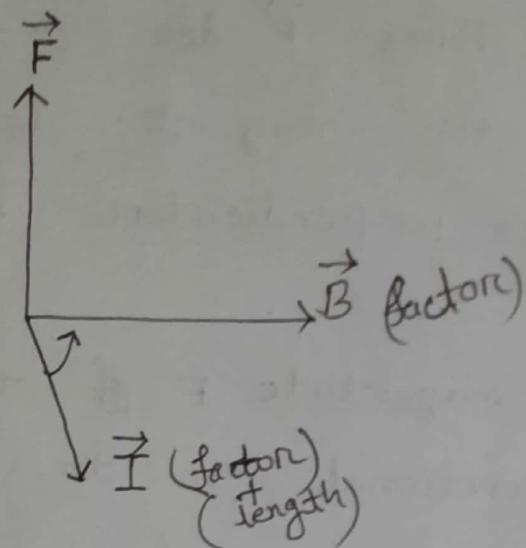
$$\vec{F} = av \vec{v} \times \vec{B}$$



The magnetic force on a current carrying conductor:



$$F = BIL \sin\theta$$



SI unit of  $B \rightarrow$  Tesla / wb-m<sup>-2</sup>

CGS unit  $B \rightarrow$  Gauss,  $[1 T = 10,000 G]$

$$\vec{F} = \alpha_0 \vec{v} \times \vec{B}$$

$$B = \frac{F}{\alpha_0 v} - (1)$$

$$1 \text{ wb m}^{-2} = 1 \text{ N C}^{-1} \text{ m}^{-1} \text{s}$$

$$\therefore F = ILB \sin\theta$$

$$l = 1 \text{ m}$$

$$= (10 \times 1 \times 1.5 \sin 30^\circ) \text{ N} \quad I = 10 \text{ A}$$

$$\theta = 30^\circ$$

$$= 7.5 \text{ N}$$

$$B = 1.5 \text{ wb m}^{-2}$$

$$\boxed{\frac{1}{2} \times 10 \times 1.5}$$

The force will be perpendicular to both  $I$  and  $B$ .

$$\text{K.E of the proton} = (5.0 \times 10^6)(1.6 \times 10^{-19} \text{ J/eV}) \\ = 8.0 \times 10^{-13} \text{ J}$$

$$K.E = \frac{1}{2} m v^2$$

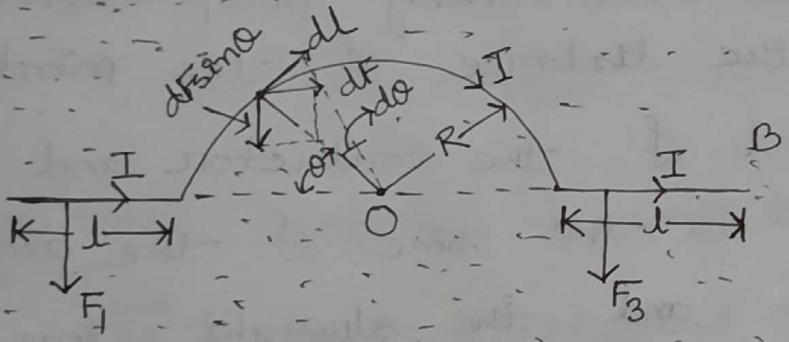
$$\Rightarrow v = \sqrt{\frac{2 K.E}{m}} \\ = \sqrt{\frac{2 \times 8.0 \times 10^{-13}}{1.67 \times 10^{-27}}}$$

$$= 3.1 \times 10^7 \text{ ms}^{-1}$$

Now,  $F = \alpha v B \sin\theta$

$$= \{(1.6 \times 10^{-19})(3.1 \times 10^7)(1.5) \times \sin 90^\circ\} N \\ = 7.4 \times 10^{-12} \text{ N}$$

Ampere



$$F_1 = F_3 = I l B$$

$$dF = I B dl \\ = I B (R d\theta)$$

$$F_2 = \int_0^\pi dF \sin\theta$$

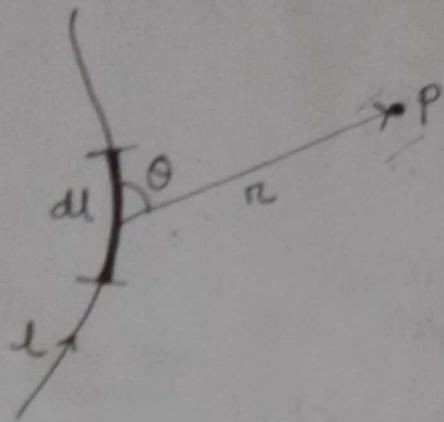
$$= \int_0^\pi (IBR d\theta) \sin\theta \\ = IBR \int_0^\pi \sin\theta d\theta$$

$$= 2IBR$$

The resultant force on the whole wire is,

$$F = F_1 + F_2 + F_3 = 2IB + 2IBR = 2IB(1+R)$$

## The Biot-Savart's Law:



Law: The magnitude of the magnetic field is at a point in a particular medium around a conductor of small length due to current flowing through it is directly proportional to the current, inversely proportional to the square of the distance of the point from the mid-point of the conductor and directly proportional to the sine of the angle between the conductor and the straight line joining the point and the mid-point of the conductor.

Explanation: If electric current  $I$  passes through a small length  $dl$  of the conductor, then the magnitude of the magnetic field  $d\vec{B}$  at a point  $P$ , which is at a distance  $r$  and at an angle

$\theta$  from the mid-point of the considered portion of the conductor would be

$$dB \propto \frac{Idl \sin \theta}{r^2}$$

$$\text{or, } dB = K \frac{Idl \sin \theta}{r^2}$$

Here,  $K$  is proportionality constant. Its magnitude depends on the units of the quantities and the magnetic properties of the medium.

Ampere's law : The line integral of a magnetic field along a closed path is equal to  $\mu_0$  times the total electric current enclosed by the area of the closed path.

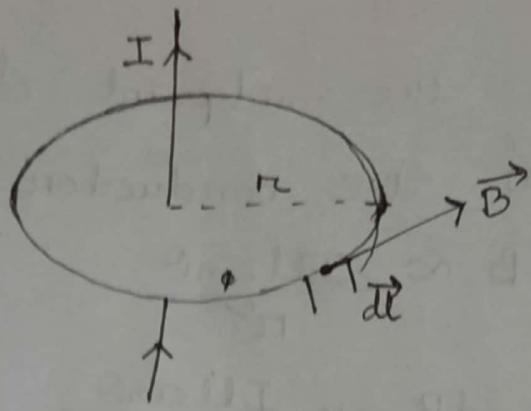
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Here,  $\mu_0$  = magnetic permeability of vacuum

$d\vec{l}$  = an infinitesimal vector element of the path.

$\oint$  = denotes integration along a closed path

Explanation:



Let  $I$  current is flowing through a conductor.

Let us consider a circular path centering the conductor. If the magnetic field is  $B$  at every points on the circumference of this circle then from experimental result it is obtained that,

$$B \propto \frac{I}{r}$$

$$\text{or, } B = \frac{\mu_0 I}{2\pi r}$$

Here,  $\frac{\mu_0}{2\pi}$  is the const. of proportionality.

$$\text{Therefore, } B 2\pi r = \mu_0 I$$

The left hand side can be written as  $\oint \vec{B} \cdot d\vec{l}$

$$\oint \vec{B} \cdot d\vec{l}$$

Here  $d\vec{l}$  is along the tangent of the circular path. The direction of  $\vec{B}$  and  $d\vec{l}$  are same.

$$\text{Therefore, } \oint \vec{B} \cdot d\vec{l} = B dl \cos 0^\circ = \oint B dl$$

$$= B \oint dl = B 2\pi r$$

So, the experimental relation between  $B$  and  $I$  is,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Avg/mean value of  $I$  or  $v$ :

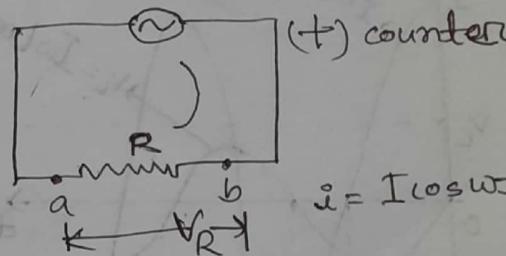
$$I_{av} = \frac{\int_0^T I_0 \sin \omega t dt}{\int_0^T dt} = -\frac{I_0}{T\omega} [\cos \omega t]_0^T = 0$$

$$\begin{aligned} I^2 &= \frac{\int_0^T I^2 dt}{\int_0^T dt} \\ &= \frac{I_0^2}{T} \int_0^T (\sin^2 \omega t) dt \\ &= \frac{I_0^2}{2T} \int_0^T (1 - \cos 2\omega t) dt \\ &= \frac{I_0^2}{2T} \left[ t - \frac{\sin 2\omega t}{2\omega} \right]_0^T \end{aligned}$$

$$= \frac{I_0^2}{2T} \left[ T - \frac{\sin \frac{2\pi}{T} \cdot T}{2\pi \cdot T} \right]$$

$$= \frac{I_0^2}{2T} \left[ T - \frac{1}{2} \right] = \frac{I_0^2}{2T} \cdot \frac{T}{2} = \frac{I_0^2}{4T} \cdot T = \frac{I_0^2}{4}$$

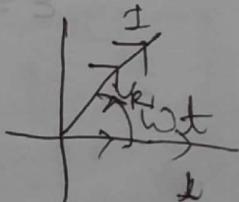
$$I_{avg} = \sqrt{\frac{I^2}{2}} = \frac{I_0}{\sqrt{2}}$$



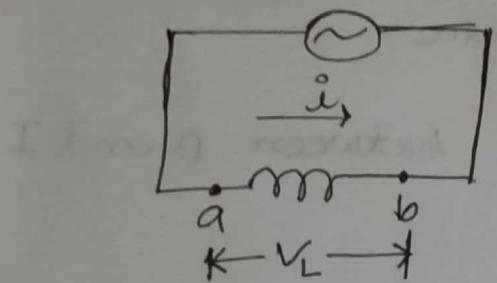
$$v_R = iR = (IR), \cos \omega t$$

$$V_R = IR$$

$$V_R = \text{AP} V_R \cos \omega t$$



Inductor in an AC circuit



$$i = I \cos \omega t$$

$$\epsilon = -L \frac{di}{dt}$$

$v_L = +L \frac{di}{dt}$ , the  $\epsilon$  is negative & the induced emf

So, we have,

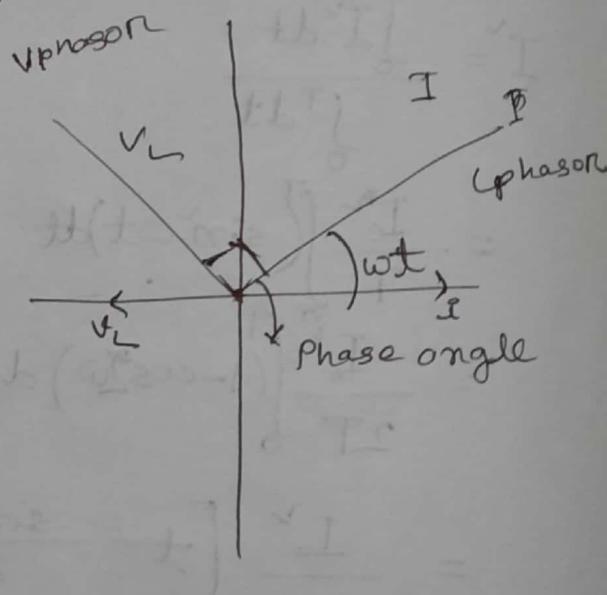
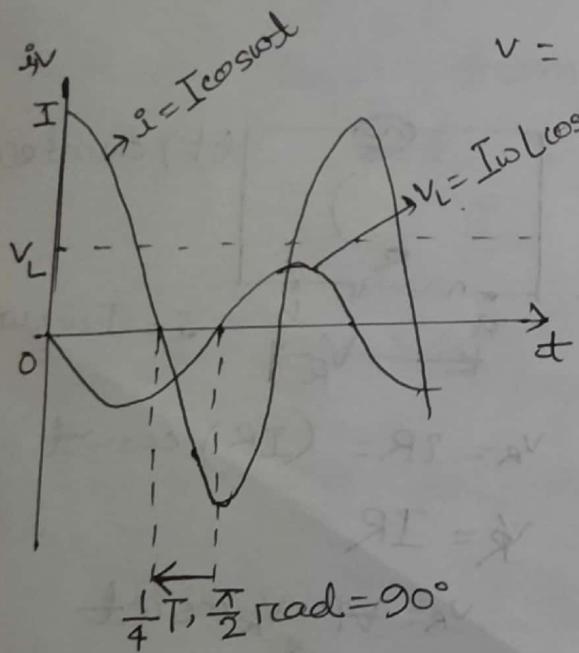
$$v_L = L \frac{di}{dt}$$

$$= L \frac{d}{dt} (I \cos \omega t)$$

$$= -I \omega L \sin \omega t \quad \dots \dots \dots (1)$$

$$v_L = I \omega L \cos(\omega t + 90^\circ)$$

$$v = V \cos(\omega t + \phi)$$

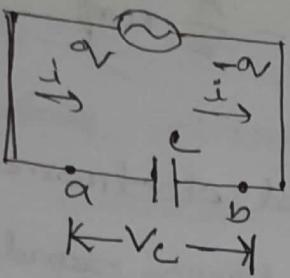


$$IR : v_L = I X_L$$

$$v_L = +L \frac{di}{dt}$$

$$\epsilon = -L \frac{di}{dt}$$

Capacitor in an AC ckt.

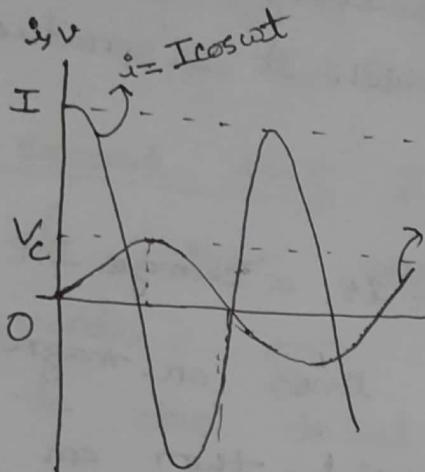


$$i = \frac{dv}{dt} = I \cos \omega t$$

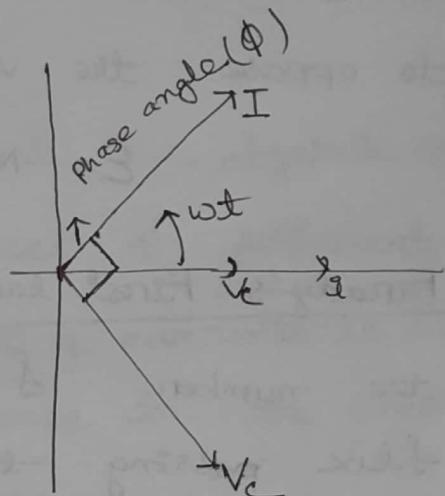
$$v = \frac{I}{\omega} \sin \omega t$$

$$v = C v_c$$

$$\therefore v_c = \frac{I}{C \omega} \sin \omega t \quad \dots \dots (1)$$



$$v_c = \frac{I}{\omega C} \sin \omega t = \frac{I}{\omega C} \cos(\omega t - 90^\circ)$$



$$3.13. \quad R = 200 \Omega$$

$$C = 5 \text{ mF} = 5.0 \times 10^{-6} \text{ F}$$

$$v_R = (1.20 \text{ V}) \cos(2500 \text{ rad/s}) t$$

$$(a) \quad v_R = \omega R \alpha; \alpha = \frac{v_R}{R} = \frac{(1.20 \text{ V}) \cos(2500 \text{ rad/s}) t}{200}$$

$$= 6.0 \times 10^{-3} \text{ A} \cos(2500 \text{ rad/s}) t$$

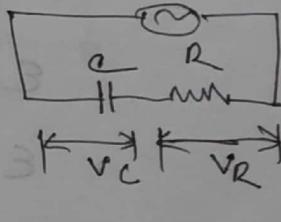
$$(b) \quad X_C = \frac{1}{\omega C} \Rightarrow \frac{1}{2500 \times 5 \times 10^{-6}} = 80 \Omega$$

$$(c) \quad v_c = I X_C \\ = 0.48 \text{ V}$$

$$\frac{X_L}{R} = 40\%$$

$$\therefore \frac{v_c}{v_R} = 40\%$$

$$\therefore v_c = v_R \times \frac{40}{100} = 0.48 \text{ V}$$



$$V_c = V_c \cos(\omega t - 90^\circ)$$

$$= (0.48v) \cos[(2500 \text{ rad/s}t - \frac{\pi}{2} \text{ rad})]$$

Lenz's law: The direction of induced electromotive force or current is such as always tends to oppose the very cause for which it is produced.

$$E = -N \frac{d\Phi}{dt}$$

Faraday's First law: whenever there is a change in the number of magnetic field lines or magnetic flux passing through a closed coil, then an electromotive force or electric current is induced in the coil. The induced electromotive force or electric current exists as long as the magnetic field lines or magnetic flux keeps changing.

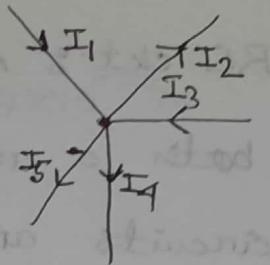
Faraday's second law: The magnitude of induced electromotive force in a closed coil is directly proportional to the negative of the rate of change of magnetic flux through the coil.

$$E \propto -\frac{\Phi_2 - \Phi_1}{t}$$

$$E = -K \frac{\Phi_2 - \Phi_1}{t}$$

## Kirchhoff's laws:

First law: In a junction of an electric circuit, the algebraic sum of current is zero.



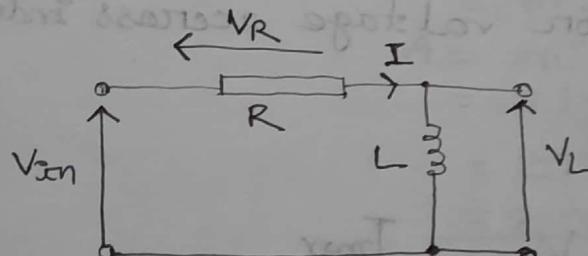
$$I_1 + I_3 - I_2 - I_4 - I_5 = 0$$

$$\Rightarrow \therefore I_1 + I_3 = I_2 + I_4 + I_5$$

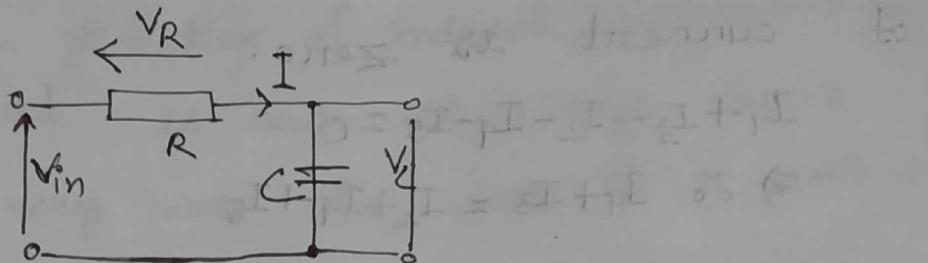
Second law: In a closed circuit, the algebraic sum of the product of resistances of different parts and their corresponding currents is equal to the total electromotive force in the circuit

$$\sum IR = \sum E$$

LR ckt: A LR series ckt consists basically of an inductor of inductance, L connected in series with a resistor of resistance, R. The resistance "R" is the DC resistive value of the wire turns or loops that goes into making up the inductor's coil.



**RC ckt:** An RC circuit is a circuit with both a resistor (R) and a capacitor (C). RC circuits are frequent element in electronic devices.



A  $48\ \Omega$  resistor is connected in series with an inductor of  $450\text{ mH}$  and a capacitor of  $9\text{ }\mu\text{F}$ .

$$\textcircled{1} \quad f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{450 \times 10^{-3} \times 9 \times 10^{-6}}} \text{ Hz}$$

$$= 79.08 \text{ Hz}$$

Here,  $L = 450\text{ mH}$   
 $= 450 \times 10^{-3}\text{ H}$   
 $C = 9\text{ }\mu\text{F}$   
 $= 9 \times 10^{-6}\text{ F}$

$$\textcircled{2} \quad I = \frac{E_0}{R} = \frac{120}{48} \text{ A}$$

$$= 2.5 \text{ A}$$

$E_0 = 120\text{ V}$   
 $R = 48\ \Omega$

$$\textcircled{3} \quad \text{For voltage across inductor } V_L = \omega LI$$

$$= 2\pi f LI$$

$$= 2\pi \times 79.08 \times 450 \times 10^{-3}$$

$$\times 2.5$$

$$= 558.98\text{ V}$$

$$V_C = \frac{I_{max}}{2\pi f C}$$

$$= \frac{79.08 \times 2.5}{2\pi \times 79.08 \times 9 \times 10^{-6}}$$

$$= 559.05\text{ V}$$

$$W_p = m_p g$$

$$= 1.6 \times 10^{-27} \times 9.8 = 1.637 \times 10^{-26} N$$

Now,  $F_p = W_p$

$$\Rightarrow \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{r^2} = 1.637 \times 10^{-26}$$

$$\Rightarrow \frac{q^2}{r^2} = \sqrt{(9 \times 10^9) (1.637 \times 10^{-26})} = \sqrt{\frac{1.637 \times 10^{-26}}{9 \times 10^9}}$$

$$\Rightarrow r = \frac{1.6 \times 10^{-19}}{4 \pi \times 10^{-8} \times 1.34 \times 10^{-18}}$$

$$\therefore r = 0.12 m$$

A path diff of 1.0 volts is applied to a 100ft length of a copper wire where diameter is 0.021 inch. calculate,

(a) The current

(b) Current density

(c) " electric field strength

(d) The rate of joule heating.

So, (a)  $R = \rho \frac{l}{A}$

$$= 1.72 \times 10^{-8} \frac{30.48}{8.17 \times 10^{-7}}$$

$$= 0.641 \Omega$$

$$V = IR$$

$$\therefore I = \frac{V}{R}$$

$$= \frac{1}{0.641} A$$

$$= 1.6 A$$

$$100 \text{ ft} = 30.48 \text{ m}$$

$$1 \text{ ft} = \frac{30.48}{100}$$

$$= 0.3048$$

$$1 \text{ inch} = 0.0254 \text{ m}$$

$$l = 100 \text{ ft}$$

$$= 30.48 \text{ m}$$

$$r = 0.02 \text{ inch}$$

$$= 5 \times 10^{-4} \text{ m}$$

$$A = \pi r^2$$

$$= \pi (5 \times 10^{-4})^2$$

$$= 2.47 \times 10^{-7} \text{ m}^2$$

$$= 8.17 \times 10^{-7} \text{ m}^2$$

$$\boxed{\rho = 1.72 \times 10^{-8} \Omega \cdot m}$$

$$(b) J = \frac{I}{A}$$

$$= \frac{1.6}{8.17 \times 10^{-7}}$$

$$= 1.96 \times 10^6 \text{ Am}^{-2}$$

$$(c) E = \frac{V}{l}$$

$$= \frac{1}{30.48} = 3.3 \times 10^{-2} \text{ N C}^{-1}$$

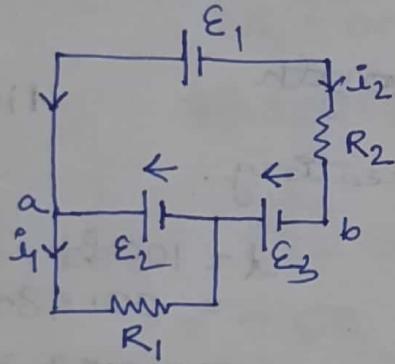
$$(d) \text{ Joule heating } P = \frac{V^2}{R(I)v}$$

$$= \frac{0.641}{0.641}$$

$$= 1.6 \text{ W}$$

KCL/KVL

- ① In the figure find the current in each resistor & potential diff betn a & b:



$$\begin{aligned} E_1 &= 6 \text{ V} \\ E_2 &= 5 \text{ V} \\ E_3 &= 4 \text{ V} \\ R_1 &= 100 \Omega \\ R_2 &= 50 \Omega \end{aligned}$$

Sol<sup>n</sup>: Applying loop rule to the lower loop

$$E_2 - i_1 R_1 = 0$$

$$\Rightarrow 5 - i_1 \times 100 = 0$$

$$\Rightarrow i_1 = 0.05 \text{ A}$$

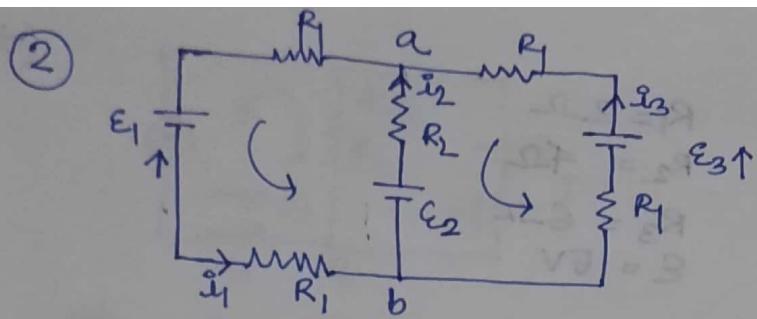
Upper loop,

$$-E_1 - i_2 R_2 + E_3 + E_2 = 0$$

$$\Rightarrow -6 - i_2 \times 50 + 4 + 5 = 0$$

$$\Rightarrow i_2 = \frac{-3}{50} = 0.06 \text{ A}$$

$$\begin{aligned} V_{ab} &= E_2 + E_3 \\ &= 5 + 4 \\ &= 9 \text{ V} \end{aligned}$$



$$R_1 = 1\Omega$$

$$R_2 = 2\Omega$$

$$E_1 = 2V$$

$$E_2 = E_3 = 4V$$

(i) Find the current in the ckt.

(ii) Find  $V_{ab}$

Sol<sup>n</sup> (i) Applying junction rule to at 'a'

$$i_2 + i_3 = i_1 \quad \text{--- (1)}$$

Applying loop rule to the left rule,

$$E_2 - i_2 R_2 - i_1 R_1 - E_1 - i_1 R_1 = 0$$

$$\Rightarrow 4 - 2i_2 - i_1 - 2 - i_1 = 0$$

$$\Rightarrow i_2 + i_1 = 1 \quad \text{--- (II)}$$

Right loop,

$$E_3 - i_3 R_3 + i_2 R_2 - i_3 R_1 - E_2 = 0$$

$$\Rightarrow 4 - i_3 + 2i_2 - i_3 - 4 = 0$$

$$\Rightarrow i_2 - i_3 = 0 \quad \text{--- (III)}$$

$$\begin{aligned} i_1 &= 1 - i_2 & i_2 &= i_3 \\ &\Rightarrow 2i_2 = i_1 && \\ &\Rightarrow i_2 = \frac{i_1}{2} && \end{aligned} \quad \left| \begin{array}{l} i_1 + 2i_1 = 2 \\ \Rightarrow 3i_1 = 2 \\ \Rightarrow i_1 = \frac{2}{3} = 0.67A \end{array} \right.$$

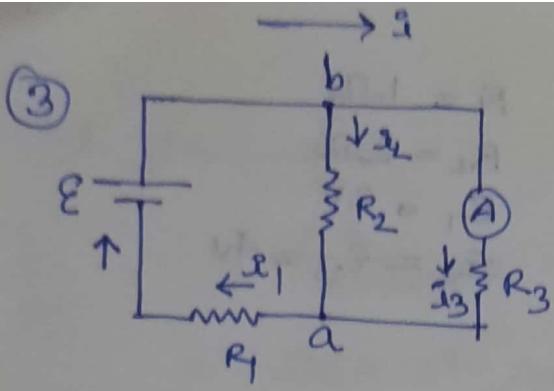
$$i_2 = 1 - \frac{2}{3} = \frac{1}{3} = 0.33A$$

$$i_3 = 0.33A$$

10(ii)  $V_{ab} = E_2 - i_2 R_2$

$$= 4 - \frac{1}{3} \times 2$$

$$= \frac{10}{3} = 3.33V$$



$$\begin{aligned}R_1 &= 2\Omega \\R_2 &= 4\Omega \\R_3 &= 6\Omega \\E &= 5V\end{aligned}$$

- (a) What will be the ammeter reads reading?  
 (b) The ammeter, the source of emf are now physically interchanged, show that the reading remain unchanged.

Sol<sup>n</sup>: (a) Applying junction rule at a,

$$i = i_3 + i_2 \quad \dots \quad (1)$$

loop rule to the left loop,

$$\begin{aligned}\varepsilon - i_2 R_2 - i_1 R_1 &= 0 \\ \Rightarrow 5 - 4i_2 - 2i_1 &= 0 \\ \Rightarrow 2(2i_2 - i_1) &= 5 \\ \Rightarrow 2i_2 - i_1 &= \frac{5}{2} \quad \dots \quad (1)\end{aligned}$$

$$\begin{aligned}5 - 4i_2 + 2i_1 &= 0 \\ \Rightarrow 2i_1 - 4i_2 &= -5 \\ \Rightarrow i_1 - 2i_2 &= -\frac{5}{2} \\ \Rightarrow 5 - 4i_2 - 2i_1 &= 0\end{aligned}$$

Right loop,

$$\begin{aligned}-i_3 R_3 + i_2 R_2 &= 0 \\ \Rightarrow -6i_3 + 4i_2 &= 0 \\ \Rightarrow i_3 &= \frac{2}{3}i_2\end{aligned}$$

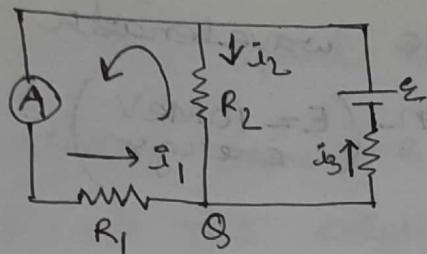
$$\begin{aligned}\Rightarrow 5 - 4i_2 - \frac{20i_2}{3} &= 0 \\ \Rightarrow 15 - 12i_2 - 20i_2 &= 0 \\ \Rightarrow -32i_2 &= -15 \\ \Rightarrow i_2 &= \frac{15}{32}\end{aligned}$$

$$\begin{aligned}i_1 &= \frac{2}{3}i_2 + i_2 \\ &= \frac{5i_2}{3}\end{aligned}$$

$$\begin{aligned}\Rightarrow 2i_2 - \frac{5i_2}{3} &= \frac{5}{2} \\ \Rightarrow i_2 &= \frac{15}{22} \\ i_3 &= \frac{15}{22} \times \frac{2}{3} = \frac{5}{11} \text{ A} \quad 0.457 \text{ A}\end{aligned}$$

$$i_1 = \frac{5}{11} + \frac{15}{22} = \frac{35}{22}$$

(b)



at junction 'a'

$$i_1 + i_2 = i_3 \quad (1)$$

Left loop:

$$-i_1 R_1 + i_2 R_2 = 0$$

$$\Rightarrow -2i_1 + 4i_2 = 0 \Rightarrow i_2 = \frac{1}{2}i_1 \quad (1')$$

Right loop:

$$E - i_2 R_2 - i_3 R_3 = 0$$

$$\Rightarrow 5 - 4i_2 - 6i_3 = 0$$

$$\Rightarrow i_1 + \frac{1}{2}i_1 = i_3$$

$$\Rightarrow \frac{3}{2}i_1 = i_3$$

$$5 - 4 \times \frac{1}{2}i_1 - 6 \times \frac{3}{2}i_1 = 0 \quad | \quad 5 - 2i_1 = 0 \Rightarrow i_1 = \frac{5}{11}$$

$$\Rightarrow 10 - 4i_1 - 18i_1 = 0$$

$$\Rightarrow 10 = 22i_1$$

$$\Rightarrow i_1 = \frac{10}{22} = \frac{5}{11}$$

$$\approx \frac{5}{11} = 0.45A$$

Q1 calculate the de broglie wavelength associated with proton ( $E = 10 \text{ MeV}$  energy)

$$\text{Soln: } E = 10 \text{ MeV} \\ = 10 \times 10^6 \text{ eV}$$

$$E = \frac{1}{2}mv^2 \\ \Rightarrow m^2v^2 = 2Em \\ \Rightarrow mv = \sqrt{2Em}$$

$$= \sqrt{2 \times 10 \times 10^6 \times 1.6 \times 10^{-19} \times 1.67 \times 10^{-27}} \\ = 1.79 \times 10^{-6} \times 7.31 \times 10^{20}$$

$$\therefore \lambda = \frac{6.63 \times 10^{-34}}{1.79 \times 10^{-6} \times 7.31 \times 10^{20}} \\ = 3.70 \times 10^{-28} \text{ m}$$

$$= 3.7 \times 10^{-18} \text{ A}^\circ$$

$$= 9.1 \times 10^{-15} \text{ m}$$

$$= 9.1 \times 10^{-5} \text{ A}^\circ$$

$$\text{Here, } E = 10 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$v = \frac{E}{h} = \frac{10 \times 10^6 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} \\ = 2.41 \times 10^{21} \text{ Hz}$$

$$\text{Now, } c = \frac{v\lambda}{\lambda} \\ \lambda = \frac{c}{v} \\ = \frac{2.41 \times 10^{21}}{3 \times 10^8} \text{ m} \\ = 8.03 \times 10^{-12} \text{ m}$$

$$\text{Now, } c = v\lambda \\ \Rightarrow \lambda = \frac{c}{v} \\ = \frac{3 \times 10^8}{2.41 \times 10^{21}} \\ = 1.24 \times 10^{-13} \text{ m} \\ = 1.24 \times 10^{-3} \text{ A}^\circ$$

Ques How far apart must two protons be if the electric repulsive force acting on either one is equal to its weight?

Soln: Hence,  $m_p = 1.67 \times 10^{-27} \text{ kg}$

$$\begin{aligned} W_p &= m_p g \\ &= 1.67 \times 10^{-27} \times 9.8 \text{ N} \\ &= 1.637 \times 10^{-26} \text{ N} \end{aligned}$$

Again,

$$\begin{aligned} F_e &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} \\ &= \frac{1}{4\pi\epsilon_0} \frac{(1.6 \times 10^{-19})^2}{r^2} \\ &= \frac{2.304 \times 10^{-28}}{r^2} \end{aligned}$$

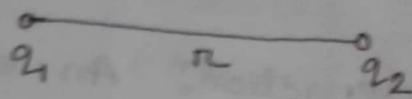
$$\begin{aligned} \text{Now, } F_e &= W_p \\ \Rightarrow \frac{2.304 \times 10^{-28}}{r^2} &= 1.637 \times 10^{-26} \end{aligned}$$

$$\Rightarrow r^2 = 0.014 \text{ m}$$

$$\therefore r = 0.14 \text{ m} \quad 0.12 \text{ m}.$$

Coulomb's law: 1785

1736-1806



$$F \propto \frac{q_1 q_2}{r^2}$$

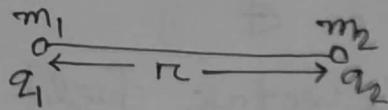
Charges should be stationary.

Medium  $\rightarrow$  isotropic & homogeneous

Coulomb's law of electrostatic forces states that the force between two point charges at rest is directly proportional to the product of the magnitude of the charges and is inversely proportional to the distance between them.

Coulomb's law = inverse square law

- \* Newton's inverse square law is always attractive
- \* Coulomb's " " " can be attractive or repulsive.



$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = k \frac{q_1 q_2}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad [\text{in vacuum}]$$

$\epsilon_0 = 8.854 \times 10^{-12} \text{ coul}^2/\text{nd-mt}$

$\Leftrightarrow [E = \text{permittivity}]$

$$q = it$$

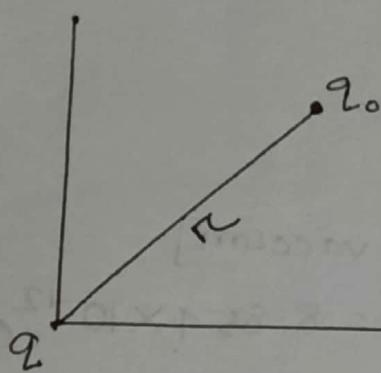
**Q1** The distance  $r$  between  $e^-$  and the  $p^+$  in the  $H$  atom is about  $5.3 \times 10^{-11} \text{ m}$ . What are the magnitude of electrical forces and gravitational forces.

**Q2** How far apart must two protons be if the electrical repulsive force acting on either one is equal to its weight?

Scope of Coulomb's law:

Limitations of Coulomb's law: Can't describe the stability of nucleus.

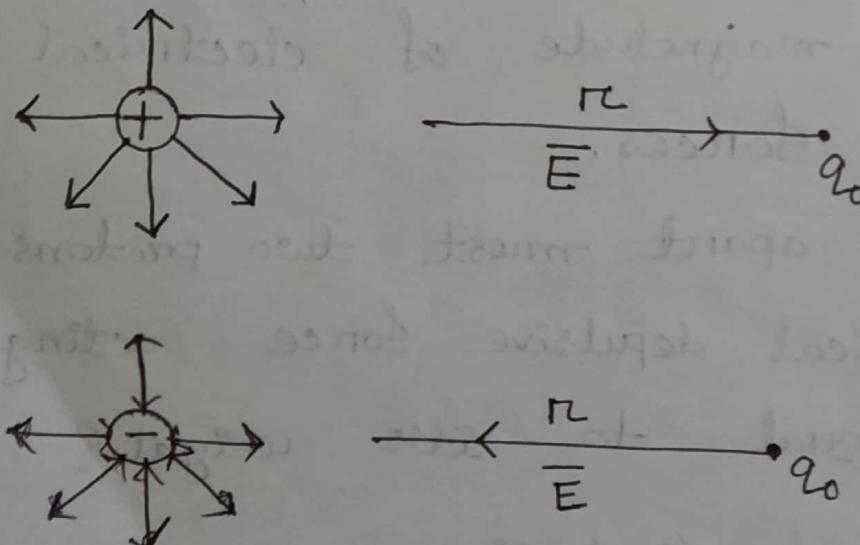
Calculate the force & Electric field  $\vec{E}$  (Electric field intensity)  
 (" " " strength)



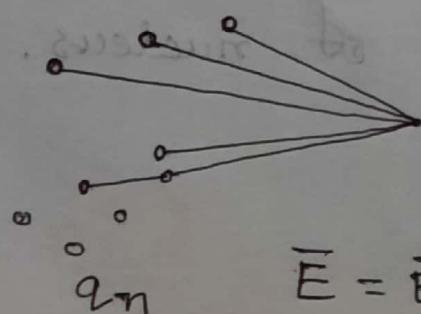
$$\vec{E} = \frac{\vec{F}}{q_0}$$

What is  $\vec{E}$  30 cm from a charge  $q = 4 \times 10^{-9} C$ ?

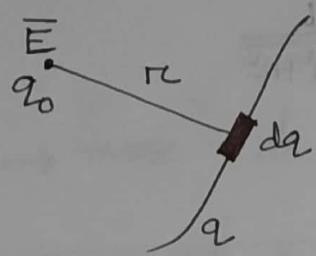
Calculation of  $\vec{E}$ .



For a group of point charges,



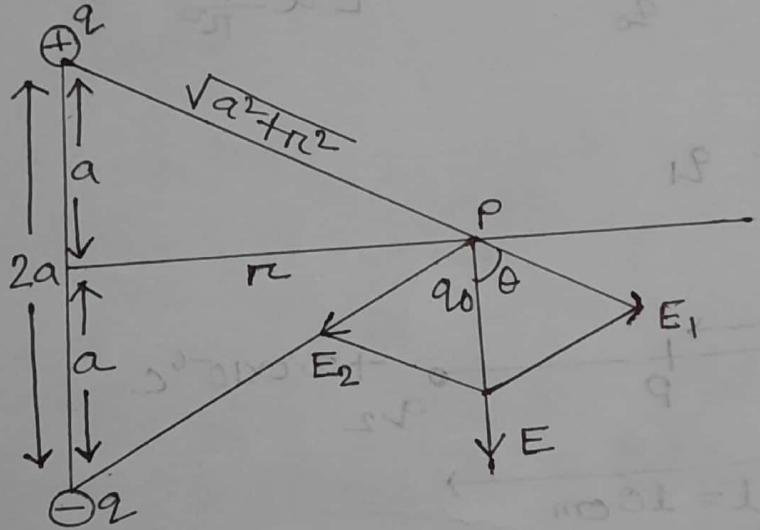
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots + \vec{E}_n$$



$$dE = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2}$$

$$E = \int dE$$

**Electric dipole:** A configuration of two charges of equal magnitude & of opposite charge.



$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{a^2+r^2}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$E = 2E_1 \cos\theta$$

$$\cos\theta = \frac{a}{\sqrt{a^2+r^2}}$$

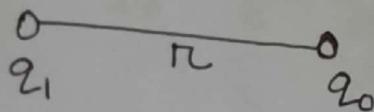
$$E = 2 \frac{1}{4\pi\epsilon_0} \frac{q}{a^2 + r^2} \frac{q}{\sqrt{a^2 + r^2}}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{2aq}{(a^2 + r^2)^{3/2}}$$

$$E \approx \frac{1}{4\pi\epsilon_0} \cdot \frac{2aq}{r^3} \quad \text{if } r \gg a; (2a)q \rightarrow \text{charges distribution}$$

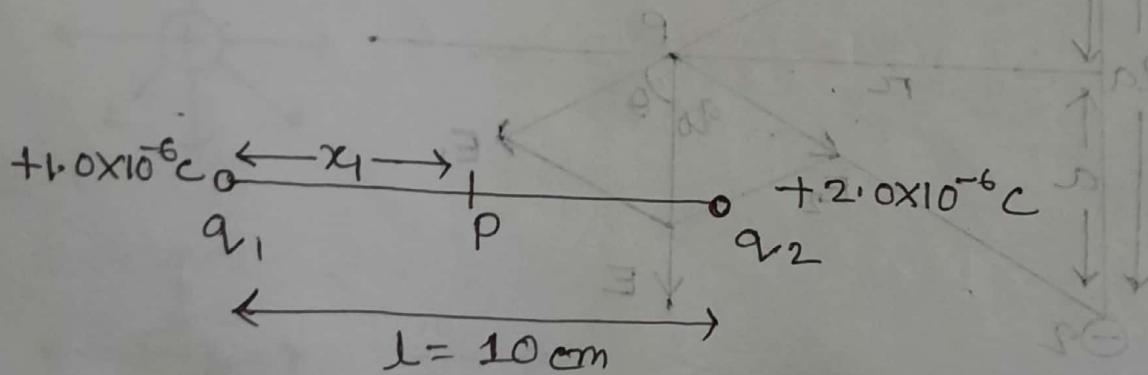
$2aq \rightarrow$  electric dipole moment

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}, \quad E \propto \frac{1}{r^3}$$



$$E \propto \frac{1}{r^2}$$

Given a charge  $q_1$



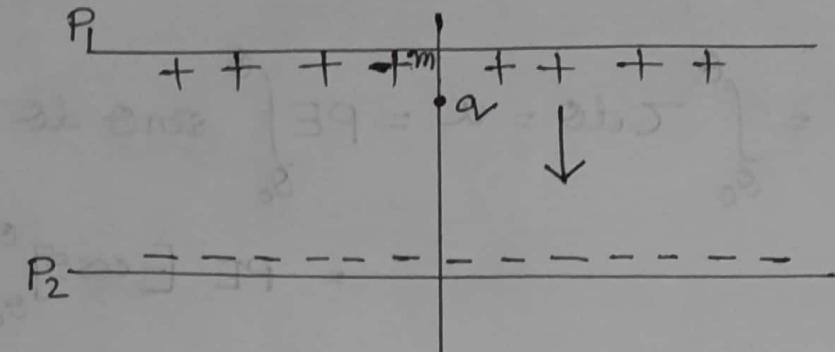
$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{x^2} \quad \frac{p}{r^3} \parallel \vec{E} = \vec{E}$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{(L-x)^2} \quad \frac{p}{r^3} \parallel \vec{E} = \vec{E}$$

$F_C \gg F_g$   
 $10^{39}$  times.

$$\bar{F} = \bar{E}q$$

$$\bar{a} = \frac{\bar{F}}{m} \rightarrow \text{mass of the charge particle}$$



$$V_0 = 0$$

$$v = at = \frac{qEt}{m}$$

$$y = \frac{1}{2}at^2$$

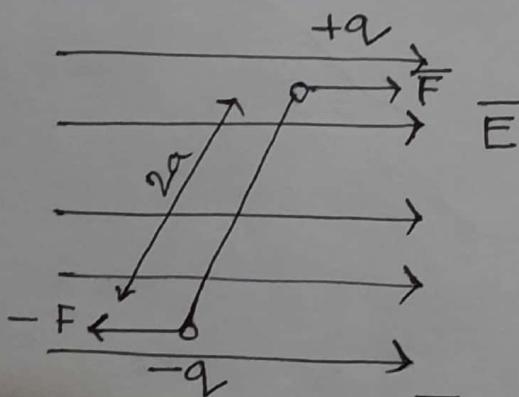
$$= \frac{qEt^2}{2m}$$

$$K.E. = \frac{1}{2}mv^2$$

$$v^2 = a^2t^2 = 2(a \cdot \frac{1}{2}at^2)$$

$$K.E. = \frac{1}{2}m \cdot \frac{2qEy}{m} = (2E)y$$

A dipole in an electric field:



$$\begin{aligned} F &= qE \\ \text{net } F &= 0 \\ \text{net torque } \tau &= 0 \end{aligned}$$

$$|P| = 2qa$$

$\hookrightarrow$  to (+)ve

$$\bar{\tau} = \bar{r} \times \bar{F}$$

$$= 2qaF \sin\theta$$

$$= 2aqE \sin\theta$$

$$\bar{T} = PE \sin \theta$$

$$\bar{T} = \bar{p} \times \bar{E}$$

$$p = 2aq$$

$$P.E = u$$

$$W = \int du = \int_{\theta_0}^{\theta} T d\theta = u = PE \int_{\theta_0}^{\theta} \sin \theta d\theta \\ = PE [E \cos \theta]_{\theta_0}^{\theta}$$

$$\theta_0 = 90^\circ$$

$$u = -PE \cos \theta + 0 \\ = -PE \cos \theta \\ = -\bar{p} \cdot \bar{E}$$

Q1 An electric dipole consists of two opposite charges of magnitude  $q = 1 \times 10^{-6} C$  separated by  $d = 2 \text{ cm}$ . Dipole is placed in an electrical field  $\bar{E} = 1 \times 10^5 \text{ N/C}$ .  $T_{\max} = ?$

Sol<sup>n</sup>:  $T = T_{\max}$

When  $\theta = 90^\circ$

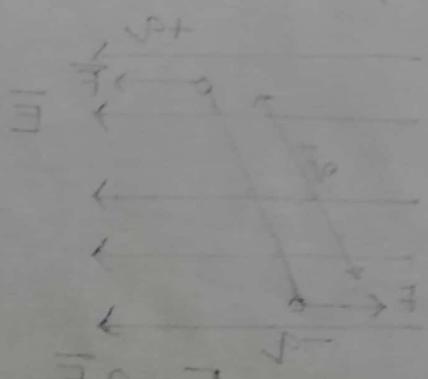
$$T = PE$$

$$qv = |\bar{q}| \\ 3v(1) \text{ or } E = 2 \times 10^3 \text{ N/C}$$

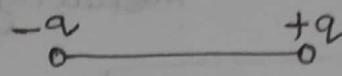
$$\sqrt{3} \times \pi = 5$$

$$3 \times \pi \times 10^{-6} =$$

$$3 \times 10^{-6} \text{ J}$$



How much work must an external agent do to turn the dipole  $n$  for  $n$  starting from a position alignment.



$$\tau = \int_{\theta_0=0}^{\theta=180} \omega = 2PE = 4 \times 10^{-3} N$$



$$\left\{ \begin{array}{l} \mu_0 = \frac{1}{4\pi} \cdot \vec{B} \\ \frac{1}{4\pi} = \vec{B} \end{array} \right.$$

$$\mu_0 \cdot 3 = \vec{B}$$

$$\mu_0 \cdot I = \vec{B}$$

lets write to  $\vec{B}$  constant

$$(\vec{B} \cdot \vec{B}) = 1 = \vec{B}$$

$$\left( \frac{1}{36\pi} + 3^2 \right) I^2 R = 1$$

$$\frac{1}{36\pi} = \vec{B}$$

$$\left( \frac{1}{36\pi} + 3^2 \right) I^2 R = 1$$

$$\frac{1}{36\pi} = \vec{B}$$

$$\left( \frac{1}{36\pi} + 3^2 \right)$$

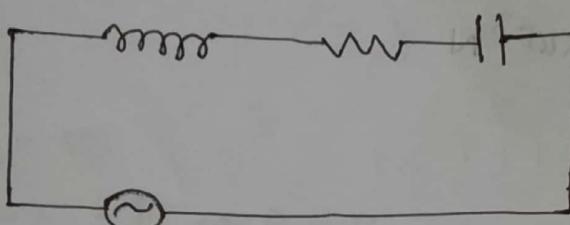
one part

then from  $R$

## Resistance & Reactance impedance

↓  
Physical significance  
meaning of  $X_L, X_C$

LCR ckt



$$V_R = IR$$

$$V_L = IX_L$$

$$V_C = IX_C$$

$$\mathcal{E} = \mathcal{E}_0 e^{j\omega t}$$

$$I = I_0 e^{j\omega t}$$

$$\left. \begin{aligned} X_L &= \omega L \\ X_C &= \frac{1}{\omega C} \end{aligned} \right\} \text{Imaginary part}$$

Impedance  $Z$  of the ckt

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = j\omega C$$

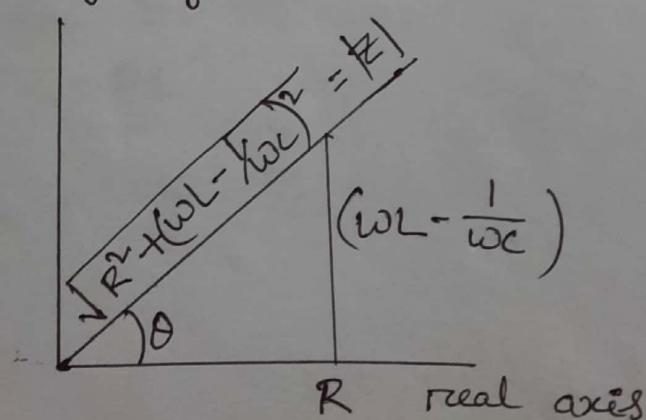
$$X_C = \frac{1}{j\omega C}$$

$$Z = R X_j \left( \omega L + \frac{1}{j^2 \omega C} \right)$$

$$= R X_j \left( \omega L - \frac{1}{\omega C} \right)$$

Imaginary axis

Imaginary axis



$$\begin{aligned}
 \text{Given } I &= \frac{\epsilon}{Z} \\
 \Rightarrow I &= \frac{\epsilon}{Z} \\
 &= \frac{\epsilon_0 e^{j\omega t}}{R + j(\omega t - 1/\omega c)} \\
 &= \frac{1}{R + j(\omega t - 1/\omega c)} \\
 &= \frac{R - j(\omega t - 1/\omega c)}{[R + j(\omega t - 1/\omega c)][R - j(\omega t - 1/\omega c)]} \\
 &= \frac{R - j(\omega t - 1/\omega c)}{R^2 + j(\omega t - 1/\omega c)^2} = \alpha - j\beta
 \end{aligned}$$

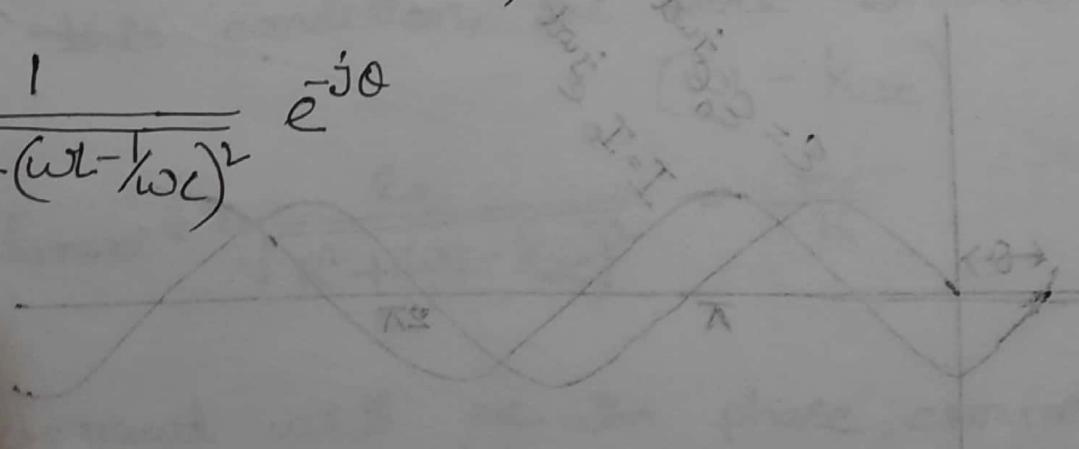
when,  $\alpha = \frac{R}{R^2 + (\omega L - 1/\omega c)^2}$

$$\beta = \frac{\omega L - 1/\omega c}{R^2 + (\omega L - 1/\omega c)^2}$$

$$\tan \theta = \frac{X}{R} = \frac{\omega L - 1/\omega c}{R}$$

$$\sin \theta = \frac{\omega L - 1/\omega c}{\sqrt{R^2 + (\omega L - 1/\omega c)^2}}; \quad \cos \theta = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega c)^2}}$$

$$\frac{1}{\sqrt{R^2 + (\omega L - 1/\omega c)^2}} e^{-j\theta}$$



$\sin \omega t$

Expression current,  $I = \frac{e_0 e^{j\omega t}}{R + j(\omega L - \frac{1}{\omega C})}$

$$= e_0 e^{j\omega t} \frac{e^{-j\theta}}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \cdot e^{-j(\omega t - \theta)}$$

$$= \frac{e_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \cdot e^{-j(\omega t - \theta)} \quad \text{--- (II)}$$

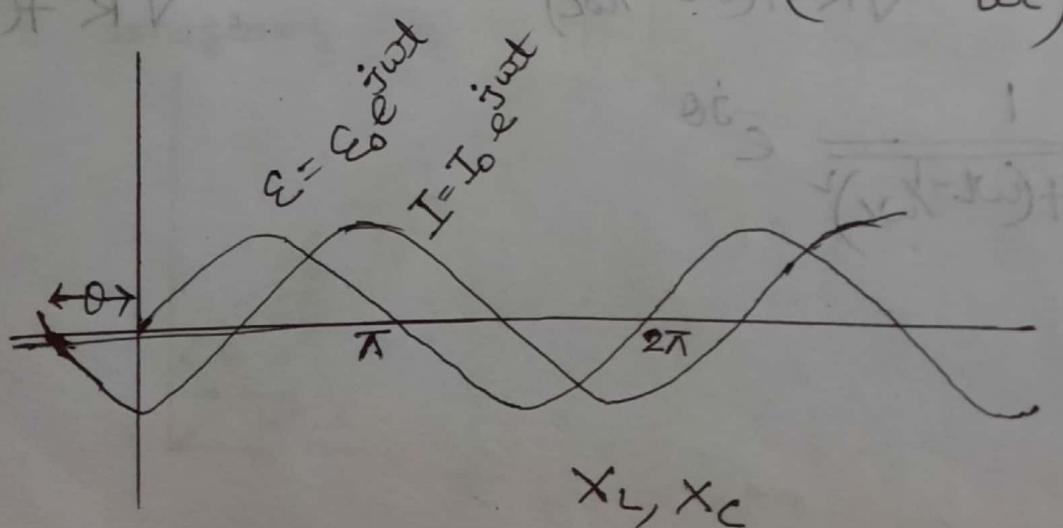
$$= I_0 e^{j(\omega t - \theta)} \quad \text{--- (III)}$$

$$I_0 = \frac{e_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \quad \text{--- (IV)}$$

From (V) impedance of  $Z$  of the ckt,

$$\frac{e}{I_0} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

eqn (III) current lags behind the applied voltage by an angle  $\theta$ , given by

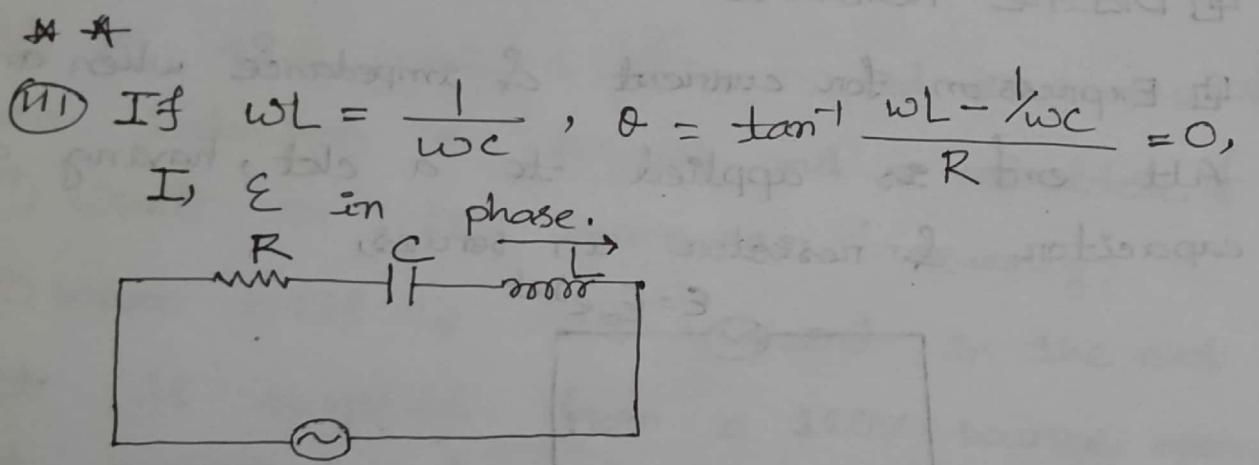


- ① If  $\omega L > \frac{1}{\omega C}$  when,  $f = \frac{\omega}{2\pi}$  is very large then  $\theta$  will be positive, current will lag behind the applied emf.

$V_L = I_0 X_L$  = Potential diff across across the inductor is greater than  $V_C = I_0 / \omega C$

$V_L \gg V_C$ , the ckt behaves as inductive ckt.

- ② If  $\omega L < \frac{1}{\omega C}$ ,  $\theta$  will be  $\pi$  negative, current will lead the emf. Ckt will behave as capacitive ckt.



Under this condition, the peak current will be max  
 $(\omega L - \frac{1}{\omega C})$

$$I_{max} = \frac{E_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{E_0}{R}$$

emf, current will be in phase, current is max

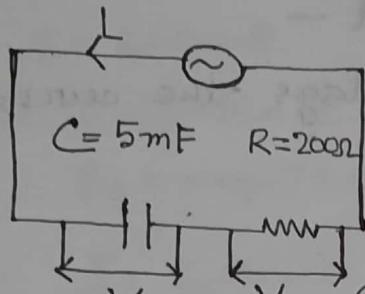
[Series Resonance ckt]

Q a. Find the current through the resistor.

Q b. " " capacitive reactance of the ckt.

Q c. " " voltage amplitude to write an expression instantaneous voltage across the capacitor.

Soln:



$$V_R = (1.20 \text{ V}) \cos(2500 \text{ rad/s})t$$

$$= V_R \cos \omega t.$$

(a)  $V_R = iR$

$$\therefore i = \frac{V_R}{R}$$

$$= \frac{1.20 \text{ V} \cos(2500 \text{ rad/s})t}{200 \Omega}$$

$$= 6 \times 10^{-3} \text{ amp } \cos(2500 \text{ rad/s})t$$

$i = I_0 \cos \omega t$

instantaneous  
current

(b)  $X_C = \frac{1}{\omega C}$

$$= 80 \Omega$$

$$\textcircled{C} \quad v_c = V_c \cos(\omega t + \phi) \quad | \quad v_c = V_c \cos(\omega t + \phi)$$

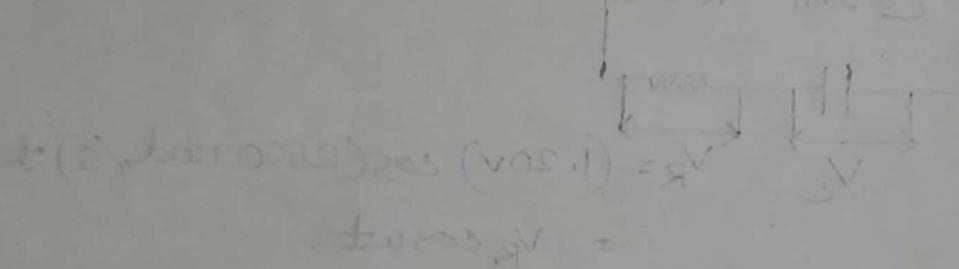
$$V_c = I X_c \quad | \quad \Rightarrow v_c = (0.48V) \cos[2500$$

$$= 6 \times 10^{-3} \text{ amp} \times 8 \Omega \quad |$$

$$= 0.48V \quad |$$

\* ac capacitive ckt -

Voltage lags the current by  $90^\circ$ .



$$I = 9V \quad \textcircled{1}$$

$$\frac{9V}{Z} = I \cdot C$$

$$I = 9V \cos(90^\circ) = 0$$

$$V = 9V \sin(90^\circ) = 9V$$

$$I = \omega C V = I$$

$$\omega C = \frac{1}{Z}$$

$$\frac{1}{9\Omega} = X \quad \textcircled{2}$$

Alternating current: An alternating current is an periodic function of time. It passes through a cycle of changes at regular intervals. Each cycle consists of 2 half cycles. The current and voltage state of an A.C ckt oscillates sinusoidally without change in amplitude.

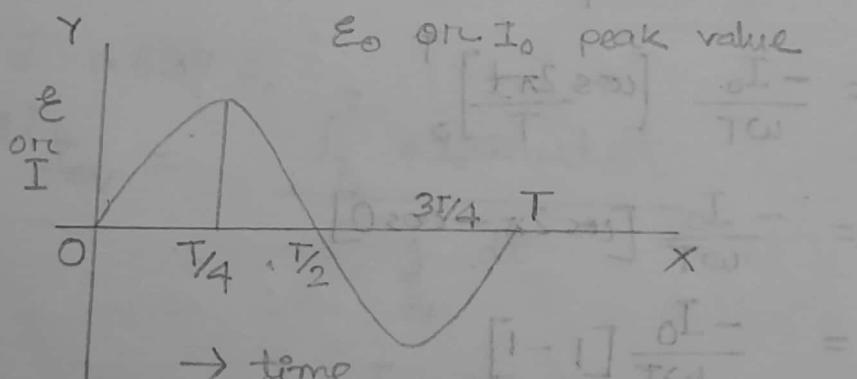
$$e = E_0 \sin \omega t$$

where  $e$  = voltage at any time  $t$

$E_0$  = amplitude or peak value of voltage

$$I = I_0 \sin \omega t$$

- \* The emf and  $I$  first rise to max in one direction and fall to zero. Then the direction quickly reverse and emf and  $I$  rise to max in opposite direction and again fall to zero.



Average / mean value of  $I$  or  $v$ : Mean value of AC is that value of steady current which sends the same amount of charge through a ckt in a certain time interval as is sent by an AC through the same ckt in the same time interval.

The value of current at any instant  $t$  is given by,

$$I = I_0 \sin \omega t$$

The avg value over one cycle is given by,

$$I_{av} = \frac{\int_0^T I_0 \sin \omega t \, dt}{\int_0^T dt}$$

$$= \frac{I_0}{\omega} \left[ \cos \omega t \right]_0^T$$

$$= -\frac{I_0}{\omega T} \left[ \cos \frac{2\pi t}{T} \right]_0^T$$

$$= -\frac{I_0}{\omega T} [\cos 2\pi - \cos 0]$$

$$= -\frac{I_0}{\omega T} [1 - 1]$$

$$= 0$$

During half cycle:

$$\text{(Pos)} \quad I_{av} = \frac{\int_0^{T/2} I_0 \sin \omega t \, dt}{\int_0^{T/2} dt}$$
$$= \frac{-I_0}{\omega \frac{T}{2}} [\cos \omega t]_0^{T/2}$$
$$= \frac{-I_0}{\frac{2\pi}{T} \cdot \frac{T}{2}} \left[ \cos \frac{2\pi}{T} t \right]_0^{T/2}$$
$$= \frac{-I_0}{\pi} [\cos \pi - \cos 0]_0^{T/2}$$
$$= \frac{2}{\pi} I_0$$
$$= 0.637 I_0$$

So, the avg over one half cycle is finite and  $0.637 I_0$

$$E_{av} = \frac{2A}{\pi} \frac{2}{\pi} E_0$$

$$= 0.637 E_0$$

$$\rightarrow \text{(Neg)} \quad I_{av} = \frac{\int_{T/2}^T I_0 \sin \omega t \, dt}{\int_{T/2}^T dt}$$

$$= -\frac{I_0}{\pi} [\cos 2\pi - \cos \pi]$$

$$= -\frac{I_0}{\pi} \times 2$$

$$= -\frac{2I_0}{\pi}$$

$$= -0.637 I_0$$

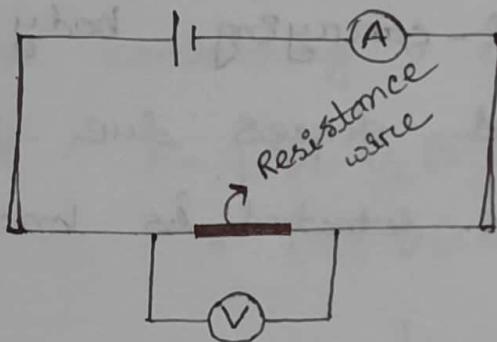
KAR(b)

□ Ohm's law: The ratio of the potential difference ( $V$ ) between any two points on a conductor to the current ( $I$ ) flowing between them, is constant provided the temperature of the conductor does not change.

In other words,  $\frac{V}{I} = \text{constant}$

$$\text{or, } \frac{V}{I} = R$$

where  $R$  is the resistance of the conductor between the two points considered.



Gauss's law: The total of the electric flux out of a closed surface is equal to the charge enclosed by divided by the permittivity.

$$\Phi_E = \frac{q}{\epsilon_0}$$

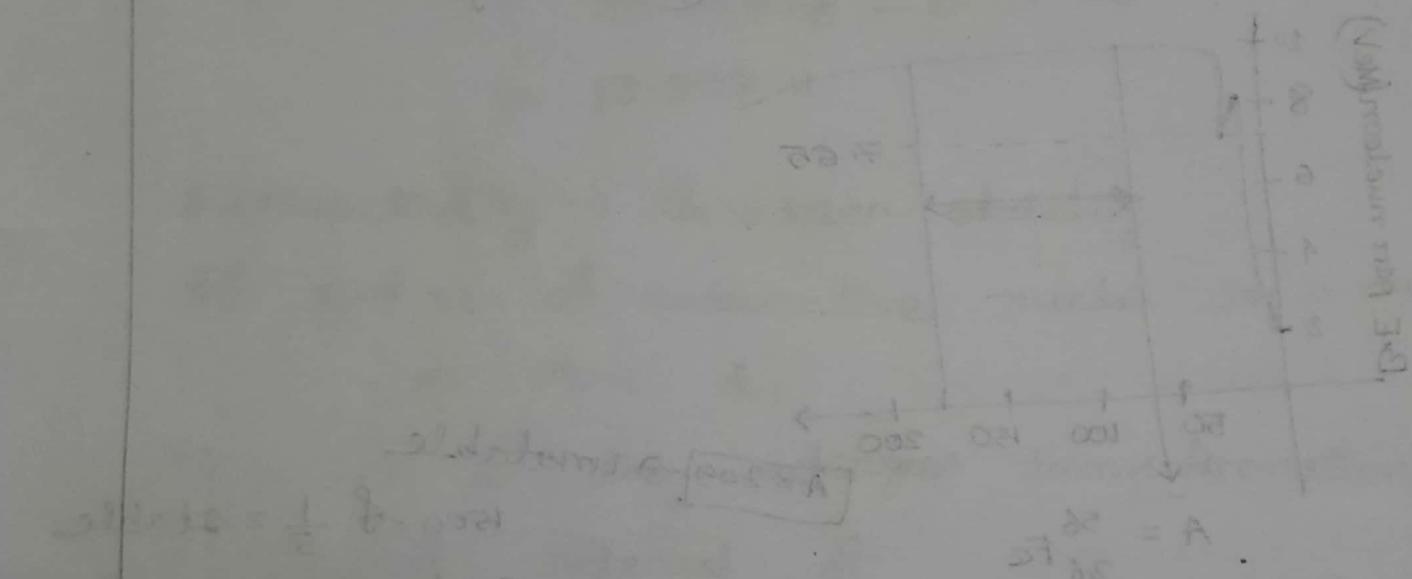
$$\Rightarrow \epsilon_0 \Phi_E = q$$

$$\therefore \epsilon_0 \oint \Phi$$

$$\therefore \epsilon_0 \oint \vec{E} \cdot d\vec{s} = q$$

$$v_c = V_c \cos(\omega t - 90^\circ)$$

$$= (0.48V) \cos[(2500 \text{ rad/s})t - \frac{\pi}{2} \text{ rad}]$$



$$\text{Half-time} = \frac{1}{2} \text{ of } 0.5$$

$$\frac{1}{2} \cdot 0.5 = 0.25$$

$$n = 10 \times 0.25 = 4 \times 0.5 = 5$$

and the current =  $\delta$

$$(x \frac{A}{5} \text{ sec} - 0.5 + 0.5) \text{ sec}$$

$$= 0.5 \rightarrow V_{\text{cell}} = 0.5 \times 0.5 = 0.25$$

## Capacitive reactance

$$X_C = \frac{1}{\omega C}$$

$\rightarrow$  C.R. is inversely proportional to  $\omega C$

$\rightarrow$  The greater the C the higher the if the smaller the  $X_C$ .

Cap. react. is the reactance due to the presence of capacitance in an AC ckt. It's the opposition of the capacitance to the AC. The opposition of current through an AC capacitor is called capacitive reactance.

Math: 31.3:  $R = 200\Omega$

$$C = \sqrt{5} \mu F = 5.0 \times 10^{-6} F$$

$$V_R = (1.20V) \cos(2500 \text{ rad/s}) +$$

(a) derive expression for ckt current.

$$(b) X_C = ?$$

(c) derive expression for voltage.

Soln: (a)  $V_R = iR ; i = \frac{V_R}{R} = \frac{(1.20V) \cos(2500 \text{ rad/s})}{200} = 6.0 \times 10^{-3} A$

$$= (6.0 \times 10^{-3} A) \cos(2500 \text{ rad/s}) +$$

$$(b) \omega = 2500 \text{ rad/s}$$

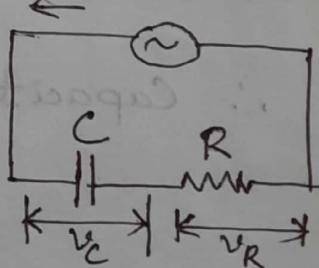
$$X_C = \frac{1}{\omega C} = 80\Omega$$

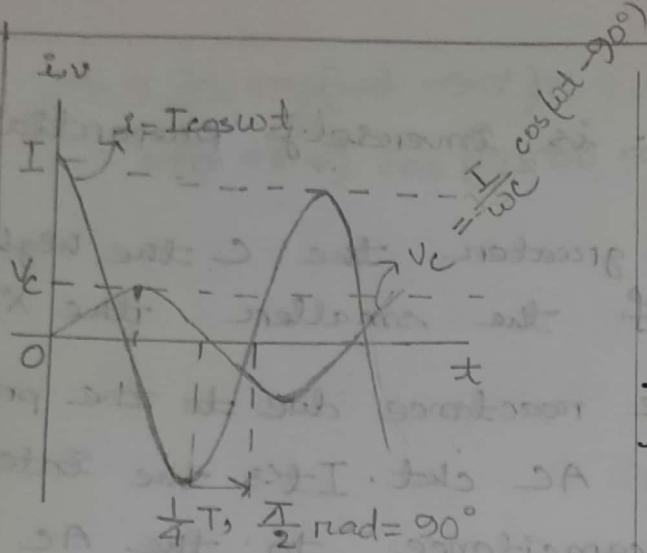
$$(c) V_C = IX_C = (6.0 \times 10^{-3} A) 80\Omega$$

$$= 0.48V$$

$$\frac{X_L}{R} = 40\% \quad \therefore \frac{V_C}{V_R} = 40\%$$

$$\therefore V_C = V_R \times \frac{40}{100} = 0.48V$$

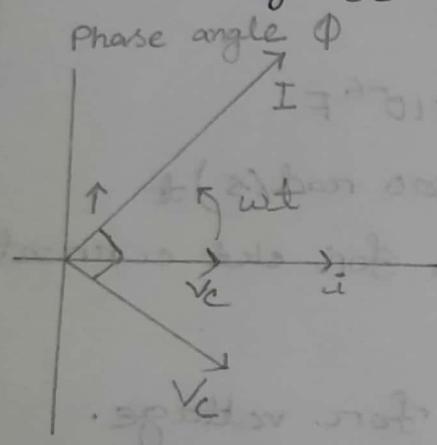




The figure shows  $V_c$  and  $i$  as function of  $t$ . The current has its greatest magnitude when the  $V_c$  curve is rising or falling most steeply.

→ It's zero when  $V_c$  curve instantaneously levels off at its max or minimum value.

Voltage curve lags currents, curve by  $90^\circ$



The phasor diagram shows phase relationship between  $V_c$  and  $i$ .

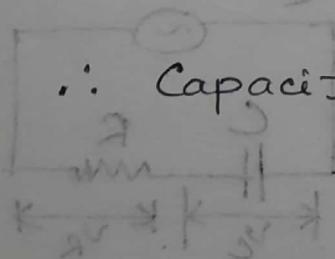
→ The peaks of  $V_c$  occur a quarter cycle after the corresponding current peaks.

$V_c$  lags  $I$  by  $90^\circ$ .

Voltage phasor lags current phasor by  $-90^\circ$

$$\therefore V_c = \frac{1}{\omega C} \cos(\omega t - 90^\circ)$$

eqtn (1) shows that  $V_c$  is  $\rightarrow V_c = \frac{I}{\omega C}$



∴ Capacitive reactance,

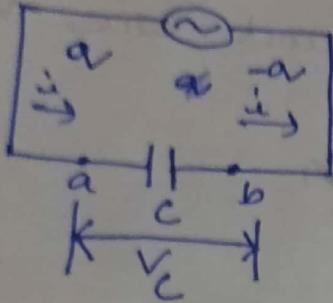
$$X_C = \frac{1}{\omega C} \quad \text{--- (3)}$$

$$\Rightarrow V_c = IX_C$$

$$\therefore X_C = \frac{V}{I}$$

$$X_C = \frac{V}{I} = \frac{V}{I}$$

Capacitor in an AC ckt



In the figure above a capacitor with capacitance  $C$ , is connected to the source of the ckt. The positive direction of  $i$  is counter clockwise around the ckt.

$V_c$  is the instantaneous voltage on the potential of point 'a' with respect to point 'b'. The charge on the left hand plate is denoted by  $q$ .

$$i = \frac{dq}{dt} = I \cos \omega t$$

Integrating this,

$$q = \frac{I}{\omega} \sin \omega t$$

$$\text{again, } q = C V_c$$

$$\therefore V_c = \frac{I}{C \omega} \sin \omega t \quad (1)$$

→  $i$  is equal to the rate of change  $\frac{dq}{dt}$  of the capacitor charge  $q$ .

→  $i$  is proportional to the rate of change of voltage.

→ Since  $X_L \propto f$ , high frequency  
V apply = small current and vice versa.

Math (31\*2) \*  $I = 250 \text{ mA} = 250 \times 10^{-3} \text{ A}$

$$* V_L = 3.60 \text{ V}$$

$$* f = (1.6 \times 10^6) \text{ Hz}$$

(a)  $X_L = ?$  (Inductive Reactance)

(b)  $X_L = ?$  and  $I = ?$  at any frequency.

Soln: (a)  $X_L = \frac{V_L}{I} = 1.44 \times 10^4 \Omega = 14.4 \text{ k}\Omega$

$$X_L = \omega L \therefore L = \frac{X_L}{\omega} = \frac{X_L}{2\pi f} = 1.43 \times 10^{-3} \text{ H}$$

(b)  $X_L = \omega L$

$$V_L = I X_L$$

$$I = \frac{V_L}{X_L}$$

$$= \frac{V_L}{\omega L}$$

$$= \frac{V_L}{2\pi f L}$$

Thus  $I$  is inversely proportional to  $f$  since  $I = 250 \text{ mA}$  at  $f = 1.60 \text{ MHz}$

→  $I$  at  $16 \text{ MHz}$  or at  $10f$  will be one tenth as great which is  $25 \text{ mA}$ .

→  $I$  at  $0.16 \text{ MHz} (\frac{f}{10})$  will be ten times as great which is  $2500 \text{ mA}$ .

$i$  = ins. current

$V_c$  = ins. voltage

$V_c$  = amplitude voltage

$\Phi$  = Phase angle = Phase of  $v$  relative to  $i$   
for a pure resistor  $\Phi = 0$ , pure L,  $\Phi = 90^\circ$

The amplitude  $V_L$  of the inductor voltage is

$$V_L = I \omega L$$

The inductive reactance  $X_L$  of an inductor is defined as

$$X_L = \omega L$$

### Inductive Reactance:

The I.R.  $X_L$  is the description of the self induced emf that oppose any change in the current through the inductor.

We know, amplitude of voltage across an inductor,  $V_L = IX_L$

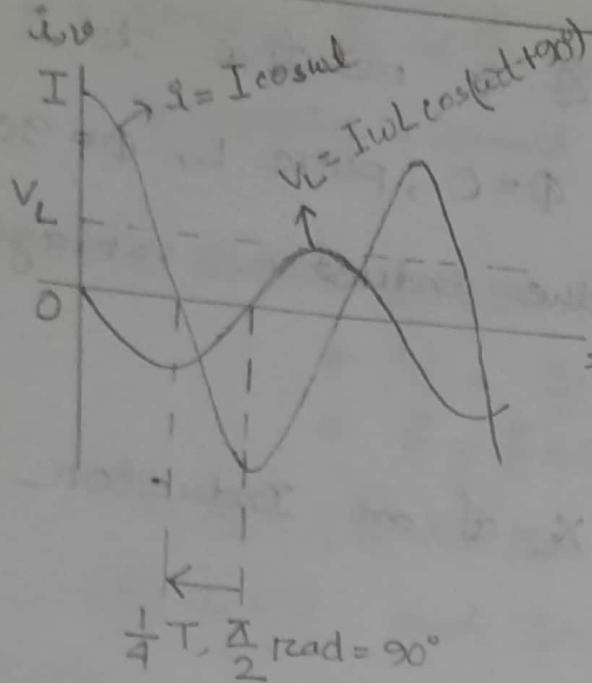
For a given  $I$ ,  $V_L = +L \frac{di}{dt}$

and the self-induced emf

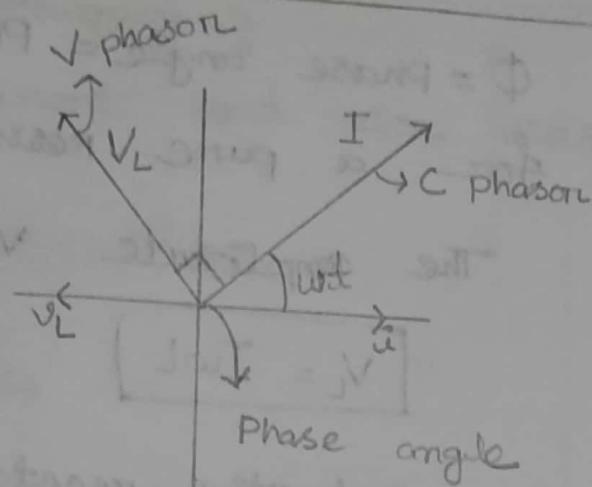
$$E = -L \frac{di}{dt} \quad (1)$$

both have an amplitude  $M$  that is directly proportional to  $X_L$ .

When an oscillating voltage of  $V_L$  amplitude is applied in an inductor  $\rightarrow$   
 $\rightarrow$  resulting current have smaller amplitude  $I$  for larger value of  $X_L$



(b) graph



(c) phasor diagram

The graph and phasor diagram above shows the relationship between  $i$  and  $v$ . They are out of step or out of phase by a quarter cycle.

The voltage leads the current by  $90^\circ$ . In the diagram  $V_L$  phasor is ahead of the current phasor by  $90^\circ$ .

Obtaining this phase relationship by rewriting eqtn (1)

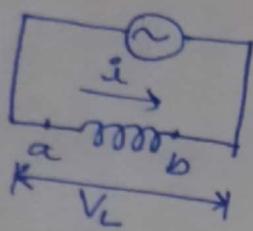
$$V_L = I w L \cos(\omega t + 90^\circ)$$

So, the voltage is a cosine function with a "head start" of  $90^\circ$  relative to the current.

If current  $i$  in a ckt is,

$$\begin{aligned} i &= I \cos \omega t \\ v &= V \cos(\omega t + \phi) \end{aligned}$$

## Inductor in an AC ckt



The fig above has only a pure Inductor with self inductance  $L$  and 0 resistance. The current  $i = I \cos \omega t$ .

There is a potential difference  $V_L$  between the inductor terminals a and b. The induced emf in the direction of  $i$  is,

$$\mathcal{E} = -L \frac{di}{dt}$$

Point a is at higher potential than is point b. The potential of point a with respect to point b is positive and is given by,

$$V_L = + L \frac{di}{dt}, \text{ the negative of the induced emf.}$$

so we have,

$$V_L = L \frac{di}{dt}$$

$$= L \frac{d}{dt} (I \cos \omega t)$$

$$= - I \omega L \sin \omega t \quad \text{--- (1)}$$

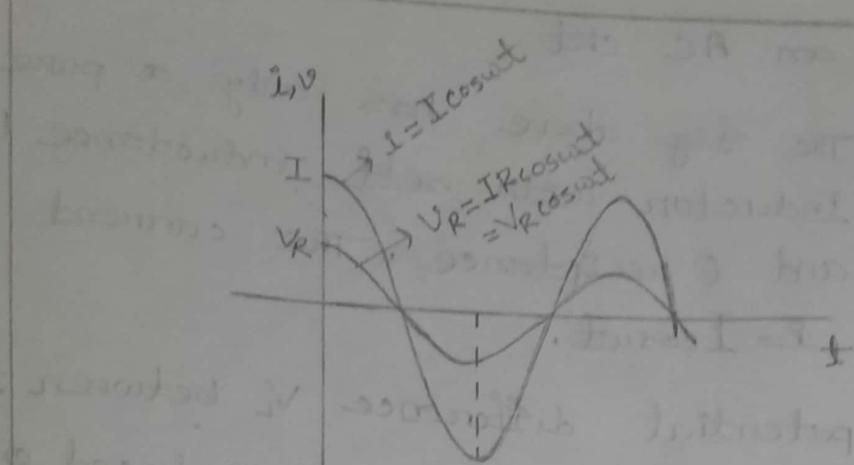
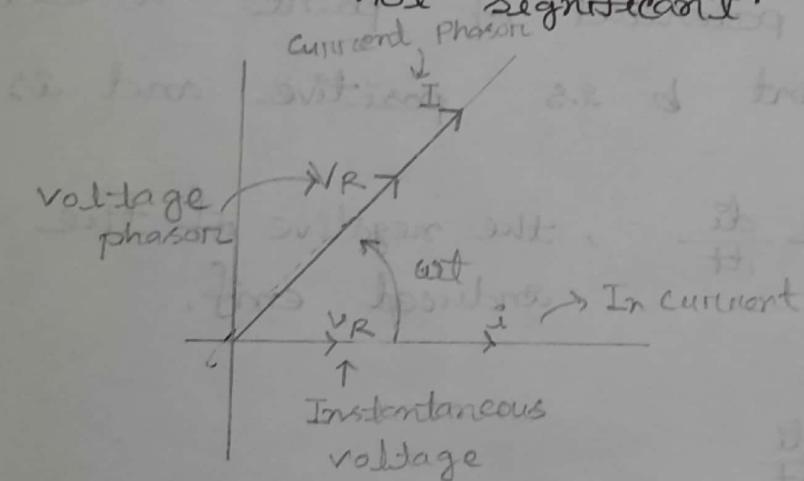


Fig: Graph of current and voltage versus time.

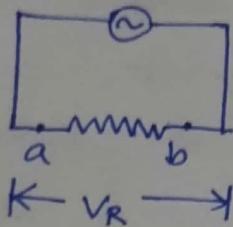
The graph above shows that  $i$  and  $v_R$  as function of time. The vertical scales for  $i$  and  $v$  are different so that the relative height of the two curves are not significant.



The figure above is the corresponding phasor diagram.

- $i$  and  $v$  are in phase
- have the same frequency
- $I$  and  $V$  rotate together.

## Resistor in AC ckt



In the figure above a resistor is considered with resistance  $R$  through which there is a sinusoidal current given by equation  $i = I \cos \omega t$ .

The positive direction of current is counter clockwise around the ckt. The current amplitude max current is  $I$ . From Ohm's law the instantaneous potential  $V_R$  of point  $a$  with respect to point  $b$  is

$$V_R = iR = (IR) \cos \omega t$$

The max voltage  $V_R$ , the voltage amplitude is the coefficient of the cosine function.

$$\boxed{V_R = IR}$$

Hence, we can write  $V_R = V_R \cos \omega t$

The current  $i$  and voltage  $V_R$  are both proportional to  $\cos \omega t$ , so the current is in phase with the voltage. The eqtn  $V_R = IR$  shows that  $i$  and  $V$  amplitudes are related in the same way as in a dc ckt.

[Current are in phase with  $V$ .  
Current and voltage occur together.  
Amplitude are in the same relationship as for a dc ckt  
 $V_R = IR$ ]

$$\begin{aligned}
 \bar{I}^2 &= \frac{\int I^2 dt}{\int dt} \\
 &= \frac{\int_0^T \int_0^t I_0^2 \sin^2 \omega t dt' dt}{T} \\
 &= \frac{I_0^2}{2T} \int_0^T (1 - \cos 2\omega t) dt \\
 &= \frac{I_0^2}{2T} \left[ t - \frac{\sin 2\omega t}{2\omega} \right]_0^T \\
 &= \frac{I_0^2}{2}
 \end{aligned}$$

Hence,  $I_{\text{eff}} = \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{2} = 0.707 I_0$

Thus, the effective or RMS value of an alternating current is  $\frac{1}{\sqrt{2}}$  times its peak value of maximum

$$E_{\text{eff}} = \frac{E_0}{\sqrt{2}} = 0.707 E_0$$

$$RI = V$$