유의 사항

▶ 조교

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▶ 제출 기간

2017. 09. 28 ~ 2017. 10. 12. 18:00 (Late. 2017. 10. 17. 18:00)
Office Hour : 10:30 ~ 18:00

▶ 제출 방법

- 아래 '2017년 2학기 이산수학 과제1'에 제공된 문제들을 별도의 용지에 번호를 표시 후, 수기로 풀어 ITBT 808호로 제출합니다. (별도의 용지에 번호를 표시할 때, 문제는 쓰지 않으셔도 됩니다.)
- 과제 제출 시, 맨 앞장은 표지/커버 입니다. (표지에 수업시간, 학번, 이름을 반드시 명시해주시기 바랍니다.)
- 각 문항에 대하여 풀이과정과 답을 명시해야하며, 가급적 볼펜 사용을 권장합니다. (연필, 샤프 등 쉽게 지워 질 수 있는 필기도구는 사용을 자제해 주시기 바랍니다.)
- 지연 제출 시 감점되며, 미제출 시 0점입니다.

▶ 감점 사항

- 각 문항에 풀이과정이 없을 경우.
- 정답을 확실하게 표시하지 않을 경우.
- 표지/커버에 수업시간, 학번, 이름이 명시되지 않은 경우.
- 지연 제출.

2017년 2학기 이산수학 과제 1

1. Write out $\exists !xP(x)$, where the universe of discourse consists of the integers 1, 2, and 3, in terms of negations, conjunctions, and disjunctions. ($\exists !xP(x)$ means "uniqueness quantification of $P(x)$.")

2. Let $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ be the statements "x is a duck," "x is one of my poultry," "x is an officer," and "x is willing to waltz," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, $R(x)$, and $S(x)$.
    a) No ducks are willing to waltz.
    b) No officers ever decline to waltz.
    c) All my poultry are ducks.
    d) My poultry are not officers.

3. Let A be a set. Show that
    a) $A \cup \varnothing = A$.
    b) $A \cap \varnothing = \varnothing$.
    c) $A \cup A = A$.
    d) $A \cap A = A$.
    e) $A - \varnothing = A$.
    f) $A \cup U = U$.
    g) $A \cap U = A$.
    h) $\varnothing - A = \varnothing$.

4. Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

5. Determine whether the symmetric difference is associative; that is, if A, B, and C are sets, does it follow that $A \oplus (B \oplus C) = (A \oplus B) \oplus C$?

6. Determine whether each of these functions from {a, b, c, d} to itself is one-to-one.
    a) $f(a) = b, f(b) = a, f(c) = c, f(d) = d$
    b) $f(a) = b, f(b) = b, f(c) = d, f(d) = c$
    c) $f(a) = d, f(b) = b, f(c) = c, f(d) = d$

7. Let $f(x) = 2x$. What is
    (Z means "Set of integers.", N means "Set of natural numbers.", R means "Set of real numbers.")
    a) $f(Z)$?
    b) $f(N)$?
    c) $f(R)$?

8. Show that if x is a real number and n is an integer, then
    a) $x \leq n$ if and only if $\lceil x \rceil \leq n$.

b) $n \leq x$ if and only if $n \leq \lfloor x \rfloor$.

9. Describe an algorithm that takes as input a list of n integers and finds the number of negative integers in the list.

10. Describe an algorithm to find the longest word in an English sentence (where a word is a string of letters and a sentence is a list of words, separated by blanks).

11. Show that $(x^3 + 2x)/(2x + 1)$ is $O(x^2)$.

12. (Calculus required) Show that if $f(x)$ and $g(x)$ are functions such that $f(x)$ is $o(g(x))$ and c is a constant, then $cf(x)$ is $o(g(x))$ where $(cf)(x) = cf(x)$.
($f(x)$ is $o(g(x))$ [read $f(x)$ is "little-oh" of $g(x)$], when $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$.)

13. Determine the number of multiplications used to find x^{2^k} starting with x and successively squaring (to gfind x^2, x^4, and so on). Is this a more efficient way to find x^{2^k} then by multiplaying x by itself the appropriate number of times?

14. Show that matrix addition is commutative; that is, show that if A and B are both $m \times n$ matrices, then $A + B = B + A$.

15. Find at least three different sequences beginning with the terms 3, 5, 7 whose terms are generated by a simple formula or rule.

16. Compute each of these double sums.

a) $\sum_{i=1}^{3} \sum_{j=1}^{2} (i + j)$

b) $\sum_{i=0}^{3} \sum_{j=0}^{2} (3i + 2j)$

c) $\sum_{i=1}^{3} \sum_{j=0}^{2} j$

d) $\sum_{i=0}^{2} \sum_{j=0}^{3} i^2 j^3$