CS181

Applied Logic and Automated Reasoning

Spring 2020

Instructor: Lucas Bang

Quick Introduction: About Me

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- Grew up in Las Vegas

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- UNLV: Math and CS

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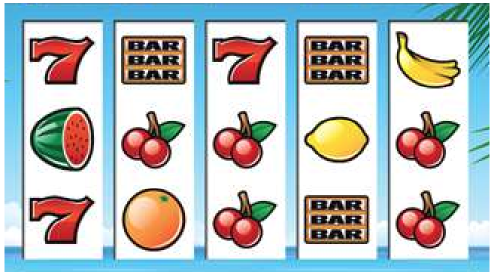

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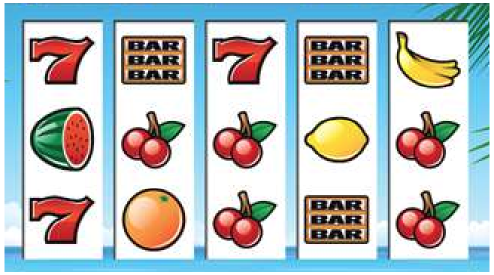
$E($  $) < 0$

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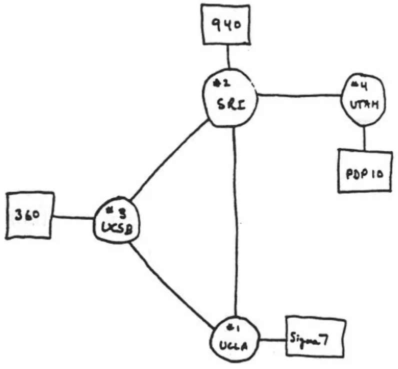

THE ARPA NETWORK

DEC 1969

4 NODES

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  Test and Verification Engineer in Casino Gaming
- PhD, UC Santa Barbara
- HMC Prof. since fall 2018

What is this class about?

By the end of this class, I hope you have an appreciation for both how logic can be used to solve complex problems and how logic fits in the broader context of culture, history, and society.

Syllabus Time!

The rest of today's class:

Example of modeling a system with logic.

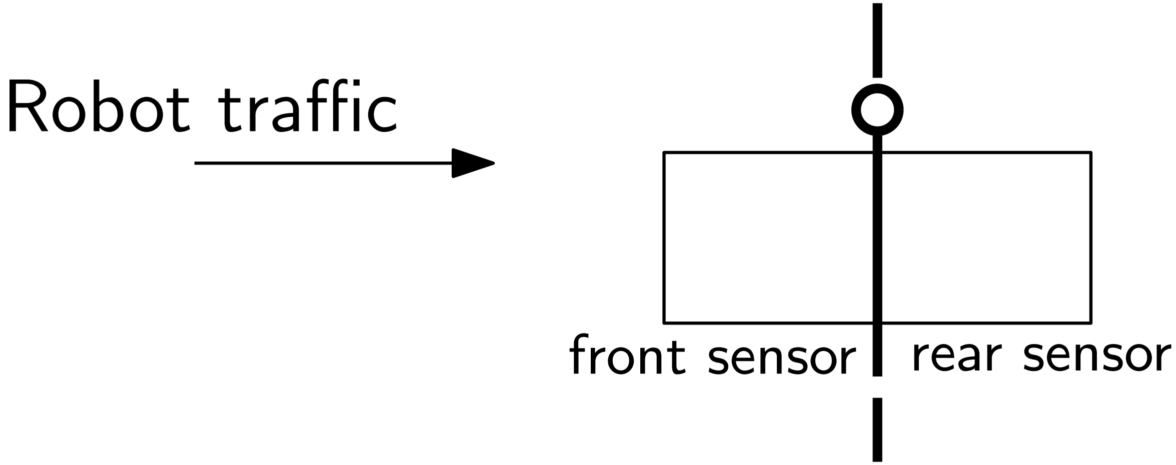
What logical properties might we care about?

Properties in temporal logic.

Using NuSMV to check properties.

Course technology and HW preview.

Example: an automatic door controller.



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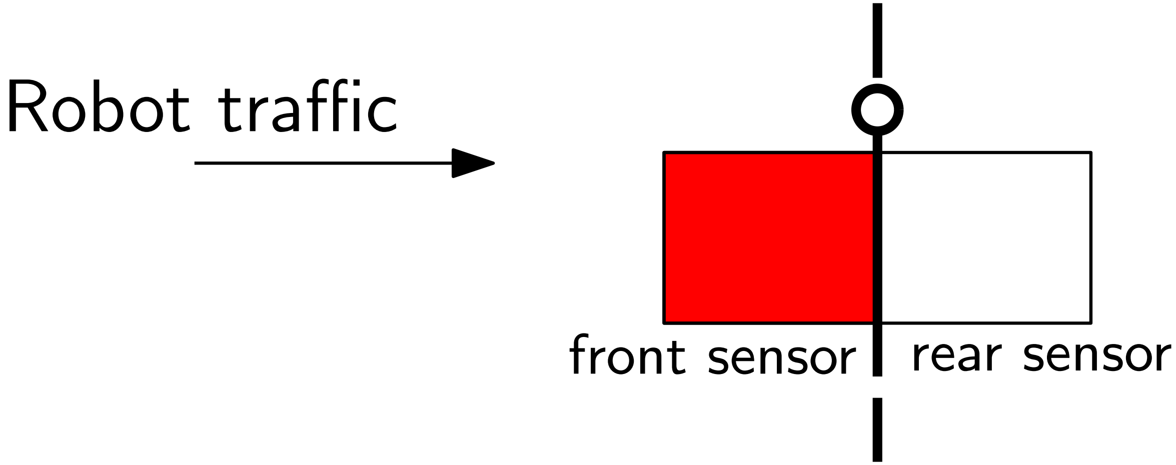

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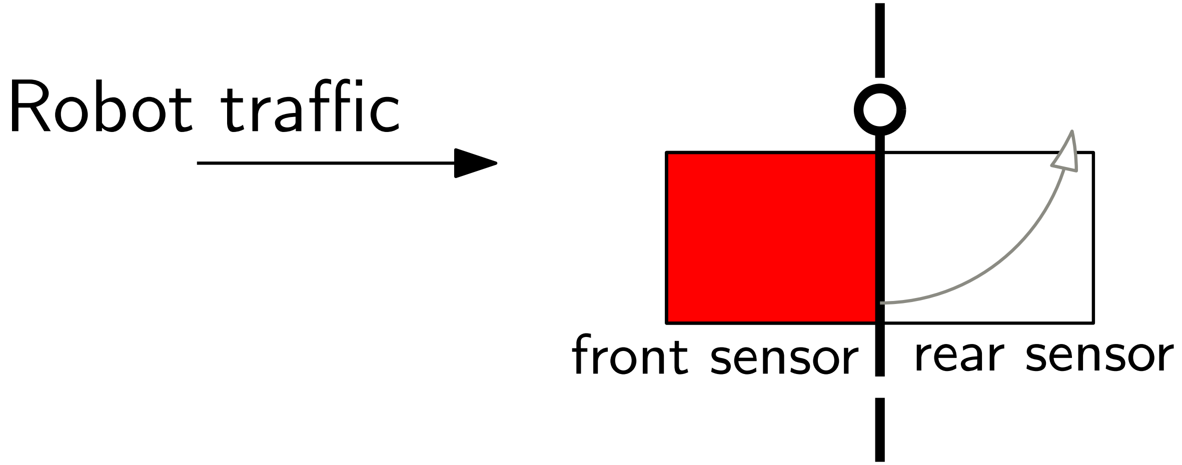
Example: an automatic door controller.



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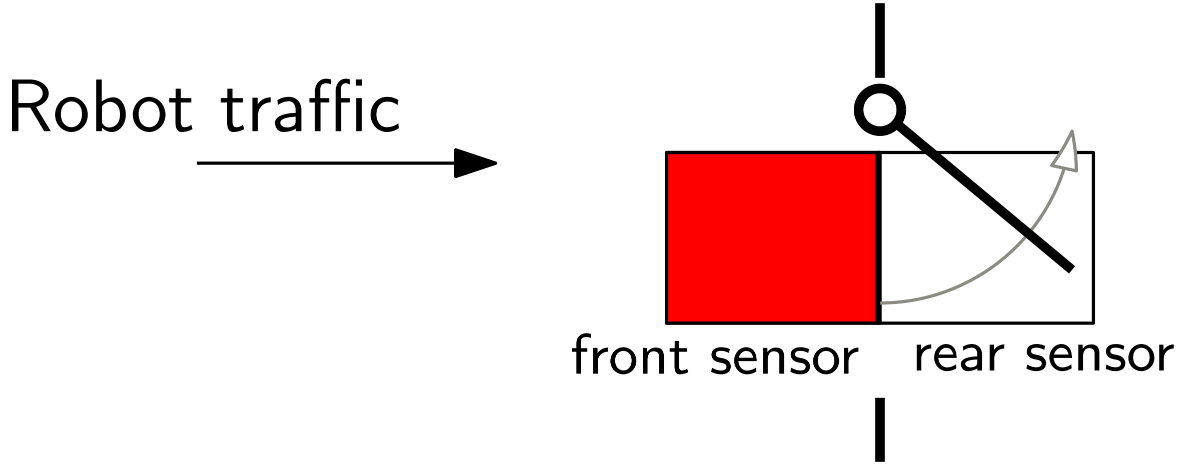

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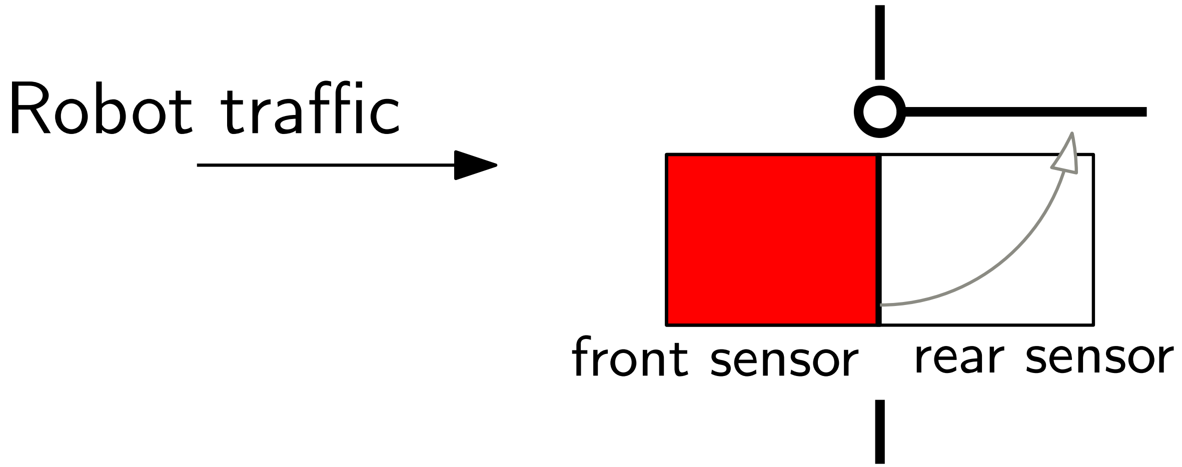

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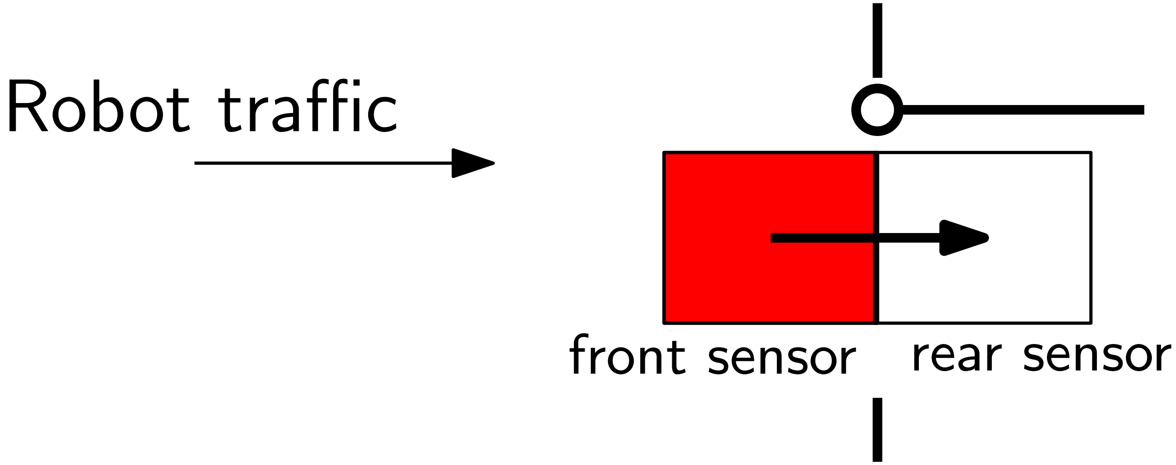

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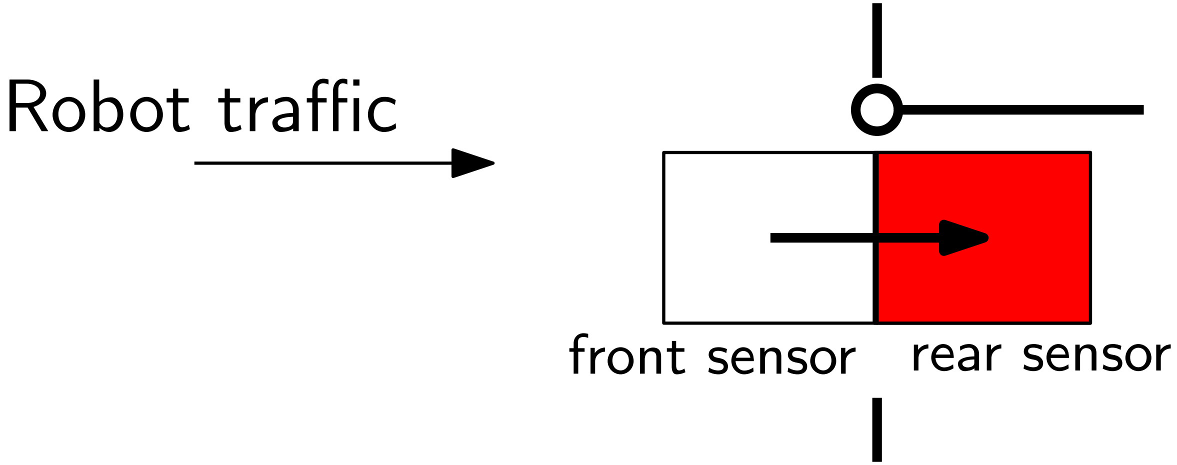

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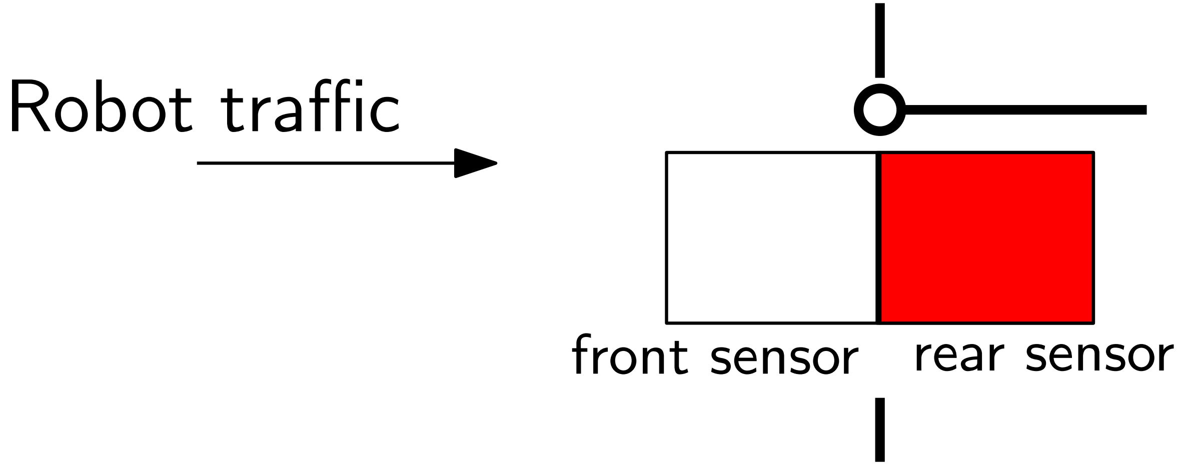

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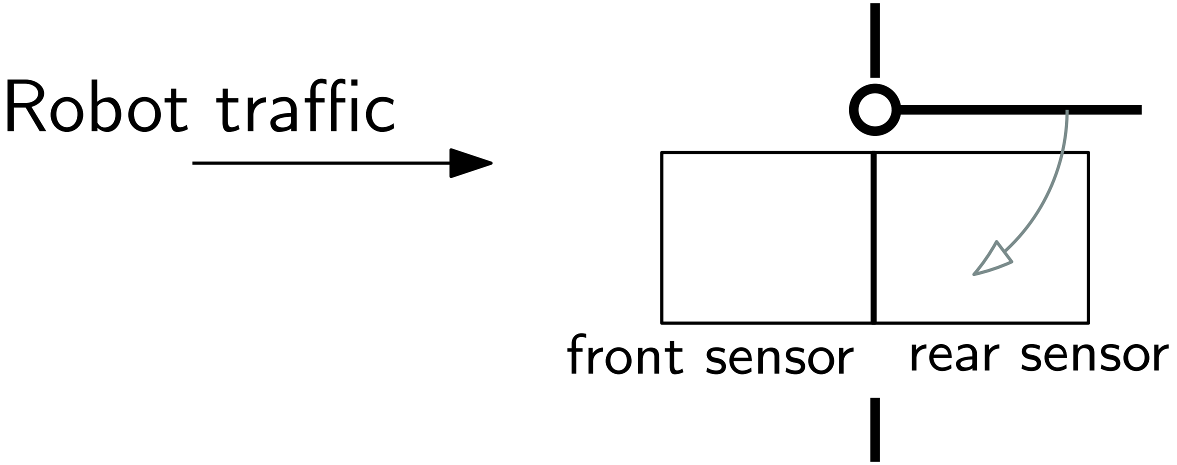

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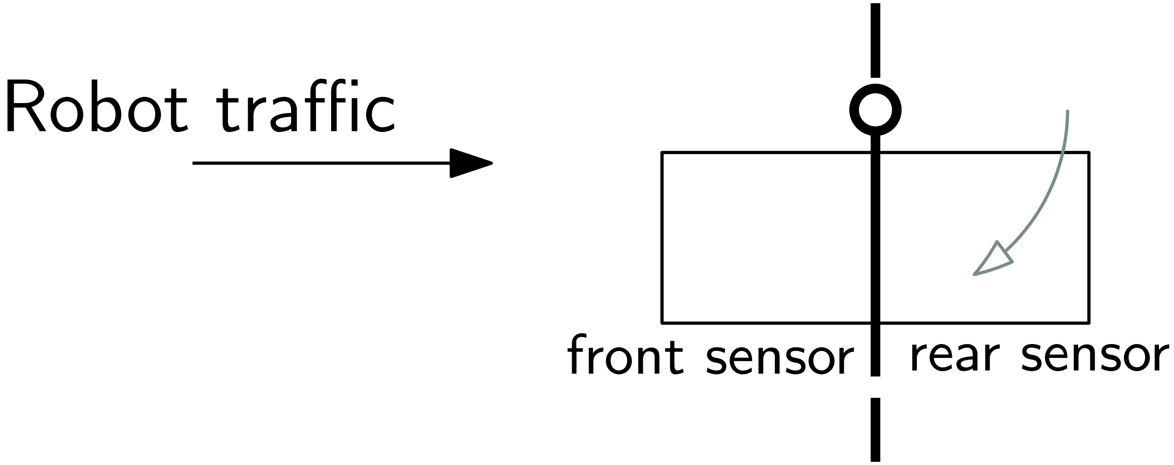

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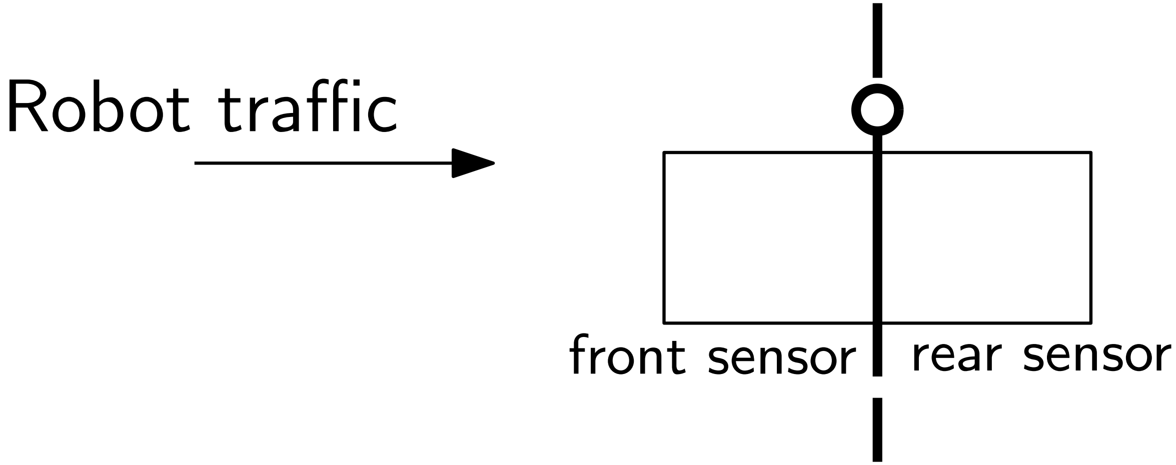

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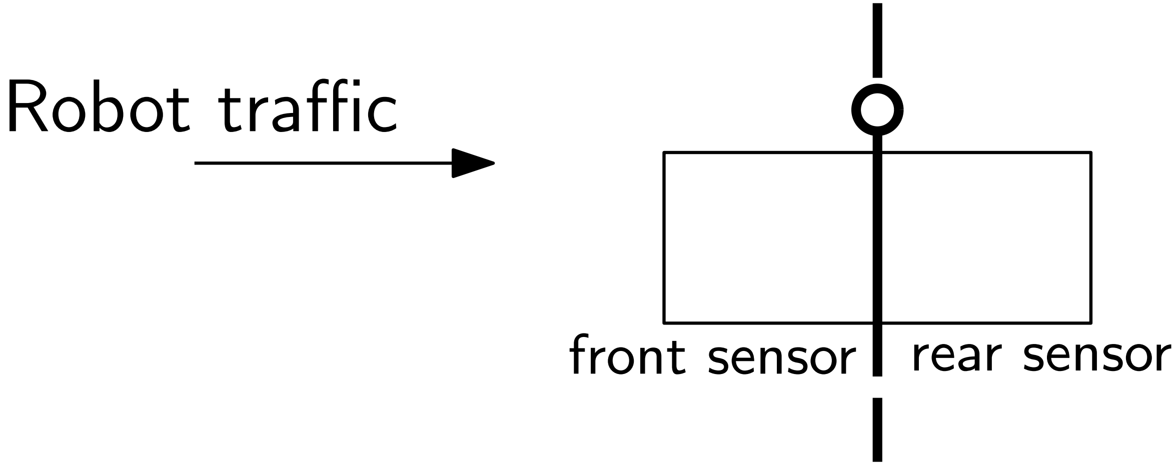
Example: an automatic door controller.



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open

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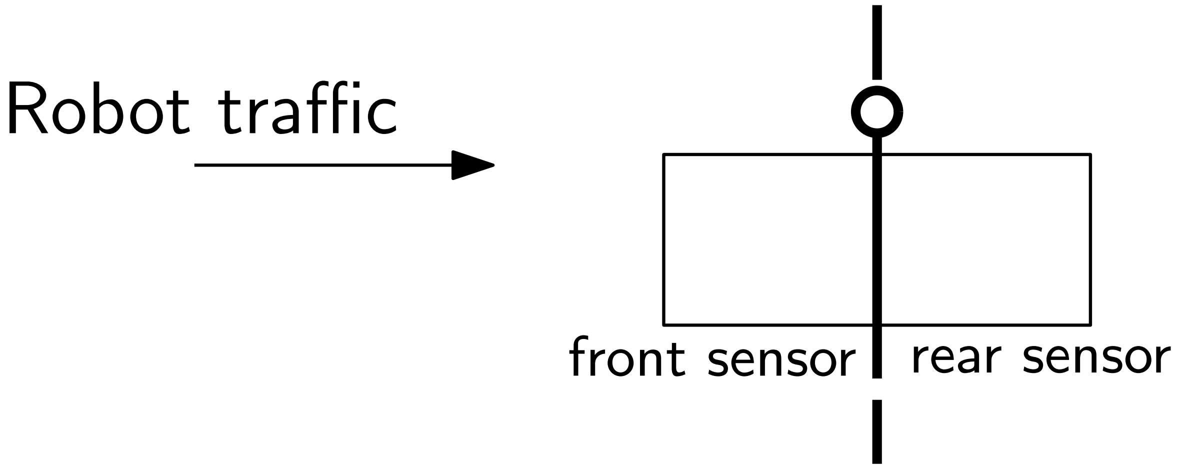

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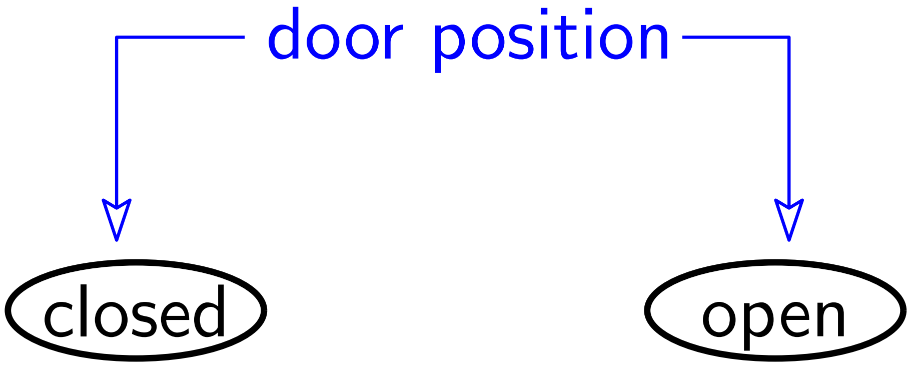

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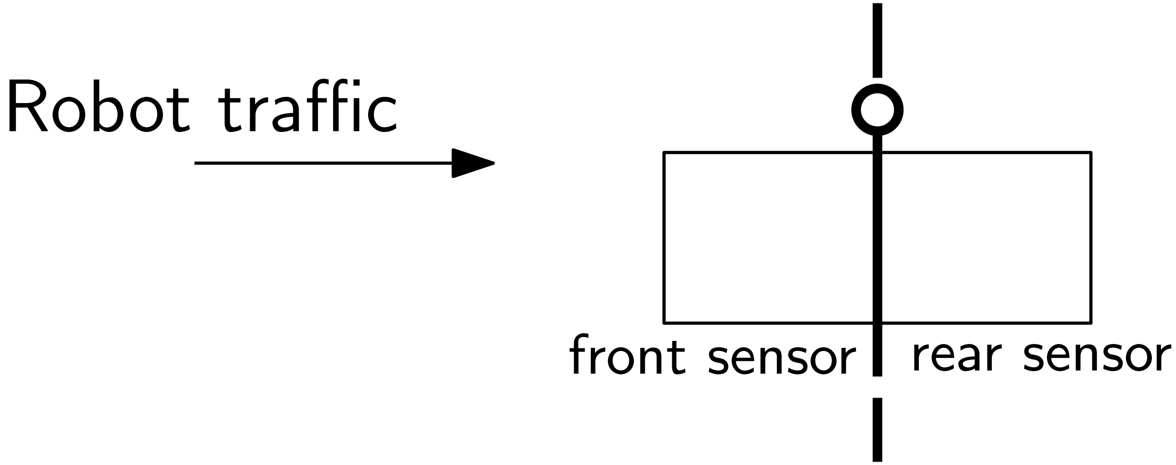

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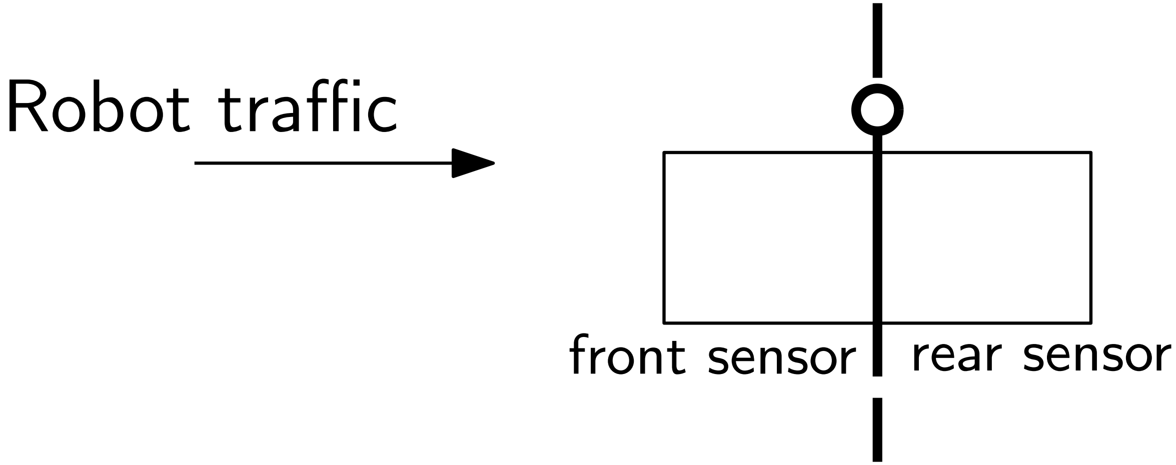

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$f \equiv$ front sensor pad active
$r \equiv$ rear sensor pad active

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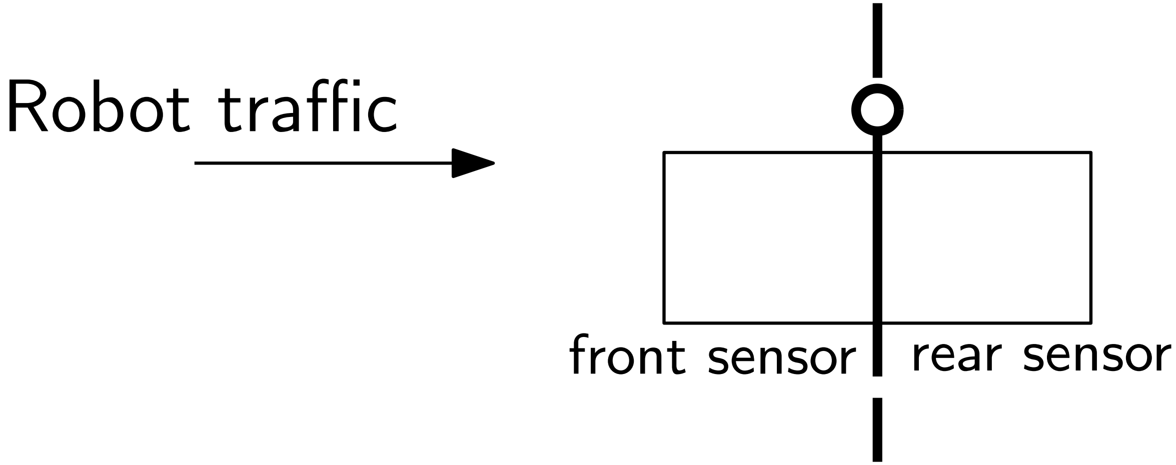

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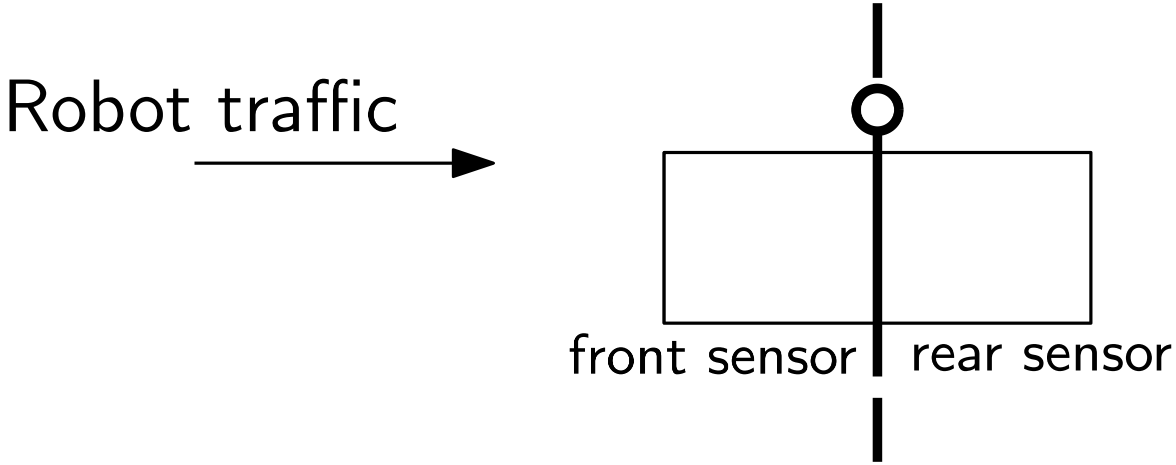

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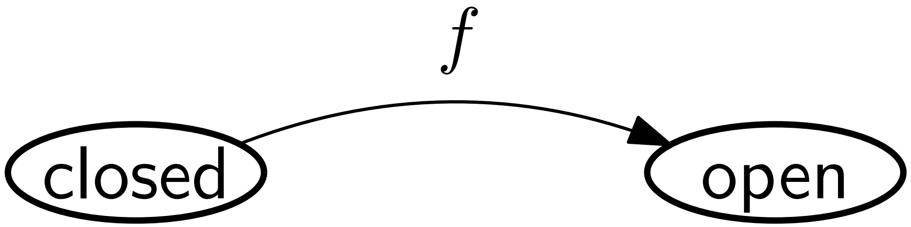

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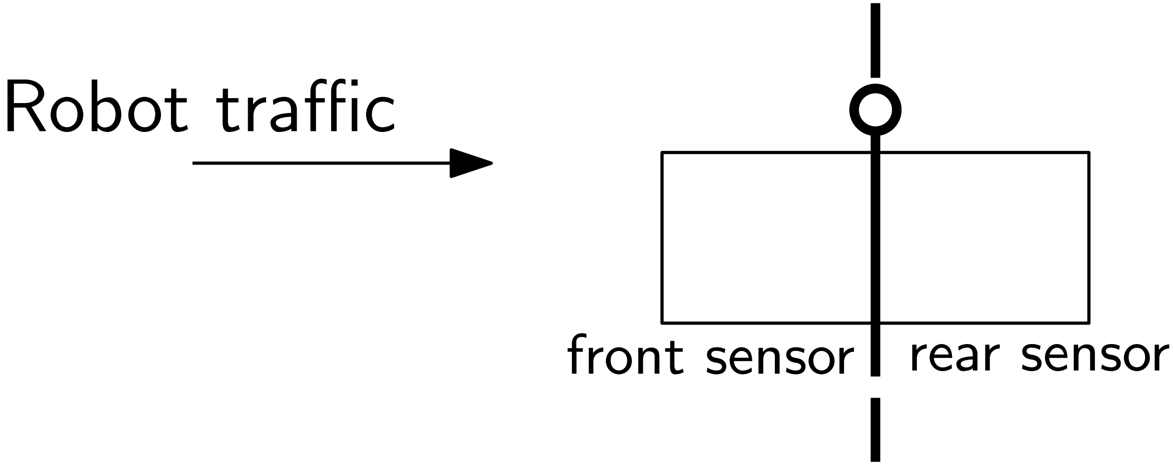

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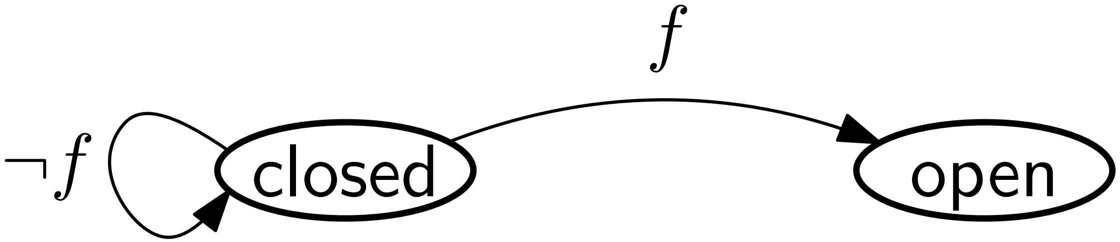

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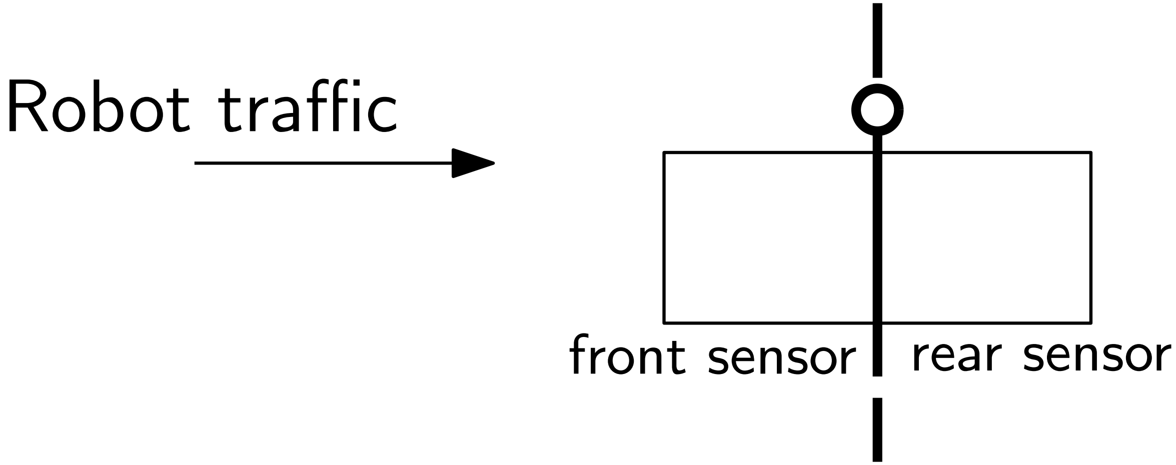

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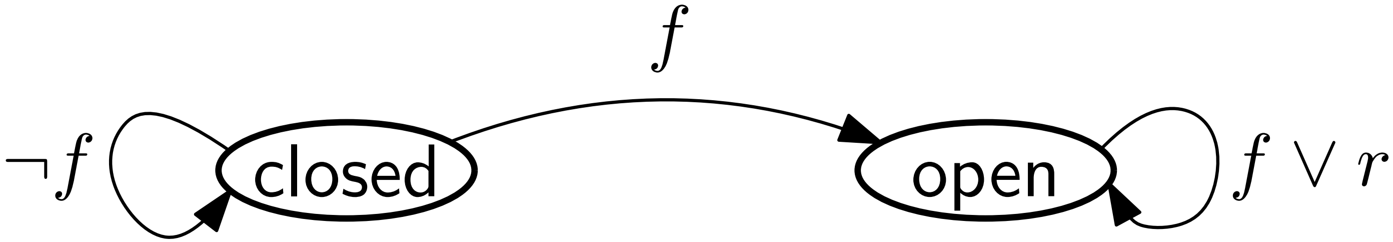

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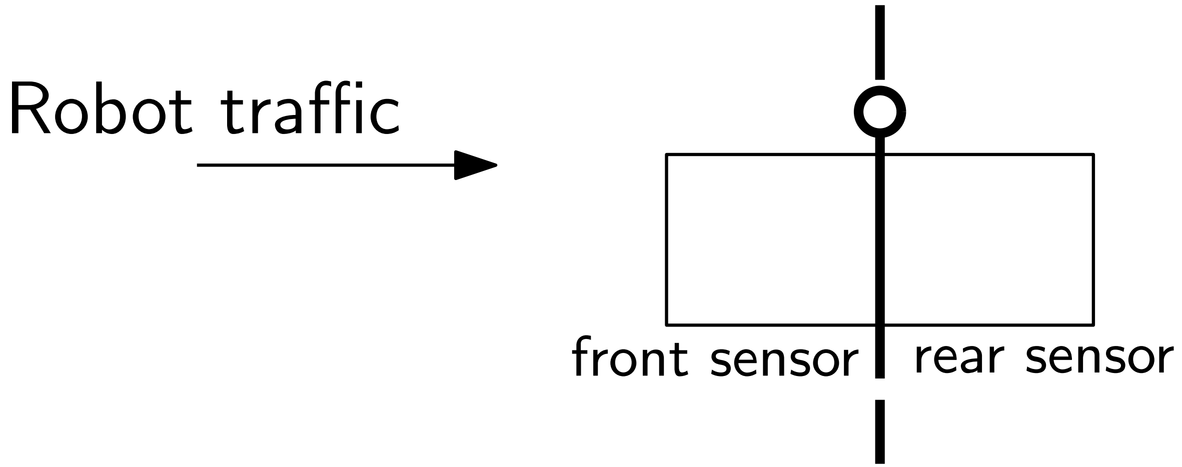

Robot traffic

front sensor

rear sensor

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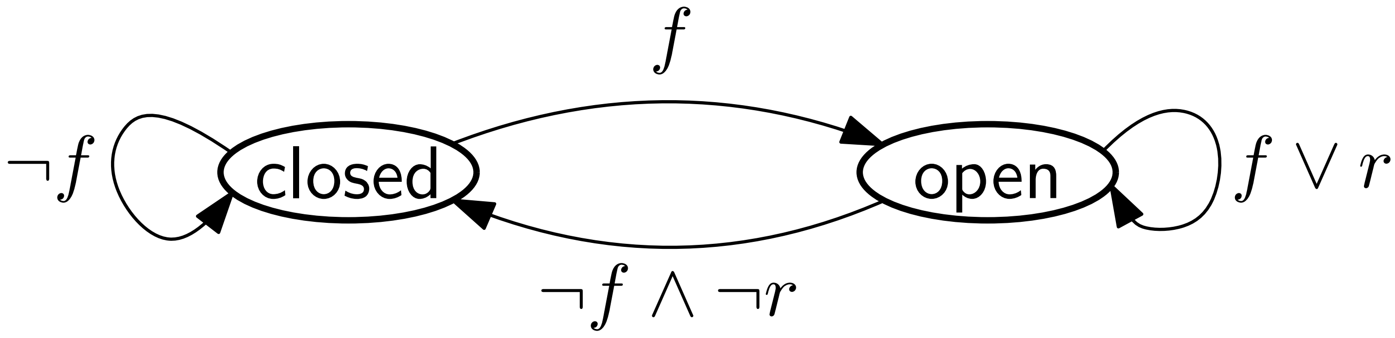

f

$\neg f$

closed

open

$f \vee r$

$\neg f \wedge \neg r$

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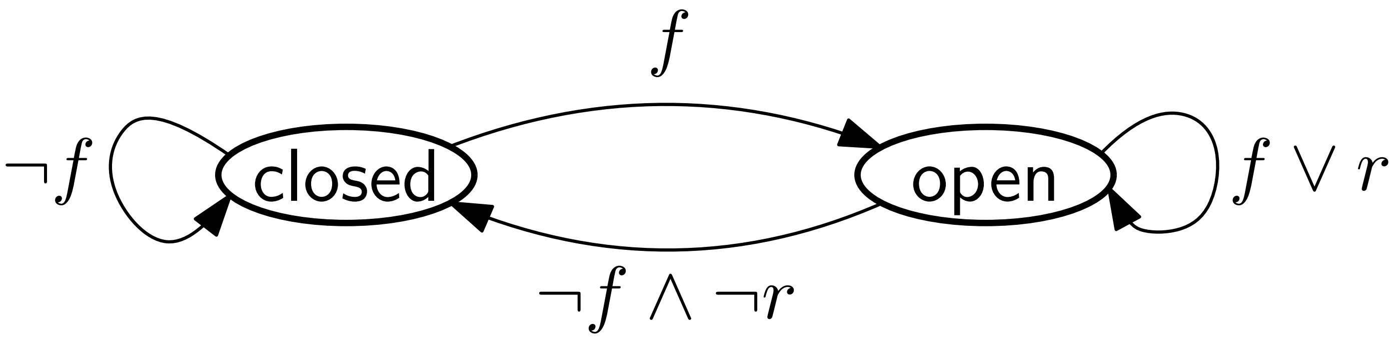

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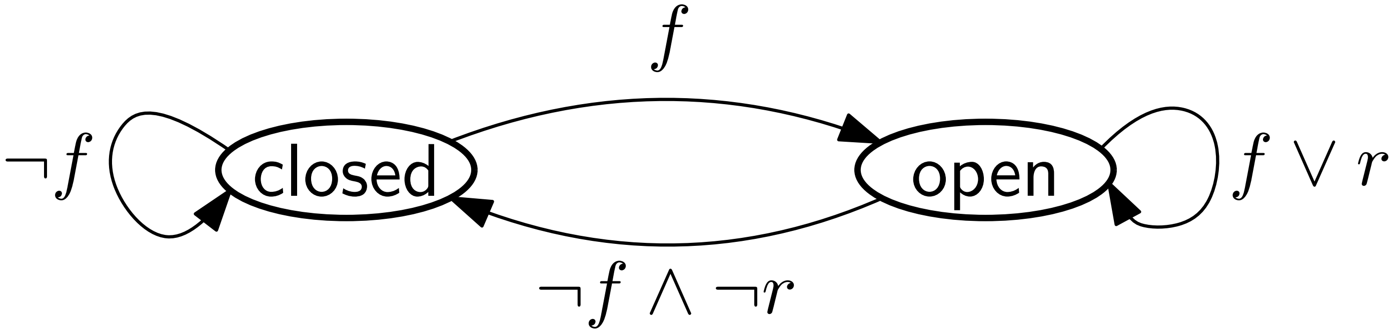

We can encode the same information in a transition table using propositional logic.

condition	next(door)

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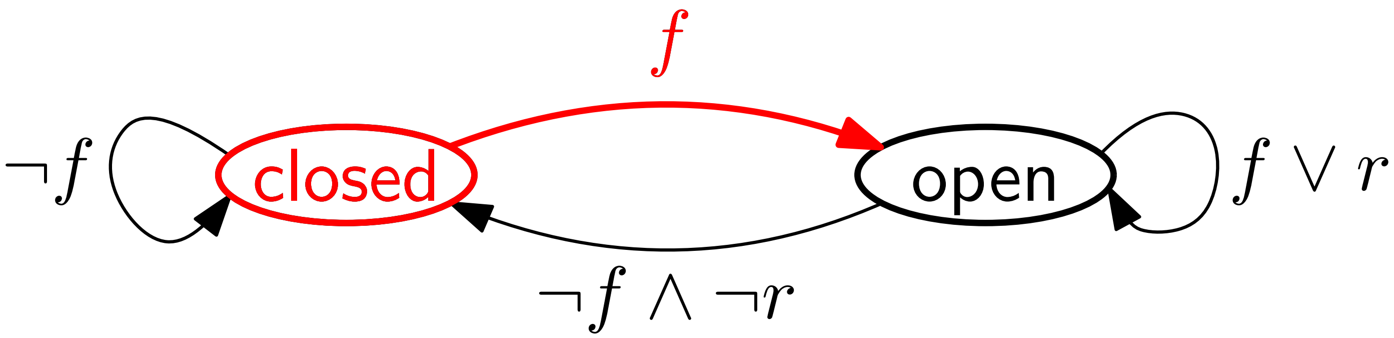

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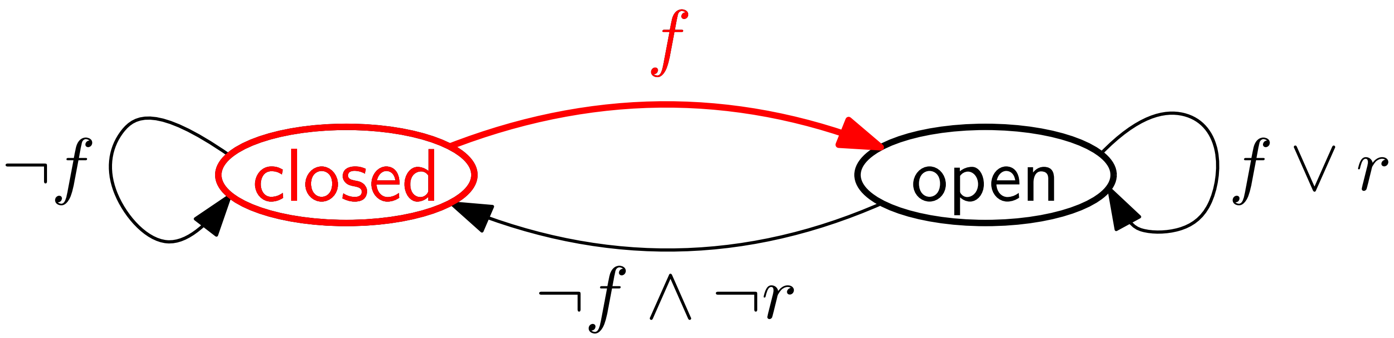

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Example: an automatic door controller.

We can further encode this in NuSMV (Symbolic Model Verifier).

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Variable declarations

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Initialization

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Transition relation
(compare with table, previous slide)

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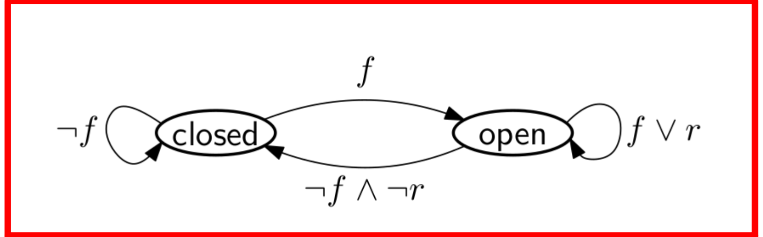
Review: The Modeling Process

Natural language description

If the robot activates front sensor then the door . . .

Unambiguous model expressed in math and logic



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A program that we can use to run or simulate our model

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What logical properties might we care about?

One important type of property

Liveness: eventually something “good” happens.

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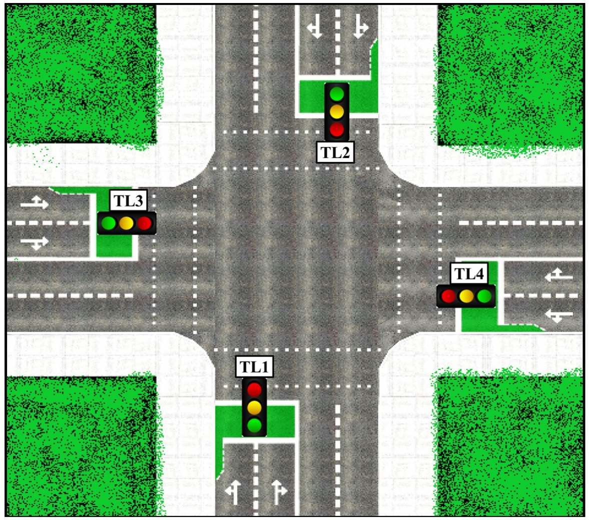

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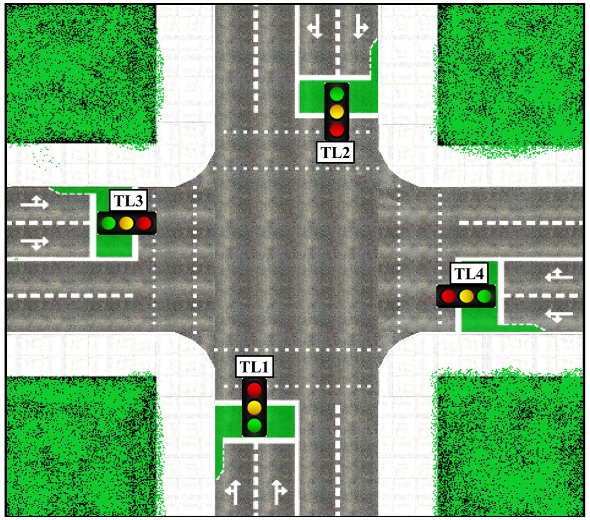
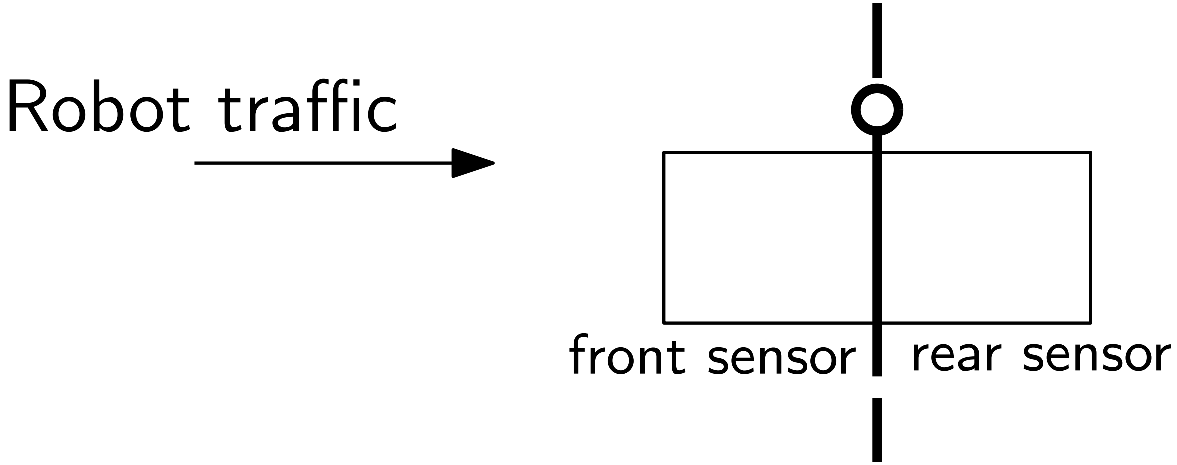


Liveness for traffic lights:
eventually one of the lights is green.

Example: an automatic door controller.



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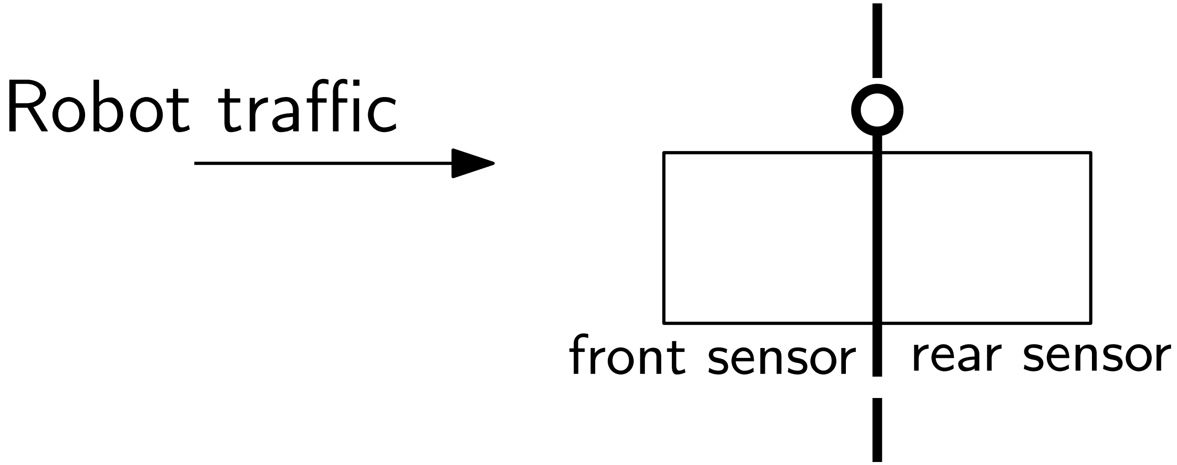

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Linear Temporal Logic

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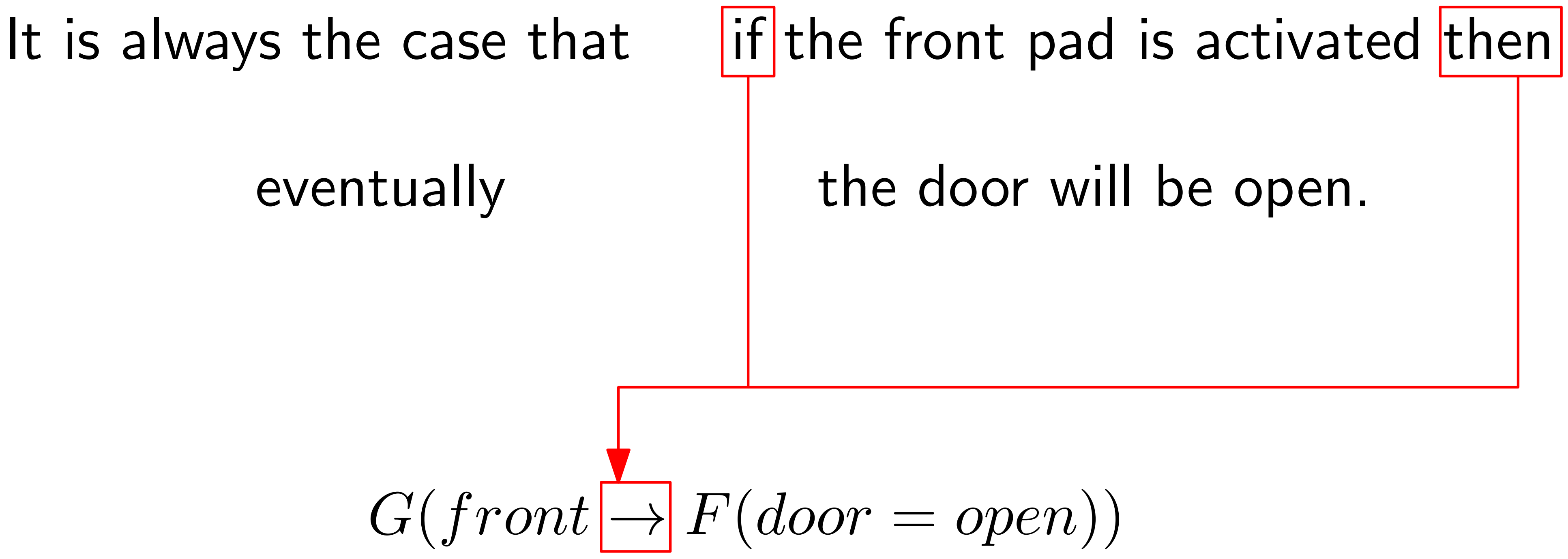

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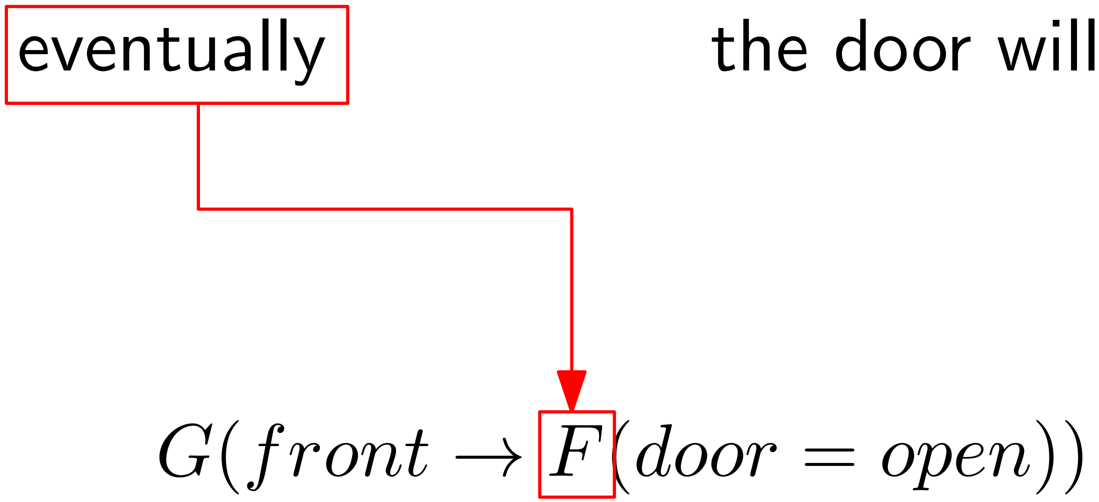

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We can check this property with NuSMV!

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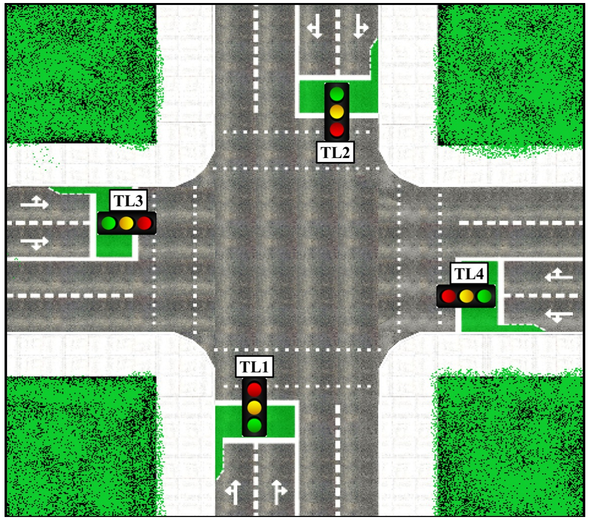

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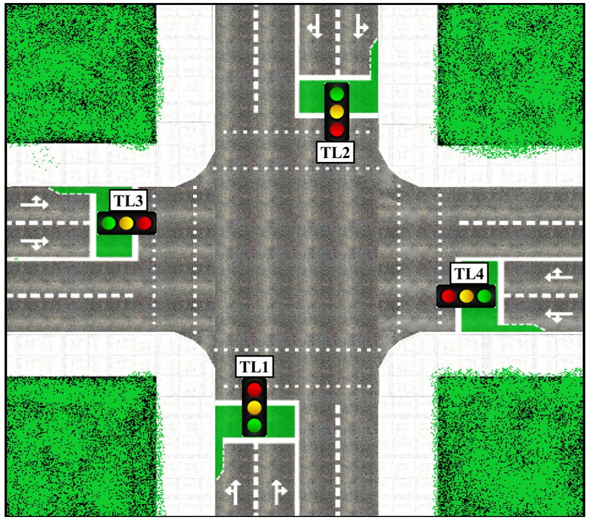

Safety: Any two perpendicular lanes never have corresponding lights that are green at the same time.

Example: an automatic door controller.

A Safety Requirement: It is always the case that if the rear pad is active and the door is closed, then in the next state if the rear pad is still active then the door remains closed.

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$$G(rear \wedge door = closed \rightarrow X(rear \rightarrow door = closed))$$

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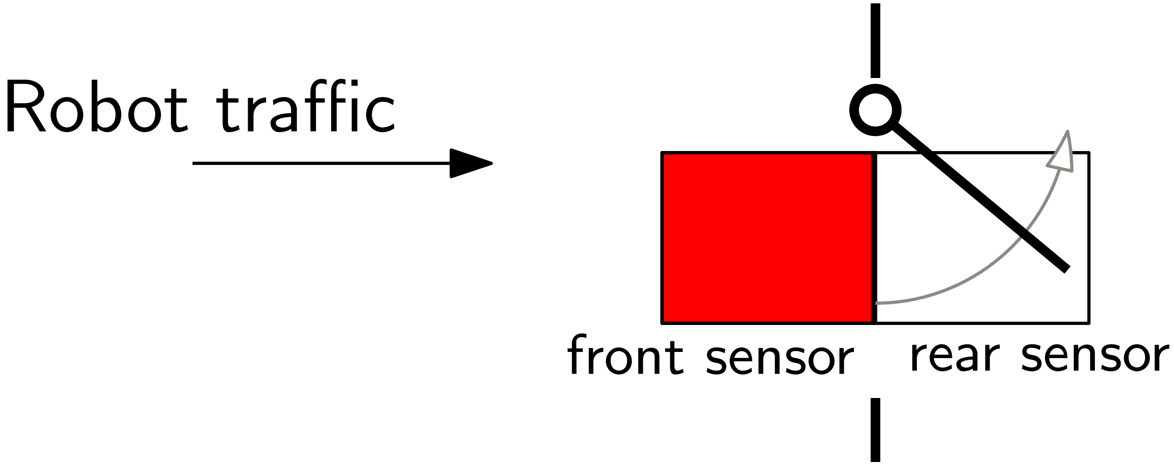
Let's also check this property with NuSMV . . .

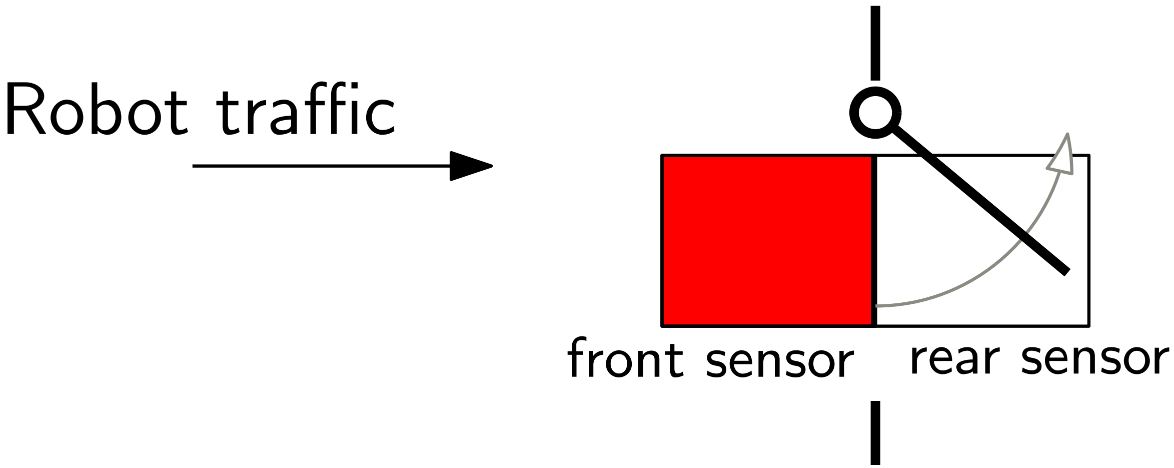
Things to consider...



We saw that our model wasn't quite right yet.

What's missing?

Things to consider...



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Do we need to model intermediate door positions?

$$door \in \{open, opening, closed, closing\} \text{ ??}$$

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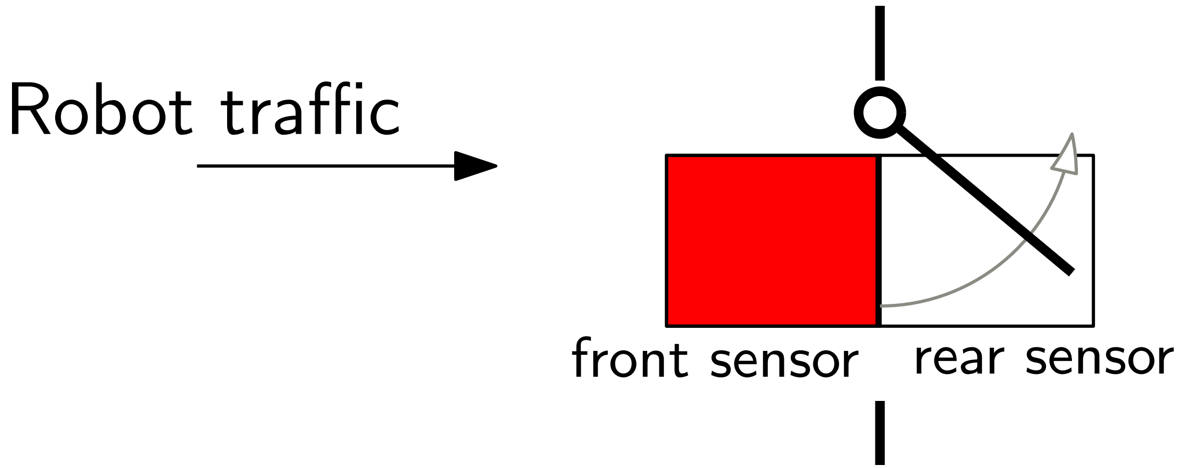

Robot traffic

front sensor

rear sensor

We saw that our model wasn't quite right yet.

What's missing?

Do we need to model intermediate door positions?

$$door \in \{open, opening, closed, closing\} \text{ ??}$$

Do we need to model the robot behavior and the sensor state??

$$next(robotPosition) := \ldots,\ next(frontSensor) := \ldots \text{ ??}$$

A common sentiment:

> “I thought I knew how my [program , proof , simulation, model] worked until I ran [NuSMV, Z3, SPIN, JPF, Alloy, etc.] on it!”

Learning automated resoning techniques forces you to think *very carefully* about what you are doing, and often exposes subtle misunderstandings.

First Few Weeks:

Propositional Logic

A python-based domain specific language for propostional logic, satisfiability checking, model counting, and data structures for logic (BDDs).

Middle part of the class:

Transition Systems

We will learn a formal system of specifying transition systems (which we often depict as a transition diagram).

Temporal Logic (LTL)

We will assign symbols for expressing temporal system requirements like *always*, *eventually*, *next*, *until*.

Temporal Logic Software

Symbolic Model Verifier (NuSMV)

Later Weeks:

Automated Theorem Proving

We will use Z3 to help us automatically prove things,
e.g. a python program doesn't have assertion violations

or give us counterexamples
e.g. inputs that cause an assertion violation

Finally:

Presentation about a logic or automated reasoning:

tool or software
theory or foundation
cultural, social, historical, cognitive, linguistic context

Now: Quick preview of HW00.