CS 181u Applied Logic

Lecture 10

F(semester = done)

Thurs April 16: Exam 2 release date

Mon April 20: Exam 2 due date

Wed April 22: last offical lecture

Mon April 27: 9:35 am CS 181 presentations (senior priority)

Wed April 29: No class

Mon May 4 - Fri May 8: Campus activities (no classed)

Mon May 11: 9am - 12pm presentations

Today's class

Linear Temporal Logic

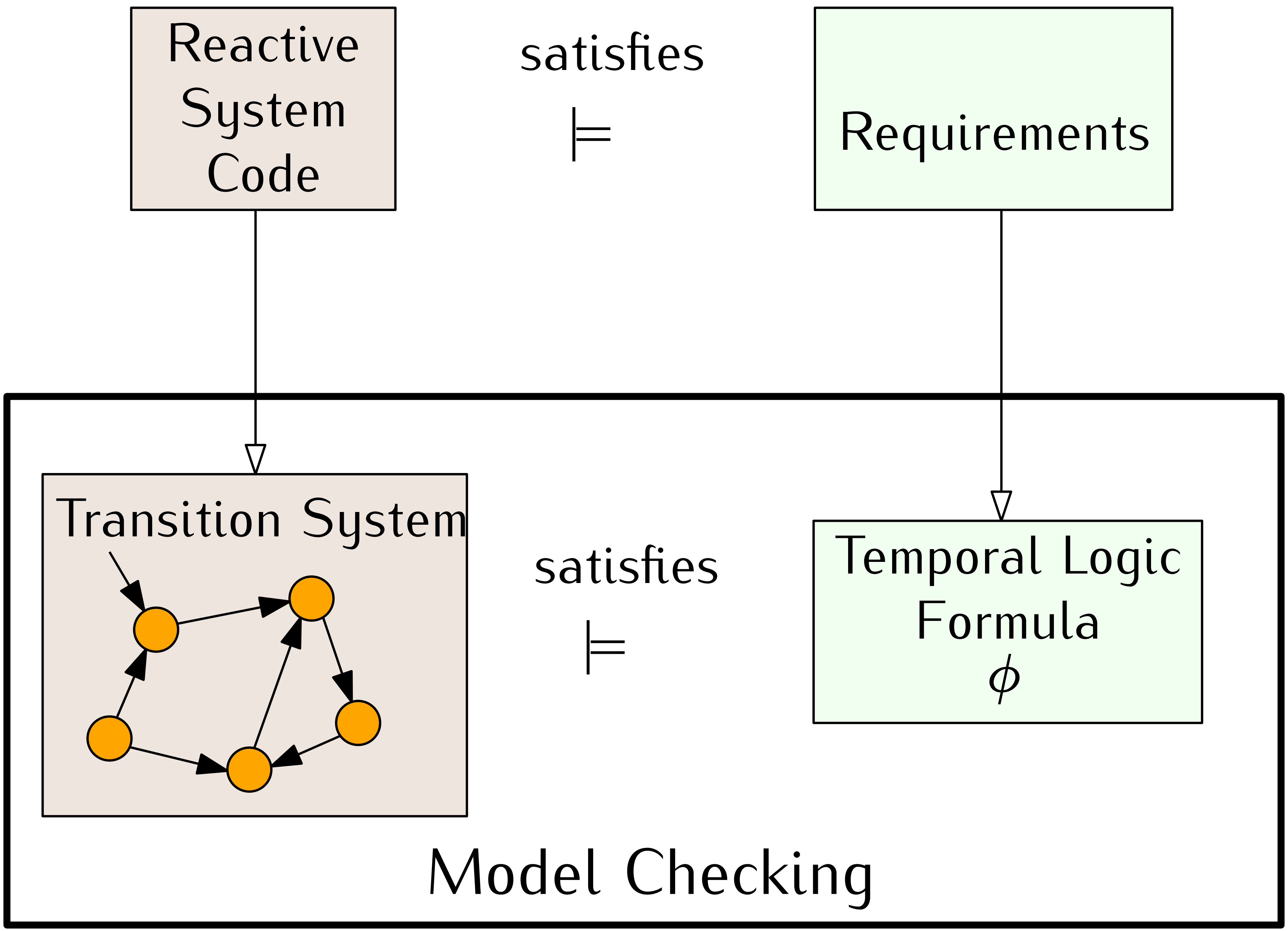
W and R operators. LTL operator basis.

Computation Tree Logic

Some practice, nested CTL formulals

LTL vs CTL

Equivalance of two temporal formulas



Reactive System Code
satisfies
$\models$
Requirements
Transition System
satisfies
$\models$
Temporal Logic Formula ϕ
Model Checking

LTL W and R operators

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Weak Until

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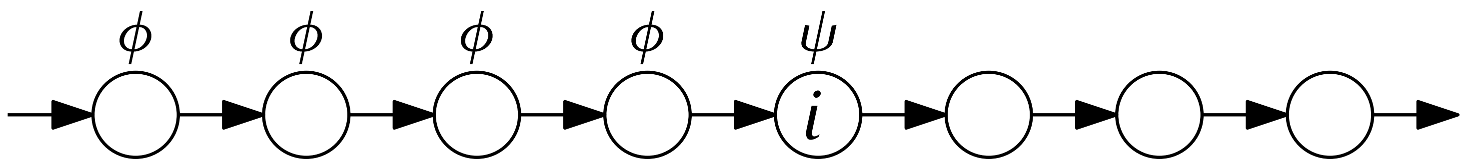

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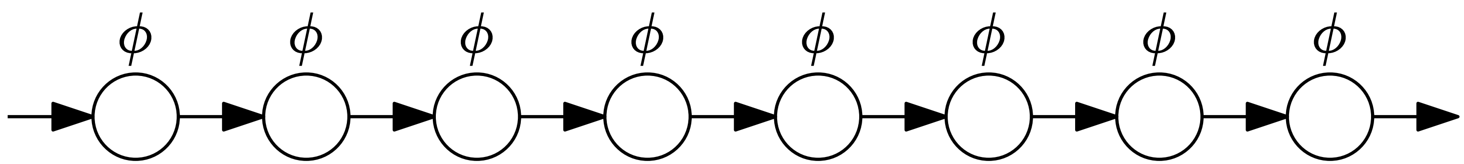

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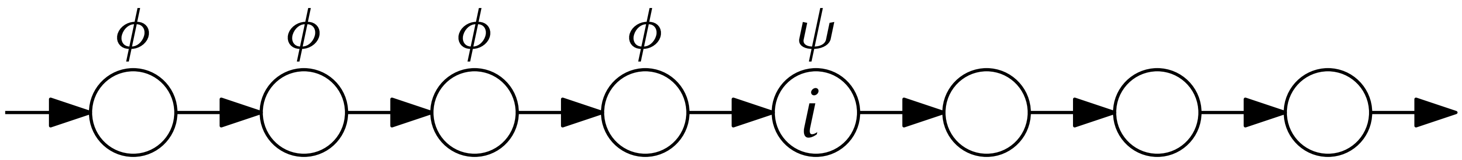

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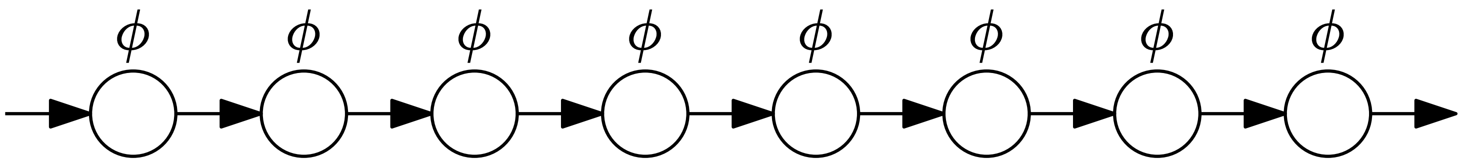

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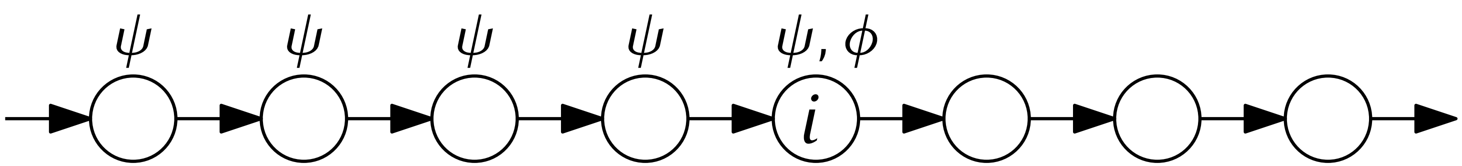

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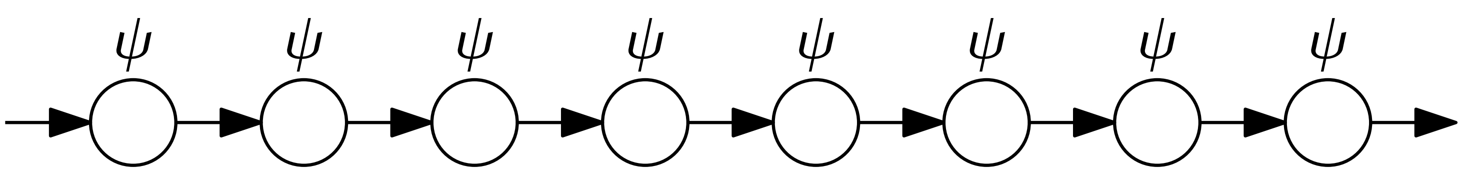

Why so many operators? G, X, U, F, W, R

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The acts of the mind, wherein it exerts its power over simple ideas, are chiefly these three: Combining several simple ideas into one compound one, and thus all complex ideas are made. The second is bringing two ideas, whether simple or complex, together, and setting them by one another so as to take a view of them at once, without uniting them into one, by which it gets all its ideas of relations. The third is separating them from all other ideas that accompany them in their real existence: this is called abstraction, and thus all its general ideas are made.

SICP by Abelson, Sussman, and Sussman quoting John Locke from his *Essay Concerning Human Understanding*

Mechanical Thinking?

If what is exactly stated can be done by a machine, the residue of the uniquely human becomes coextensive with the linguistic qualities that interfere with precise specification—ambiguity, metaphoric play, multiple encoding, and allusive exchanges between one symbol system and another. The uniqueness of human behavior thus becomes assimilated to the ineffability of language, and the common ground that humans and machines share is identified with the univocality of an instrumental language that has banished ambiguity from its lexicon.

-N. Katherine Hayles

How we Became Posthuman: Virtual Bodies in Cybernetics, Literature, and Informatics

Some history

Aristotle (350ish BCE): “All philosophers are mortal, Socrates is a philosopher, therefore Socrates is mortal.”

Leibniz (1685?): “The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us calculate [calculemus], without further ado, to see who is right”

Boole(1800s): Law’s of thought (symbolic logic)

Newell and Simon (1976): “A physical symbol system has the necessary and sufficient means for general intelligent action.”

Present day: biased ML algorithms, software errors cause human suffering

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$\{\wedge, \neg\}$	$\{\lambda, (f \;\; e)\}$	$\{S, K, I\}$
Propositional logic	Lambda calc	Combinatory Logic

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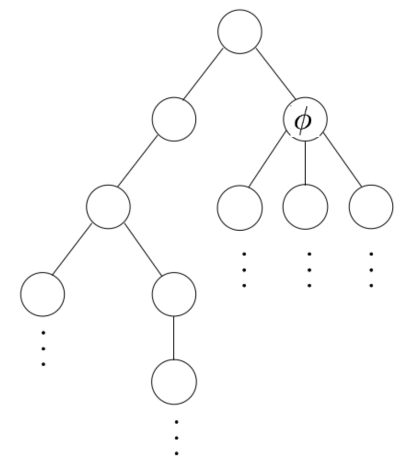
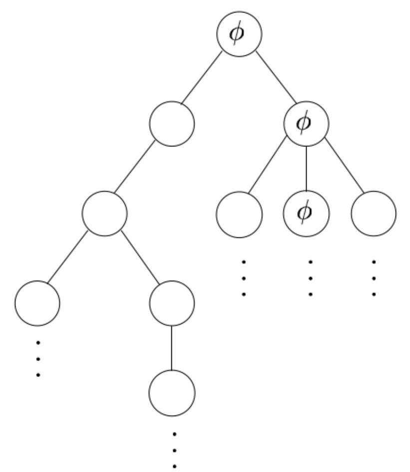
Similar reasoning shows that $\{R, X\}$
and $\{W, X\}$ are also bases.

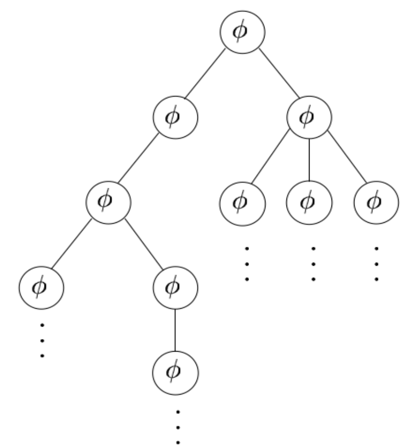
CTL vs LTL

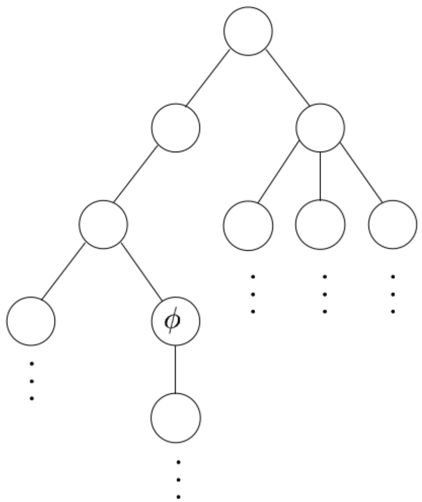
We would like to say something about the difference in expressiveness between CTL and LTL. Let's put that on hold for the moment and get a better grip on CTL first.

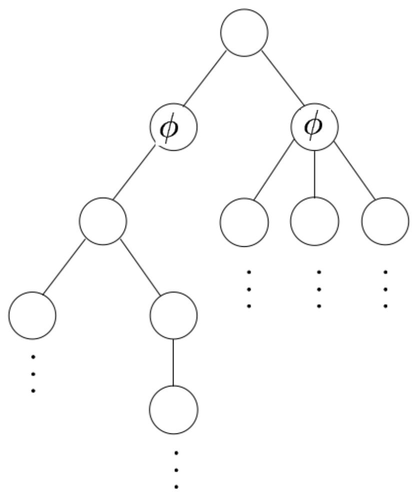
CTL review $AG\phi$ $EG\phi$ $AF\phi$ $EF\phi$ $AX\phi$ $EX\phi$

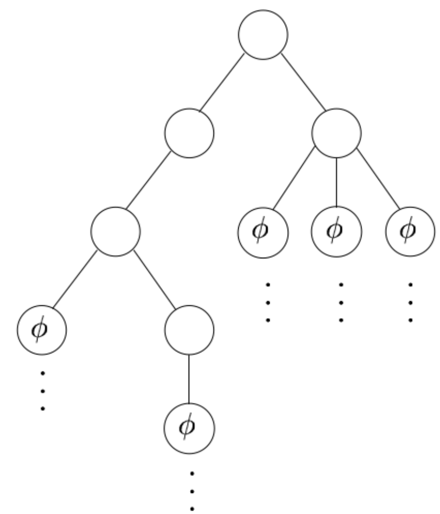












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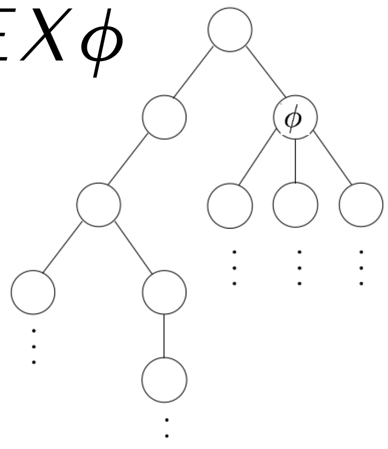

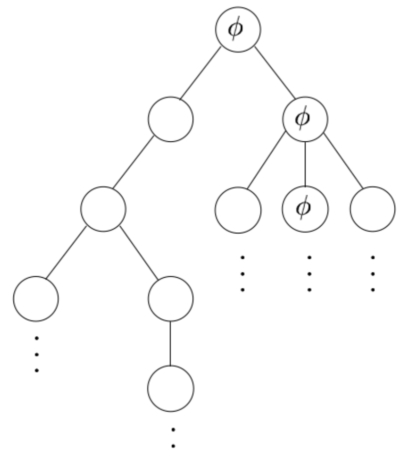

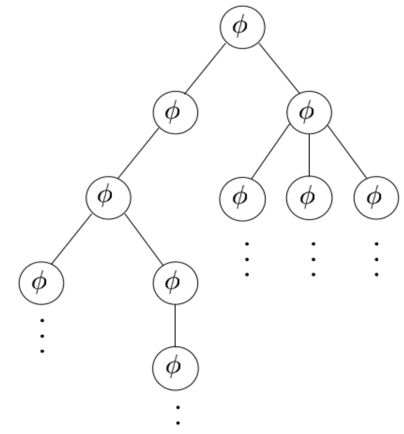

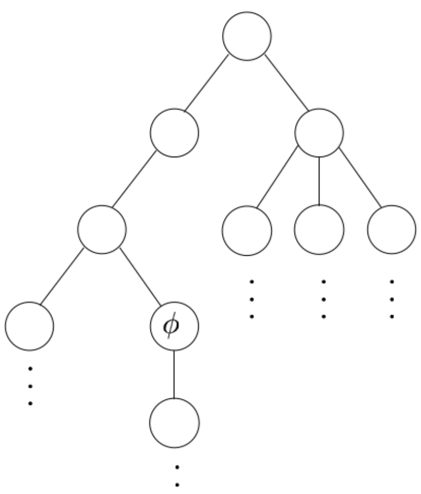

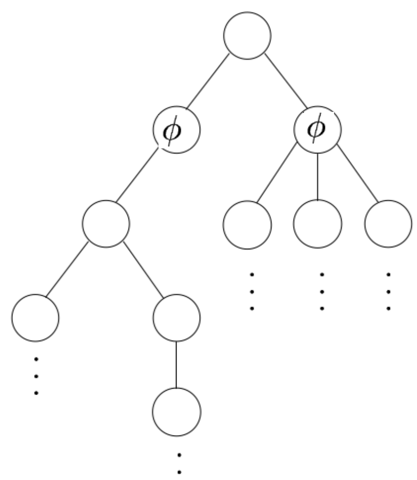

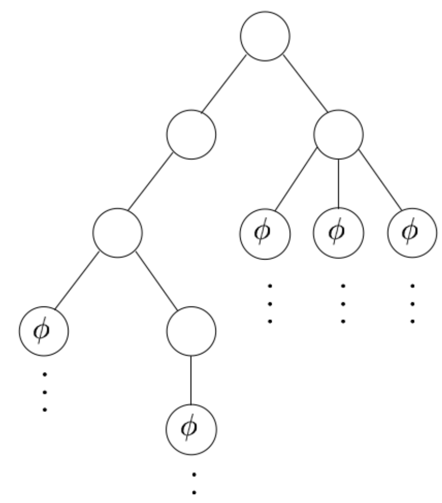

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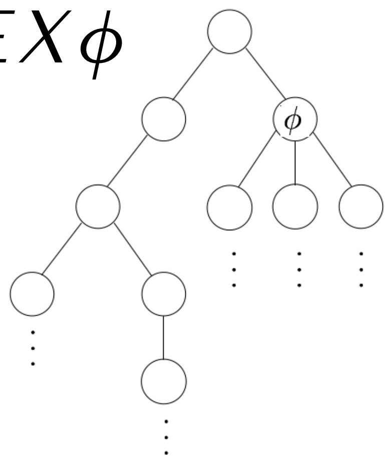
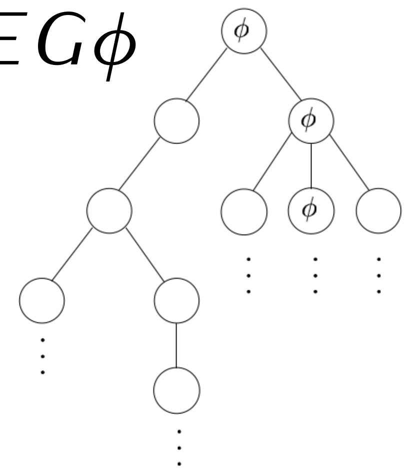
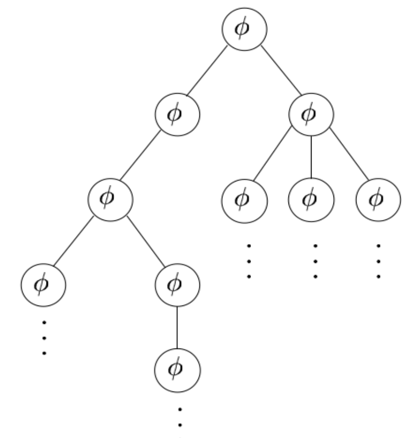
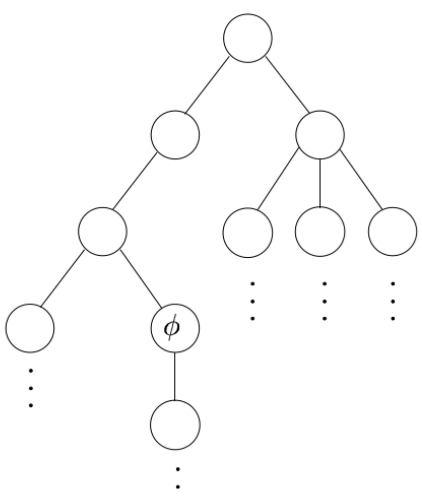
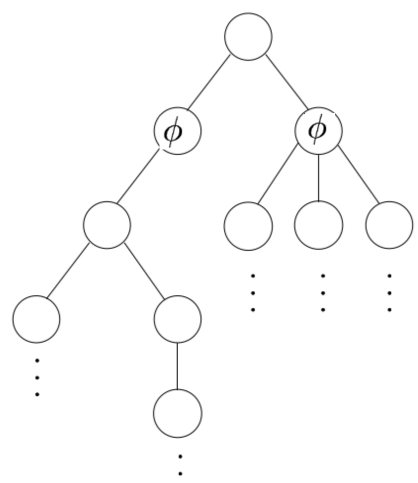


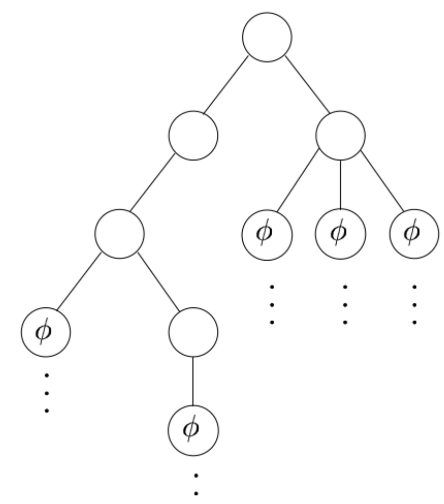
$EG\phi$











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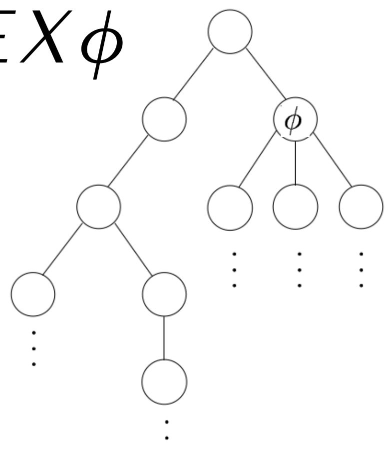
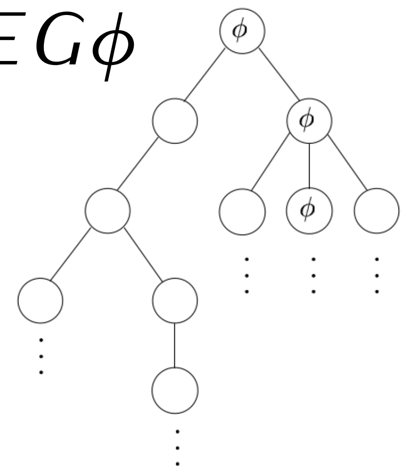
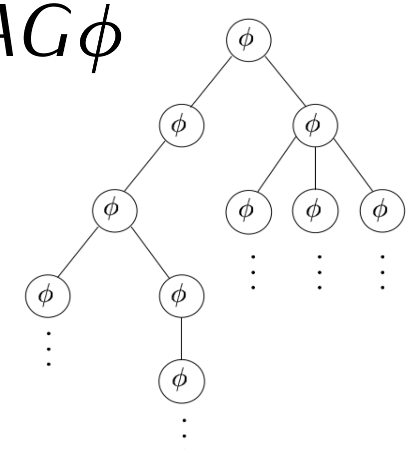
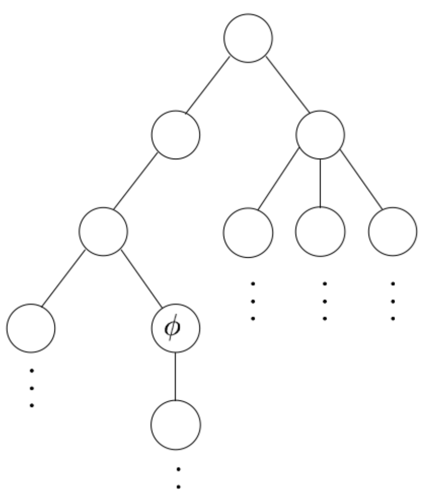


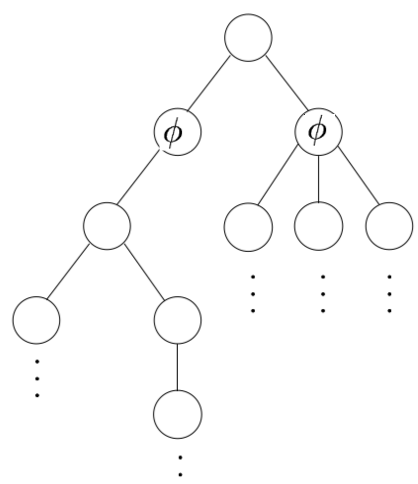
$EG\phi$

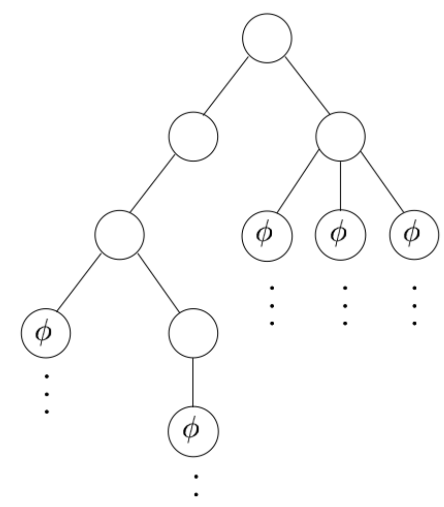


$AG\phi$









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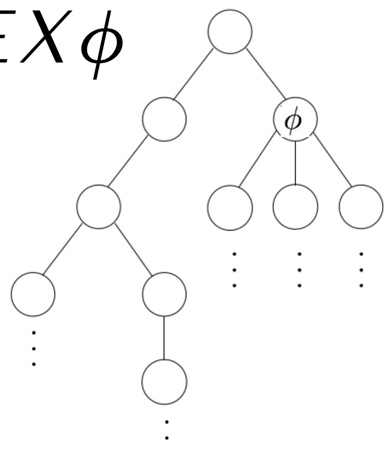
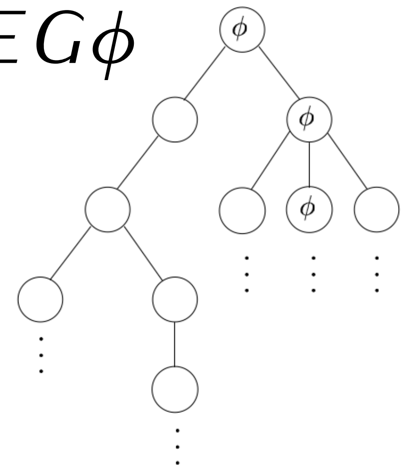


$EG\phi$



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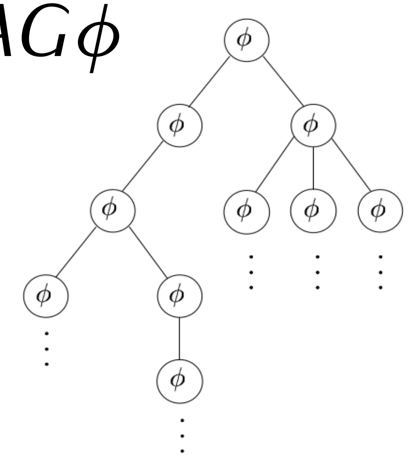
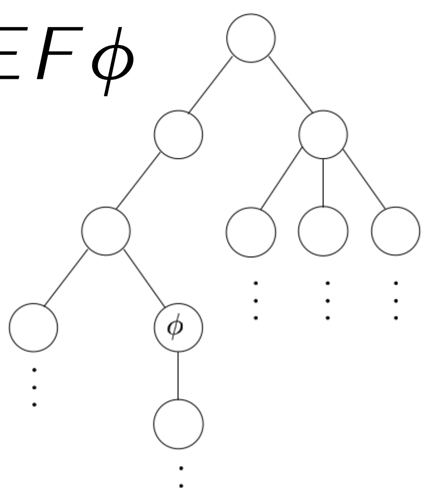
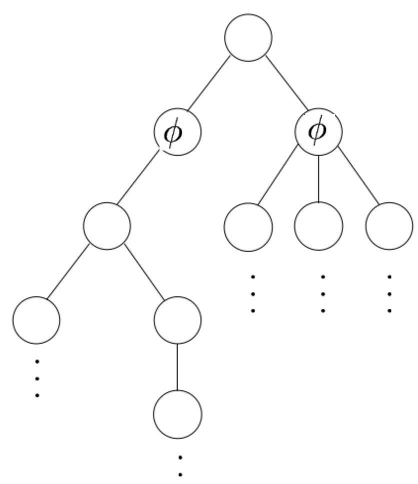
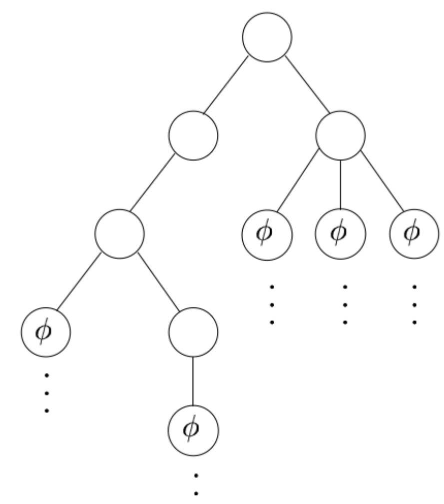


$EF\phi$







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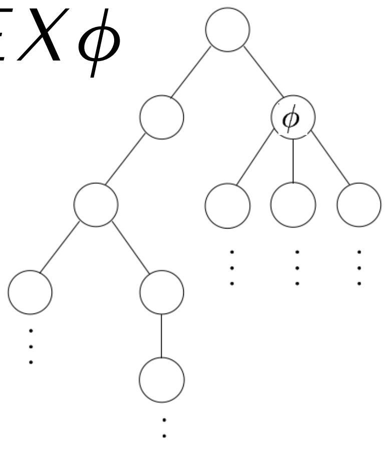
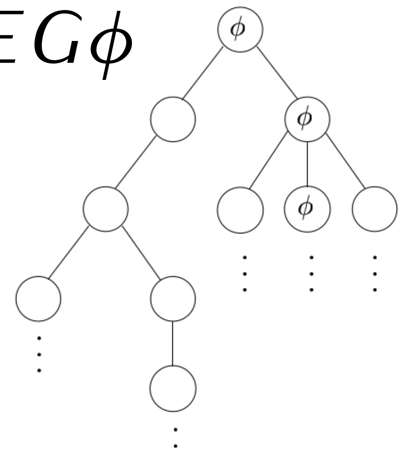
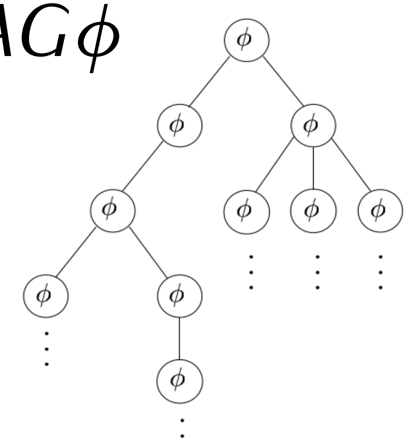


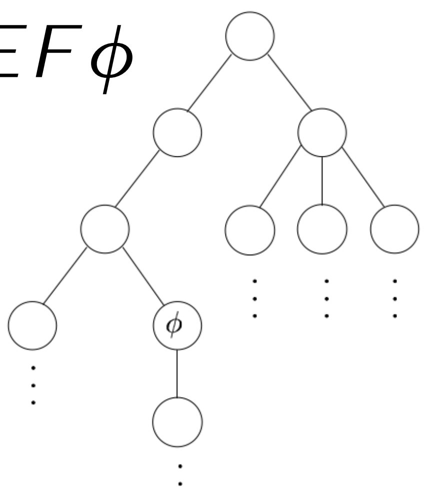
$EG\phi$



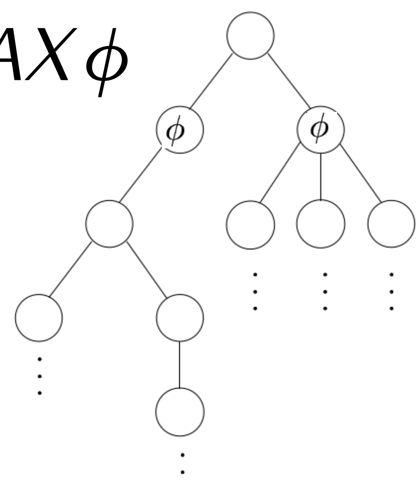
$AG\phi$

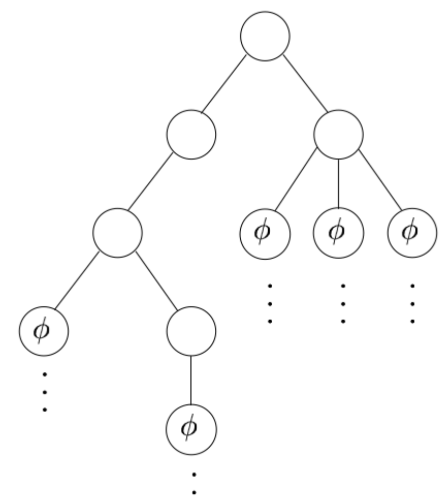


$EF\phi$



$AX\phi$





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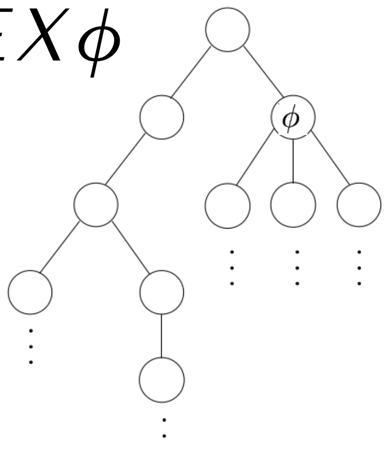

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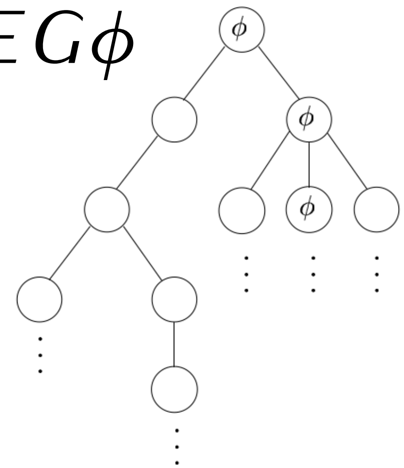
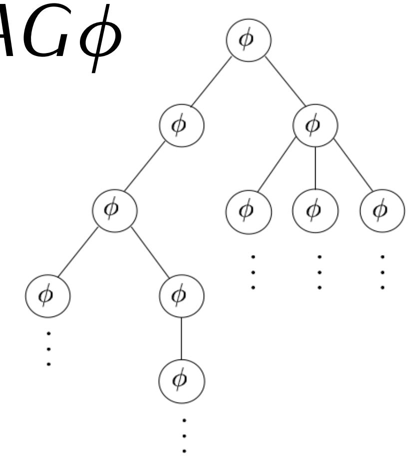
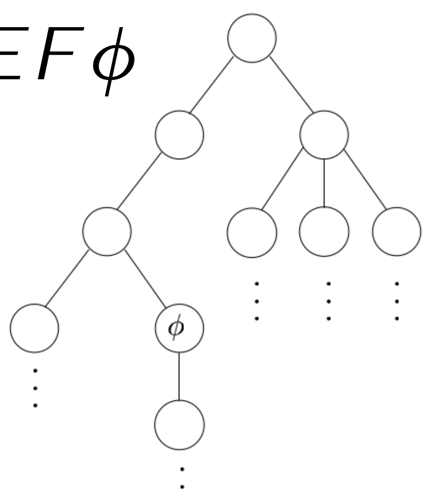


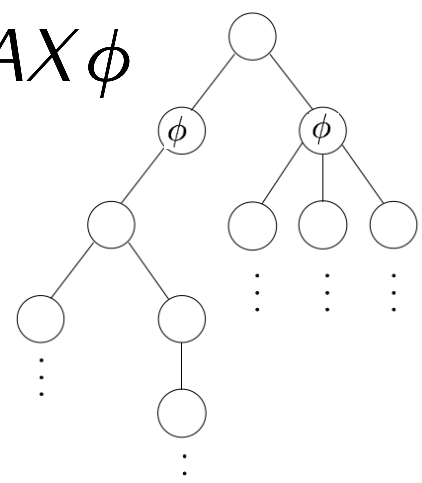
$AG\phi$



$EF\phi$



$AX\phi$



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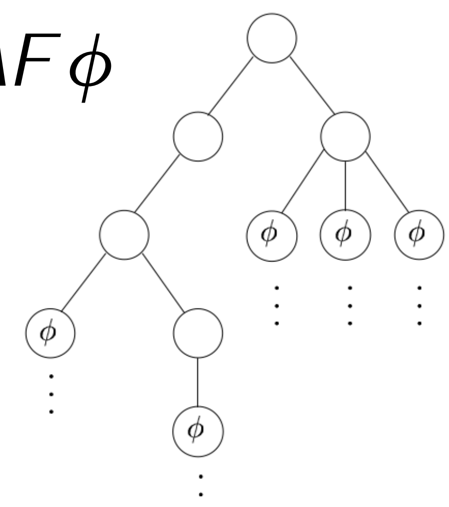
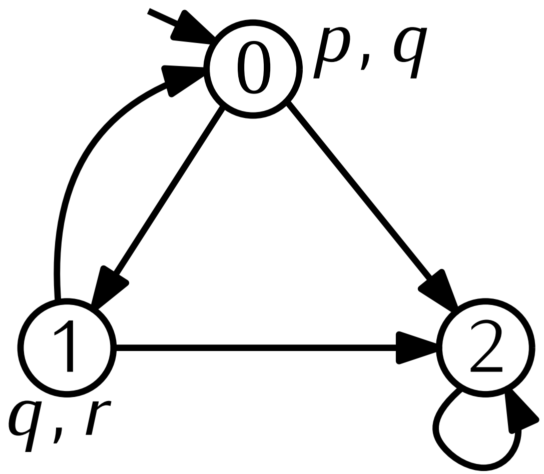
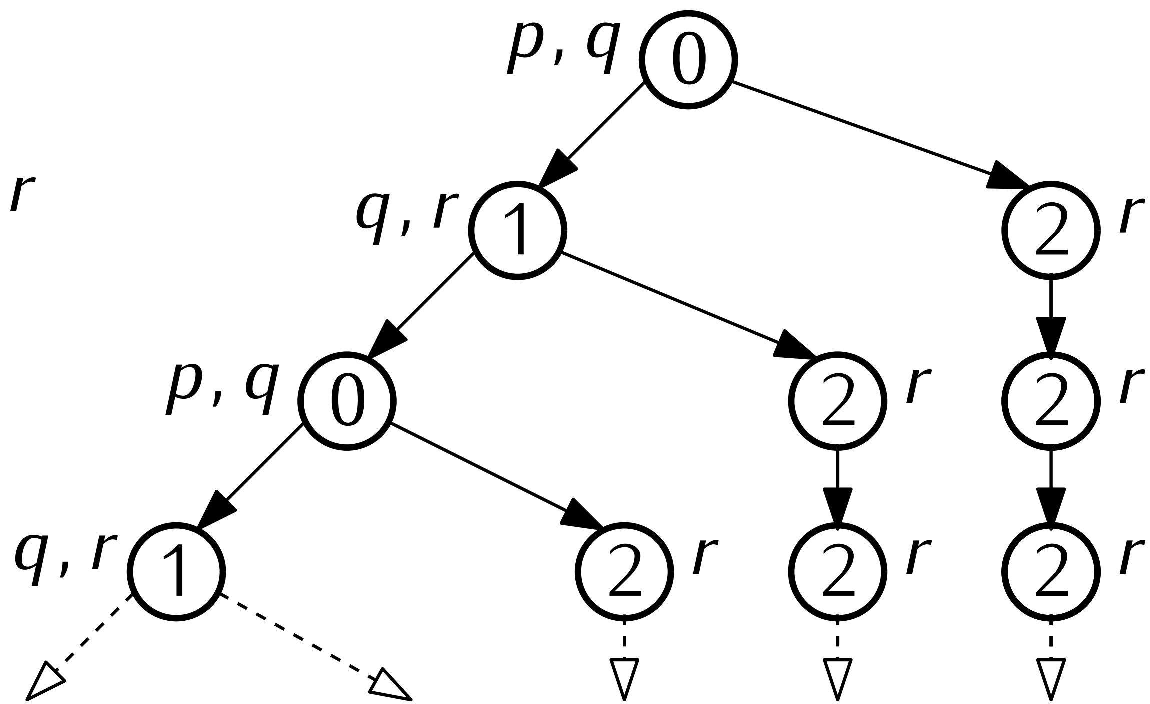


Computation Tree Logic Example Properties



Computation tree for $\mathcal{M}$



$\mathcal{M} \models p \wedge q$?

$\mathcal{M} \models \neg r$?

$\mathcal{M} \models EX(q \wedge r)$?

$\mathcal{M} \models AX(q \wedge r)$?

$\mathcal{M} \models \neg AX(q \wedge r)$?

$\mathcal{M} \models \neg EF(p \wedge r)$?

$\mathcal{M} \models EG \neg r$?

$\mathcal{M} \models AFq$?

$\mathcal{M} \models p\ AU\ r$?

$\mathcal{M} \models \neg(p \wedge q)\ EU\ r$?

Nested CTL operators

Consider the following sentence:

> No matter what, at some point the transition system reaches a state where there exists a path such that from that point on, p holds forever.

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Inevitably Eventually Possibly Always p

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A typical HW problem: translate a few sentences into CTL.

A basis for CTL

The CTL operators AG, AF, AX, EF, and EG can all be written using only negations ($\neg$), and EX, EU, and AU.

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$$AX\phi \equiv \ ????$$

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A typical HW problem: show that CTL operators can be written using only $\{\neg, EX, EU, AU\}$.

Another interesting property of CTL formulas

Any CTL operator $\oplus$ can be written using only $\oplus$, AX, EX, and Boolean connectives.

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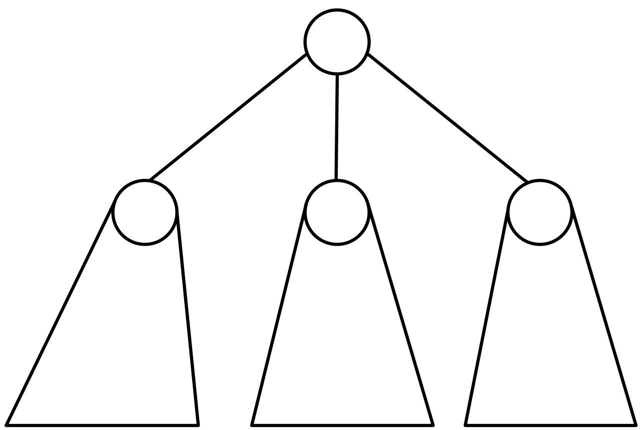

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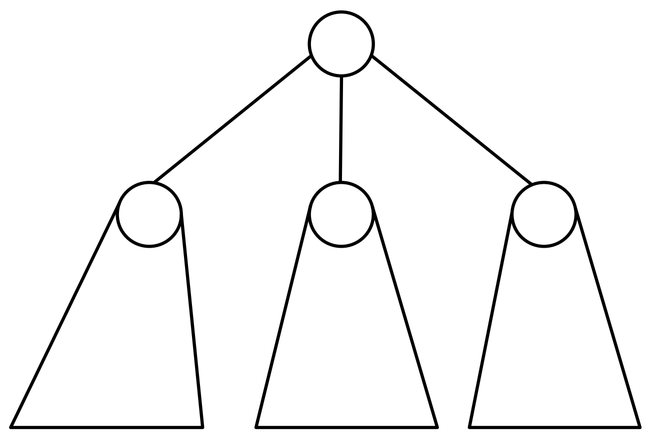

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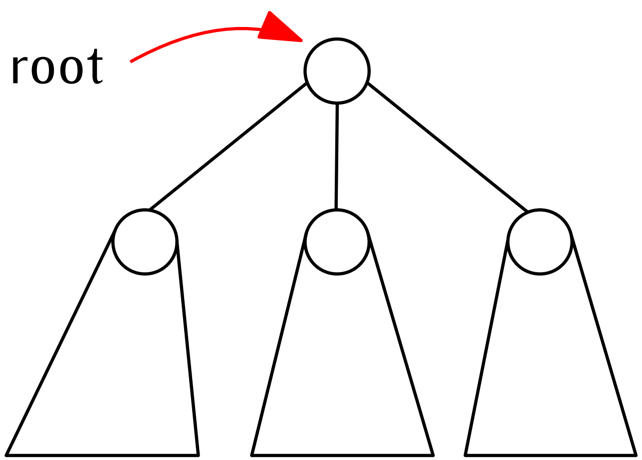

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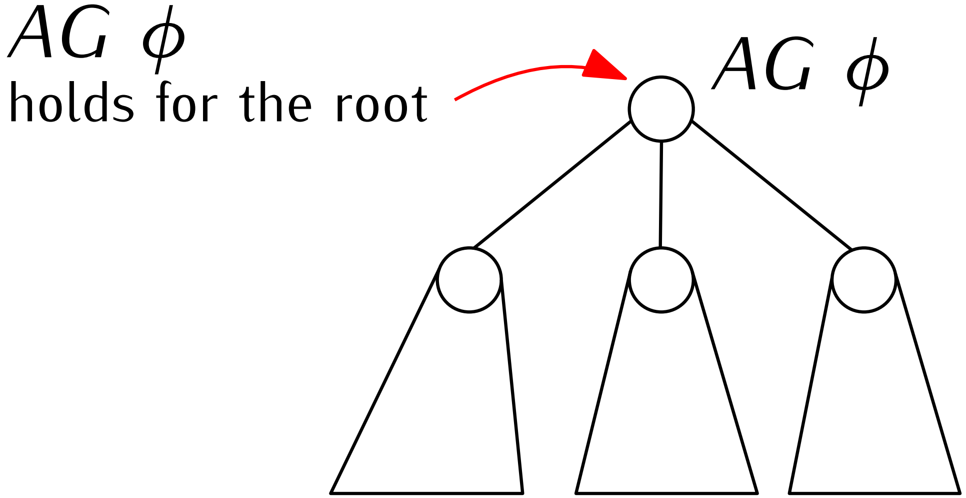

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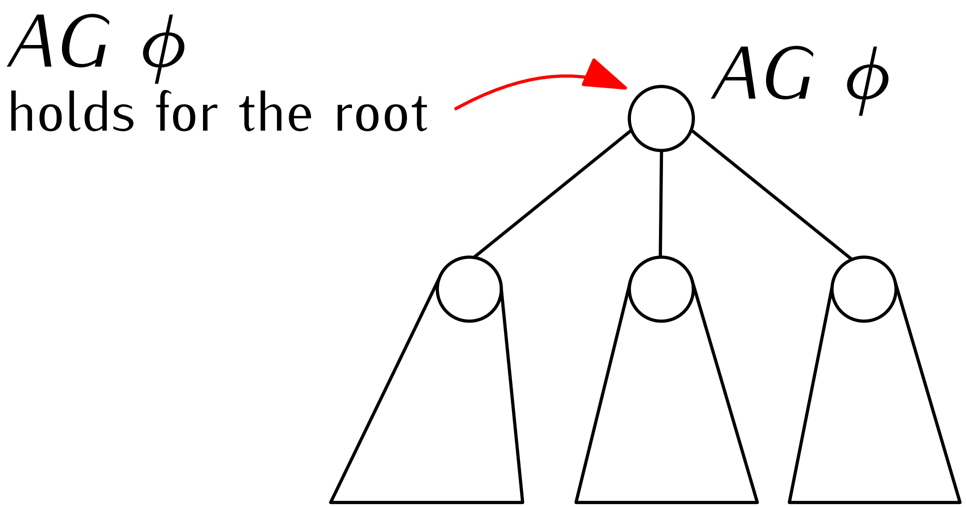

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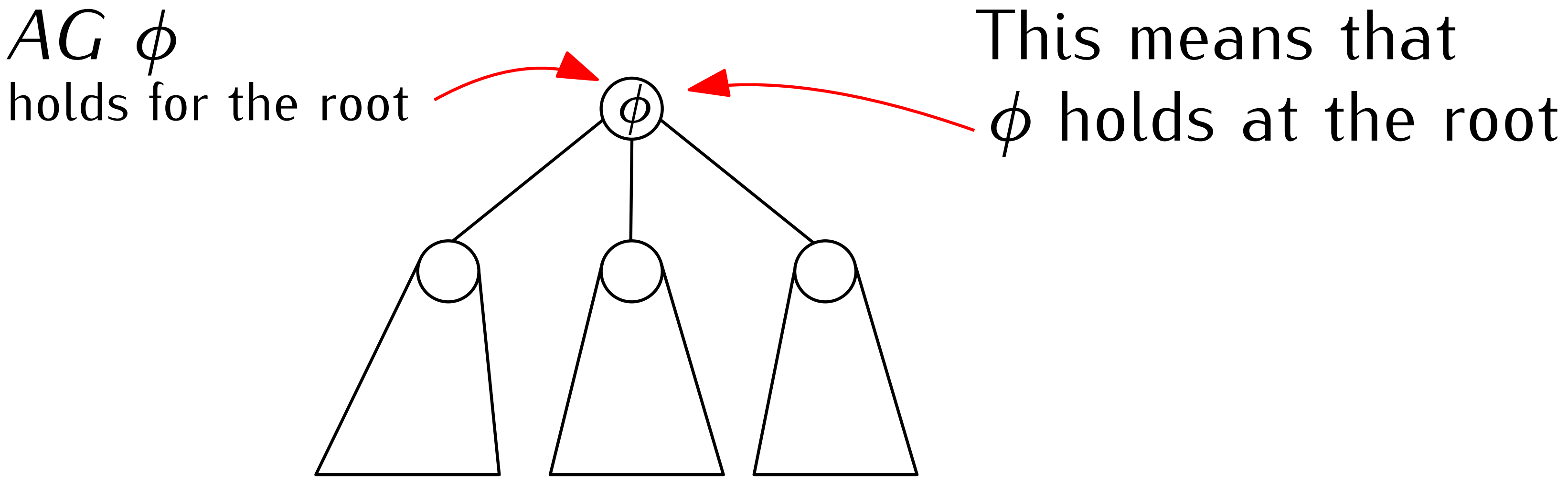

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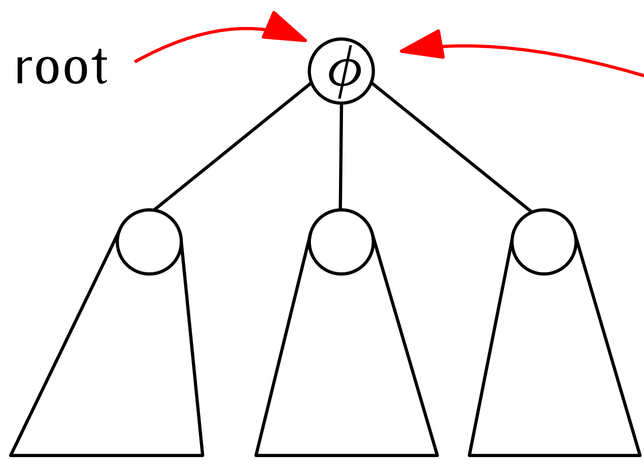

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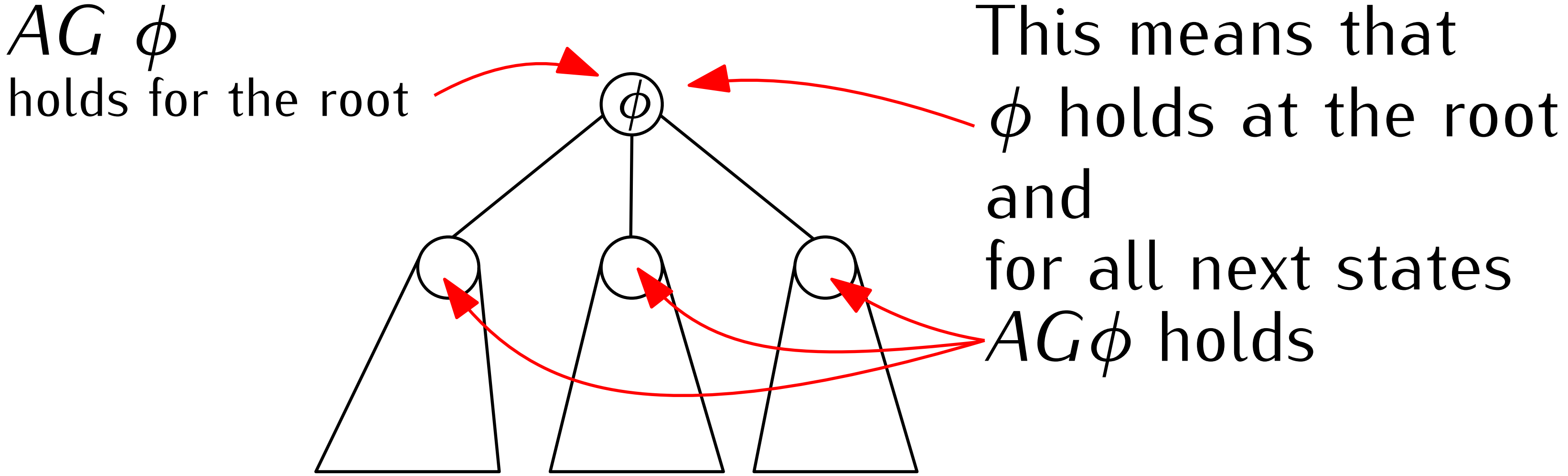

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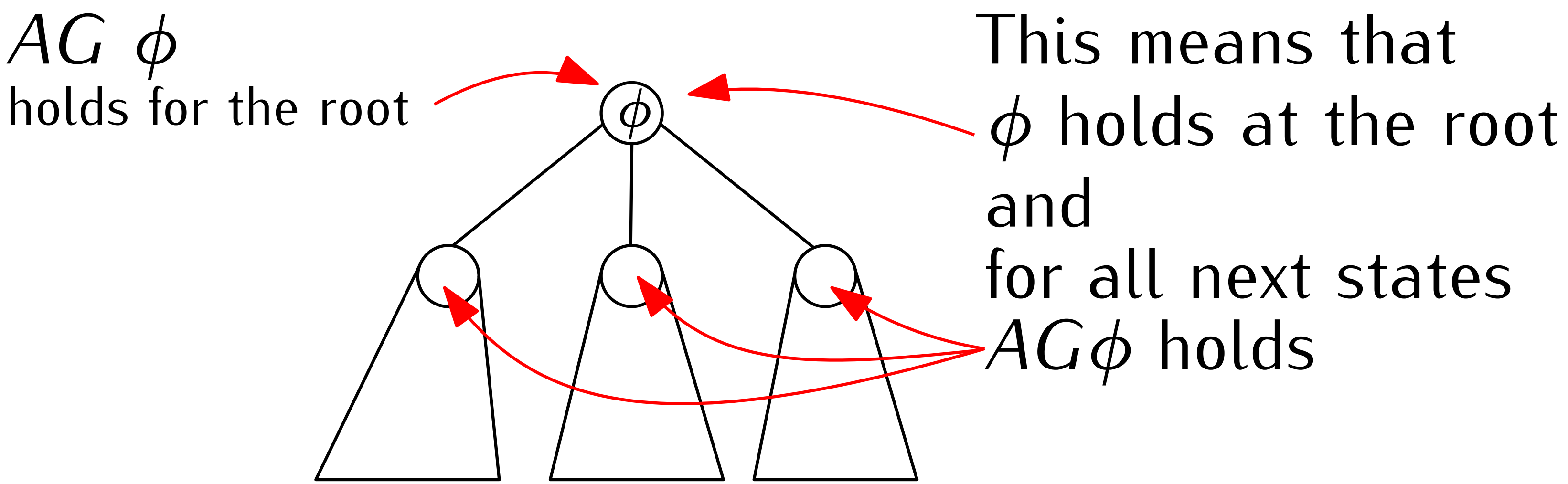

$$AG\ \phi \equiv \phi \wedge (AX\ AG\ \phi)$$

A typical HW problem: Do the same for EG, EF, AF, EU, AU.

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Given two temporal logic formulas α and β, when can we say that the two formulas are equivalent?

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In words, two formulas are not equivalent if we can find a transition system that satisifes one formula but not the other.

Showing that $\alpha \not\equiv \beta$

Consider these two temporal formulas

$F\ G\ p$ $\qquad\qquad\qquad\qquad$ $AF\ AG\ p$

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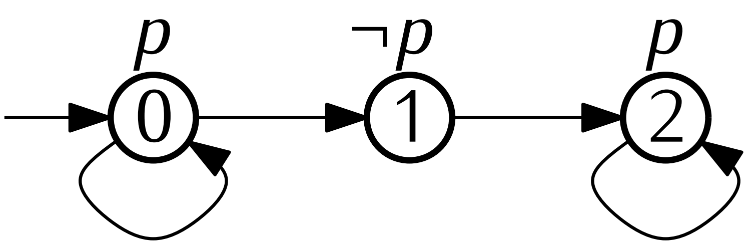

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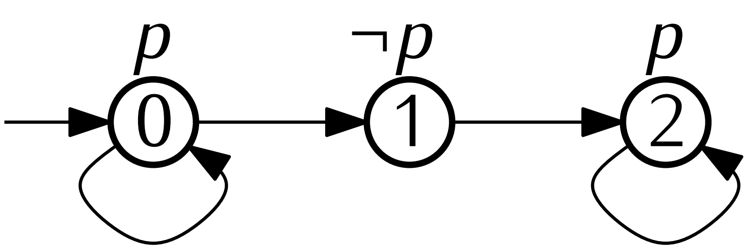

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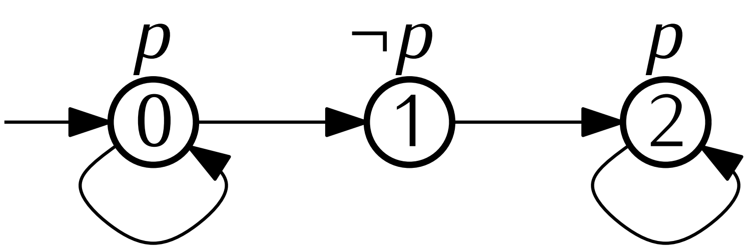

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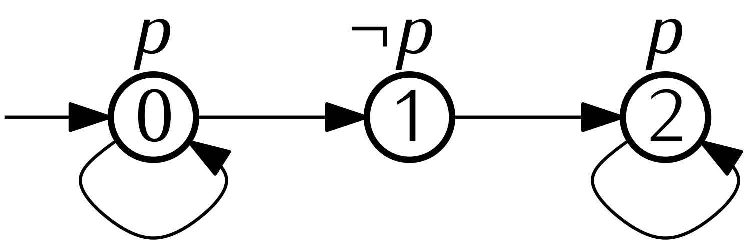

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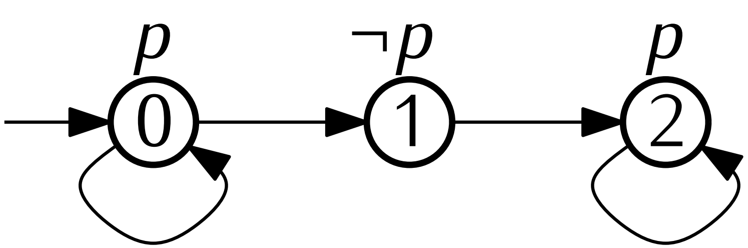

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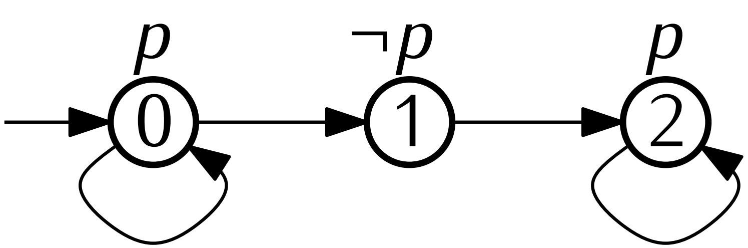

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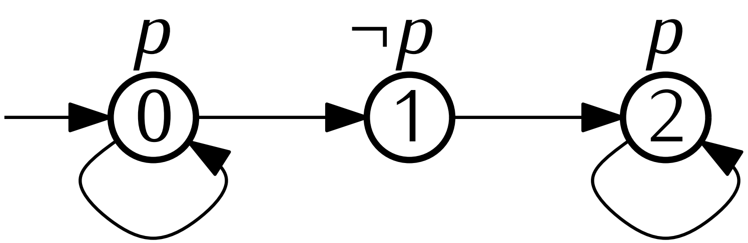

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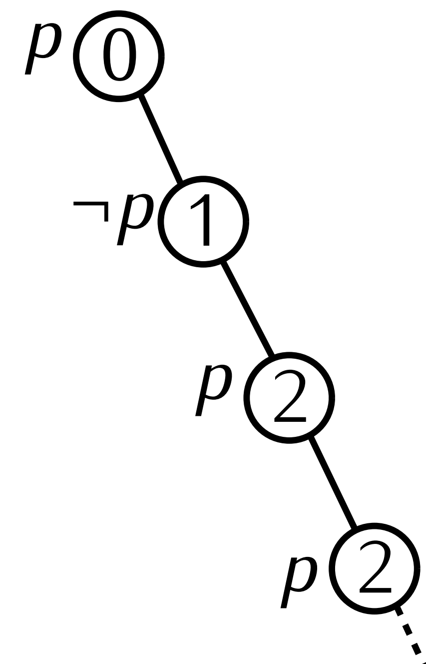

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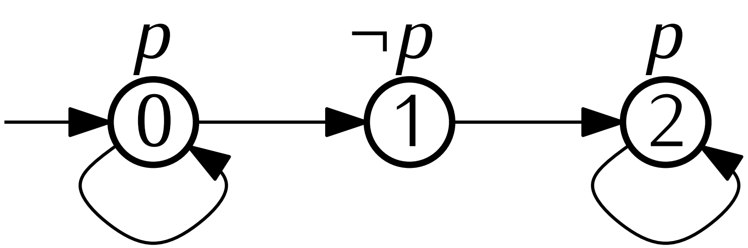

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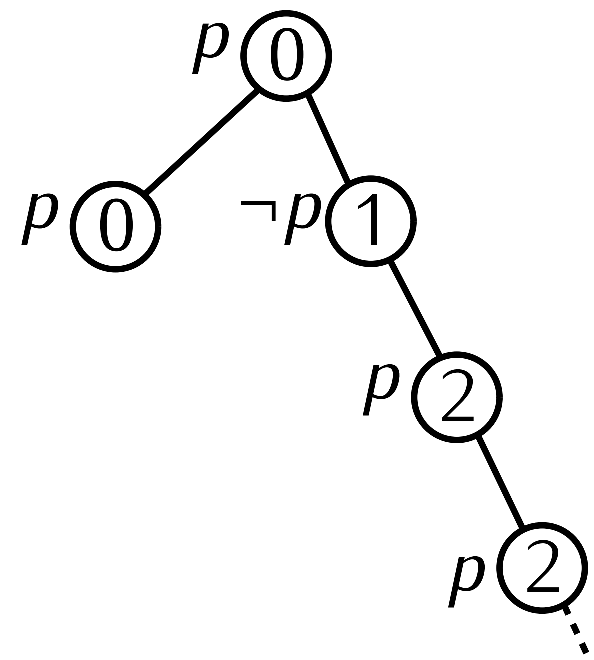

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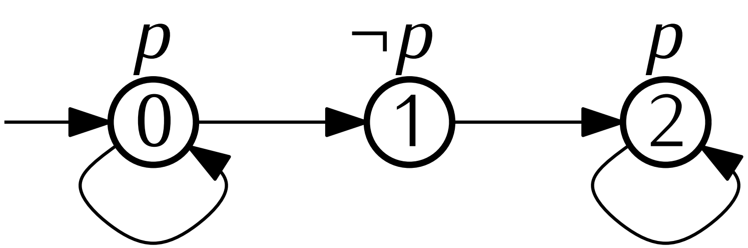

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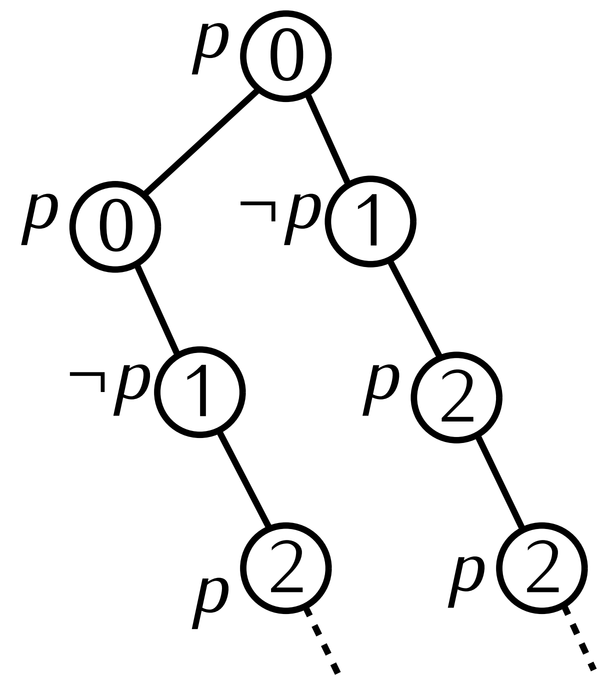

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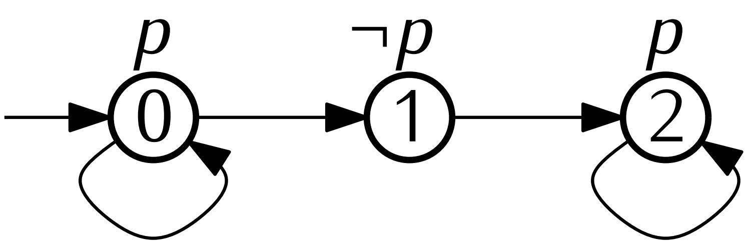

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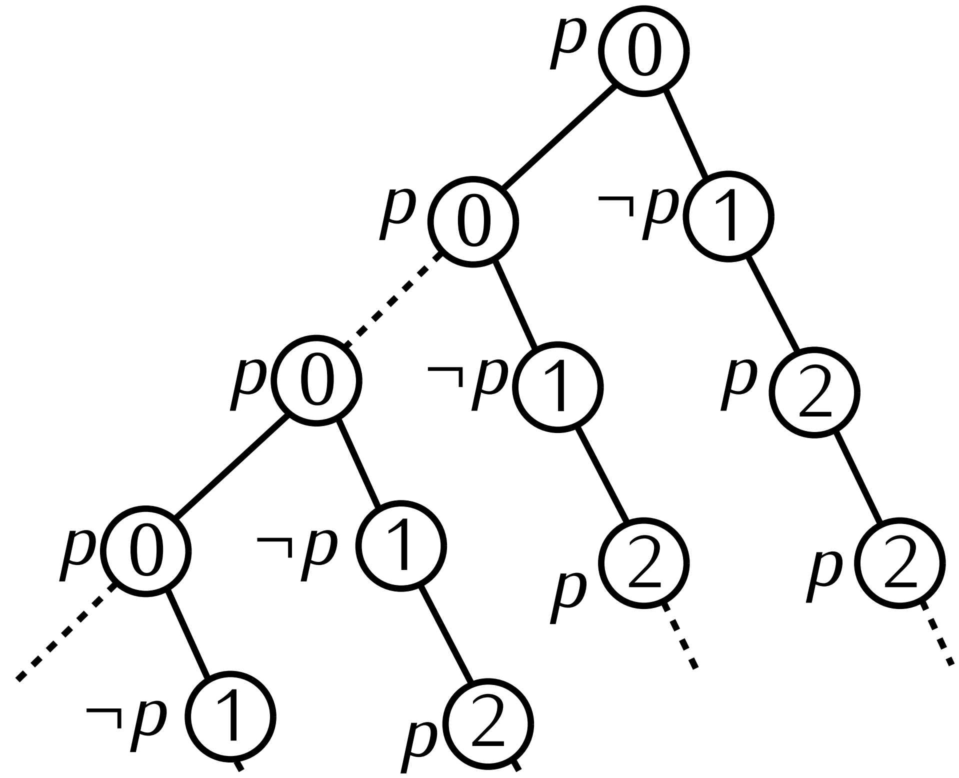

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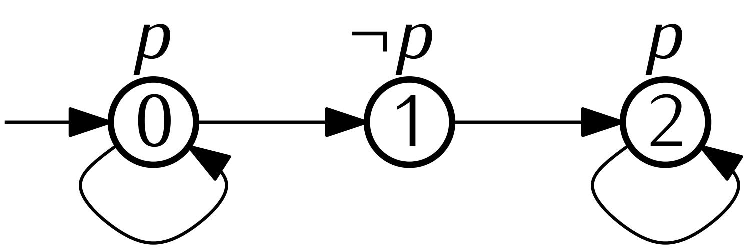

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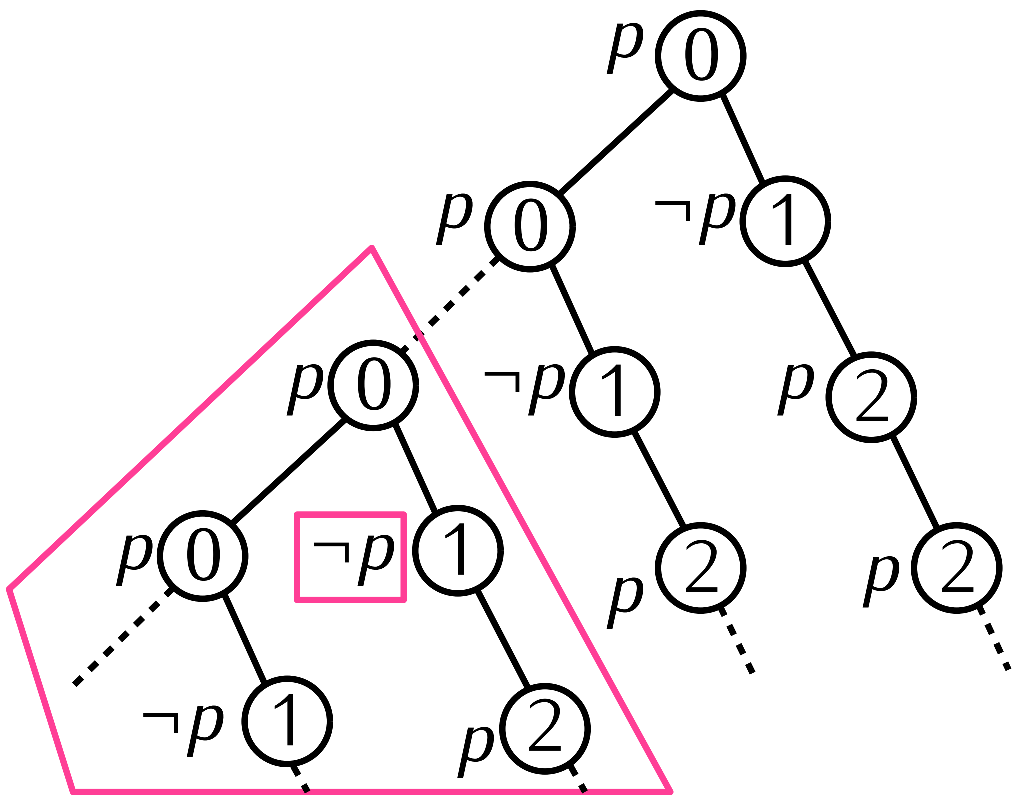

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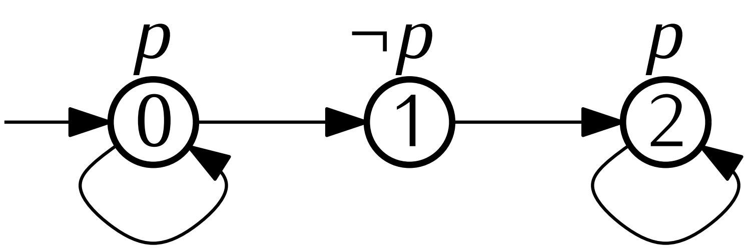

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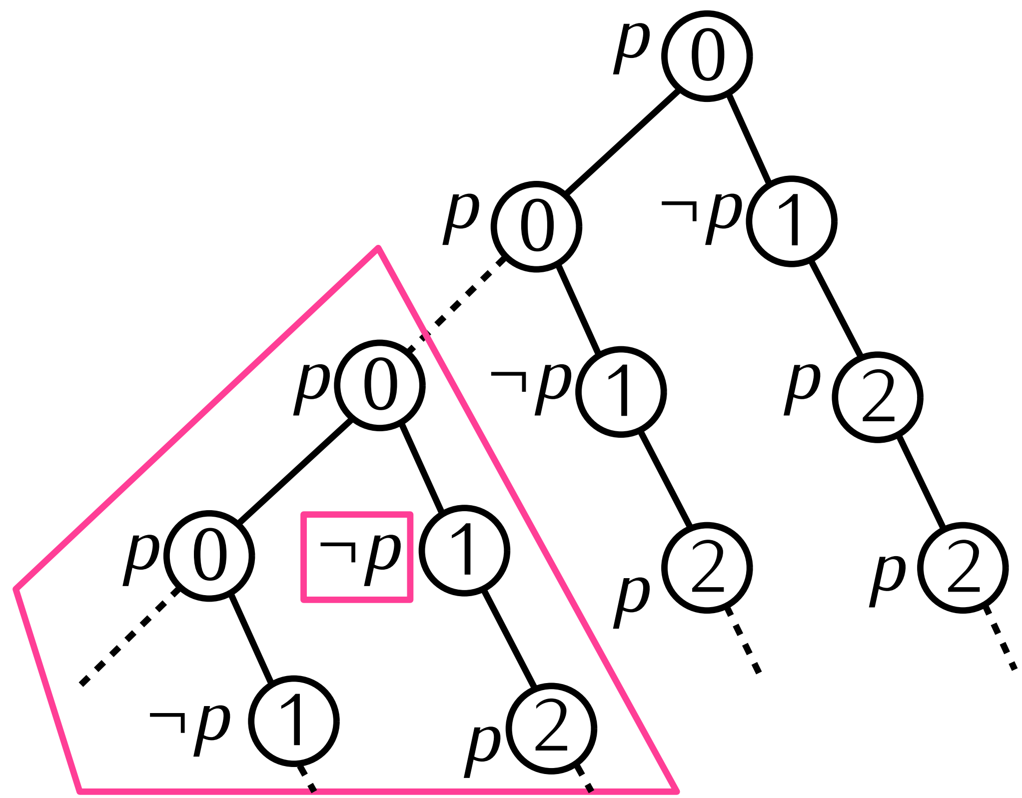

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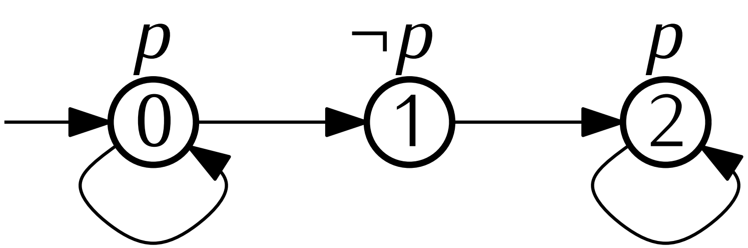

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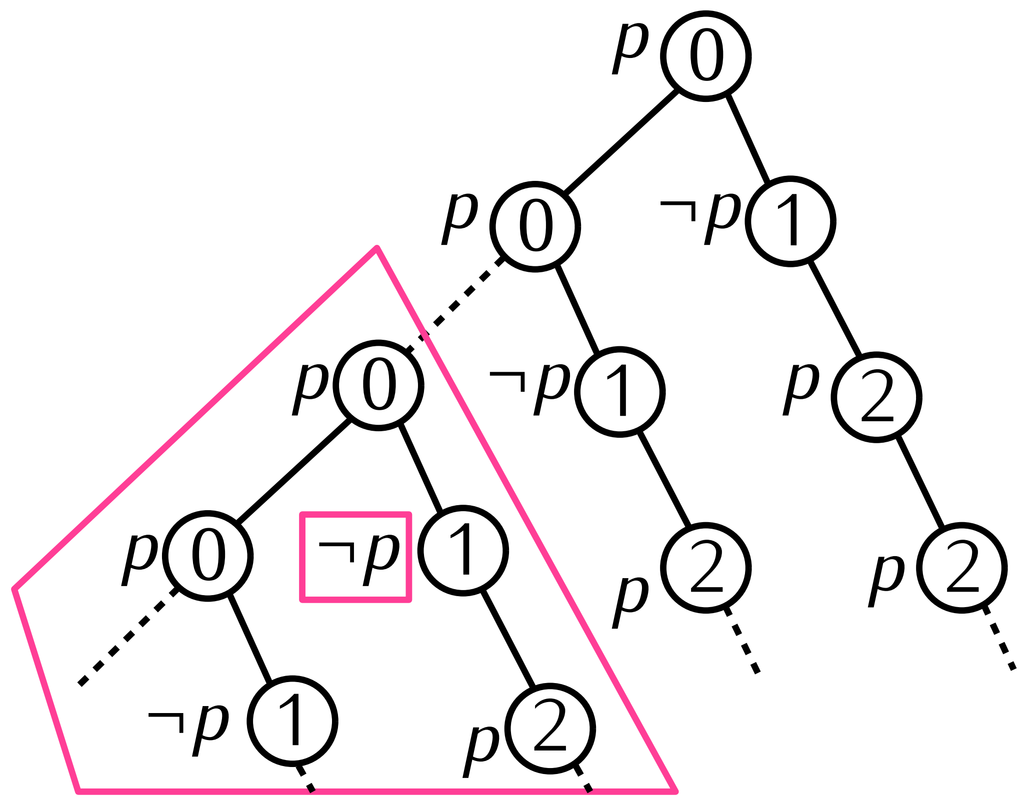

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