Appendix A. Proof of Lemma 1

Proof. The FTC mechanism is based on appropriate design of ν followed by $\mathbf{u}^{\mathrm{FTC}}$ stabilizing $\{\phi, \theta, z\}$ of the faulty quadrotor. Different forms of ν are presented by different FTC techniques as listed in Table A.6.

Controller	Auxiliary control input ν
PD [14]	$[\ddot{\phi}_d\ \ \ddot{\theta}_d\ \ \ddot{z}_d]^{\mathrm{T}} + K_{\mathrm{D}}\dot{\mathbf{e}} + K_{\mathrm{P}}\mathbf{e}$
SMC [12]	$[\ddot{\phi}_d\ \ \ddot{\theta}_d\ \ \ddot{z}_d]^{\mathrm{T}} + \lambda_{\mathrm{SMC}}\dot{\mathbf{e}} + K_{\mathrm{SMC}}\mathrm{sign}[s_\phi, s_\theta, s_z]^{\mathrm{T}}$
BS [15]	$[\hat{\ddot{\phi}}_d\ \ \hat{\ddot{\theta}}_d\ \ \hat{\ddot{z}}_d]^{\mathrm{T}} + K_{\mathrm{BSC}}\hat{\dot{\mathbf{e}}} + \mathbf{e}$

Table A.6: Auxiliary control designs by different FTC techniques

Here, $K_{\mathrm{P}}, K_{\mathrm{D}}, K_{\mathrm{SMC}}, \lambda_{\mathrm{SMC}}, K_{\mathrm{BSC}} > 0$ are gain parameters for the individual control designs. $s_\phi = \dot{e}_\phi + \lambda_{\mathrm{SMC}} e_\phi$, $s_\theta = \dot{e}_\theta + \lambda_{\mathrm{SMC}} e_\theta$, $s_z = \dot{e}_z + \lambda_{\mathrm{SMC}} e_z$ are the sliding surfaces, and sign$(\cdot)$ is the signum function for SMC. The hat symbol $\hat{\cdot}$ denotes that the command filter is used for estimating the virtual control derivative in BSC. Evidently, all the listed auxiliary control designs are akin to (13), resulting in $\mathbf{u}^{\mathrm{FTC}}$ by (12). If $\mathbf{u}^{\mathrm{FTC}} \in \mathbf{U}$, we can substitute (12) for $\mathbf{u}$ in (11), which yields the following tracking error dynamics:

$$\ddot{\mathbf{e}} = -K_1\dot{\mathbf{e}} - K_2\mathbf{e}. \tag{A.1}$$

Now, let us define a positive definite function V of the tracking errors and their derivatives.

$$V(\mathbf{e}, \dot{\mathbf{e}}) := \frac{1}{2}(\epsilon K_1 + K_2)\,\mathbf{e}^{\mathrm{T}}\mathbf{e} + \epsilon\dot{\mathbf{e}}^{\mathrm{T}}\mathbf{e} + \frac{1}{2}\dot{\mathbf{e}}^{\mathrm{T}}\dot{\mathbf{e}}$$

where $1 \gg \epsilon > 0$ is small value. Then, the time derivative of V is:

$$\dot{V}(\mathbf{e}, \dot{\mathbf{e}}) = (\epsilon K_1 + K_2)\,\mathbf{e}^{\mathrm{T}}\dot{\mathbf{e}} + \epsilon\ddot{\mathbf{e}}^{\mathrm{T}}\mathbf{e} + \epsilon\dot{\mathbf{e}}^{\mathrm{T}}\dot{\mathbf{e}} + \ddot{\mathbf{e}}^{\mathrm{T}}\dot{\mathbf{e}}. \tag{A.2}$$

And plugging (A.1) to (A.2) gives:

$$\dot{V}(\mathbf{e}, \dot{\mathbf{e}}) = -\epsilon K_2\mathbf{e}^{\mathrm{T}}\mathbf{e} - (K_1 - \epsilon)\,\dot{\mathbf{e}}^{\mathrm{T}}\dot{\mathbf{e}}.$$

Since is ϵ is sufficiently small such that $K_1 > \epsilon$, this immediately follows that:

$$\dot{V}(\mathbf{e}, \dot{\mathbf{e}}) < 0, \quad \forall \mathbf{e}, \dot{\mathbf{e}} \neq \mathbf{0}. \tag{A.3}$$

Thus, if $\mathbf{u}^{\mathrm{FTC}} \in \mathbf{U}$ so that $\mathbf{u} \equiv \mathbf{u}^{\mathrm{FTC}}$ and (A.1) hold, the existence of the positive definite Lyapunov function V satisfying (A.3) guarantees the asymptotic stability of $\mathbf{e}$ by the Lyapunov theorem. ■

Appendix B. Derivation of Eq. (21)

Proof. At the PE, the dynamics of yaw rate r from (3) can be rewritten as $(-k_r r_{\mathrm{PE}} + u_{r,\mathrm{PE}})/I_z = 0$ which gives the relationship between r_{PE} and $u_{r,\mathrm{PE}}$:

$$r_{\mathrm{PE}} = \frac{u_{r,\mathrm{PE}}}{k_r}. \tag{B.1}$$

Recalling (6), $u_{r,\mathrm{PE}}$ can be determined by:

$$u_{r,\mathrm{PE}} = b\Big(u_{f,\mathrm{PE}} - \frac{2}{l}u_{p,\mathrm{PE}}\Big) \tag{B.2}$$

where $u_{f,\mathrm{PE}}$ and $u_{p,\mathrm{PE}}$ at can be derived from (24):

$$u_{f,\mathrm{PE}} = \frac{mg}{\cos(\phi_{\mathrm{PE}})}, \qquad u_{p,\mathrm{PE}} = (I_z - I_y) r_{\mathrm{PE}}^2 \tan(\phi_{\mathrm{PE}}). \tag{B.3}$$

Substituting (B.2) and (B.3) to (B.1), we obtain a quadratic equation in terms of r_{PE}:

$$\frac{2}{l}(I_z - I_y) r_{\mathrm{PE}}^2 \tan(\phi_{\mathrm{PE}}) + \frac{k_r}{b} r_{\mathrm{PE}} - \frac{mg}{\cos(\phi_{\mathrm{PE}})} = 0. \tag{B.4}$$

Solving (B.4) and using the positive solution results:

$$r_{\mathrm{PE}} = \frac{-\frac{k_r l}{b} + l\sqrt{(\frac{k_r}{b})^2 + \frac{8mg(I_z - I_y)\tan(\phi_{\mathrm{PE}})}{l\cos(\phi_{\mathrm{PE}})}}}{4(I_z - I_y)\tan(\phi_{\mathrm{PE}})}. \tag{B.5}$$

One can notice that (B.5) has a singularity when $\phi_{\mathrm{PE}} = 0$. For this case, we set $\phi_{\mathrm{PE}} = 0$ into (B.3) resulting in:

$$u_{f,\mathrm{PE}} = mg, \qquad u_{p,\mathrm{PE}} = 0. \tag{B.6}$$

Substituting (B.2) and (B.6) to (B.1) yields the solution for r_{PE} when $\phi_{\mathrm{PE}} = 0$:

$$r_{\mathrm{PE}} = \frac{bmg}{k_r}.$$

■

Appendix C. Proof of Proposition 1

Proof. Applying the boundary condition (26) to the trajectory (25) results in the following system of equations:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & T & T^2 & T^3 & T^4 & T^5 \\ 0 & 1 & 2T & 3T^2 & 4T^3 & 5T^4 \\ 0 & 0 & 2 & 6T & 12T^2 & 20T^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} \phi(t_0) \\ \dot{\phi}(t_0) \\ \ddot{\phi}(t_0) \\ \phi_{\mathrm{PE}} \\ 0 \\ 0 \end{bmatrix}. \tag{C.1}$$

Solving (C.1), the coefficients of (25) that comply with the boundary condition can be obtained.

■

Appendix D. Proof of Lemma 2

Proof. To determine the upper and lower bounds of $r(t)$, namely $\overline{r}(t)$ and $\underline{r}(t)$, we turn to the dynamics of the yaw rate as described by (3). Specifically, these bounds can be extracted by employing the constant upper and lower limits of $u_r(t)$, represented as $\overline{u}_r$ and $\underline{u}_r$, respectively. This relationship is captured by the following differential equations:

$$\dot{\overline{r}} = (-k_r\overline{r} + \overline{u}_r)/I_z, \quad \dot{\underline{r}} = (-k_r\underline{r} + \underline{u}_r)/I_z. \tag{D.1}$$

With the aforementioned bounds $\overline{u}_r$ and $\underline{u}_r$ computed, we can solve (D.1) to achieve:

$$\overline{r}(t) = \frac{\overline{u}_r}{k_r}\left(1 - e^{-\frac{k_r}{I_z}t}\right), \quad \underline{r}(t) = \frac{\underline{u}_r}{k_r}\left(1 - e^{-\frac{k_r}{I_z}t}\right). \tag{D.2}$$

Utilizing (6) and (24), the upper bound of $u_r(t)$ can be computed as follows:

$$\overline{u}_r = b\left(\max(u_f(t)) - \frac{2}{l}\min(u_p(t))\right) = \frac{bmg}{\cos(\max(|\phi_a(t)|))}. \tag{D.3}$$

On the other hand, deducing the lower bound for $u_r(t)$ is more involved due to $\max(u_p(t))$ being unknown. Hence, the constant upper bound of $u_p(t)$ is obtained instead of $\max(u_p(t))$. By utilizing the second inequality condition from (24) and $\max(\overline{r}(t)) = \overline{u}_r/k_r$ from (D.2) and (D.3), the following upper bound is achieved:

$$\overline{u}_p = I_x\Big(\frac{I_z - I_y}{I_x}(\frac{\overline{u}_r}{k_r})^2 \tan(\max(\phi_a(t))) + \ddot{\phi}_a(t_{c1})\Big).$$

We can then proceed to compute the lower bound of $u_r(t)$:

$$\underline{u}_r = b\left(\min(u_f(t)) - \frac{2}{l}\overline{u}_p\right) = b\left(\frac{mg}{\cos(\min(|\phi_a(t)|))} - \frac{2}{l}\overline{u}_p\right).$$

■

Appendix E. Proof of Proposition 2

Proof. The second inequality condition (24) at the PE state yields $0 \le (I_z - I_y)r_{\mathrm{PE}}^2 \tan\phi_{\mathrm{PE}} \le l\overline{F}$ from which we can deduce the lower bound for the roll angle at the PE state:

$$0 \le \phi_{\mathrm{PE}}. \tag{E.1}$$

By leveraging the inequality condition (E.1), along with Assumption 1, and the ASMP trajectory given by (25) with coefficients (27), we can obtain $\max(\phi_a(t)) = \phi_{\mathrm{PE}}$. Drawing the first inequality condition from (24), we can deduce the upper bound of the roll angle at the PE:

$$\phi_{\mathrm{PE}} \le \cos^{-1}(\frac{mg}{2\overline{F}}).$$

The second inequality condition in (24) is separated into upper and lower bounds by applying the bounds of $r(t)$ from Lemma 2:

$$\begin{aligned} I_x\left(\frac{I_z - I_y}{I_x}\bar{r}(t)^2 \tan(\phi_a(t)) + \ddot{\phi}_a(t_{c1})\right) &\leq l\bar{F}, \\ 0 \leq \frac{I_z - I_y}{I_x}\underline{r}(t)^2 \tan(\phi_a(t)) + \ddot{\phi}_a(t_{c3}). \end{aligned} \tag{E.2}$$

Given the ASMP trajectory with coefficients (27), the critical time incidents defined in (28) can be computed by:

$$\begin{aligned} t_{c1} &= (3 - \sqrt{3})T/6, \\ t_{c2} &= T/2, \\ t_{c3} &= (3 + \sqrt{3})T/6. \end{aligned}$$

To design the trajectory that always guarantees the first inequality condition from (E.2), the maximum values for each element are plugged in as:

$$I_x\left(\frac{I_z - I_y}{I_x}(\frac{\bar{u}_r}{k_r})^2 \tan(\phi_{\mathrm{PE}}) + \ddot{\phi}_a(t_{c1})\right) \leq l\bar{F}.$$

It is worth noting that $u_p(t)$ monotonically increases, so $\frac{I_z - I_y}{I_x}\underline{r}(t)^2 \tan(\phi_a(t))$ at t_{c2} must be greater than the minimum value of $\ddot{\phi}_a(t)$ to guarantee the second inequality condition from (E.2):

$$0 \leq \frac{I_z - I_y}{I_x}\underline{r}(t_{c2})^2 \tan(\phi_a(t_{c2})) + \ddot{\phi}_a(t_{c3}).$$

■

Appendix F. Proof of Proposition 3

Proof. The yaw angle during $[t_p, t_f]$ can be approximated as:

$$\psi(t_p) \approx \psi(t_0) + \dot{\psi}_{\mathrm{PE}}(t_p - t_0), \quad \psi(t) \approx \psi(t_p) + \dot{\psi}_{\mathrm{PSMP}}(t - t_p). \tag{F.1}$$

Substituting (F.1) and u_f from (31) into (32) yields:

$$
\begin{aligned}
\dot{v}_x(t) &= -\frac{k_t}{m}v_x(t) + g\tan(\phi_d)\sin(\psi(t_p) + \dot{\psi}_{\text{PSMP}}(t - t_p)),\\
\dot{v}_y(t) &= -\frac{k_t}{m}v_y(t) - g\tan(\phi_d)\cos(\psi(t_p) + \dot{\psi}_{\text{PSMP}}(t - t_p)), \quad \forall t \in [t_p, t_f].
\end{aligned} \tag{F.2}
$$

Due to the nonlinearity posed by $\tan(\phi_d)$, obtaining an exact analytical solution for the differential equation is difficult. To handle this, we employ the Taylor series expansion:

$$
\tan(\phi_d) = \tan(\phi_{\text{PE}} + a[\cos(\omega_p(t - t_p)) - 1]) \approx \tan(\phi_{\text{PE}}) + \frac{a[\cos(\omega_p(t - t_p)) - 1]}{\cos^2(\phi_{\text{PE}})}. \tag{F.3}
$$

Substituting (F.3) into (F.2), the expanded differential equations can be obtained:

$$
\begin{aligned}
\dot{v}_x(t) = &-\frac{k_t}{m}v_x(t) + \left[g\tan(\phi_{\text{PE}}) - \frac{ga}{\cos^2(\phi_{\text{PE}})}\right]\sin(\dot{\psi}_{\text{PSMP}}(t - t_p) + \psi(t_p))\\
&+ \frac{ga}{2\cos^2(\phi_{\text{PE}})}\Big[\cos((\omega_p - \dot{\psi}_{\text{PSMP}})t - (\omega_p - \dot{\psi}_{\text{PSMP}})t_p - \psi(t_p) + \frac{\pi}{2})\\
&- \cos((\omega_p + \dot{\psi}_{\text{PSMP}})t - (\omega_p + \dot{\psi}_{\text{PSMP}})t_p + \psi(t_p) + \frac{\pi}{2})\Big],\\
\dot{v}_y(t) = &-\frac{k_t}{m}v_y(t) - \left[g\tan(\phi_{\text{PE}}) - \frac{ga}{\cos^2(\phi_{\text{PE}})}\right]\cos(\dot{\psi}_{\text{PSMP}}(t - t_p) + \psi(t_p))\\
&- \frac{ga}{2\cos^2(\phi_{\text{PE}})}\Big[\sin((\omega_p - \dot{\psi}_{\text{PSMP}})t - (\omega_p - \dot{\psi}_{\text{PSMP}})t_p - \psi(t_p) + \frac{\pi}{2})\\
&+ \sin((\omega_p + \dot{\psi}_{\text{PSMP}})t - (\omega_p + \dot{\psi}_{\text{PSMP}})t_p + \psi(t_p) + \frac{\pi}{2})\Big], \quad \forall t \in [t_p, t_f].
\end{aligned} \tag{F.4}
$$

Recalling (19), the orbital period of the faulty quadrotor at PSMP can be approximated as $T_{\text{PSMP}} \approx \frac{2\pi}{|\dot{\psi}_{\text{PSMP}}|}$. This implies that the macroscopic behavior of the faulty quadrotor will be straight and level flight only if the condition $\omega_p = \dot{\psi}_{\text{PSMP}}$ holds. Incorporating this condition, (F.4) can be reformulated as follows:

$$
\begin{aligned}
\dot{v}_x(t) = &-\frac{k_t}{m}v_x(t) + \left[g\tan(\phi_{\text{PE}}) - \frac{ga}{\cos^2(\phi_{\text{PE}})}\right]\sin(\dot{\psi}_{\text{PSMP}}(t - t_p) + \psi(t_p))\\
&- \frac{ga}{2\cos^2(\phi_{\text{PE}})}\Big[\cos(2\dot{\psi}_{\text{PSMP}}(t - t_p) + \psi(t_p) + \frac{\pi}{2}) + \cos(\psi(t_p) - \frac{\pi}{2})\Big],\\
\dot{v}_y(t) = &-\frac{k_t}{m}v_y(t) - \left[g\tan(\phi_{\text{PE}}) - \frac{ga}{\cos^2(\phi_{\text{PE}})}\right]\cos(\dot{\psi}_{\text{PSMP}}(t - t_p) + \psi(t_p)) - \frac{ga}{2\cos^2(\phi_{\text{PE}})}\\
&\Big[\sin(2\dot{\psi}_{\text{PSMP}}(t - t_p) + \psi(t_p) + \frac{\pi}{2}) + \sin(\psi(t_p) - \frac{\pi}{2})\Big], \quad \forall t \in [t_p, t_f].
\end{aligned} \tag{F.5}
$$

The solution to the differential equation (F.5) is shown as follows:

$$
\begin{aligned}
v_{\{x,y\}}(t) = & \frac{1}{(A(A^2+C^2)(A^2+F^2))}\Big[A^5 v(t_p)e^{At} + A^4(He^{At} - E\cos(G+Ft) - B\sin(D+Ct) - H \\
& + Ee^{At}\cos(G) + Be^{At}\sin(D)) + A^3(C^2 v(t_p)e^{At} + F^2 v(t_p)e^{At} - BC\cos(D+Ct) \\
& + EF\sin(G+Ft)BCe^{At}\cos(D) - EFe^{At}\sin(G)) + A^2(-C^2H - C^2E\cos(G+Ft) \\
& - BF^2\sin(D+Ct) - F^2H + C^2He^{At} + F^2He^{At} + C^2Ee^{At}\cos(G) + BF^2e^{At}\sin(D)) \\
& + A(C^2EF\sin(G+Ft) + C^2F^2v(t_p)e^{At} - BCF^2\cos(D+Ct) - C^2EFe^{At}\sin(G) \\
& + BCF^2e^{At}\cos(D)) + C^2F^2He^{At} - C^2F^2H\Big], \quad \forall t \in [t_p, t_f].
\end{aligned}
$$

with the coefficients for $v_x(t)$ and $v_y(t)$ defined as:

$$
\begin{cases}
A_x = -\dfrac{k_t}{m} \\
B_x = g\tan(\phi_{\text{PE}}) - \dfrac{ga}{\cos^2(\phi_{\text{PE}})} \\
C_x = \dot{\psi}_{\text{PSMP}} \\
D_x = -\dot{\psi}_{\text{PSMP}} t_p + \psi(t_p) \\
E_x = -\dfrac{ga}{2\cos^2(\phi_{\text{PE}})} \\
F_x = 2\dot{\psi}_{\text{PSMP}} \\
G_x = -2\dot{\psi}_{\text{PSMP}} t_p + \psi(t_p) + \dfrac{\pi}{2} \\
H_x = \dfrac{ga\cos\left(\psi(t_p) - \frac{\pi}{2}\right)}{2\cos^2(\phi_{\text{PE}})},
\end{cases}
\qquad
\begin{cases}
A_y = -\dfrac{k_t}{m} \\
B_y = -\dfrac{ga}{2\cos^2(\phi_{\text{PE}})} \\
C_y = 2\dot{\psi}_{\text{PSMP}} \\
D_y = -2\dot{\psi}_{\text{PSMP}} t_p + \psi(t_p) + \dfrac{\pi}{2} \\
E_y = -g\tan(\phi_{\text{PE}}) + \dfrac{ga}{\cos^2(\phi_{\text{PE}})} \\
F_y = \dot{\psi}_{\text{PSMP}} \\
G_y = -\dot{\psi}_{\text{PSMP}} t_p + \psi(t_p) \\
H_y = \dfrac{ga\sin\left(\psi(t_p) - \frac{\pi}{2}\right)}{2\cos^2(\phi_{\text{PE}})}
\end{cases}.
$$

This provides the mean velocity of the faulty quadrotor at the PE state:

$$
v_{x,\text{PSMP}} = \frac{mga\cos\left(\psi(t_p) - \frac{\pi}{2}\right)}{2k_t\cos^2(\phi_{\text{PE}})}, \quad v_{y,\text{PSMP}} = \frac{mga\sin\left(\psi(t_p) - \frac{\pi}{2}\right)}{2k_t\cos^2(\phi_{\text{PE}})}. \tag{F.6}
$$

Expressing (F.6) in polar coordinates yields:

$$
|v_{\text{PSMP}}| = \frac{mga}{2k_t\cos^2(\phi_{\text{PE}})}, \quad \angle v_{\text{PSMP}} = \psi(t_p) - \frac{\pi}{2}. \tag{F.7}
$$

Considering the fact that the quadrotor is initially at the PE state and its orbit radius is substantially smaller than the distance that the quadrotor will traverse during the PSMP phase, we can approximate $x(t_p) \approx x(t_0)$ and $y(t_p) \approx y(t_0)$. Using (F.1) and the direction of the velocity in (F.7), the time t_p directing the faulty quadrotor toward the safe zone can be computed as follows:

$$t_p = t_0 + \frac{1}{\dot{\psi}_{\text{PE}}}\left[\tan^{-1}\left(\frac{y_d - y(t_0)}{x_d - x(t_0)}\right) - \psi(t_0) + \frac{\pi}{2}\right] + \frac{2\pi n}{\dot{\psi}_{\text{PSMP}}}, \quad n \in \{0, 1, 2, \cdots\}, \quad \text{s.t. } t_p \geq t_0$$

which satisfies $\angle v_{\text{PSMP}} = \tan^{-1}\left(\frac{y_d - y(t_0)}{x_d - x(t_0)}\right)$. Based on the magnitude of the velocity in (F.7) and the time period $[t_p, t_f]$, the parameter a can be computed such that the faulty quadrotor reaches the safe zone at t_f:

$$a = \frac{2k_t \cos^2(\phi_{\text{PE}})}{mg(t_f - t_p)} \sqrt{\left(x_d - x(t_p)\right)^2 + \left(y_d - y(t_p)\right)^2}.$$

■

Appendix G. Proof of Proposition 4

Proof. Applying (30) $\forall t \in [t_p, t_f]$, the inequality condition regarding u_p in (31) can be rewritten as:

$$0 \leq I_x\left(\frac{I_z - I_y}{I_x} r^2_{\text{PSMP}} \tan(\phi_{\text{PE}} + \phi_p) + \ddot{\phi}_p\right) \leq l\overline{F}, \quad \forall t \in [t_p, t_f]. \tag{G.1}$$

And the second-order time derivative of (33) is given by:

$$\ddot{\phi}_p(t) = -a\dot{\psi}^2_{\text{PSMP}} \cos(\dot{\psi}_{\text{PSMP}}(t - t_p)). \tag{G.2}$$

Since (33) and (G.2) are sinusoidal signals, respectively having amplitudes a and $a\dot{\psi}^2_{\text{PSMP}}$ with offsets $-a$ and 0, their bounds are:

$$\begin{aligned} -2a \leq \phi_p \leq 0, \\ -a\dot{\psi}^2_{\text{PSMP}} \leq \ddot{\phi}_p \leq a\dot{\psi}^2_{\text{PSMP}}. \end{aligned} \tag{G.3}$$

Substituting the upper and lower bounds from (G.3) into (G.1), the upper and lower bounds of u_p at the PSMP phase can be determined as:

$$
\begin{aligned}
&I_x\Big(\frac{I_z - I_y}{I_x} r_{\mathrm{PSMP}}^2 \tan(\phi_{\mathrm{PE}}) + \dot{\psi}_{\mathrm{PSMP}}^2 a\Big) \le l\overline{F}, \\
&0 \le \frac{I_z - I_y}{I_x} r_{\mathrm{PSMP}}^2 \tan(\phi_{\mathrm{PE}} - 2a) - \dot{\psi}_{\mathrm{PSMP}}^2 a.
\end{aligned}
\tag{G.4}
$$

However, it is not possible to solve for a in the second inequality of (G.4) as a closed form expression, due to the non-algebraic nature of the tangent function. Hence, the Taylor series expansion is applied as follows:

$$
0 \le \frac{I_z - I_y}{I_x} r_{\mathrm{PSMP}}^2 \big(\tan(\phi_{\mathrm{PE}}) - 2a \sec^2(\phi_{\mathrm{PE}})\big) - \dot{\psi}_{\mathrm{PSMP}}^2 a. \tag{G.5}
$$

Rearranging the first inequalities of (G.4) and (G.5) with respect to the tuning parameter a gives:

$$
\begin{aligned}
&a \le \frac{l\overline{F} - (I_z - I_y) r_{\mathrm{PSMP}}^2 \tan(\phi_{\mathrm{PSMP}})}{I_x \dot{\psi}_{\mathrm{PSMP}}^2}, \\
&a \le \frac{(I_z - I_y) r_{\mathrm{PSMP}}^2 \tan(\phi_{\mathrm{PSMP}})}{2(I_z - I_y) r_{\mathrm{PSMP}}^2 \sec^2(\phi_{\mathrm{PSMP}}) - I_x \dot{\psi}_{\mathrm{PSMP}}^2}
\end{aligned}
\tag{G.6}
$$

which can be rewritten as the inequality condition (35). ■