Modeling Irregular Time Series

Hongyuan Mei
2018—2021 Bloomberg Data Science Ph.D. Fellow
Johns Hopkins University

Lecture Structure

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- Lecture-1: concepts

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  - Intensity, point process, MLE, thinning, …

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- Lecture-1: concepts
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- Lecture-2: fancy models
  - Hawkes, NHP, Neural ODE, …
- Lecture-3: advanced topics
  - NCE, Datalog, marks, …
  - Useful techniques to other kind of data

Time Series

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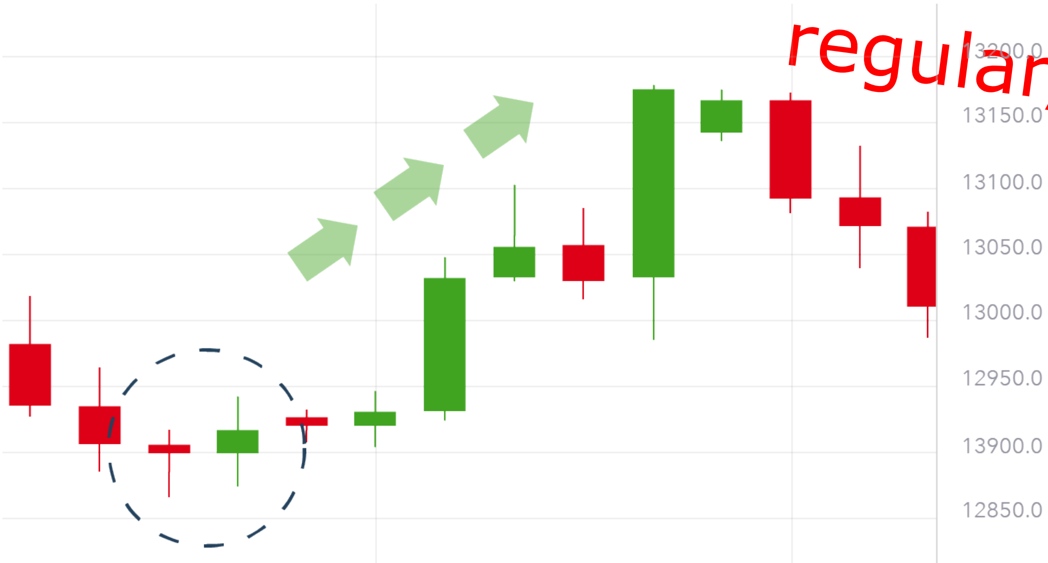
e.g., 1 min chart

http://bburl/tpp-slides-p1
http://bburl/tpp-lab-p1

Time Series

e.g., 1 min chart

regular/constant intervals



Time Series

e.g., 1 min chart

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I **like** **working** **at** **Bloomberg**

w_1 w_2 w_3 w_4 w_5

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Irregular Time Series

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http://bburl/tpp-lab-p1

Irregular Time Series

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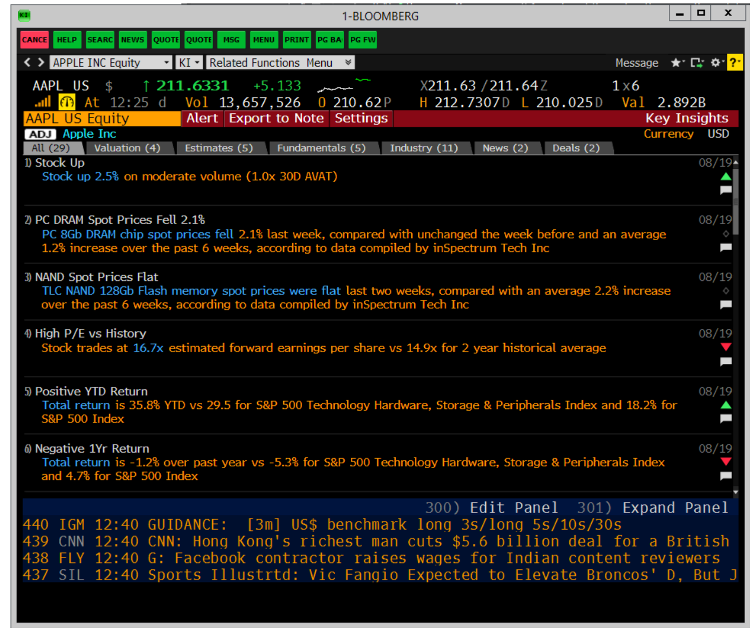

Irregular Time Series

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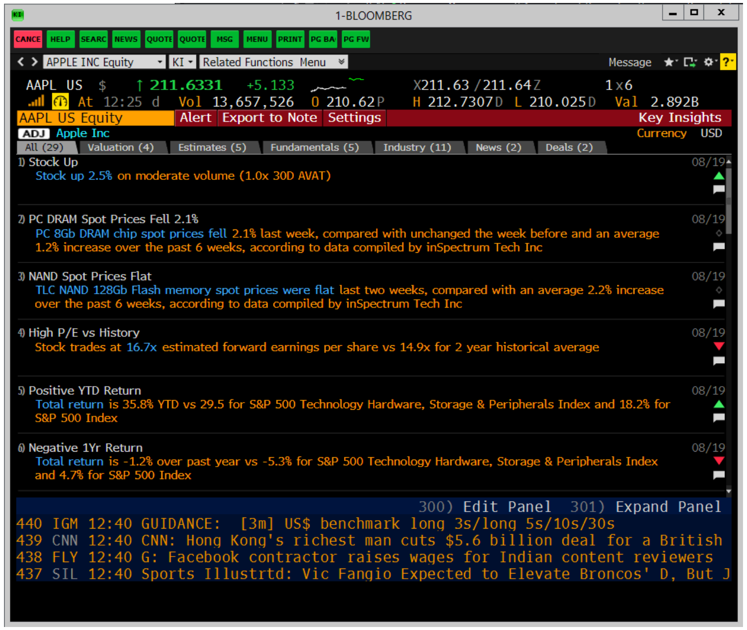
stock movement

earning announcement

rating change

implied volatility spike

...



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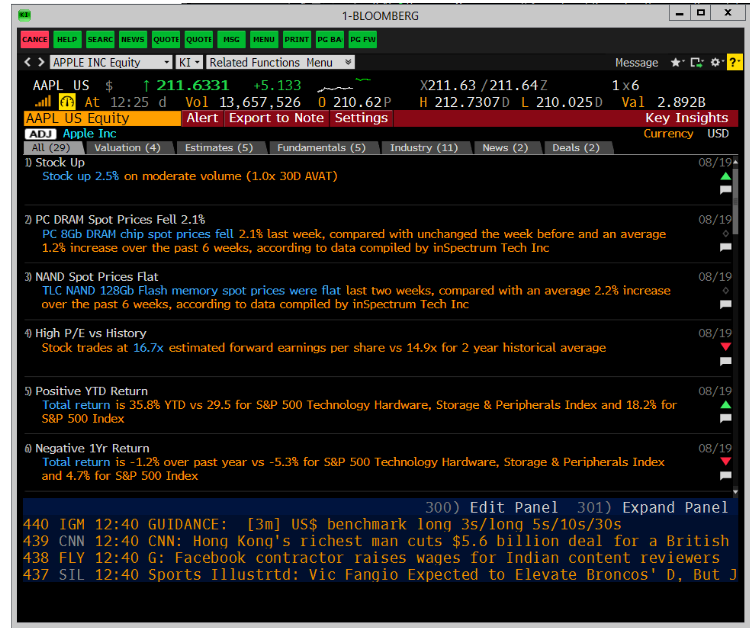

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Market

t

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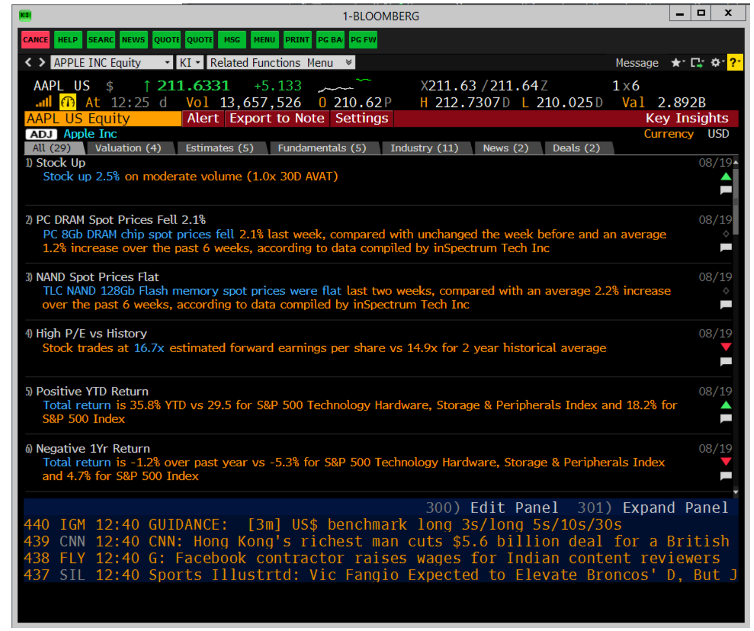

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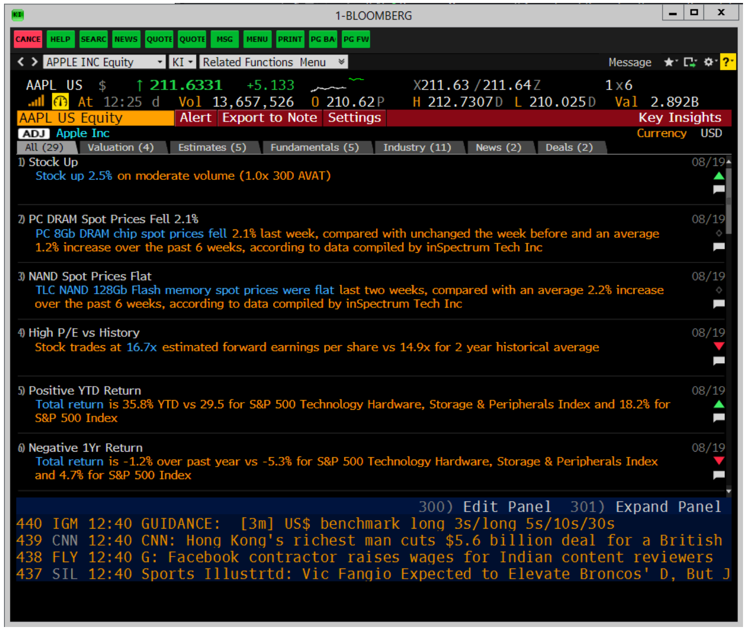

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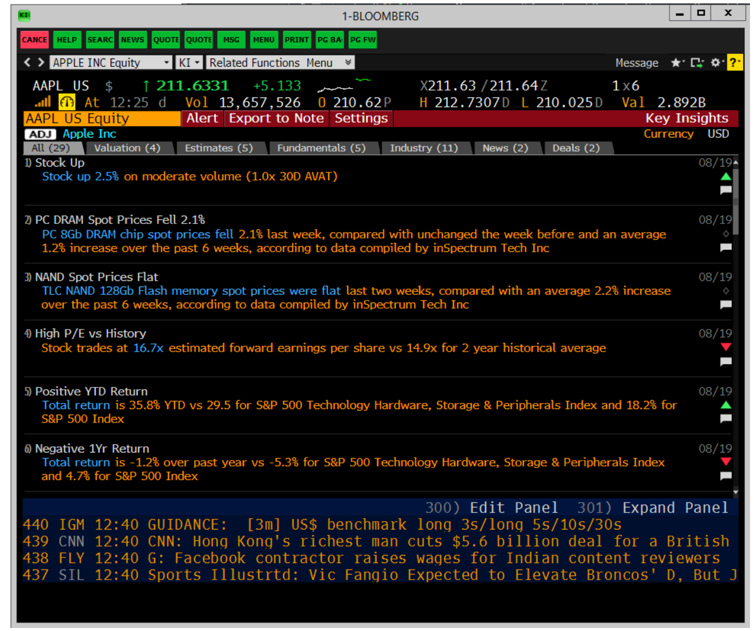

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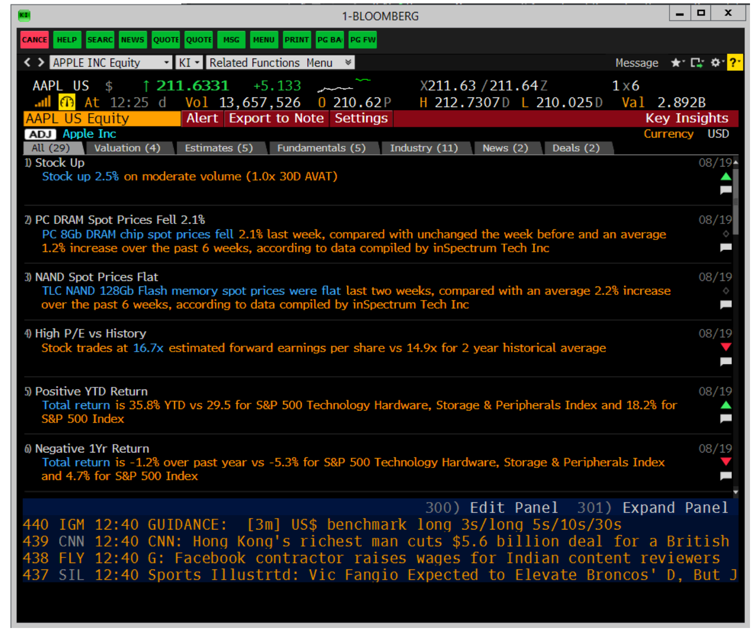

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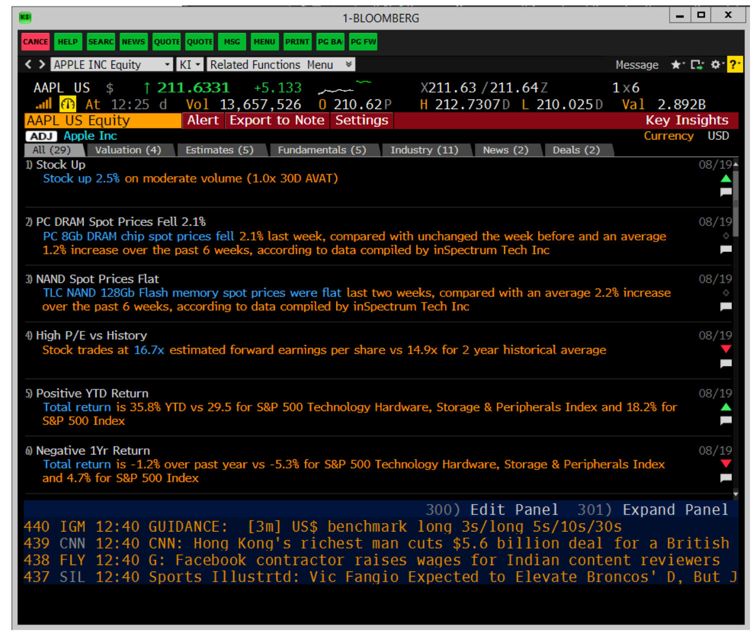

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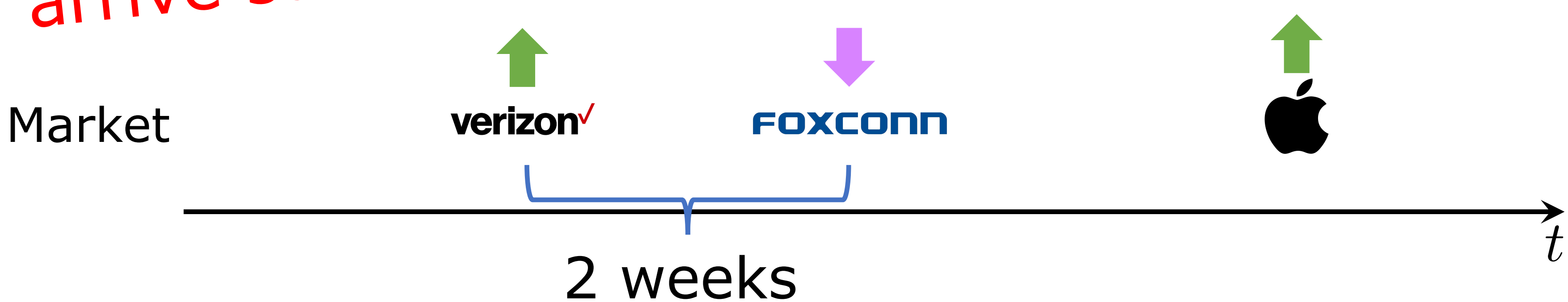

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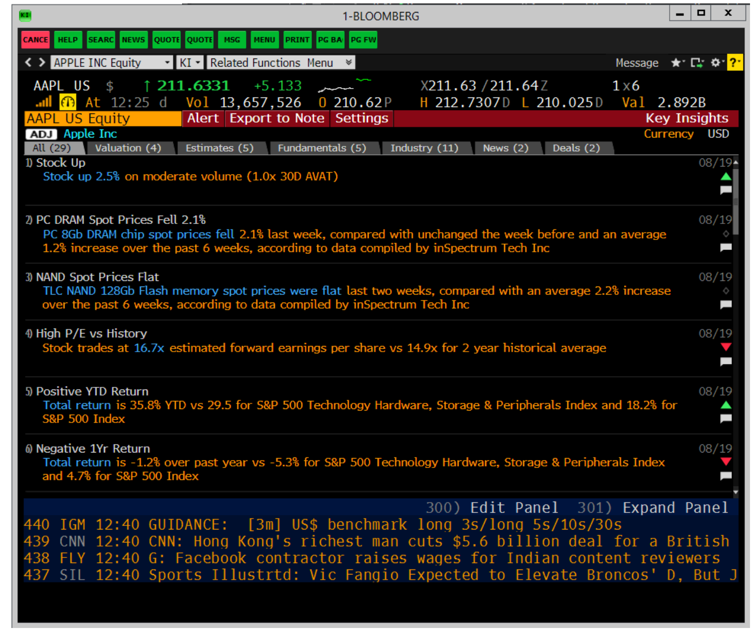

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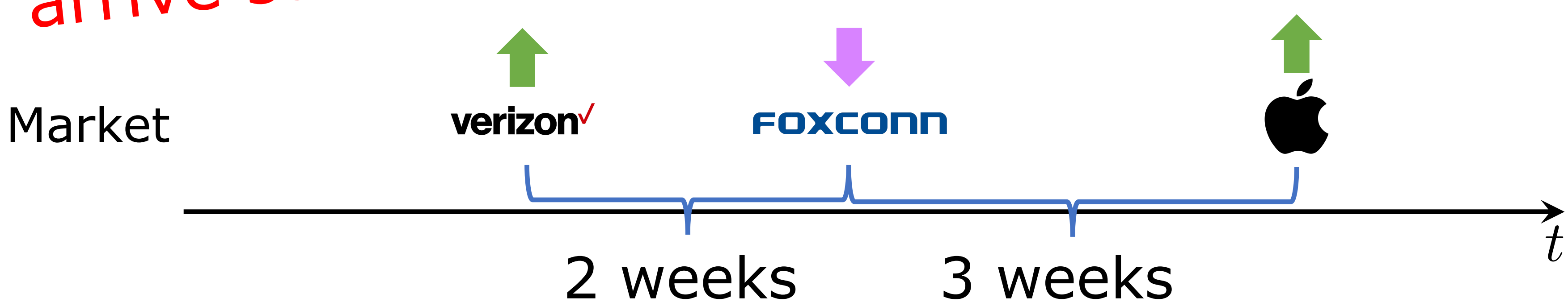

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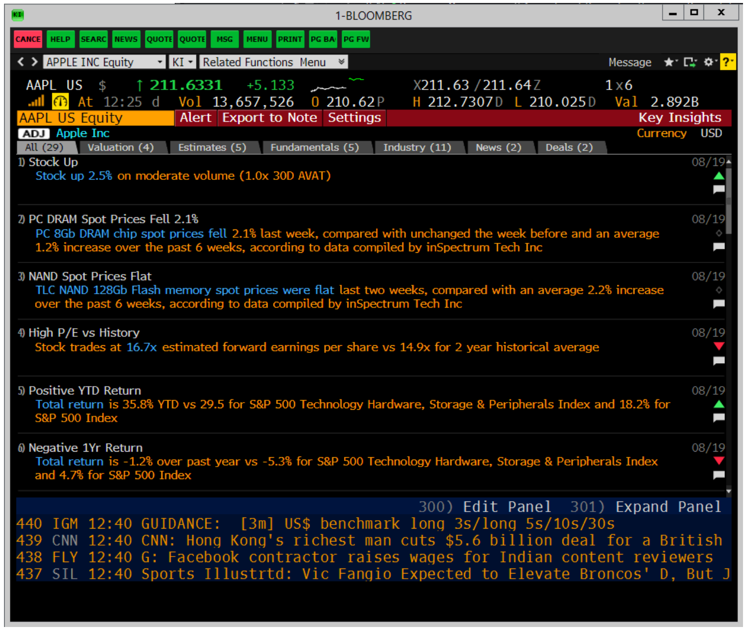

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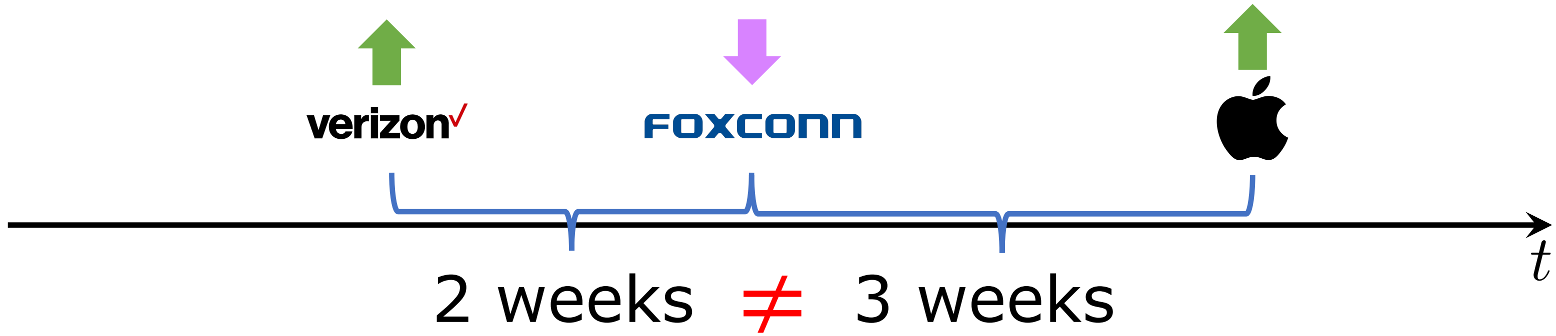

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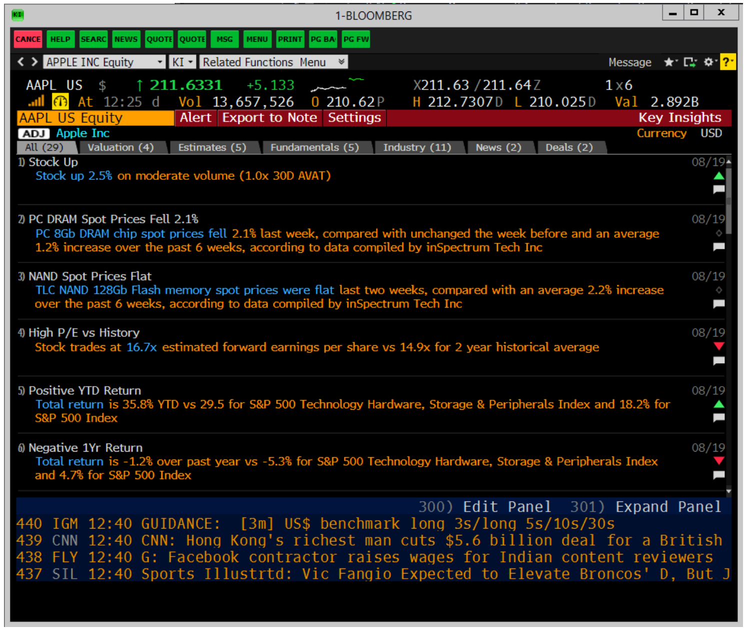

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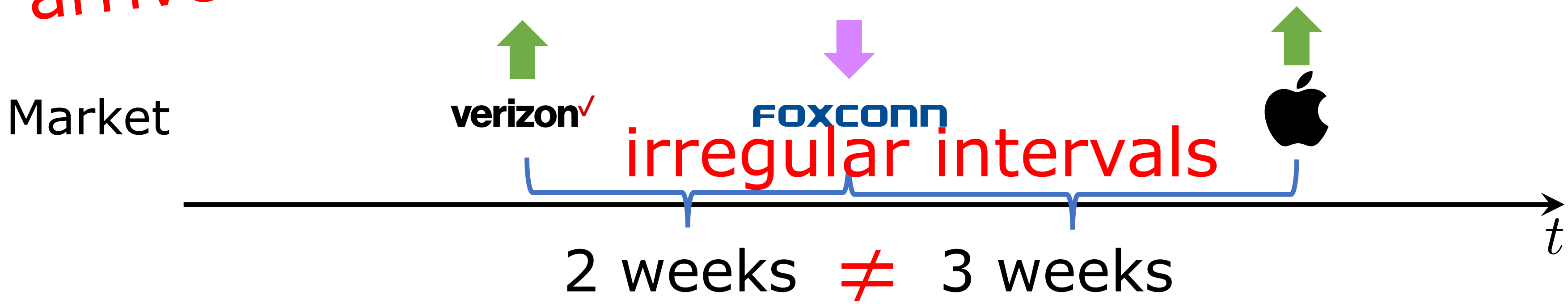

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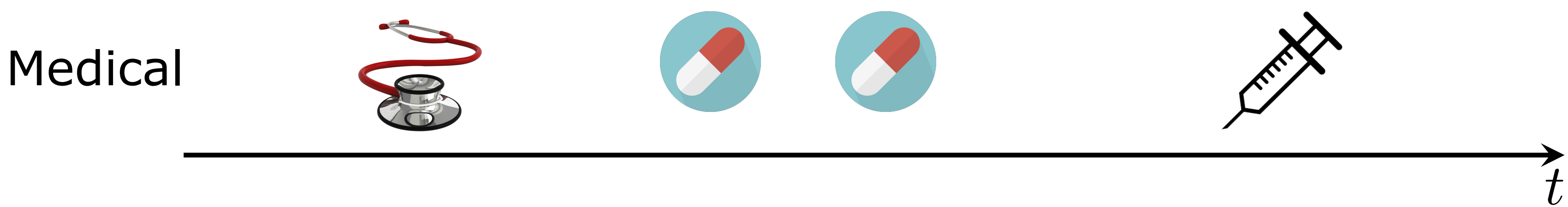

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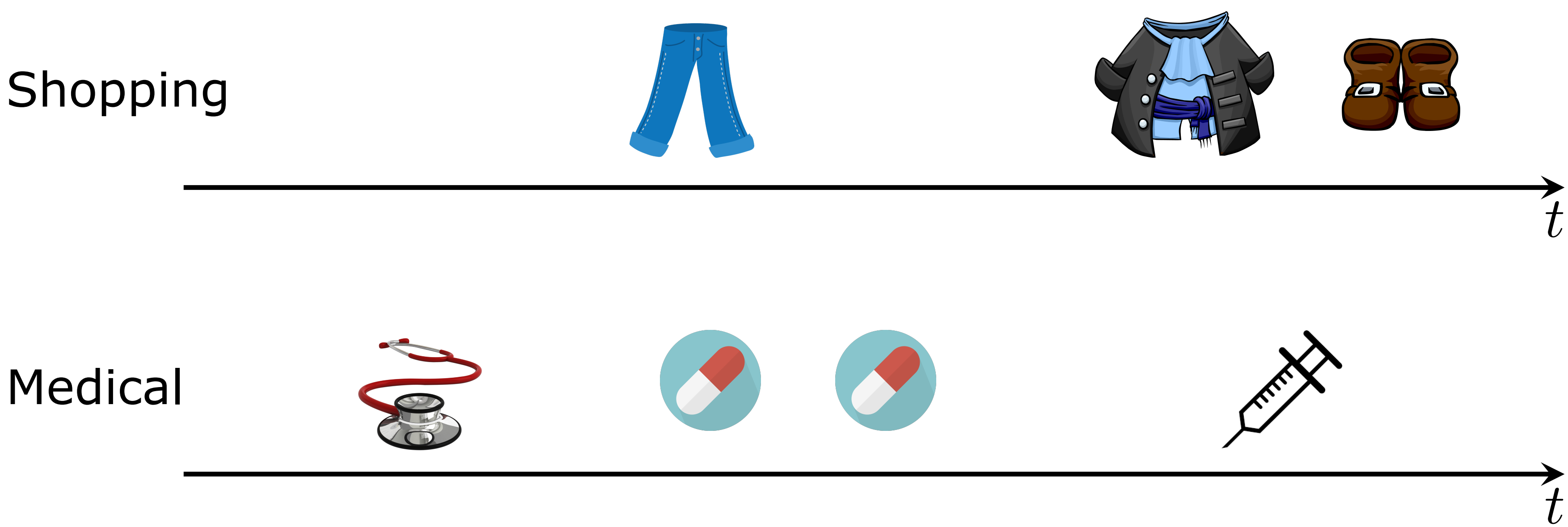

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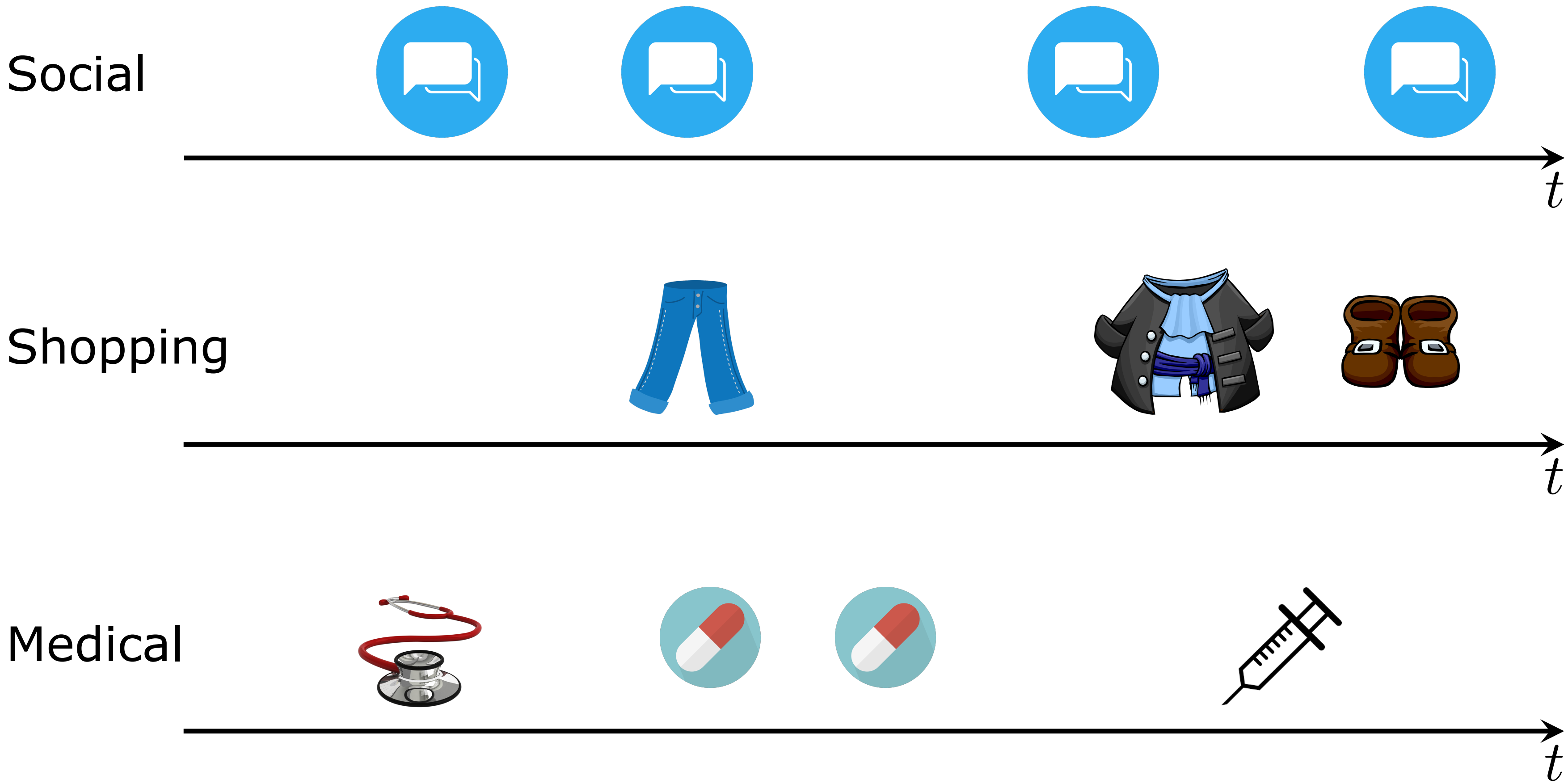

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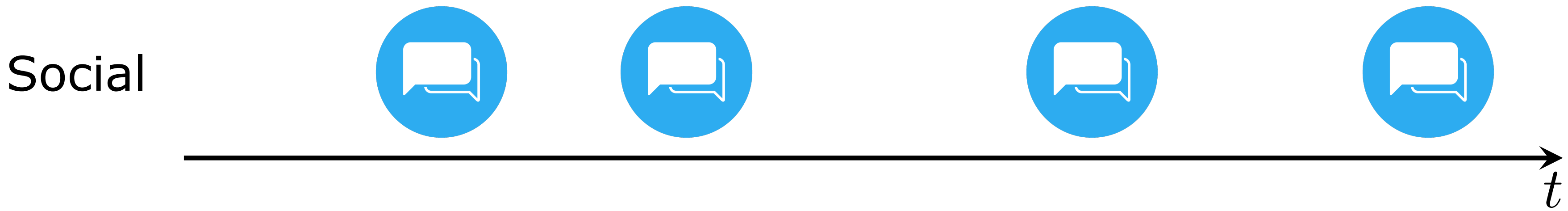

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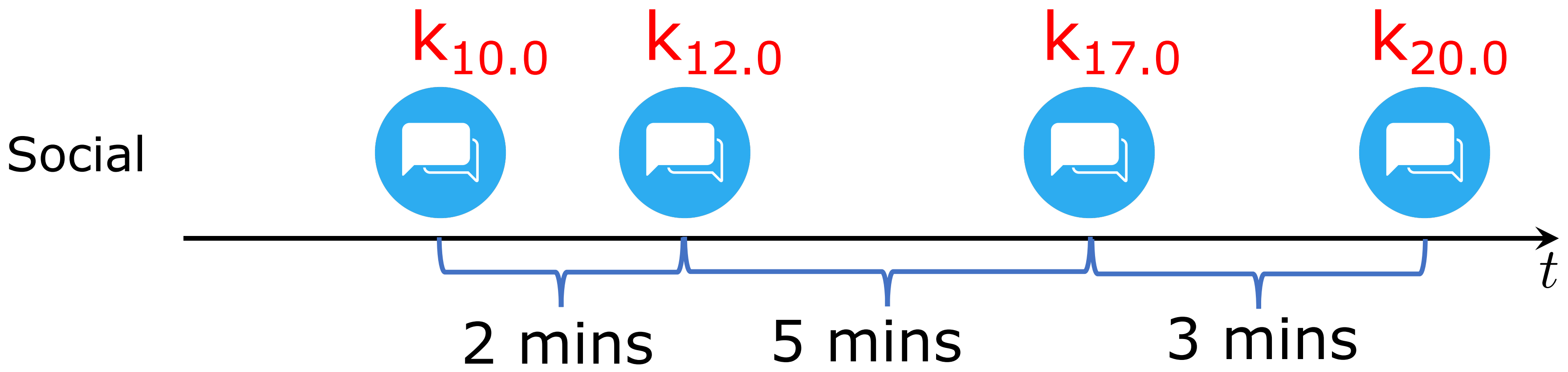

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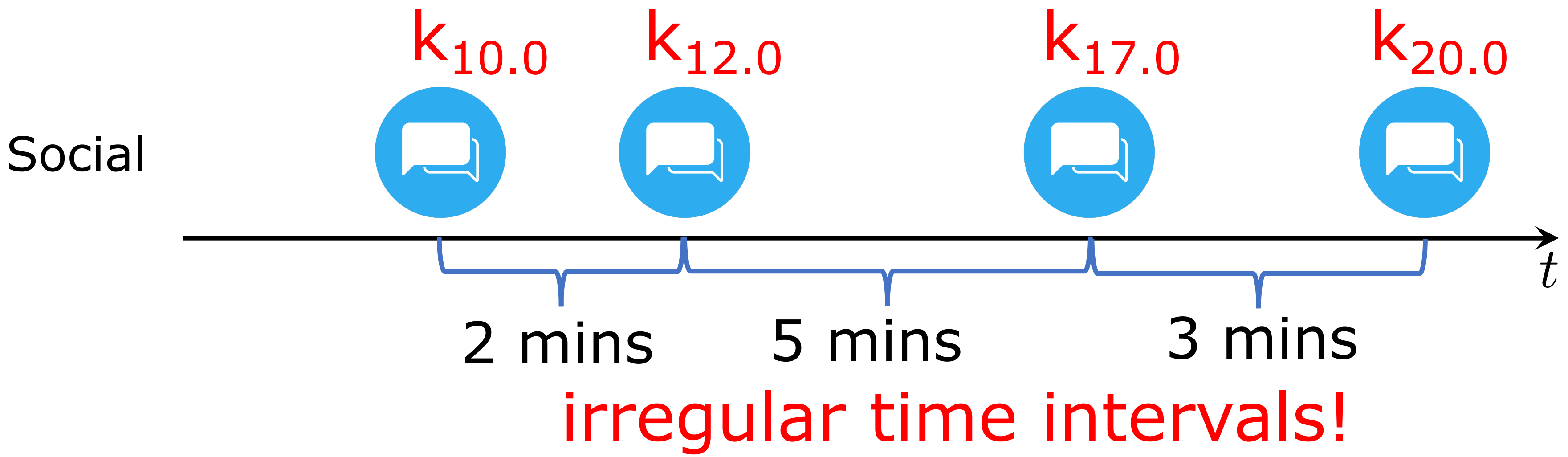

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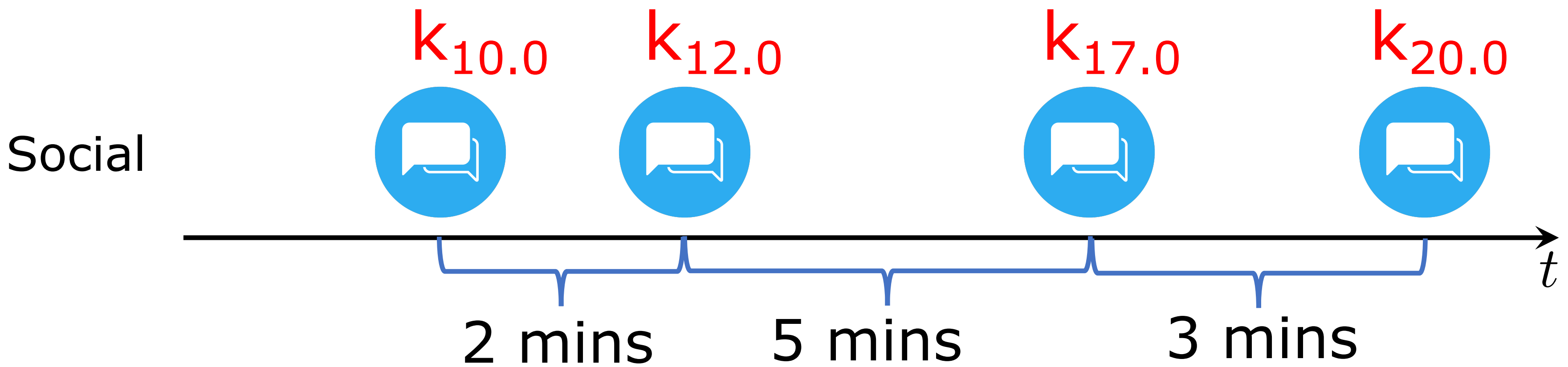

irregular time intervals!

I like working at Bloomberg

w_1 w_2 w_3 w_4 w_5

regular intervals

Any Questions?

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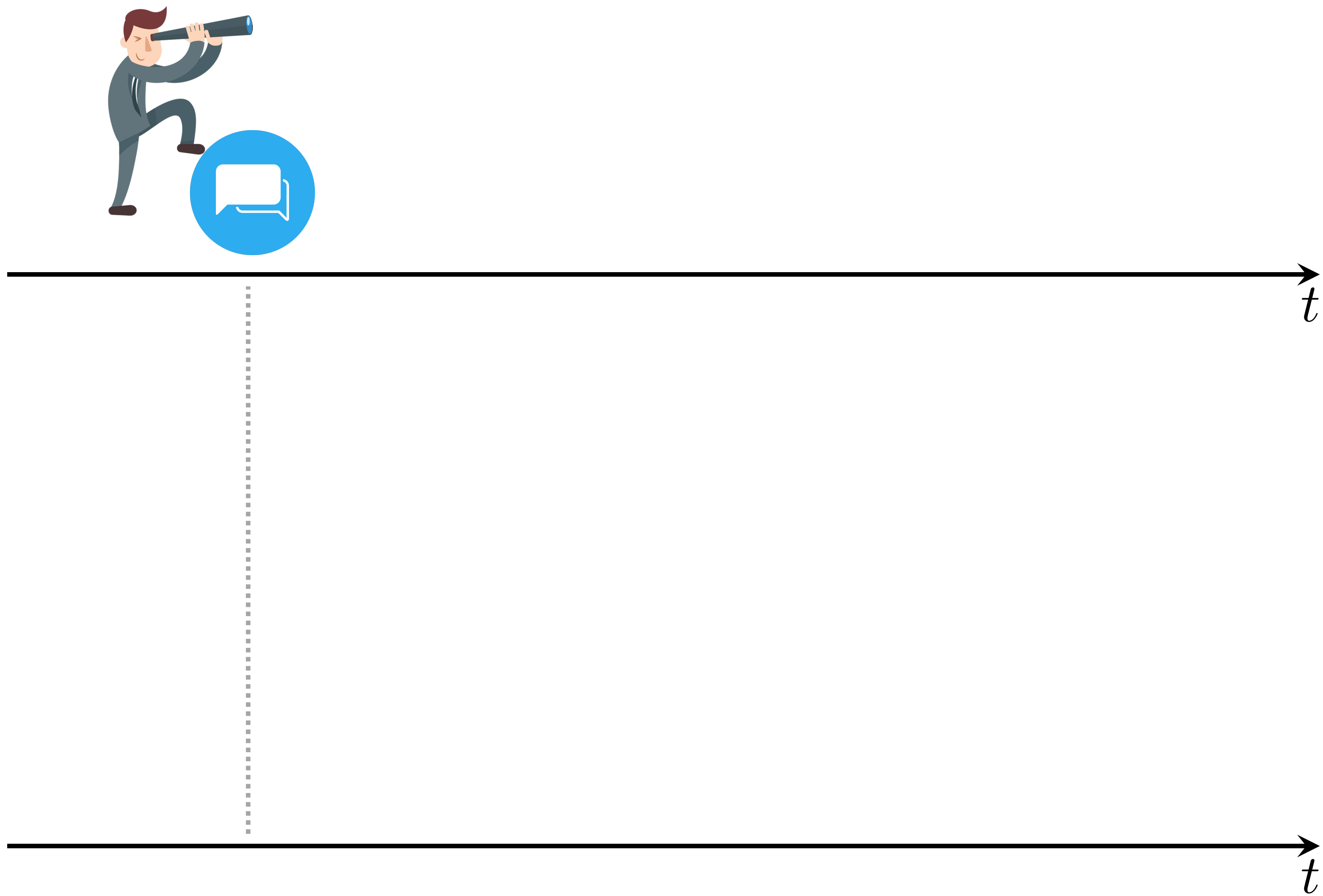
Generative Story of Irregular Time Series

Social



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Generative Story of Irregular Time Series

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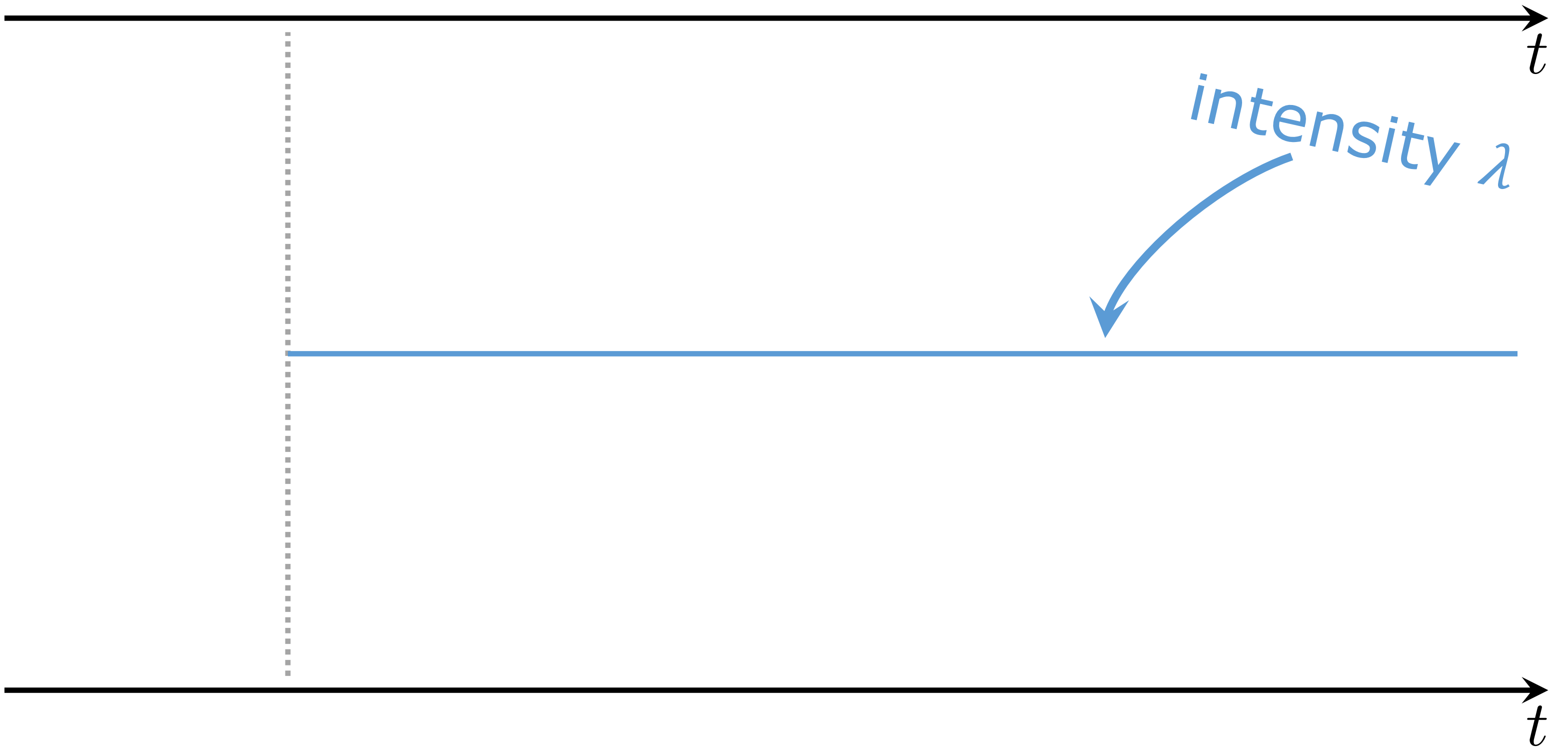
intensity =
rate of events per unit time



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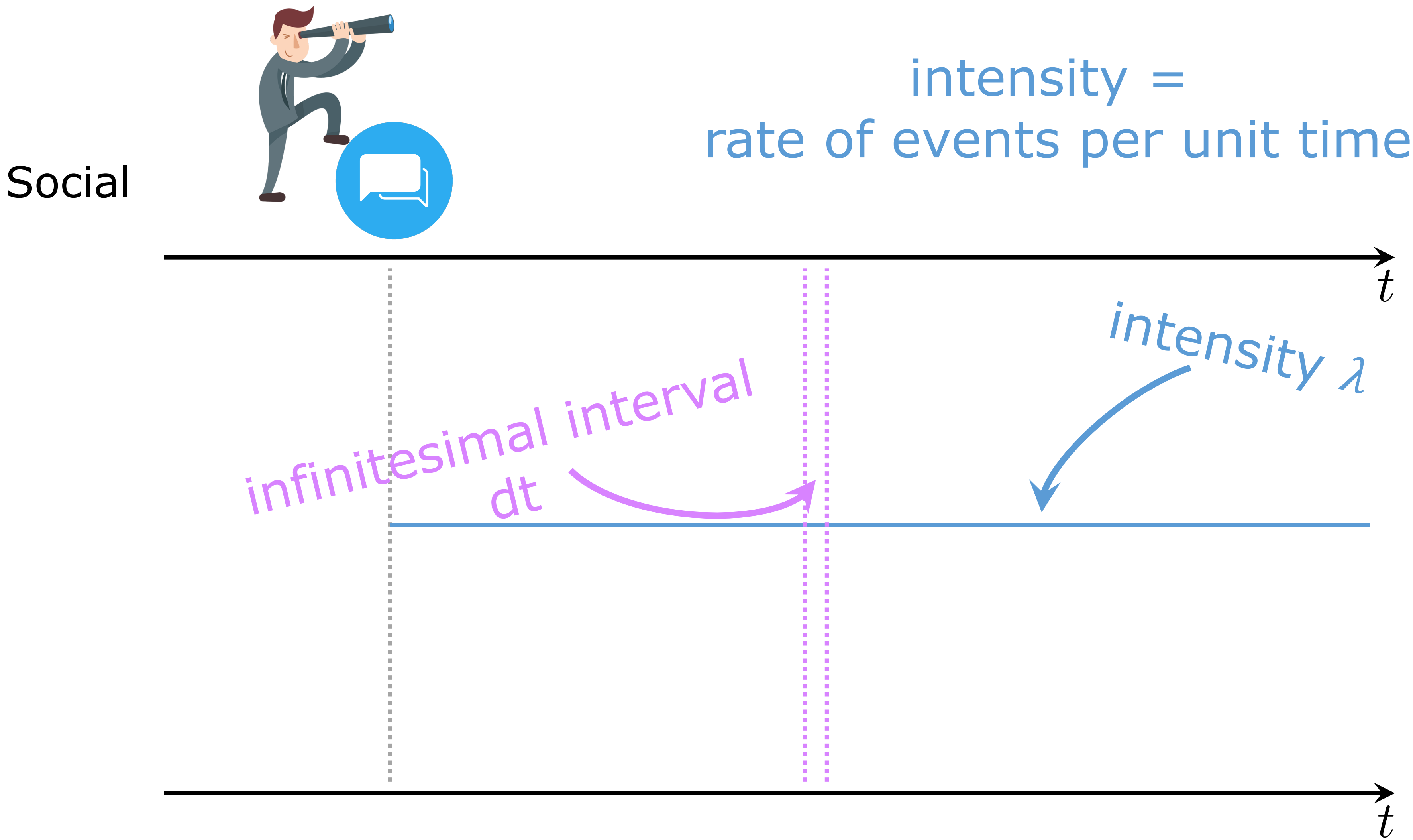

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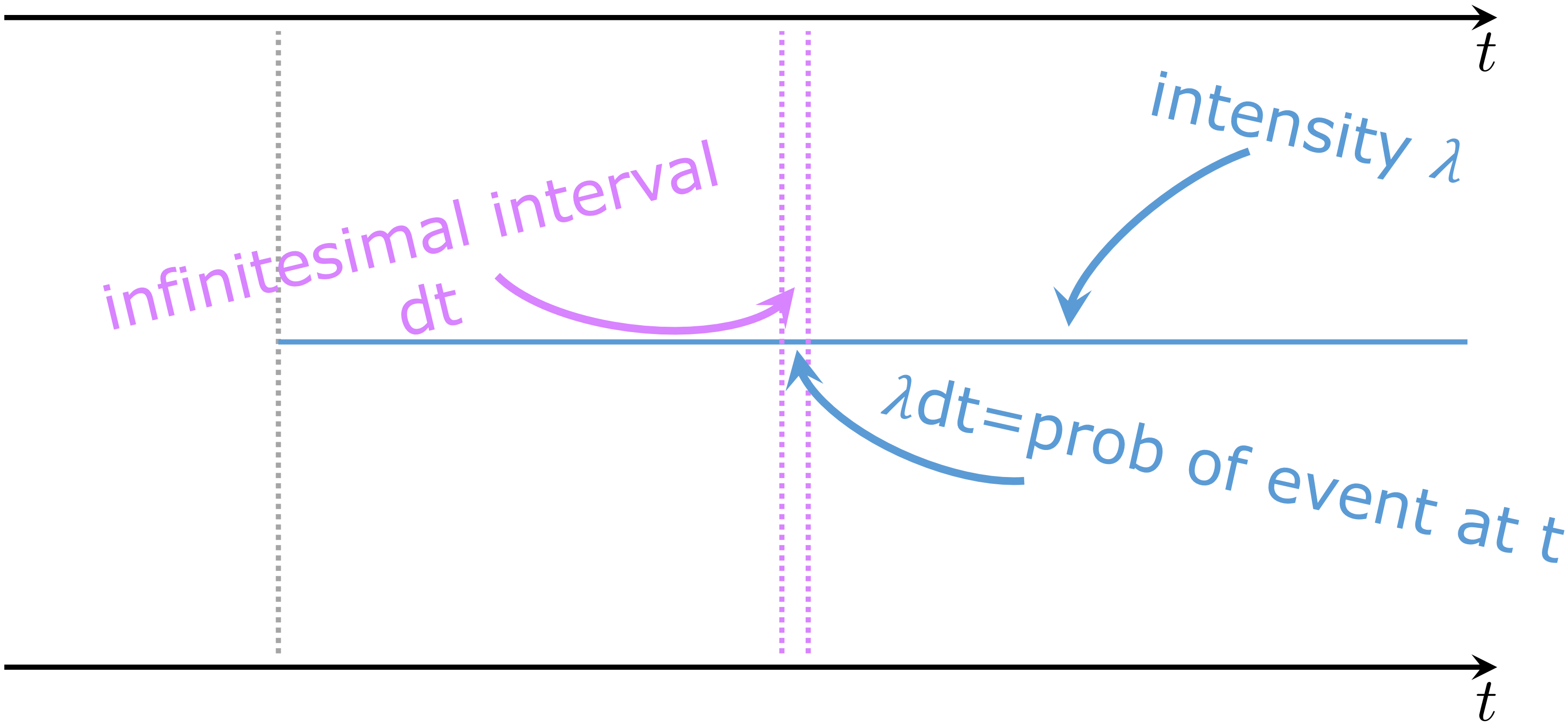

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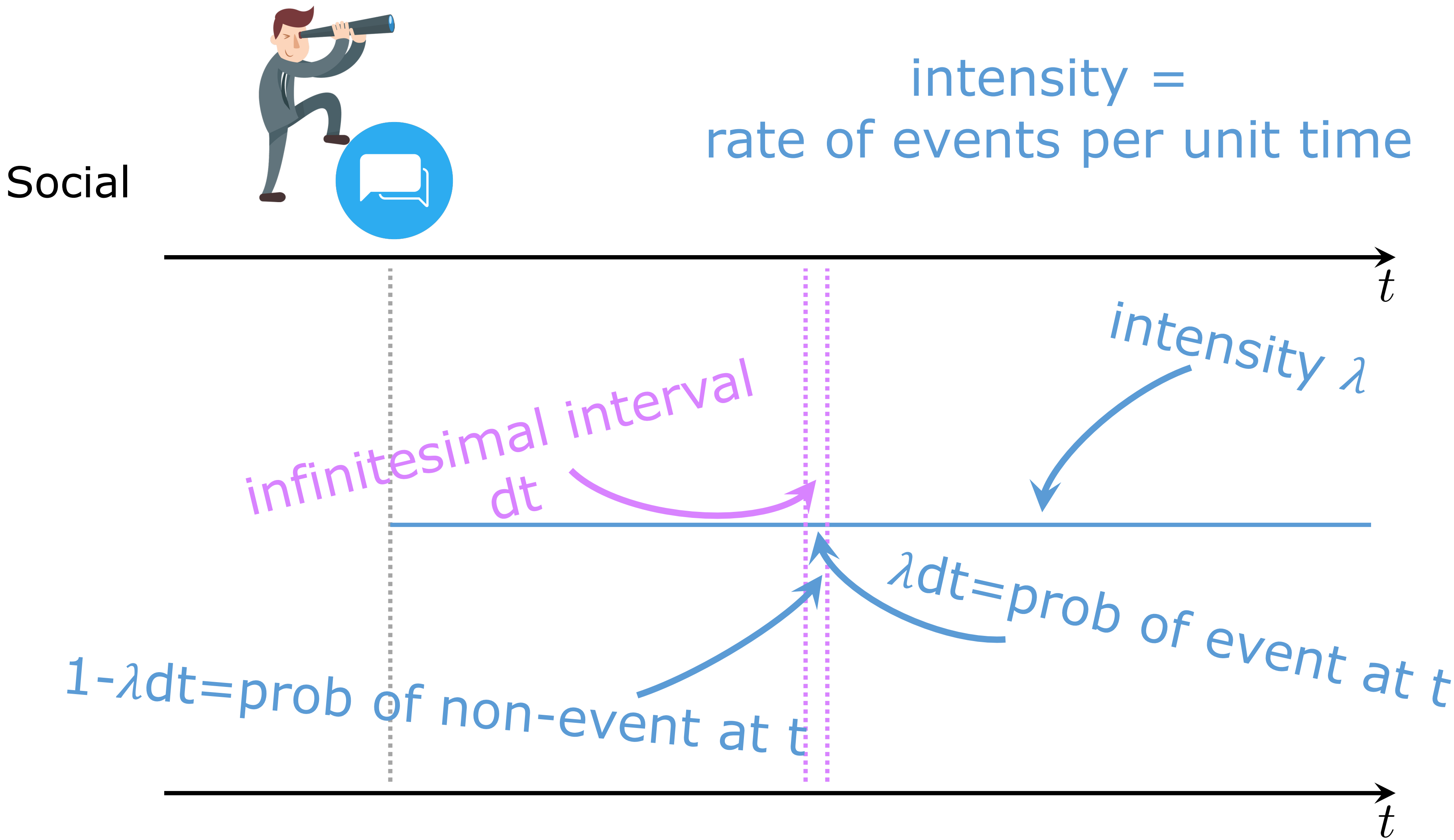

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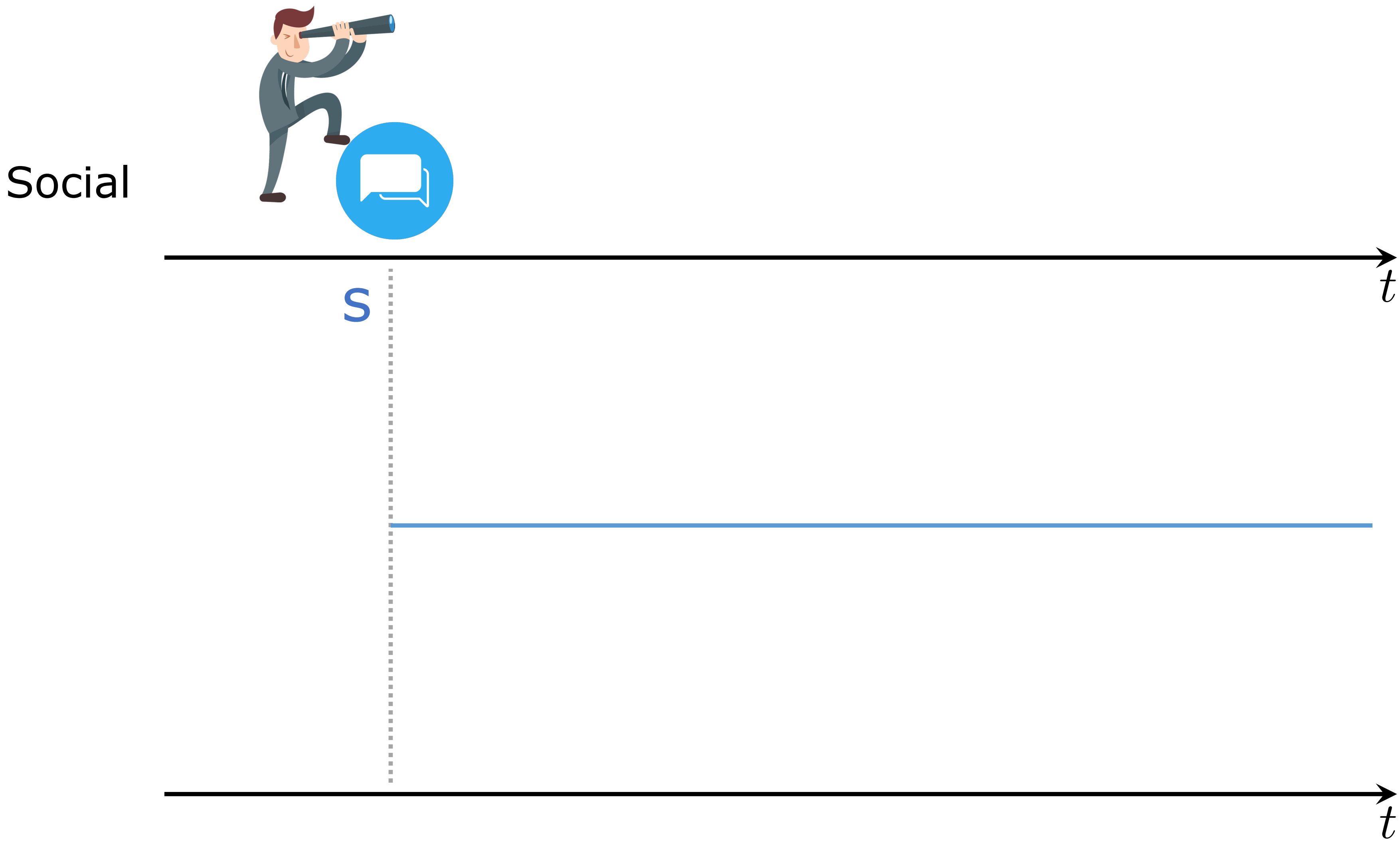

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Generative Story of Irregular Time Series

prob density p(t) of *next* event ?

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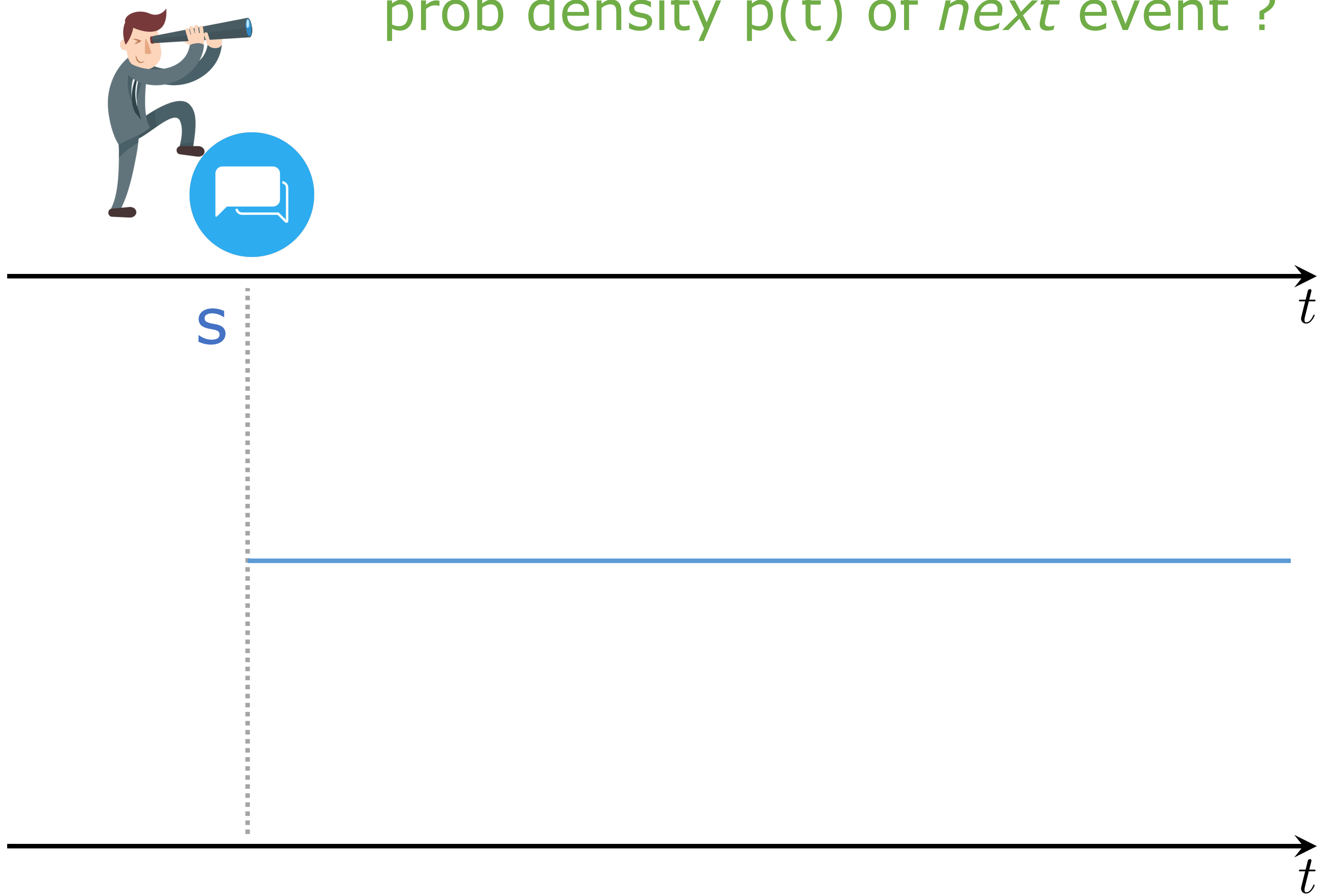

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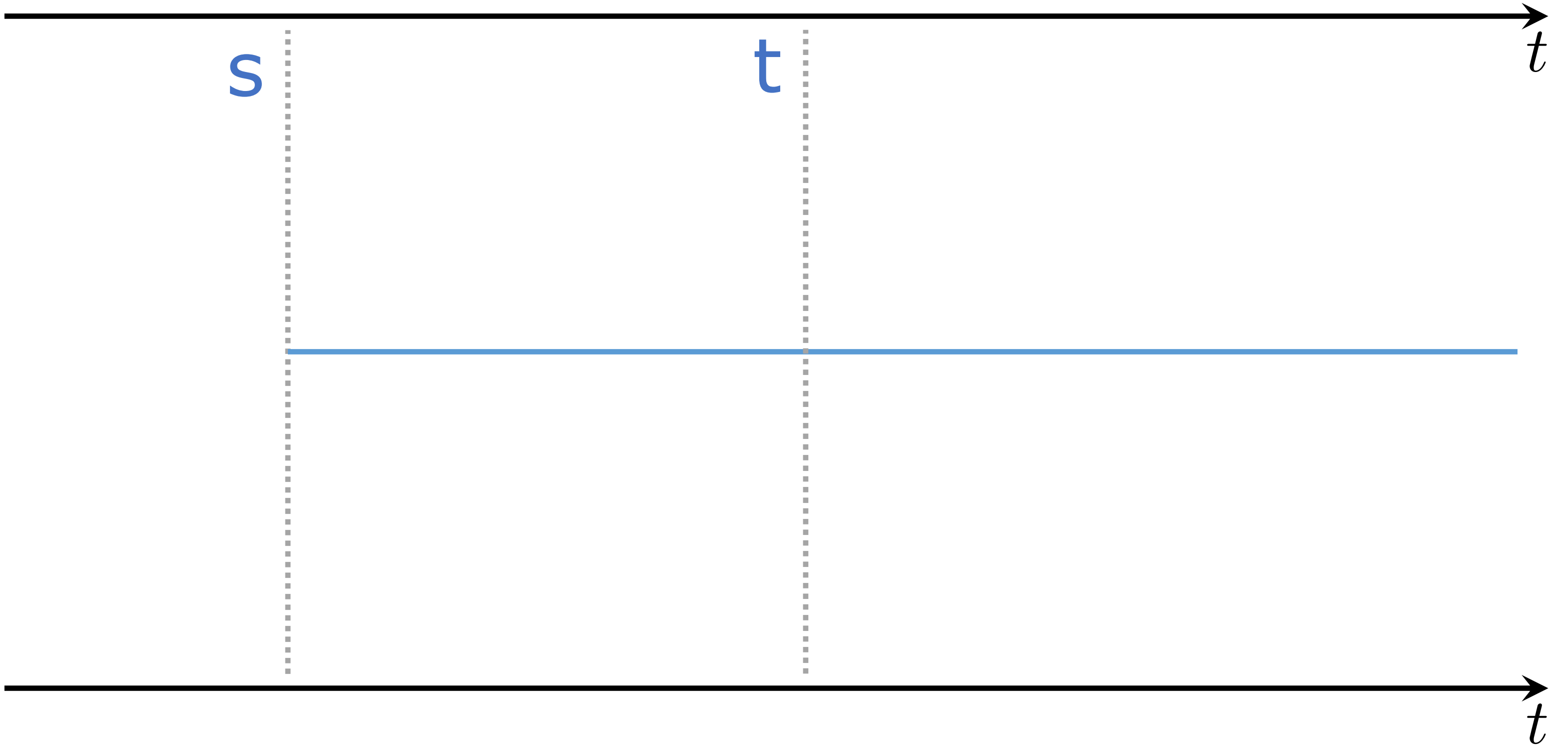

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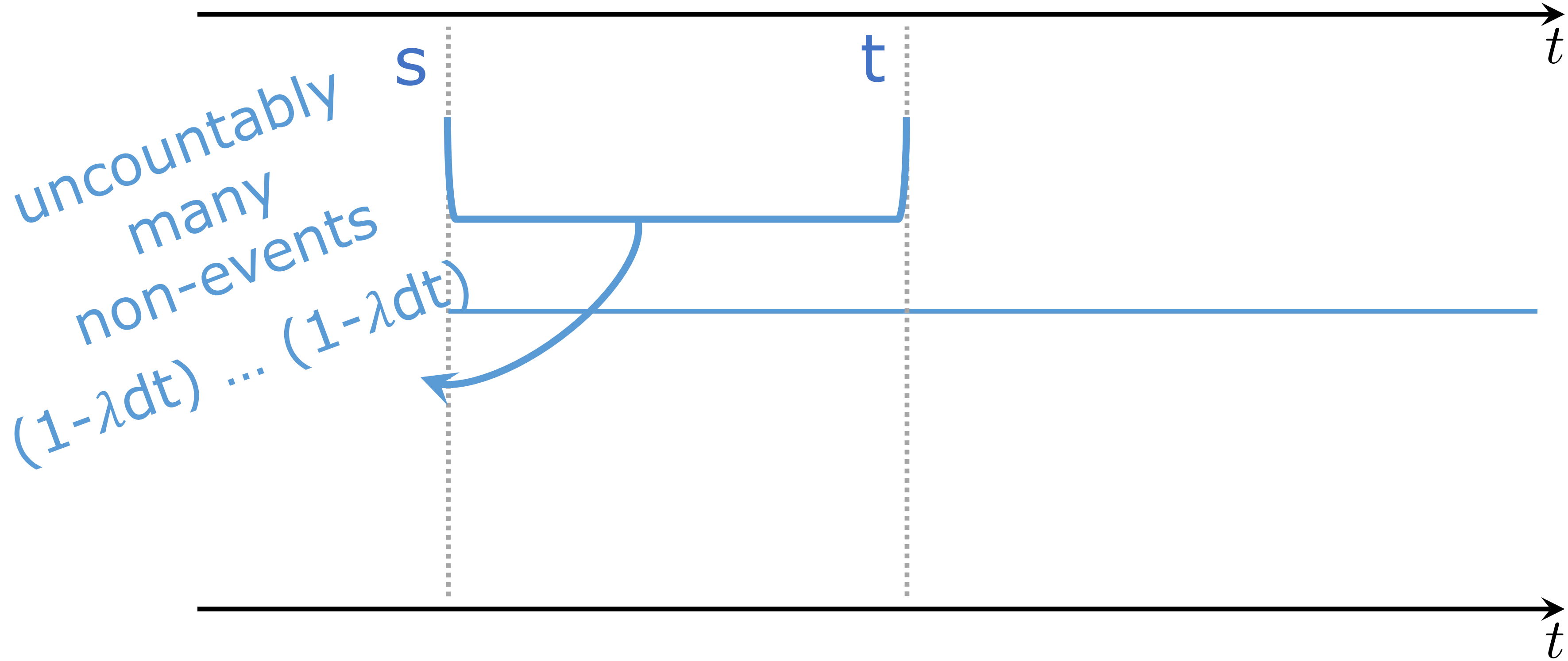

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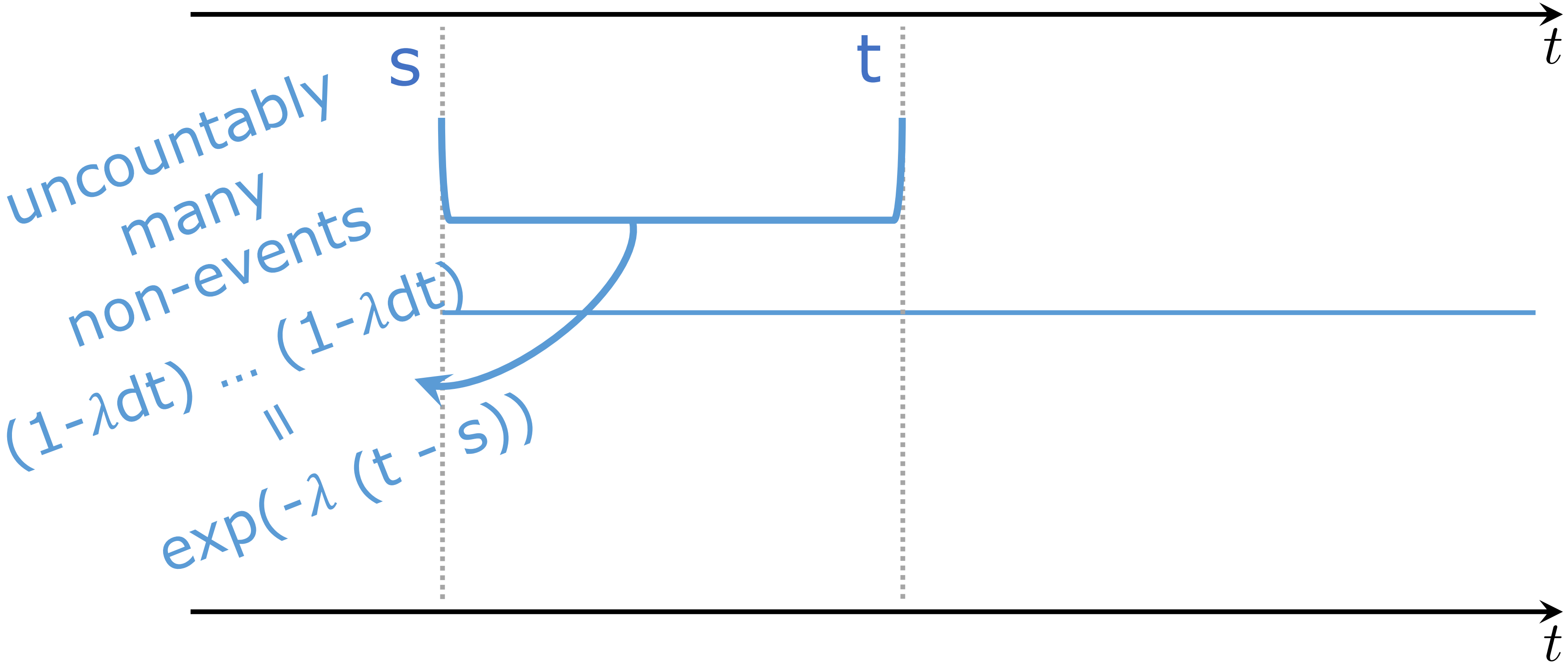

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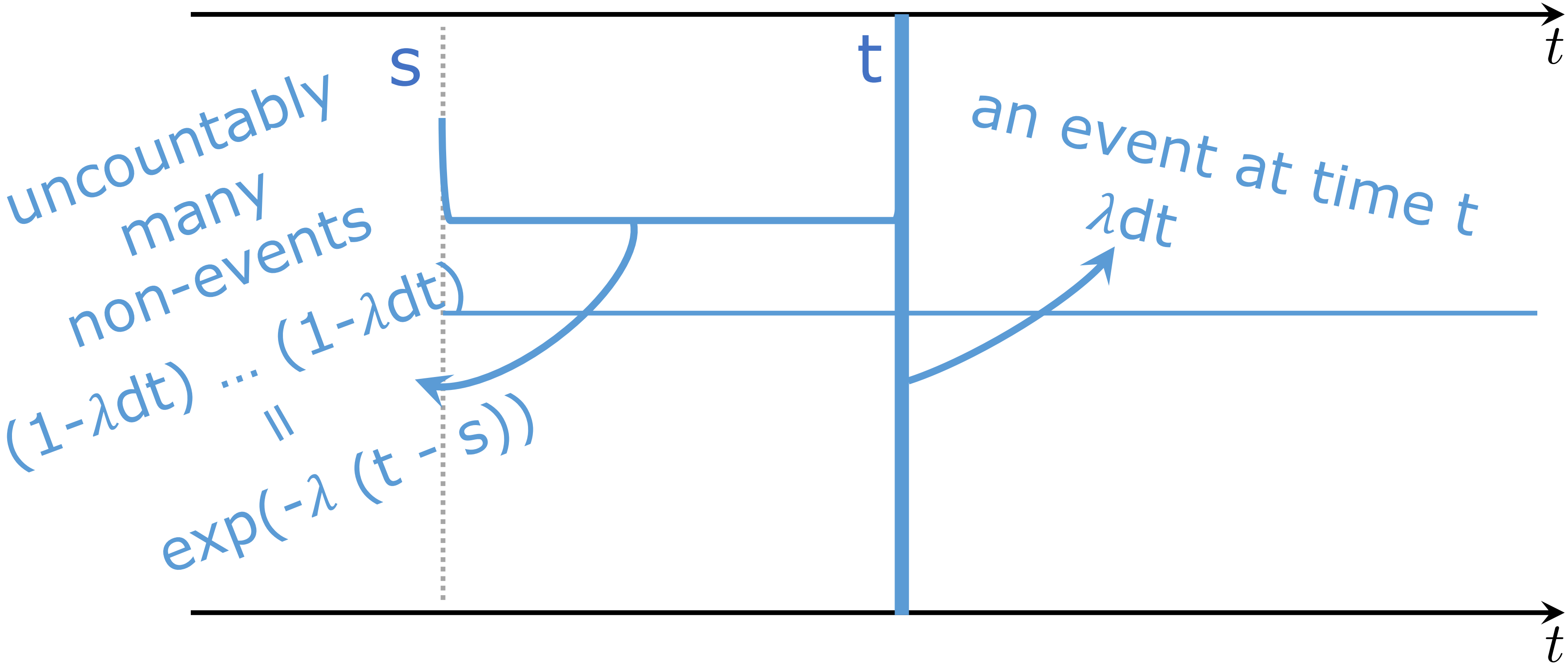
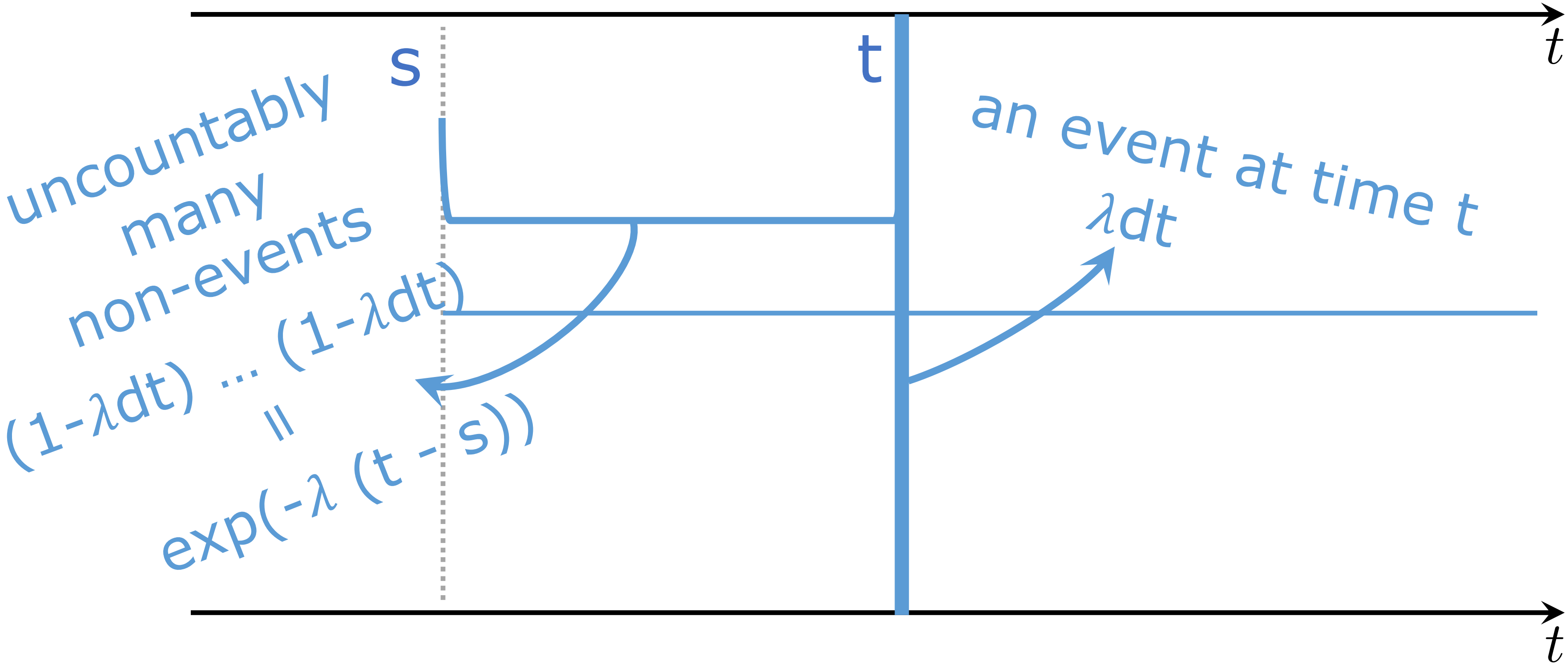


Generative Story of Irregular Time Series

prob density p(t) of *next* event ?

$p(t) = \lambda \exp(-\lambda\,(t - s))$

Social



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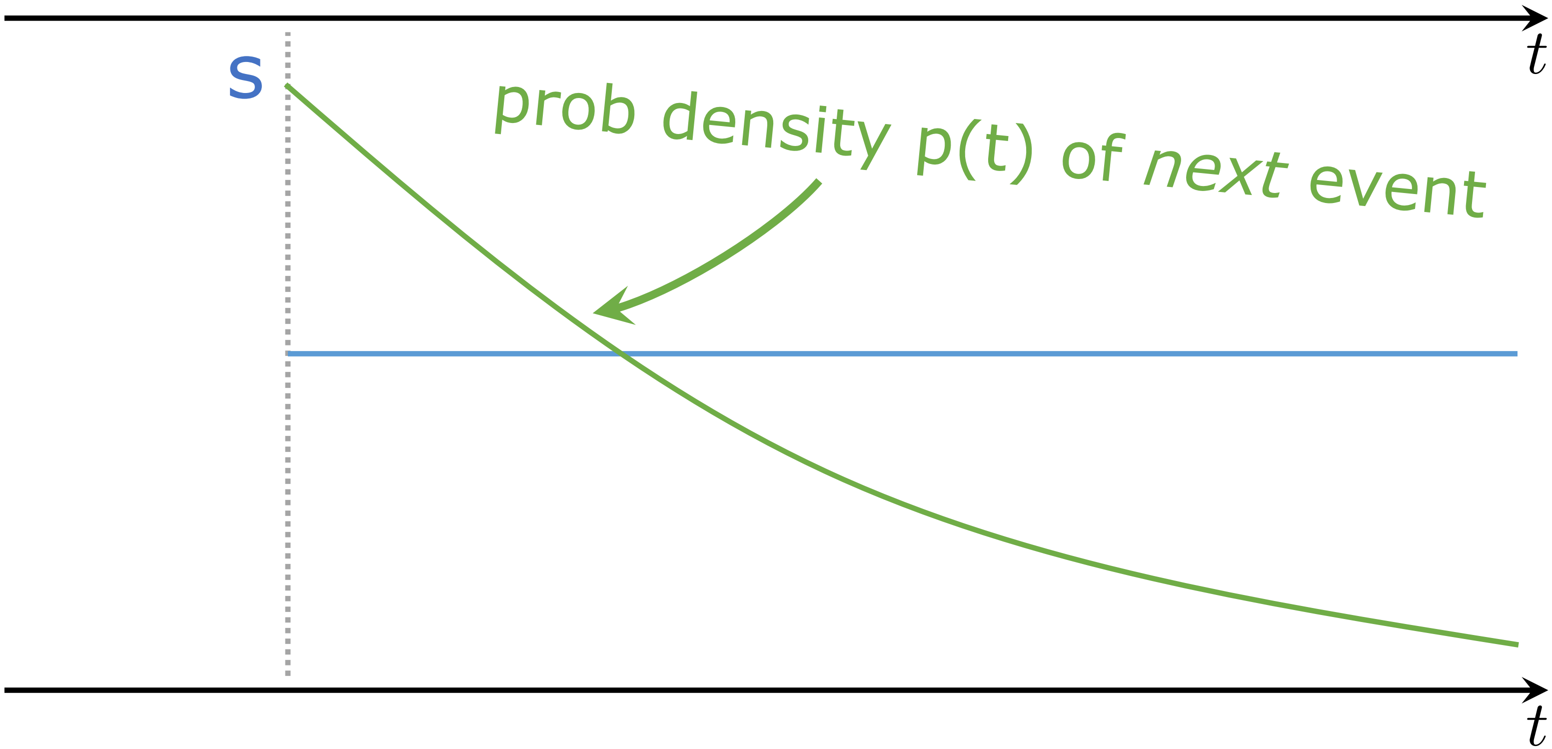
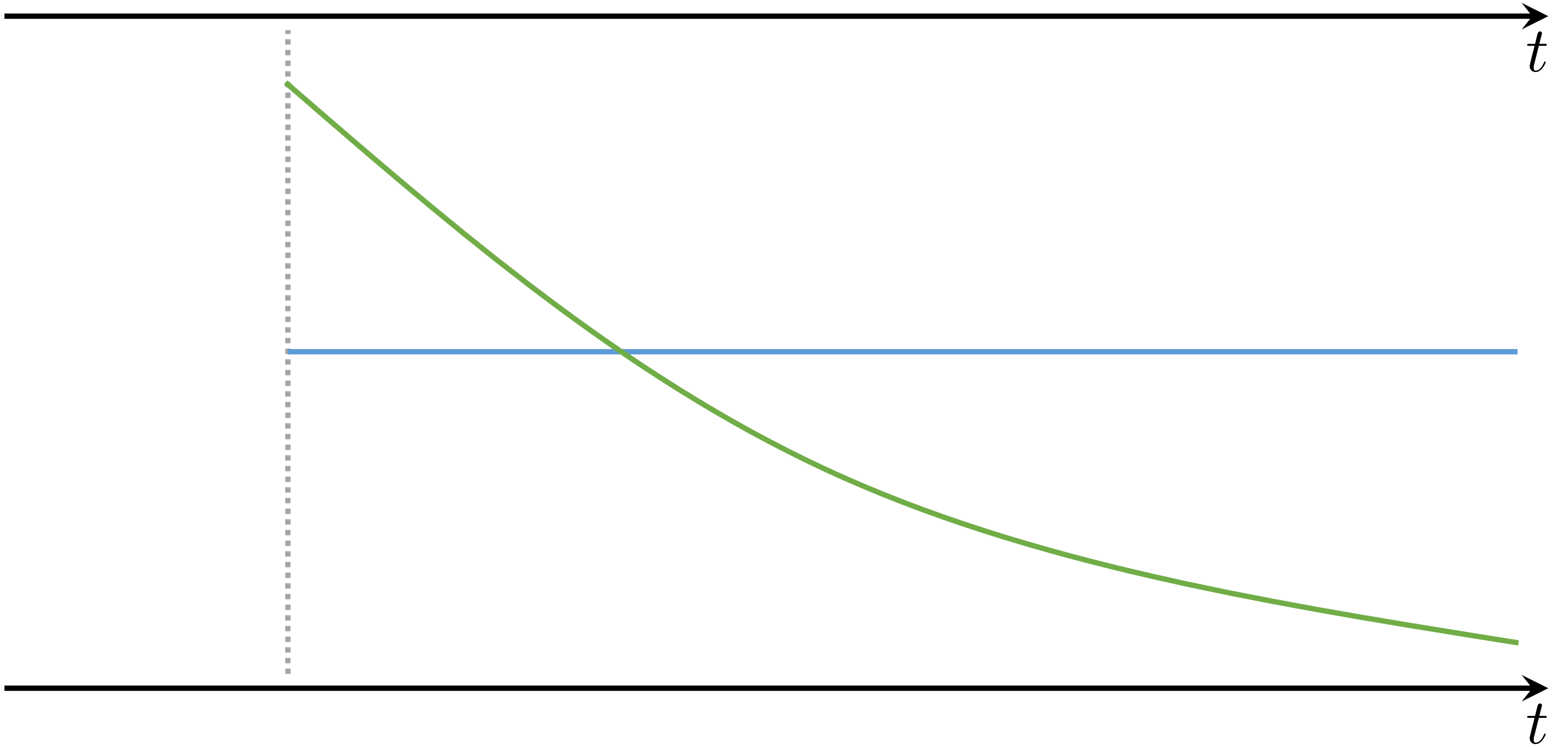


Generative Story of Irregular Time Series

$p(t) = \lambda \exp(-\lambda (t - t_1))$



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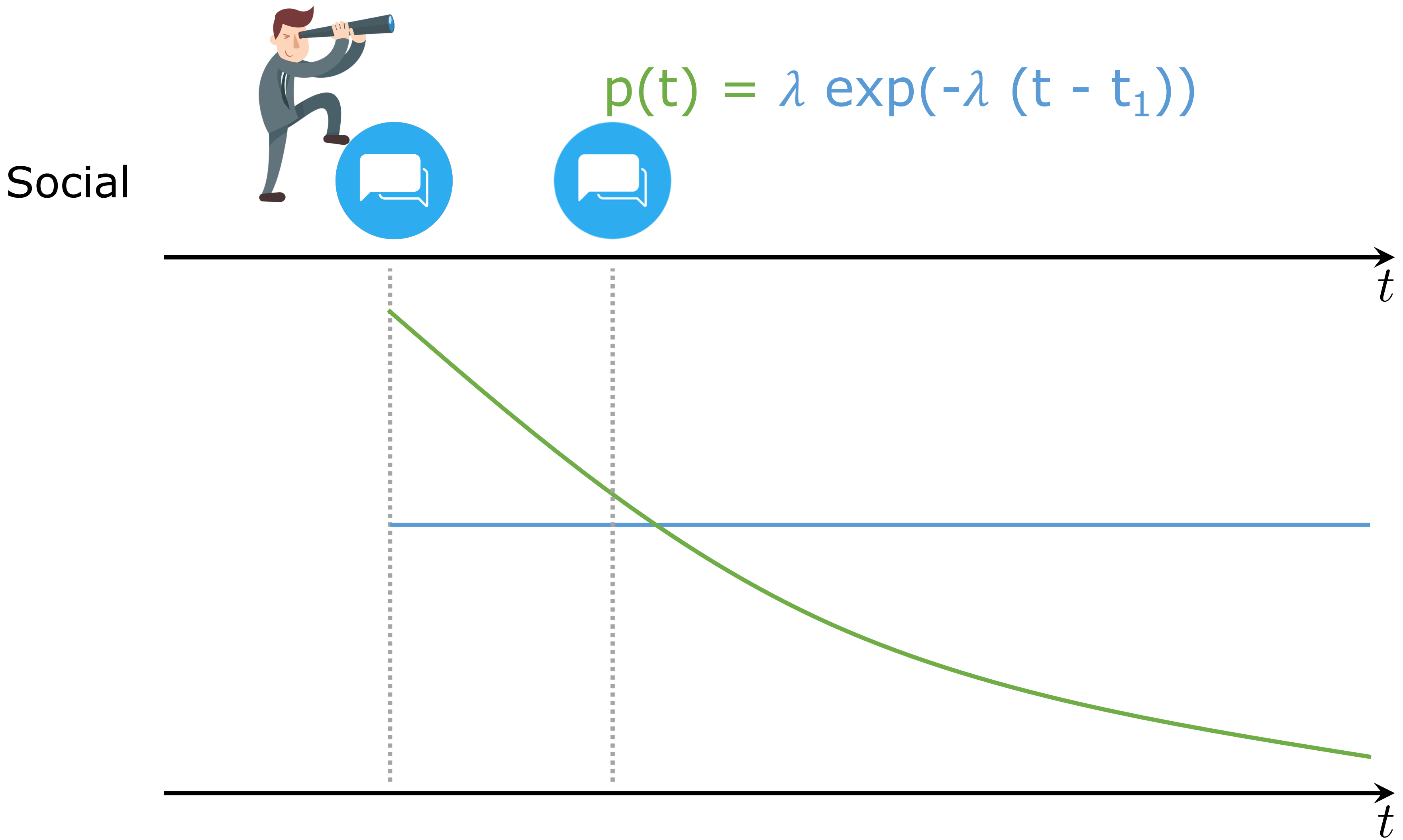

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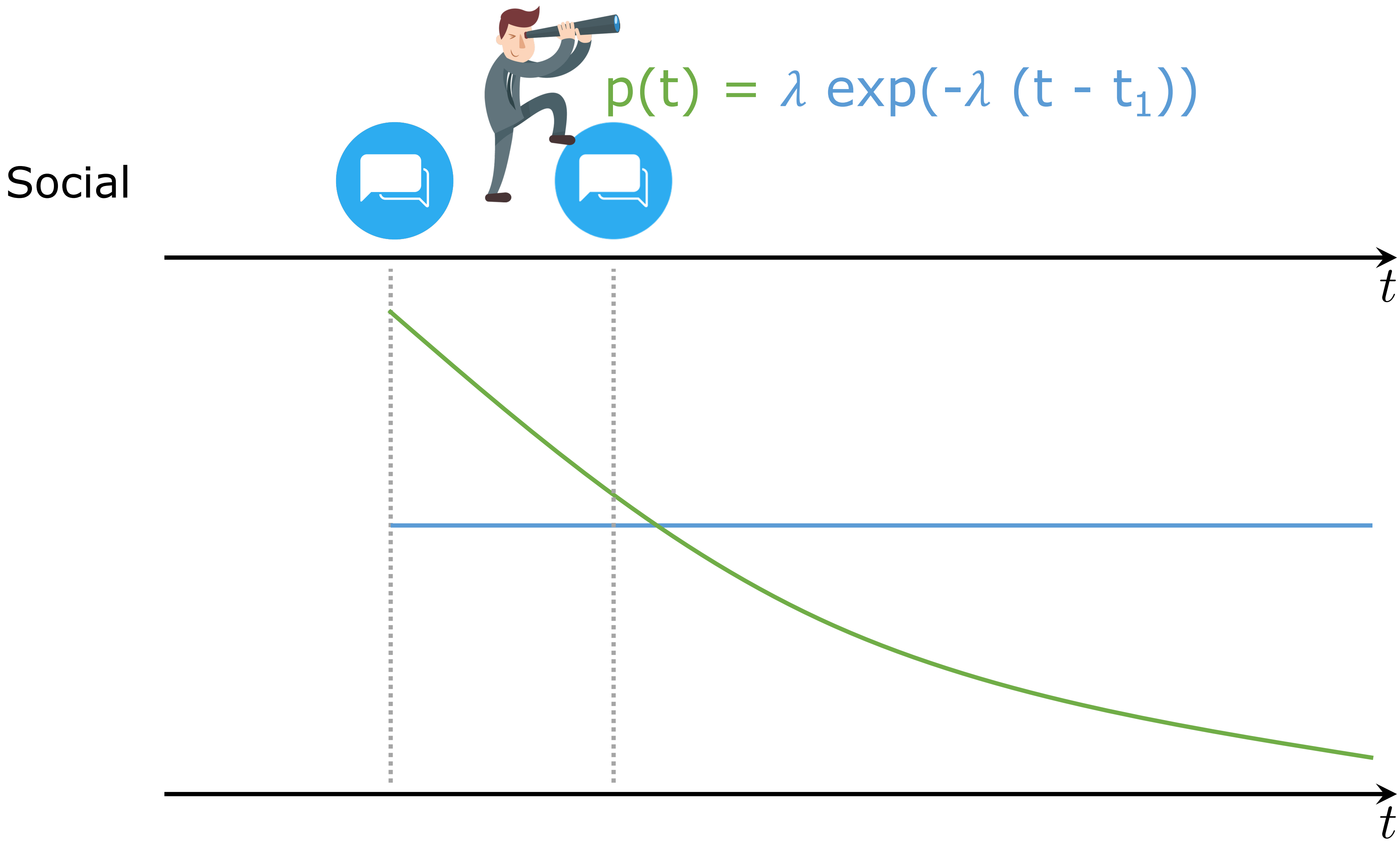

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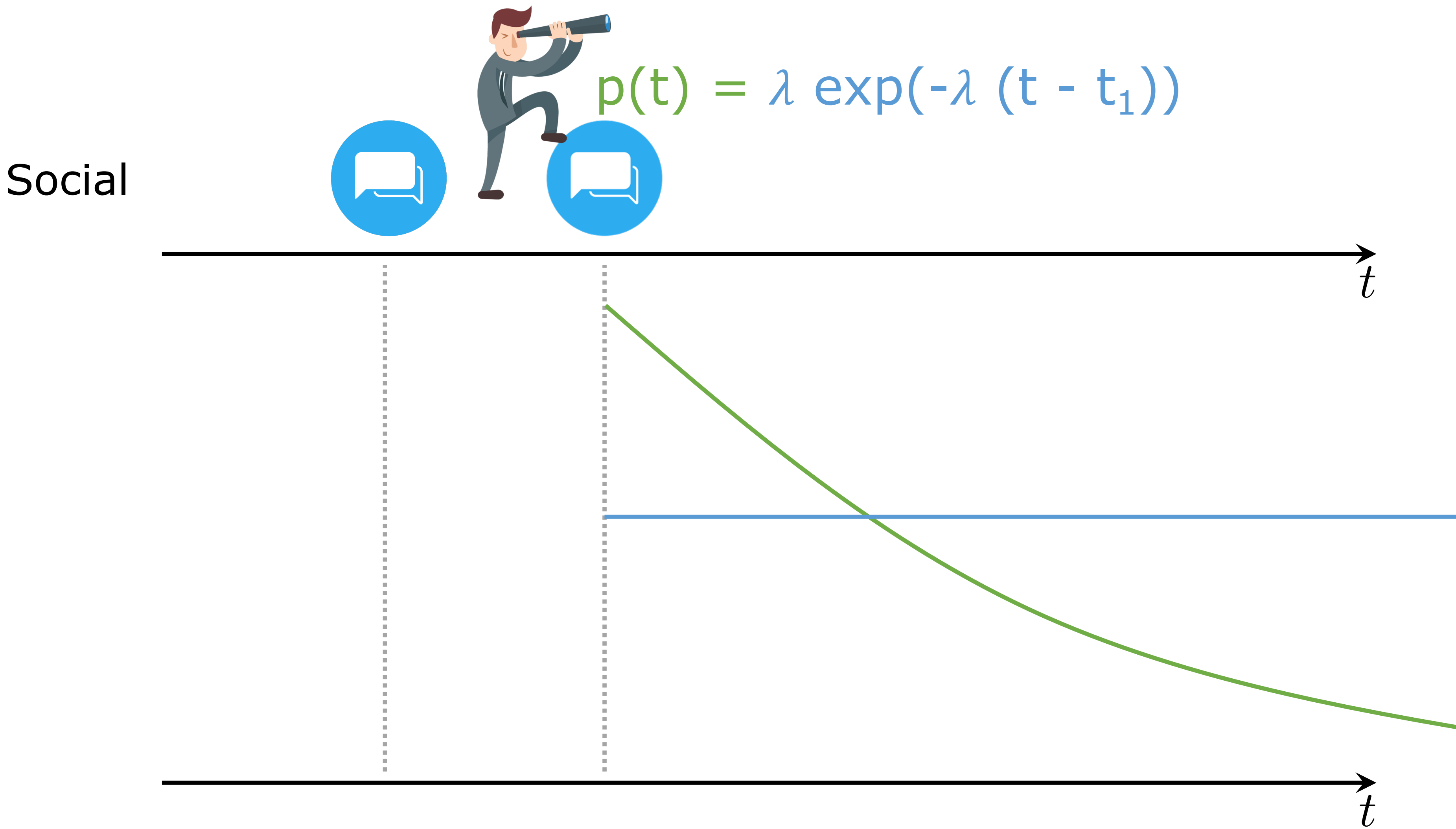

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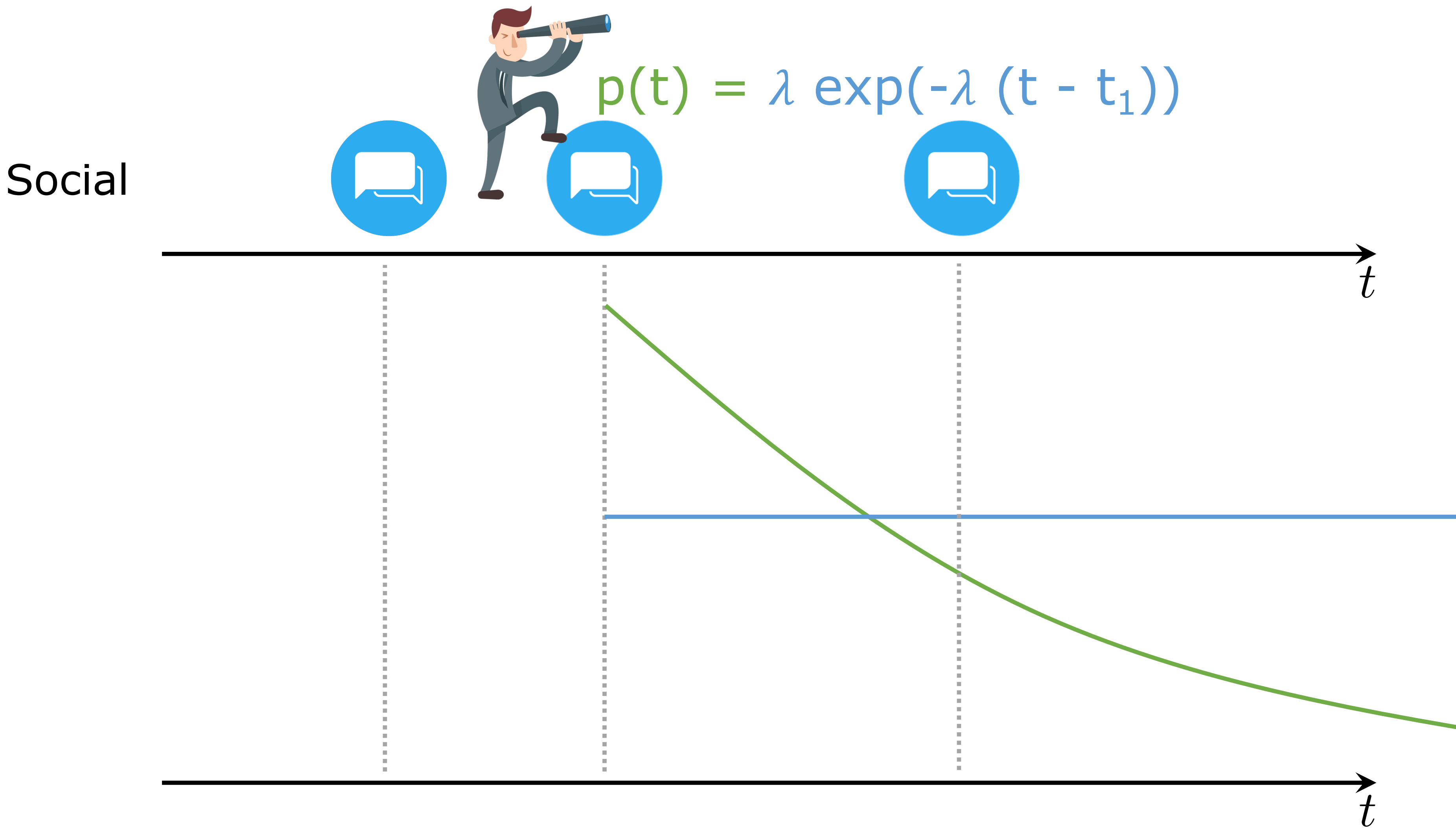

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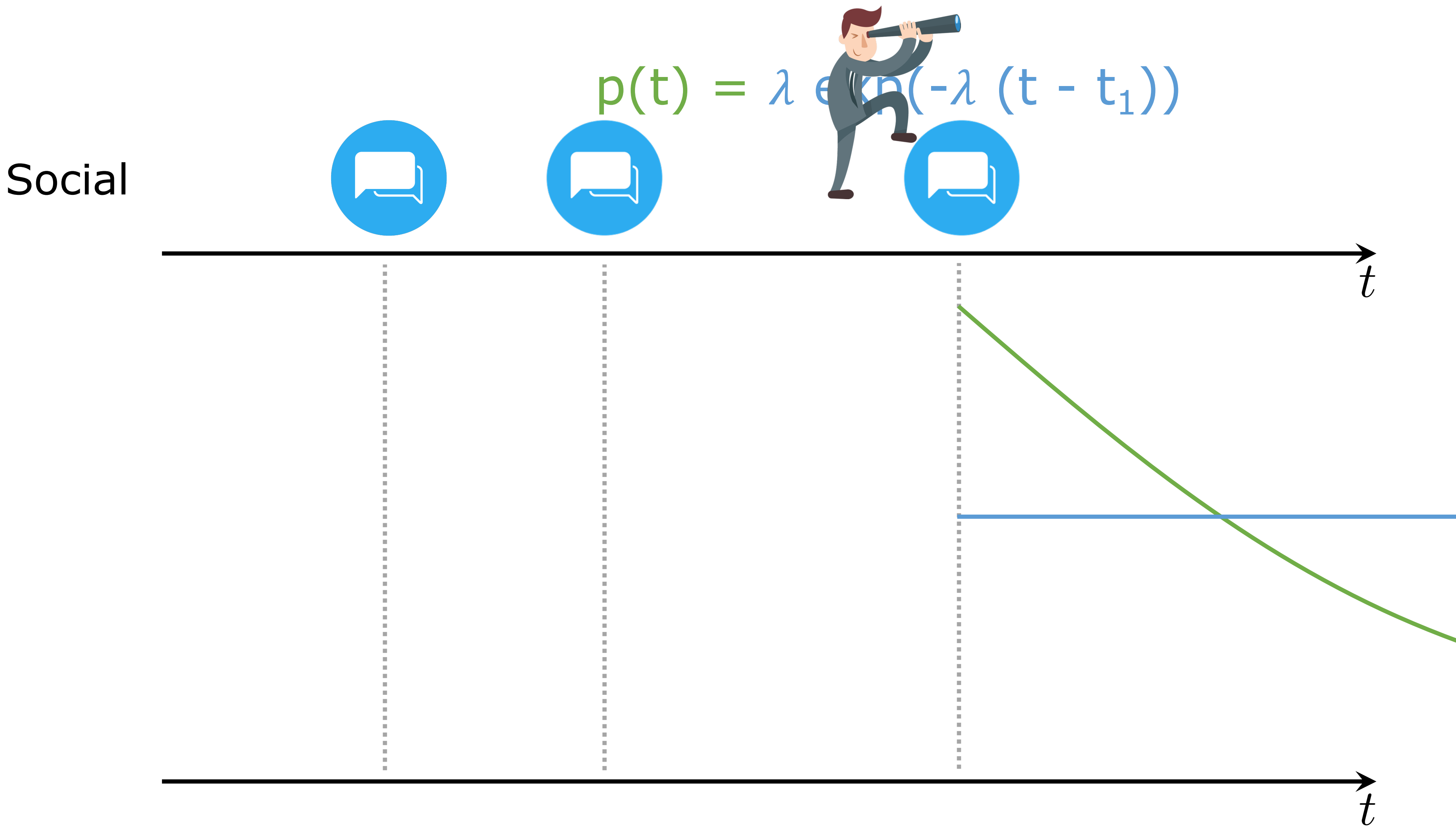

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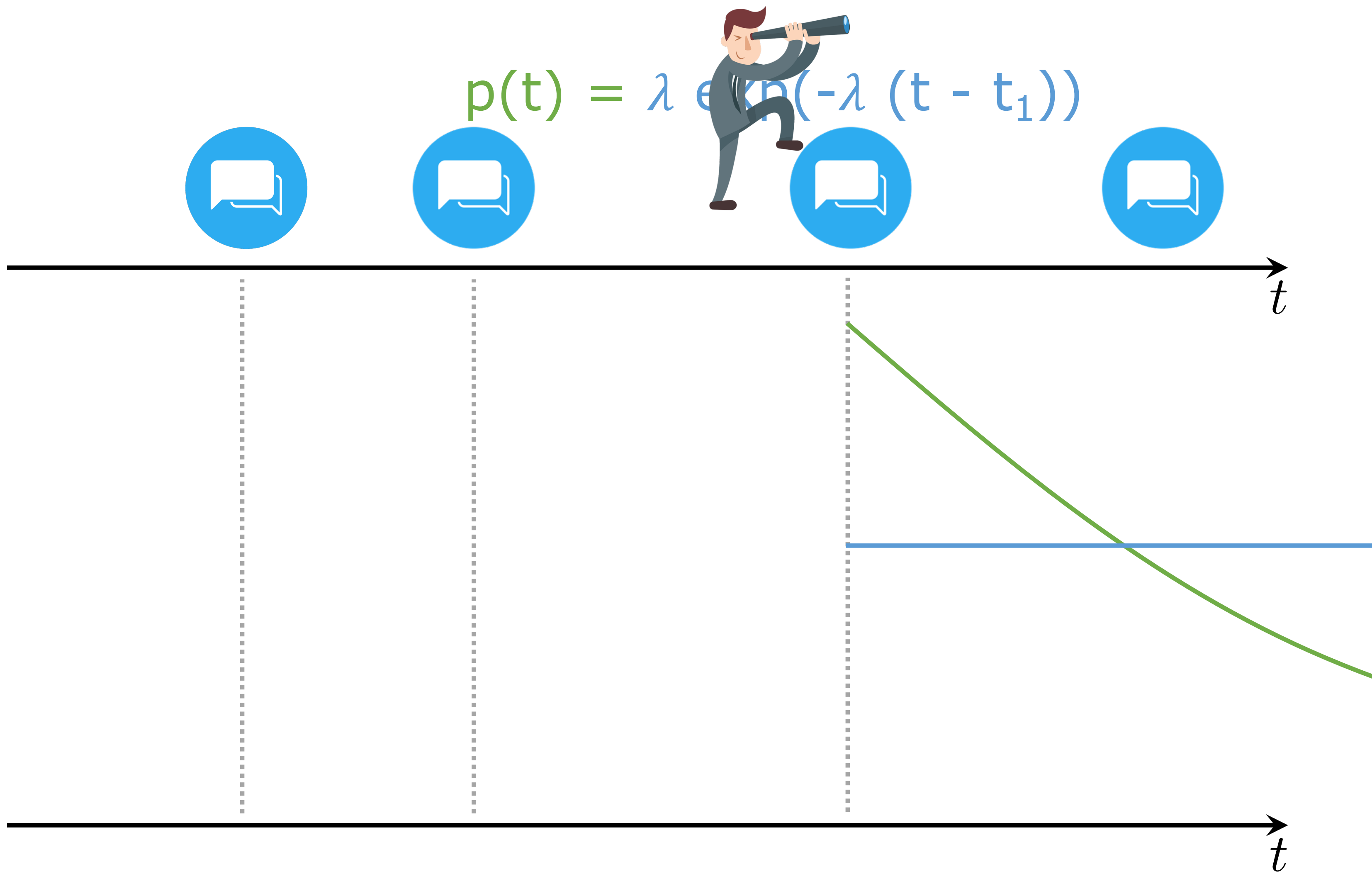
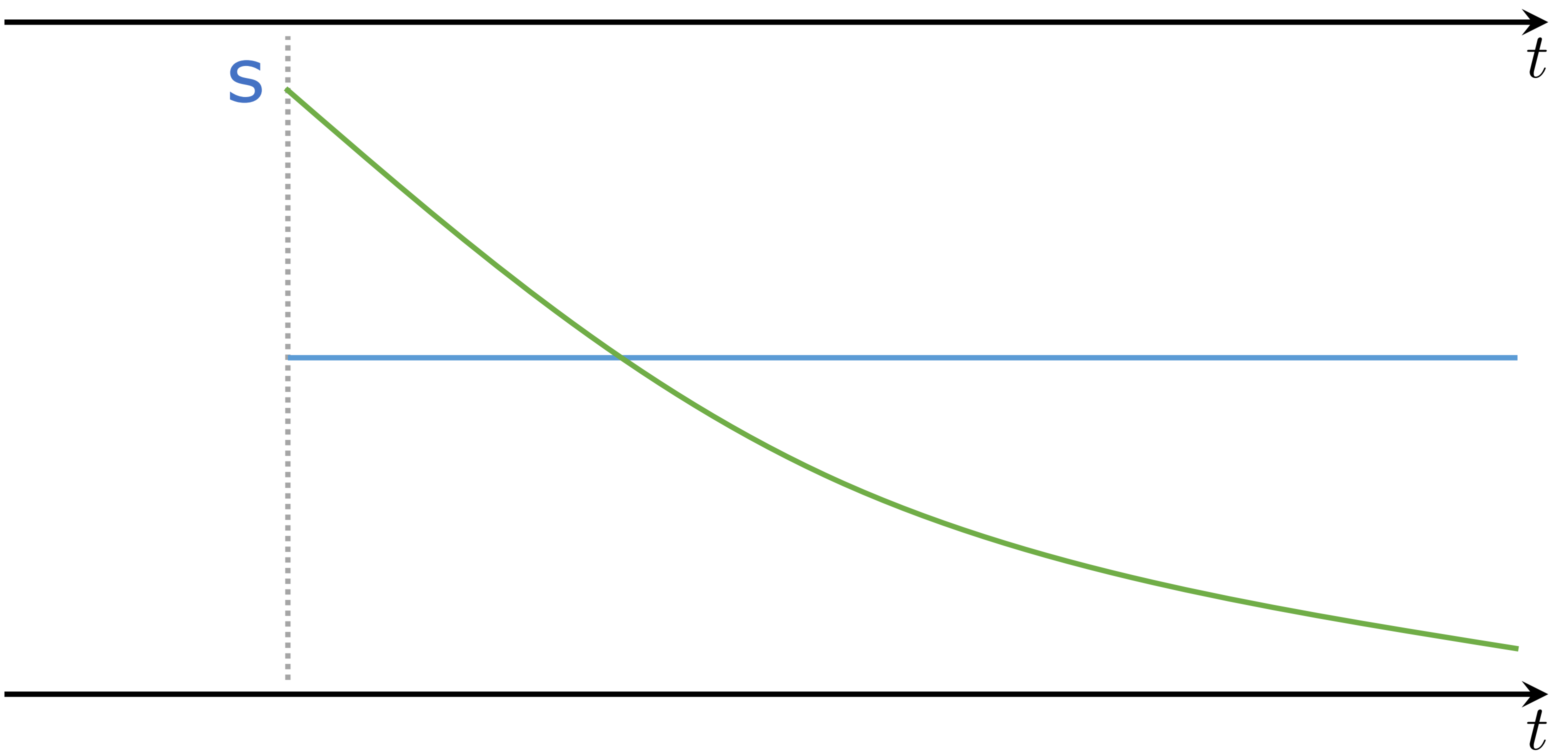


Draw Next Event: Inversion Sampling

$p(t) = \lambda \exp(-\lambda (t - s))$

Social



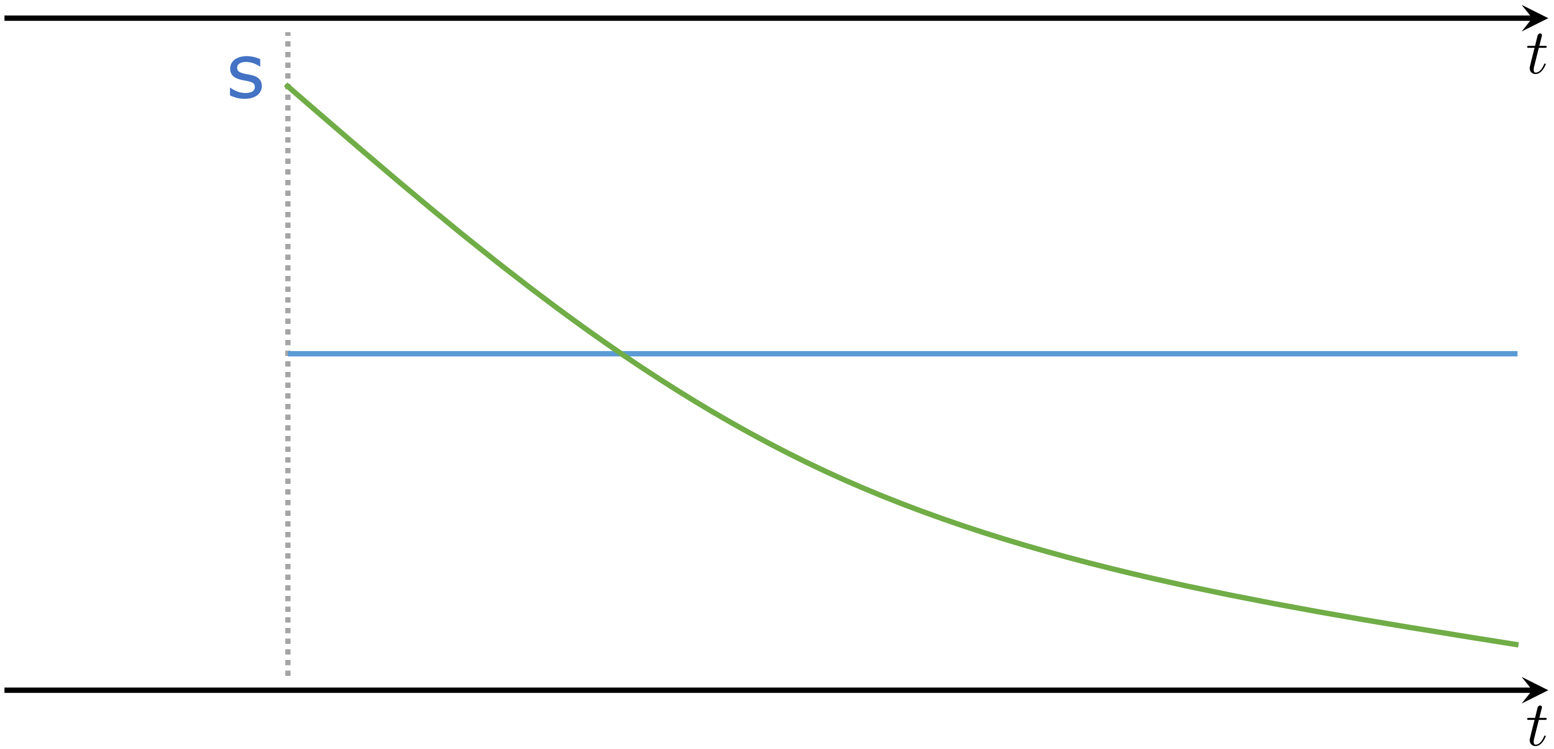
Draw Next Event: Inversion Sampling

$p(t) = \lambda \exp(-\lambda (t - s))$

$F(t) = \int_s^t p(t')dt' \in [0, 1]$

cumulative density function (CDF)

Social



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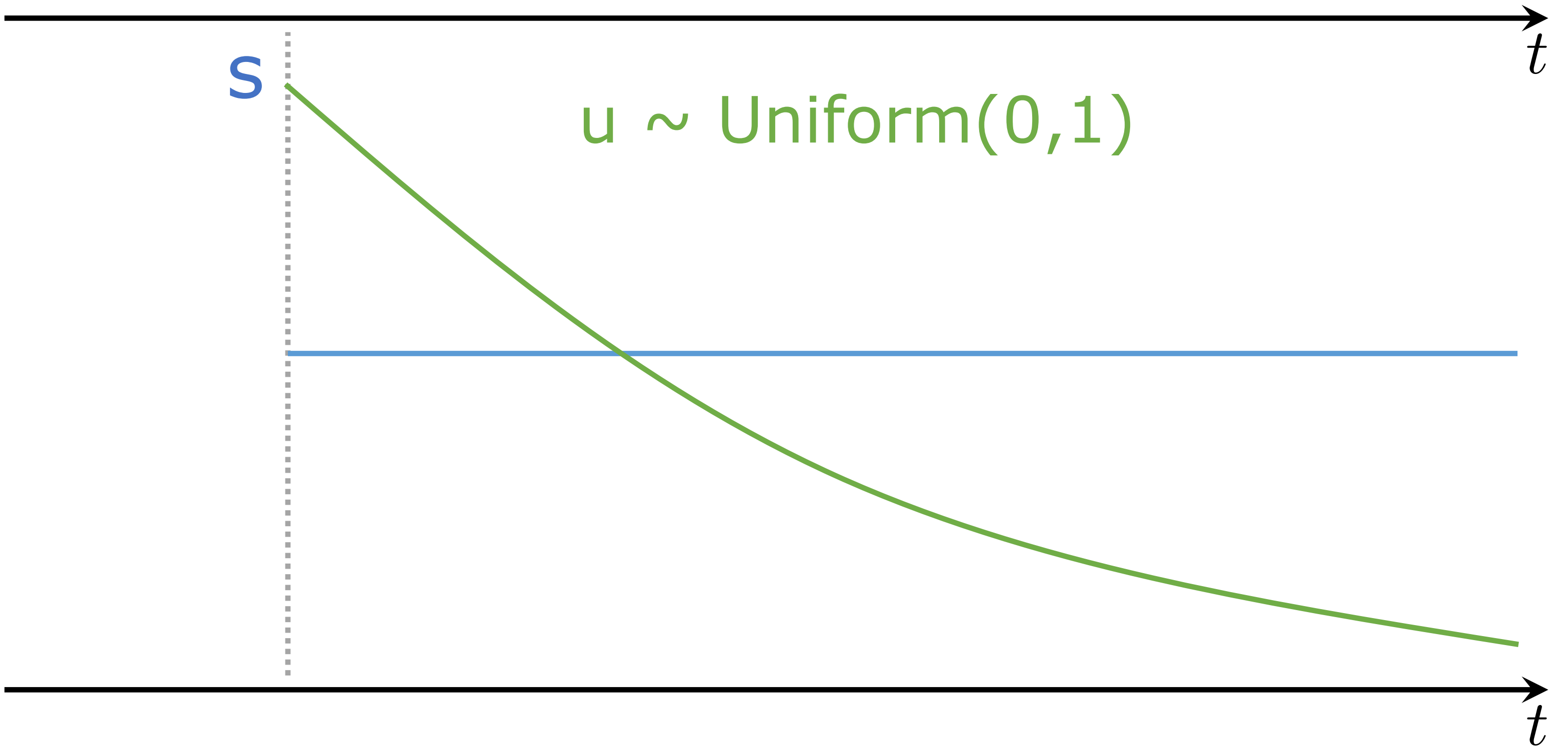

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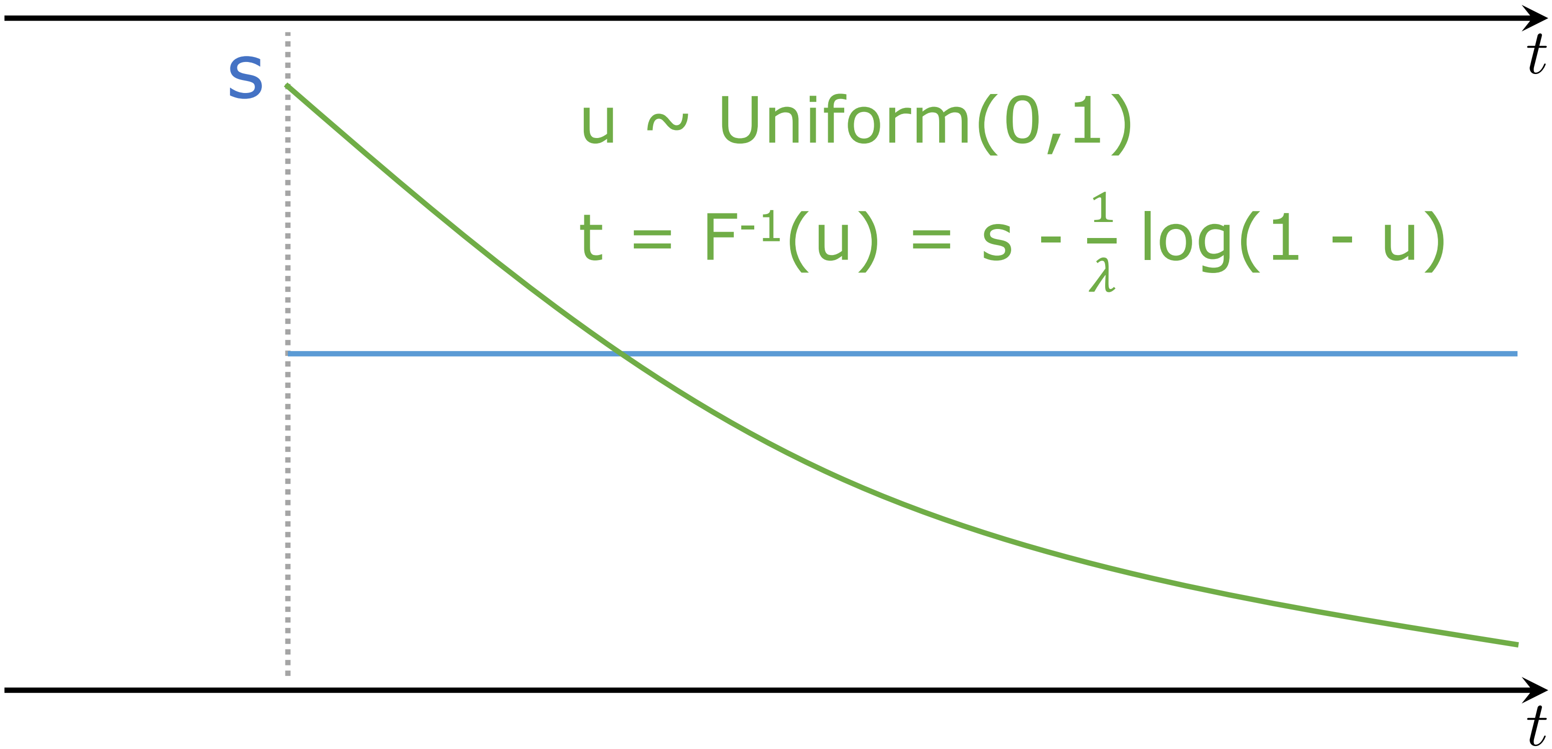


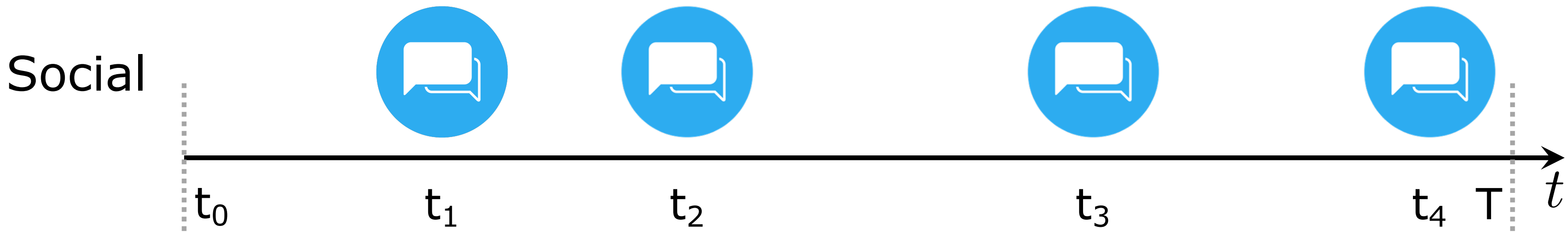
$$u \sim \text{Uniform}(0,1)$$

$$t = F^{-1}(u) = s - \frac{1}{\lambda} \log(1 - u)$$

Let's write some code

http://bburl/tpp-lab-p1

Estimating Intensity: MLE



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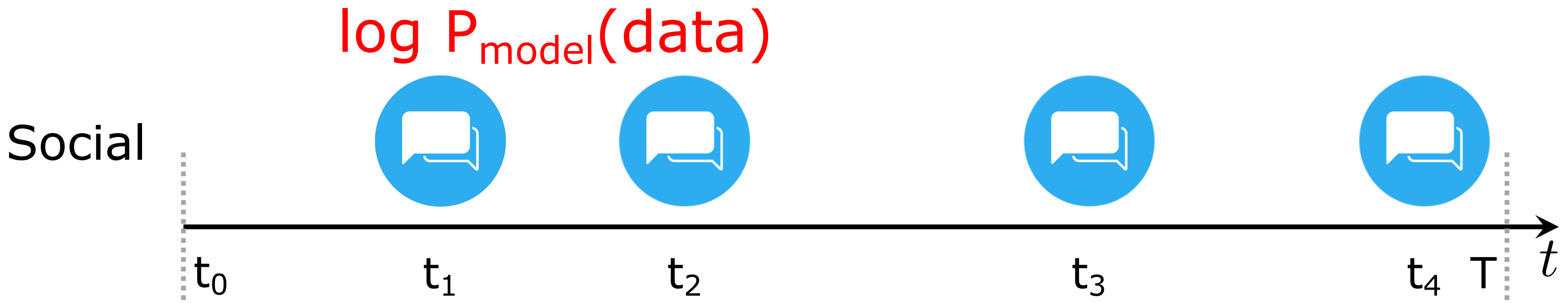
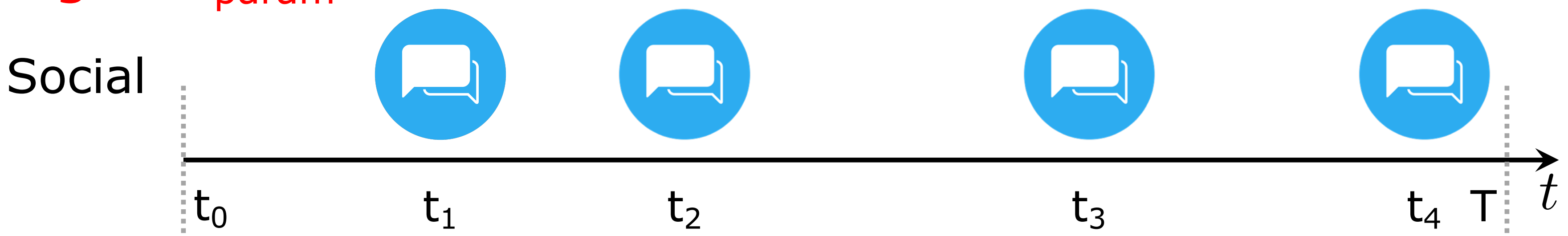


Estimating Intensity: MLE

$\text{argmax}_{\text{param}} \log P_{\text{model}}(\text{data})$



Estimating Intensity: MLE

maximum log-likelihood estimation

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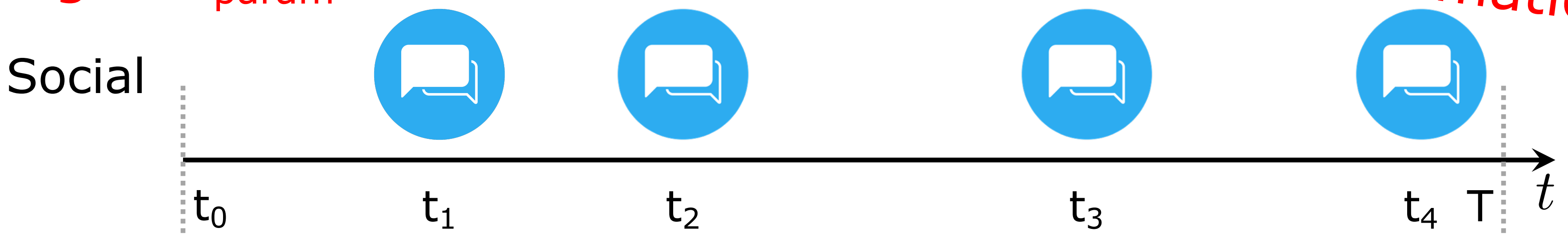

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$\lambda \exp(-\lambda\,(t_1 - t_0))$

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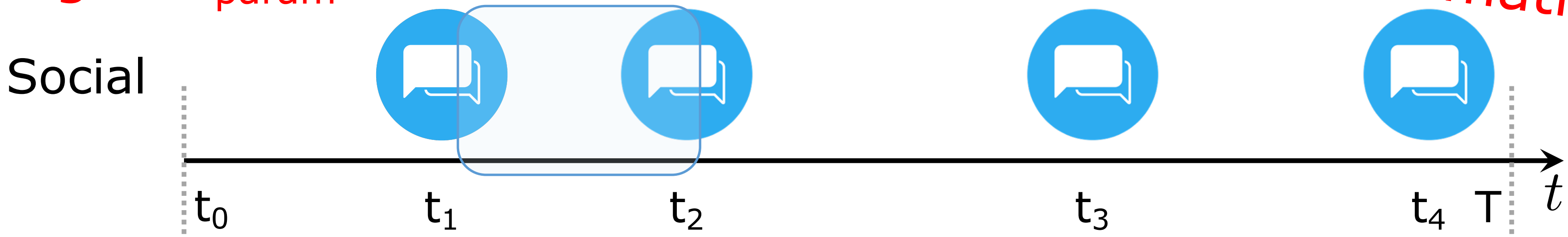


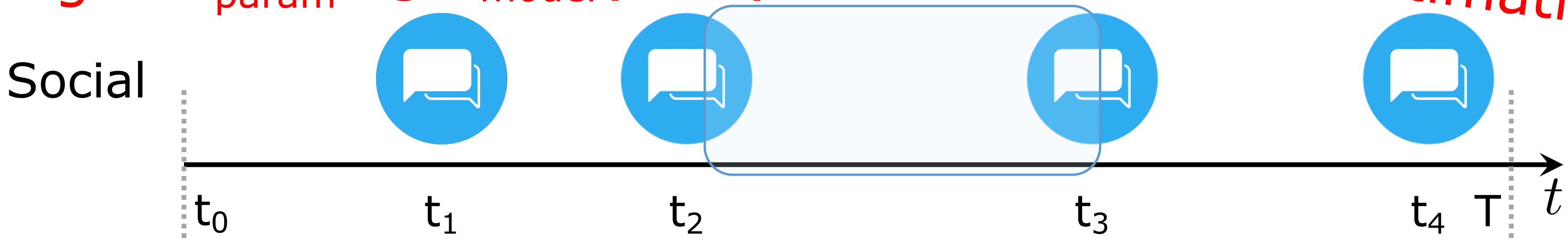
$\lambda \exp(-\lambda\,(t_1 - t_0))$

$\times\ \lambda \exp(-\lambda\,(t_2 - t_1))$

Estimating Intensity: MLE

maximum log-likelihood estimation

$\mathrm{argmax}_{\mathrm{param}} \log P_{\mathrm{model}}(\mathrm{data})$



$\lambda \exp(-\lambda\,(t_1 - t_0))$

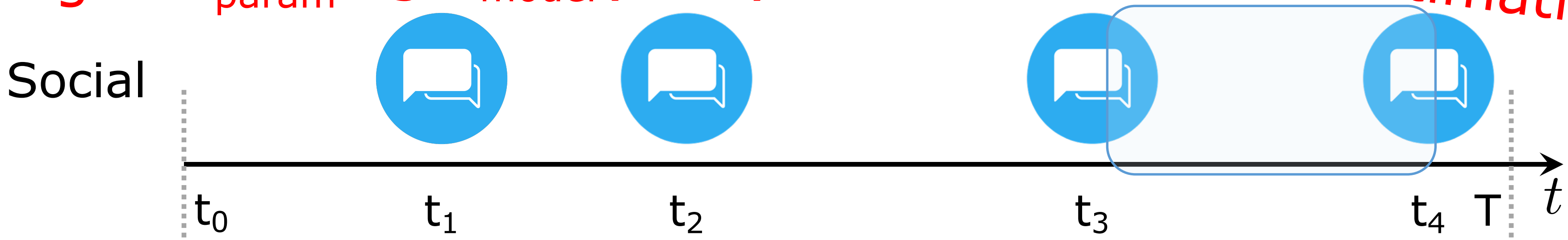
$\times\ \lambda \exp(-\lambda\,(t_2 - t_1))$

$\times\ \lambda \exp(-\lambda\,(t_3 - t_2))$

Estimating Intensity: MLE

maximum log-likelihood estimation

$\mathrm{argmax}_{param} \log P_{model}(data)$



$$\lambda \exp(-\lambda\,(t_1 - t_0))$$
$$\times\ \lambda \exp(-\lambda\,(t_2 - t_1))$$
$$\times\ \lambda \exp(-\lambda\,(t_3 - t_2))$$
$$\times\ \lambda \exp(-\lambda\,(t_4 - t_3))$$

Estimating Intensity: MLE

maximum log-likelihood estimation

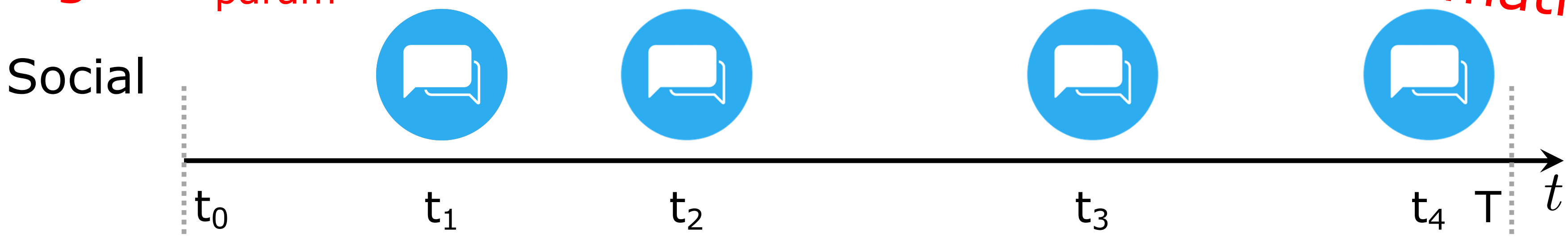
$\mathrm{argmax}_{param} \log P_{model}(\mathrm{data})$

$$\lambda \exp(-\lambda\,(t_1 - t_0))$$
$$\times\ \lambda \exp(-\lambda\,(t_2 - t_1))$$
$$\times\ \lambda \exp(-\lambda\,(t_3 - t_2))$$
$$\times\ \lambda \exp(-\lambda\,(t_4 - t_3))$$
$$\times\ \exp(-\lambda\,(T - t_4))$$

Estimating Intensity: MLE

maximum log-likelihood estimation

$$\operatorname{argmax}_{param} \log P_{model}(\text{data})$$



$$
\begin{aligned}
& \lambda \exp(-\lambda\,(t_1 - t_0)) \\
\times\ & \lambda \exp(-\lambda\,(t_2 - t_1)) \\
\times\ & \lambda \exp(-\lambda\,(t_3 - t_2)) \\
\times\ & \lambda \exp(-\lambda\,(t_4 - t_3)) \\
\times\ & \exp(-\lambda\,(T - t_4)) \\
=\ & \lambda^4 \exp(-\lambda\,(T - t_0))
\end{aligned}
$$

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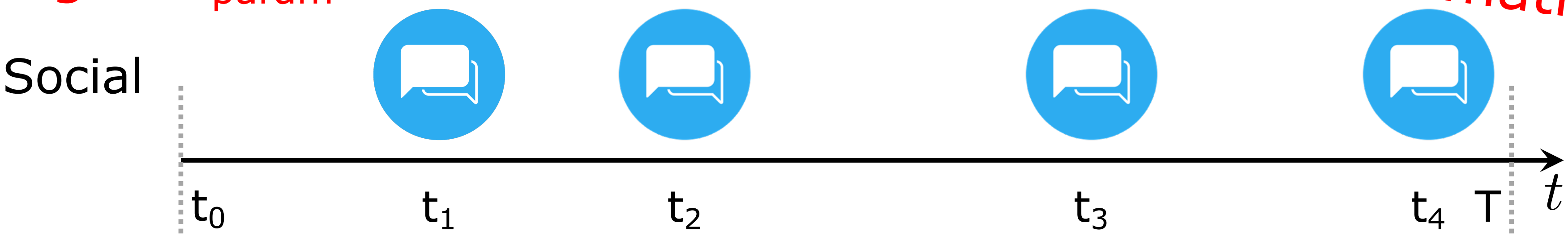
http://bburl/tpp-lab-p1

Estimating Intensity: MLE

maximum log-likelihood estimation

$\mathrm{argmax}_{\mathrm{param}} \log P_{\mathrm{model}}(\mathrm{data})$



$$
\begin{aligned}
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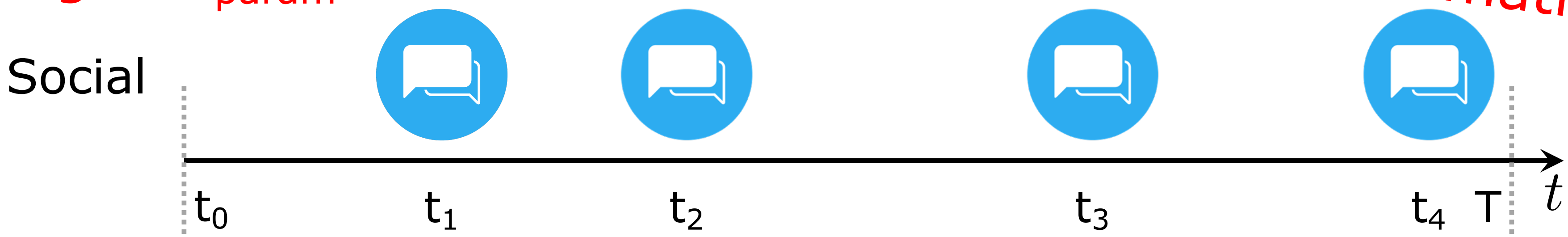
\times\ & \lambda \exp(-\lambda\,(t_2 - t_1)) \\
\times\ & \lambda \exp(-\lambda\,(t_3 - t_2)) \\
\times\ & \lambda \exp(-\lambda\,(t_4 - t_3)) \\
\times\ & \exp(-\lambda\,(T - t_4)) \\
=\ & \lambda^4 \exp(-\lambda\,(T - t_0))
\end{aligned}
$$

$$\mathrm{argmax}_{\lambda} \log \lambda^4 \exp(-\lambda\,(T - t_0))$$

Estimating Intensity: MLE

maximum log-likelihood estimation

$\mathrm{argmax}_{\mathrm{param}} \log P_{\mathrm{model}}(\mathrm{data})$



$$\begin{aligned}
&\lambda \exp(-\lambda\,(t_1 - t_0)) \\
\times\ &\lambda \exp(-\lambda\,(t_2 - t_1)) \\
\times\ &\lambda \exp(-\lambda\,(t_3 - t_2)) \\
\times\ &\lambda \exp(-\lambda\,(t_4 - t_3)) \\
\times\ &\exp(-\lambda\,(T - t_4)) \\
=\ &\lambda^4 \exp(-\lambda\,(T - t_0))
\end{aligned}$$

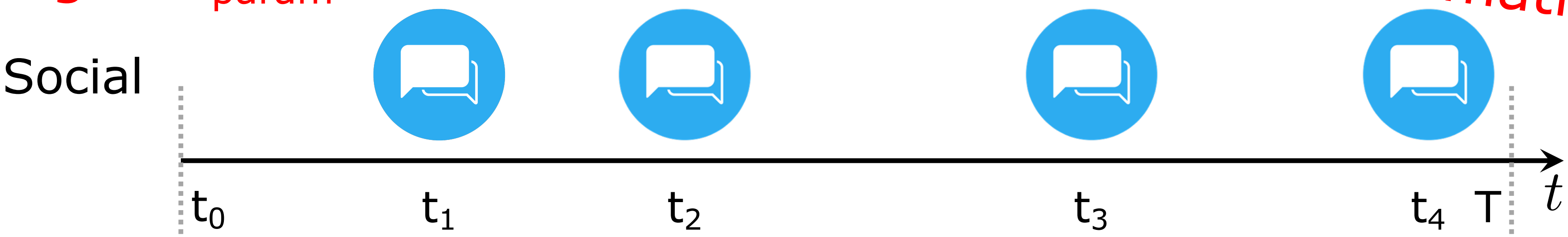
$$\mathrm{argmax}_{\lambda} \log \lambda^4 \exp(-\lambda\,(T - t_0))$$

$$\mathrm{argmax}_{\lambda}\ 4 \log \lambda - \lambda\,(T - t_0)$$

Estimating Intensity: MLE

$\text{argmax}_{\text{param}} \log P_{\text{model}}(\text{data})$

maximum log-likelihood estimation



$$\begin{aligned}
&\lambda \exp(-\lambda\ (t_1 - t_0)) \\
\times\ &\lambda \exp(-\lambda\ (t_2 - t_1)) \\
\times\ &\lambda \exp(-\lambda\ (t_3 - t_2)) \\
\times\ &\lambda \exp(-\lambda\ (t_4 - t_3)) \\
\times\ &\exp(-\lambda\ (T - t_4)) \\
=\ &\lambda^4 \exp(-\lambda\ (T - t_0))
\end{aligned}$$

$$\text{argmax}_\lambda \log \lambda^4 \exp(-\lambda\ (T - t_0))$$

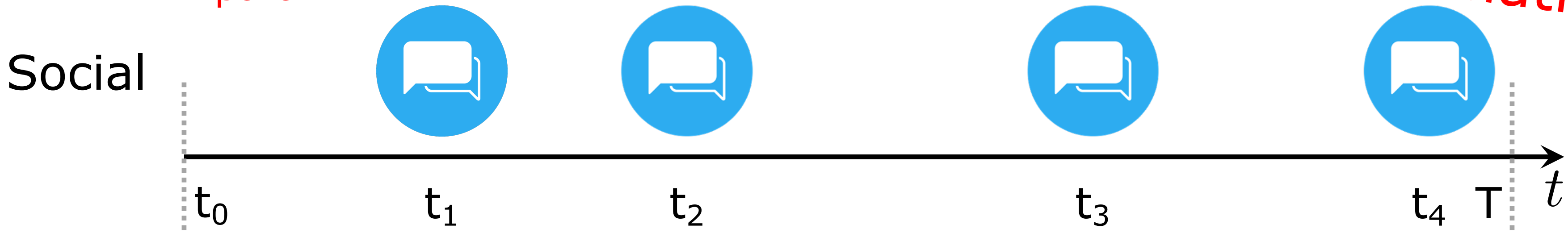
$$\text{argmax}_\lambda\ 4 \log \lambda - \lambda\ (T - t_0)$$

$$\text{argmax}_\lambda f(\lambda) \leftrightarrow \nabla_\lambda f(\lambda) = 0$$

Estimating Intensity: MLE

maximum log-likelihood estimation

$\text{argmax}_{\text{param}} \log P_{\text{model}}(\text{data})$



$$\begin{aligned}
&\lambda \exp(-\lambda\,(t_1 - t_0)) \\
\times\ &\lambda \exp(-\lambda\,(t_2 - t_1)) \\
\times\ &\lambda \exp(-\lambda\,(t_3 - t_2)) \\
\times\ &\lambda \exp(-\lambda\,(t_4 - t_3)) \\
\times\ &\exp(-\lambda\,(T - t_4)) \\
=\ &\lambda^4 \exp(-\lambda\,(T - t_0))
\end{aligned}$$

$$\text{argmax}_{\lambda} \log \lambda^4 \exp(-\lambda\,(T - t_0))$$

$$\text{argmax}_{\lambda}\ 4 \log \lambda - \lambda\,(T - t_0)$$

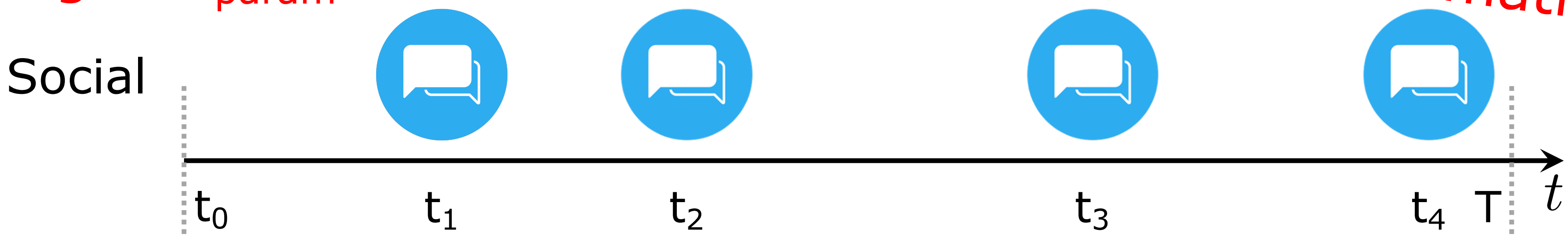
$$\text{argmax}_{\lambda} f(\lambda) \leftrightarrow \nabla_{\lambda} f(\lambda) = 0$$

$$\nabla_{\lambda}(4 \log \lambda - \lambda\,(T - t_0)) = 0$$

Estimating Intensity: MLE

maximum log-likelihood estimation

$\text{argmax}_{\text{param}} \log P_{\text{model}}(\text{data})$



$$
\begin{aligned}
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\times\ & \lambda \exp(-\lambda\,(t_3 - t_2)) \\
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\end{aligned}
$$

$$\text{argmax}_\lambda \log \lambda^4 \exp(-\lambda\,(T - t_0))$$

$$\text{argmax}_\lambda\ 4 \log \lambda - \lambda\,(T - t_0)$$

$$\text{argmax}_\lambda f(\lambda) \leftrightarrow \nabla_\lambda f(\lambda) = 0$$

$$\nabla_\lambda(4 \log \lambda - \lambda\,(T - t_0)\ ) = 0$$

$$\lambda = 4 \,/\, (T - t_0)$$

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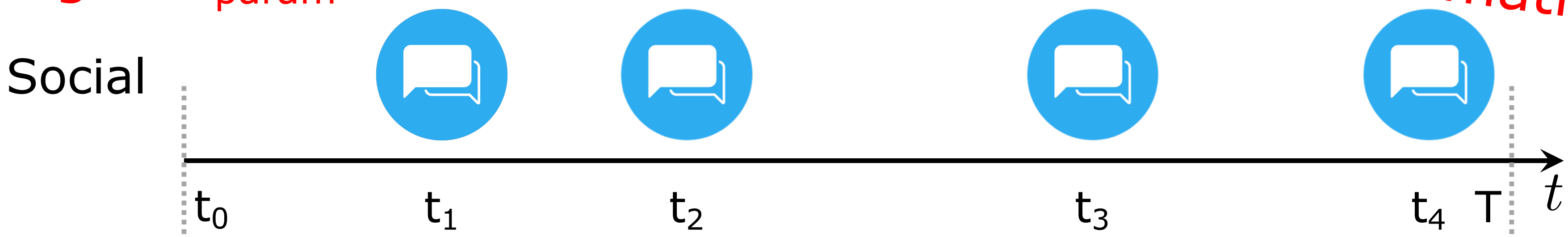
Estimating Intensity: MLE

maximum log-likelihood estimation

$\text{argmax}_{\text{param}} \log P_{\text{model}}(\text{data})$



$$\begin{aligned}
&\lambda \exp(-\lambda\,(t_1 - t_0)) \\
\times\ &\lambda \exp(-\lambda\,(t_2 - t_1)) \\
\times\ &\lambda \exp(-\lambda\,(t_3 - t_2)) \\
\times\ &\lambda \exp(-\lambda\,(t_4 - t_3)) \\
\times\ &\exp(-\lambda\,(T - t_4)) \\
=\ &\lambda^4 \exp(-\lambda\,(T - t_0))
\end{aligned}$$

$$\text{argmax}_\lambda \log \lambda^4 \exp(-\lambda\,(T - t_0))$$

$$\text{argmax}_\lambda\ 4 \log \lambda - \lambda\,(T - t_0)$$

$$\text{argmax}_\lambda f(\lambda) \leftrightarrow \nabla_\lambda f(\lambda) = 0$$

$$\nabla_\lambda(4 \log \lambda - \lambda\,(T - t_0)\,) = 0$$

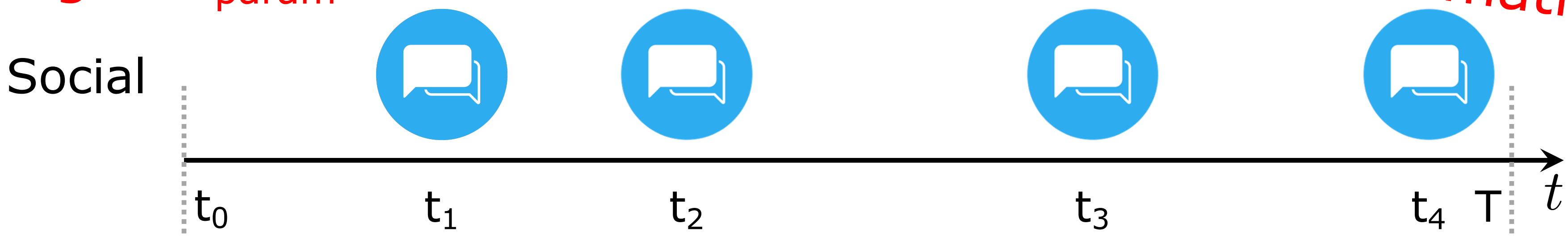
$$\lambda = 4\,/\,(T - t_0)$$

# of events

Estimating Intensity: MLE

maximum log-likelihood estimation

$\text{argmax}_{\text{param}} \log P_{\text{model}}(\text{data})$



$$
\begin{aligned}
&\lambda \exp(-\lambda\,(t_1 - t_0)) \\
\times\ &\lambda \exp(-\lambda\,(t_2 - t_1)) \\
\times\ &\lambda \exp(-\lambda\,(t_3 - t_2)) \\
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\times\ &\exp(-\lambda\,(T - t_4)) \\
=\ &\lambda^4 \exp(-\lambda\,(T - t_0))
\end{aligned}
$$

$$\text{argmax}_\lambda \log \lambda^4 \exp(-\lambda\,(T - t_0))$$

$$\text{argmax}_\lambda\ 4 \log \lambda - \lambda\,(T - t_0)$$

$$\text{argmax}_\lambda f(\lambda) \leftrightarrow \nabla_\lambda f(\lambda) = 0$$

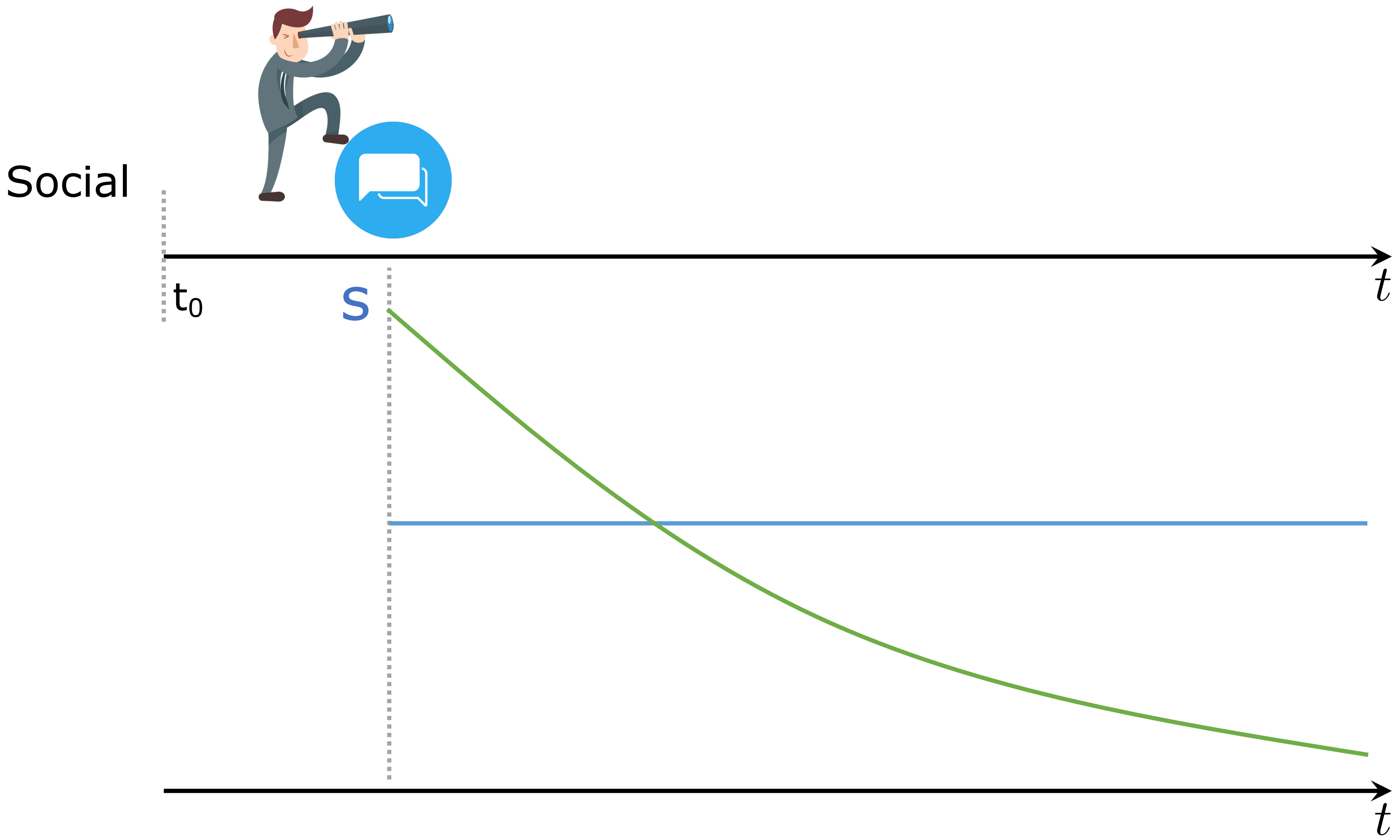
$$\nabla_\lambda(4 \log \lambda - \lambda\,(T - t_0)\,) = 0$$

$$\lambda = 4\,/\,(T - t_0)$$

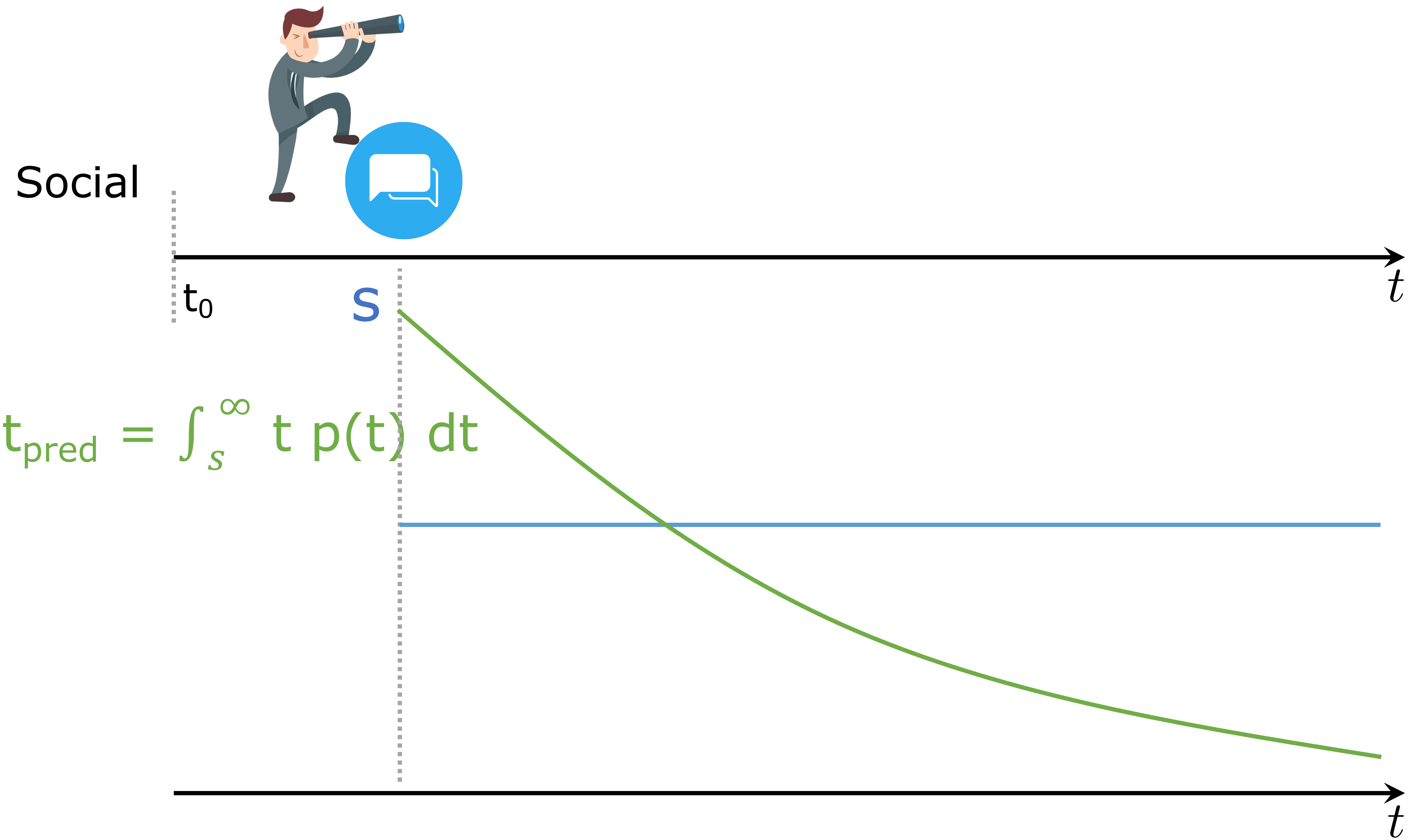
total time

# of events

Predicting Next Event Time: MBR

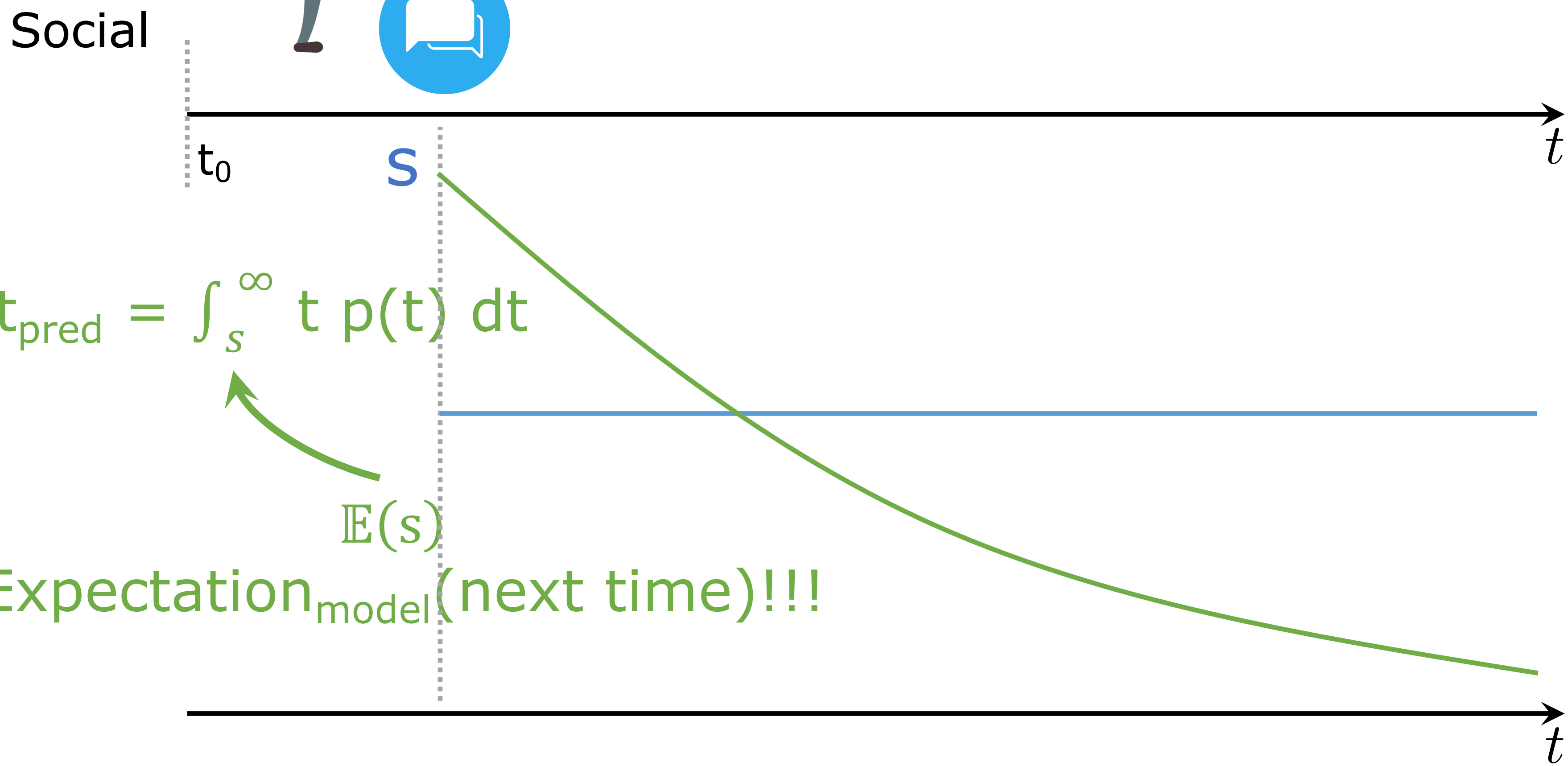


Predicting Next Event Time: MBR



$$t_{pred} = \int_s^\infty t\, p(t)\, dt$$

Predicting Next Event Time: MBR



$t_{pred} = \int_s^{\infty} t\, p(t)\, dt$

$\mathbb{E}(s)$

$\text{Expectation}_{\text{model}}(\text{next time})$!!!

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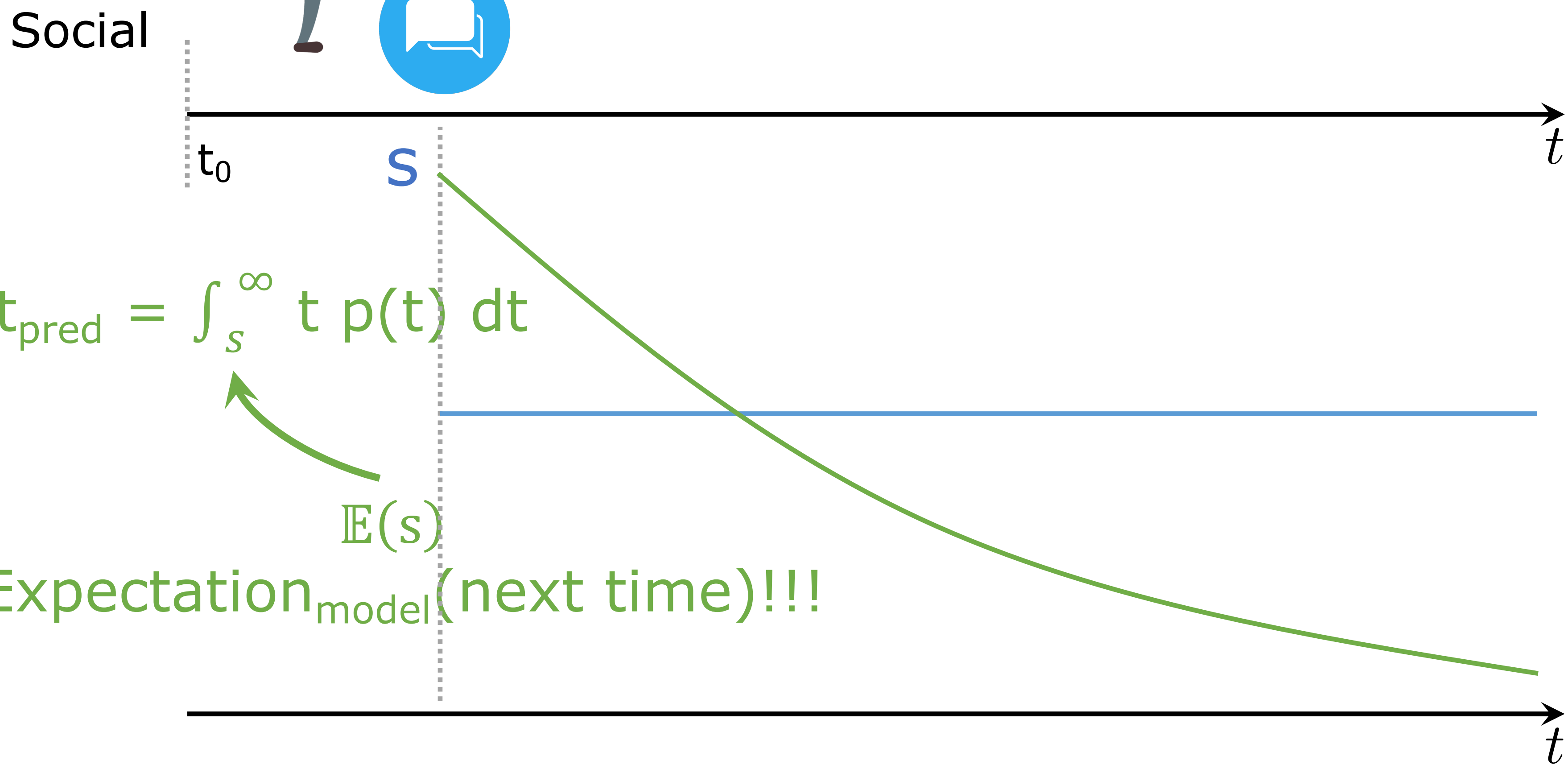
http://bburl/tpp-lab-p1

Predicting Next Event Time: MBR

minimum Bayes risk



$t_{pred} = \int_s^\infty t\, p(t)\, dt$

$\mathbb{E}(s)$

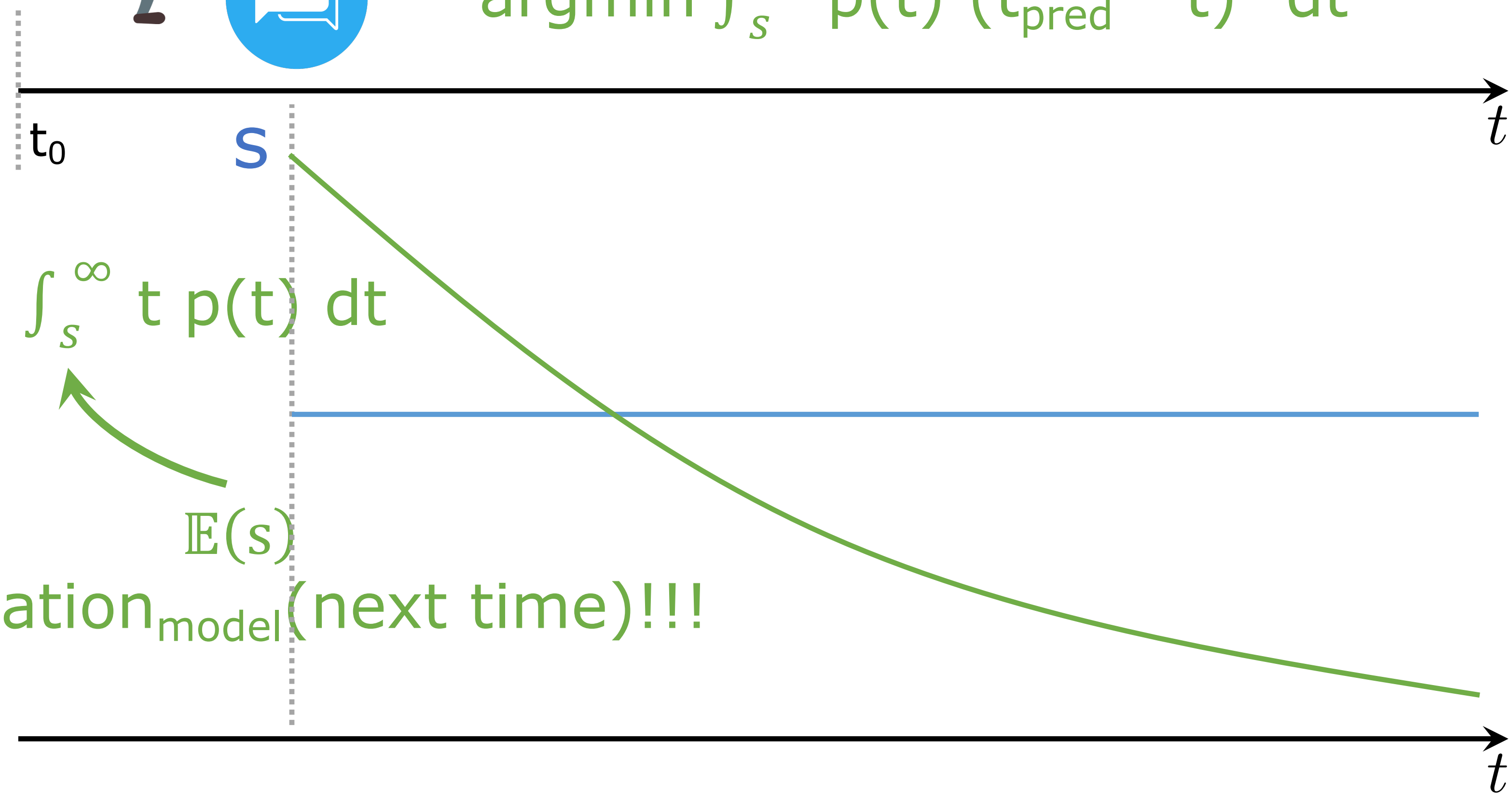
$\text{Expectation}_{\text{model}}(\text{next time})$!!!

Predicting Next Event Time: MBR

minimum Bayes risk

$$\operatorname{argmin} \int_s^\infty p(t)\,(t_{pred} - t)^2\,dt$$

Social



$$t_{pred} = \int_s^\infty t\,p(t)\,dt$$

$\mathbb{E}(s)$

Expectation$_{model}$(next time)!!!

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http://bburl/tpp-lab-p1

Predicting Next Event Time: MBR

minimum Bayes risk

$\operatorname{argmin} \int_{s}^{\infty} p(t)\,(t_{pred} - t)^2\,dt$

Social

t_0

s

t

$\operatorname{argmin} f(t) \leftrightarrow \nabla_t f(t) = 0$

$t_{pred} = \int_{s}^{\infty} t\, p(t)\, dt$

$\mathbb{E}(s)$

$\text{Expectation}_{\text{model}}(\text{next time})!!!$

t

http://bburl/tpp-slides-p1
http://bburl/tpp-lab-p1

Predicting Next Event Time: MBR

minimum Bayes risk

$$\operatorname{argmin} \int_s^\infty p(t)\,(t_{pred} - t)^2\, dt$$

Social

t_0 s t

$$\operatorname{argmin} f(t) \leftrightarrow \nabla_t f(t) = 0$$

$$t_{pred} = \int_s^\infty t\, p(t)\, dt$$

$$2\, t \int_{t_1}^\infty p(s)\, ds = 2 \int_{t_1}^\infty s\, p(s)\, ds$$

$\mathbb{E}(s)$

Expectation$_{model}$(next time)!!!

t

Any Questions?

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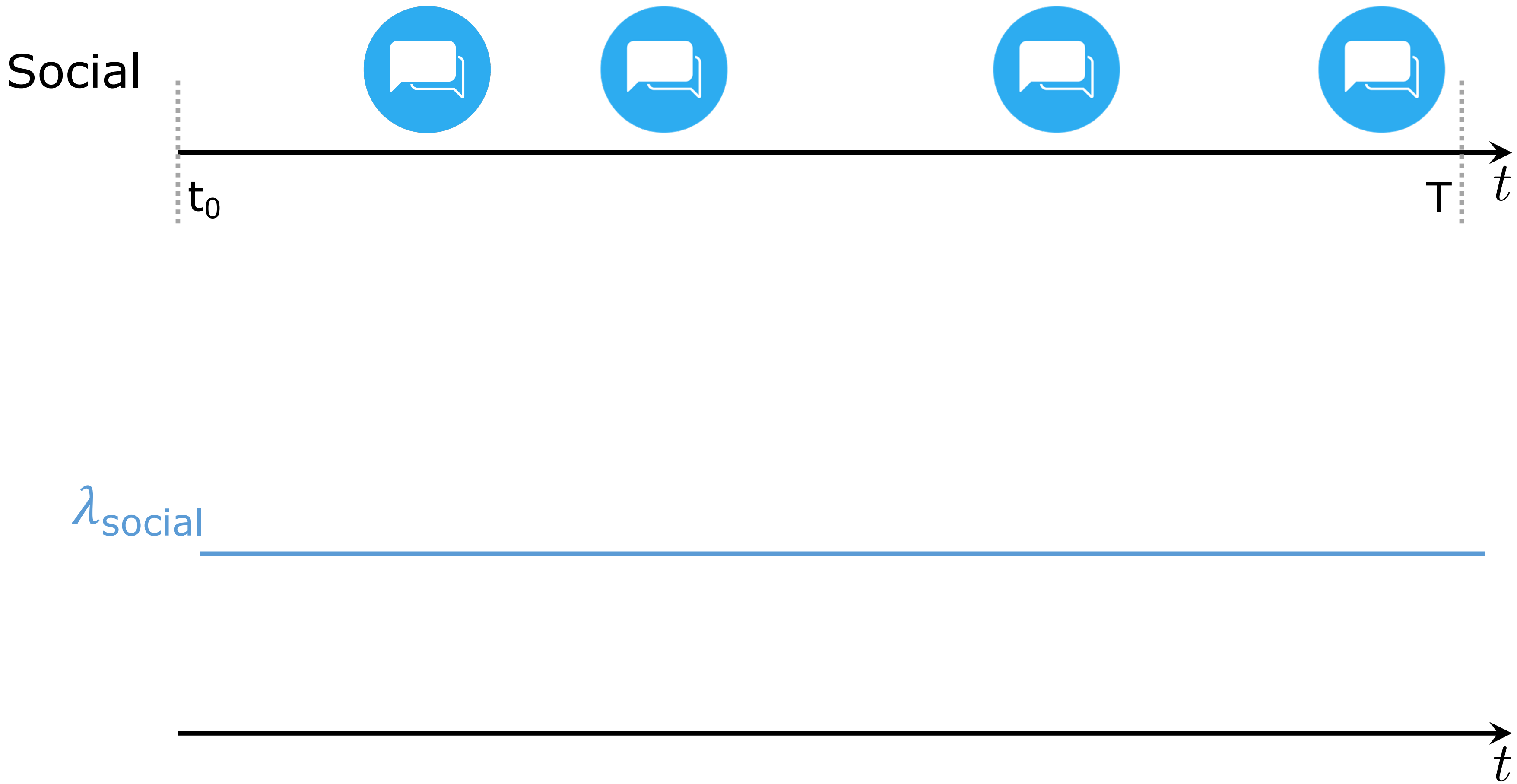
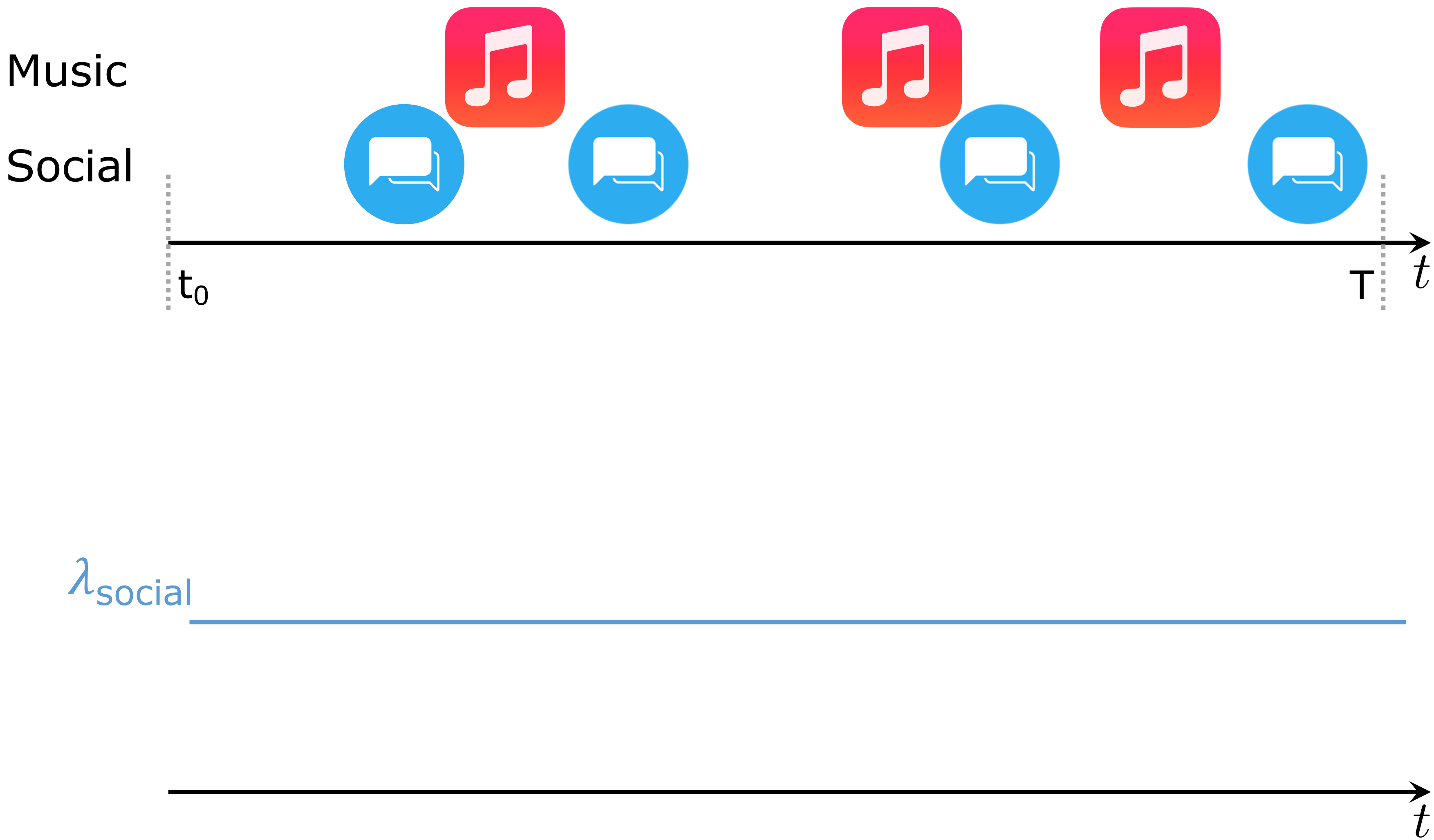
http://bburl/tpp-slides-p1
http://bburl/tpp-lab-p1

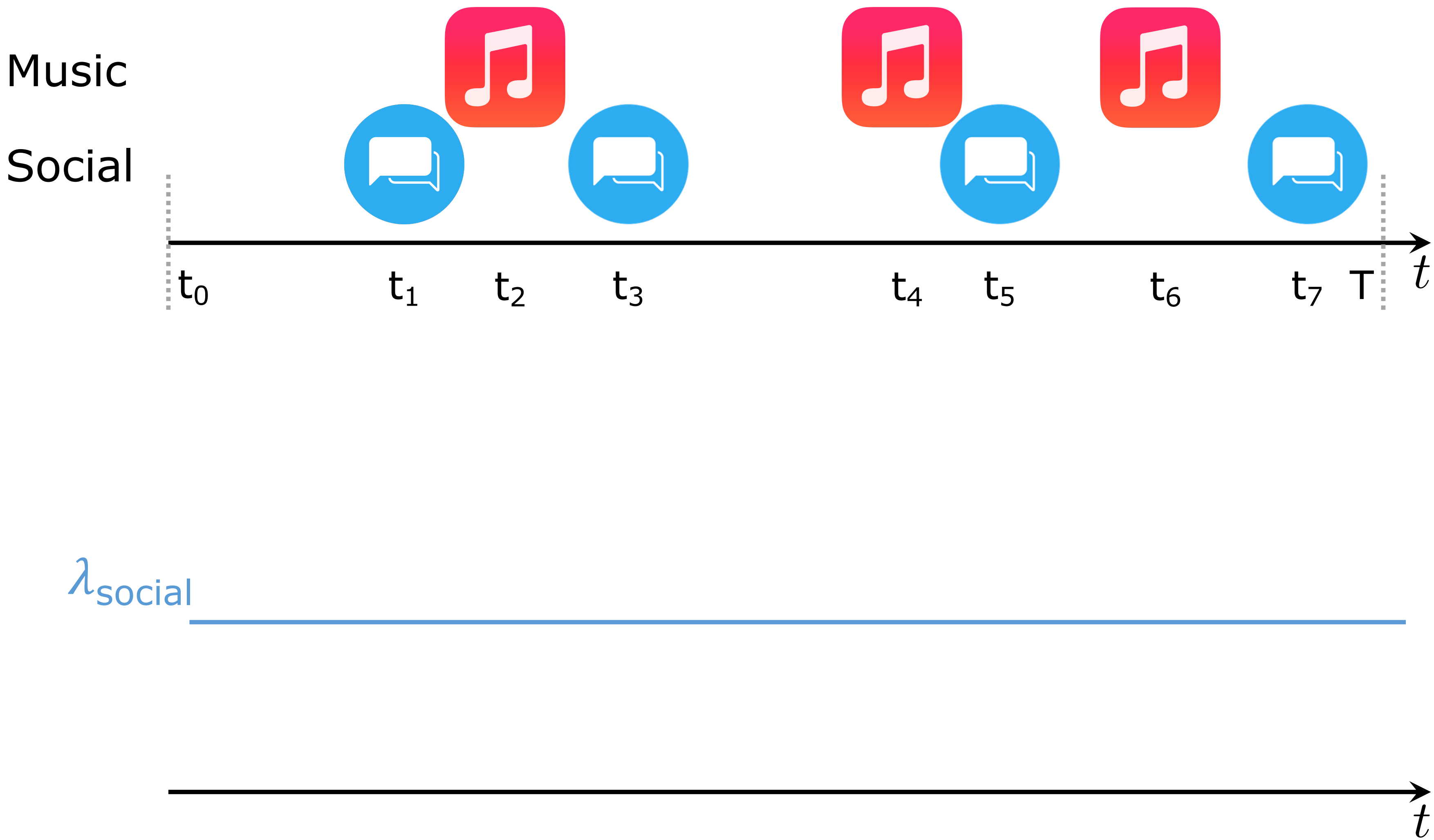
Multivariate Time Series



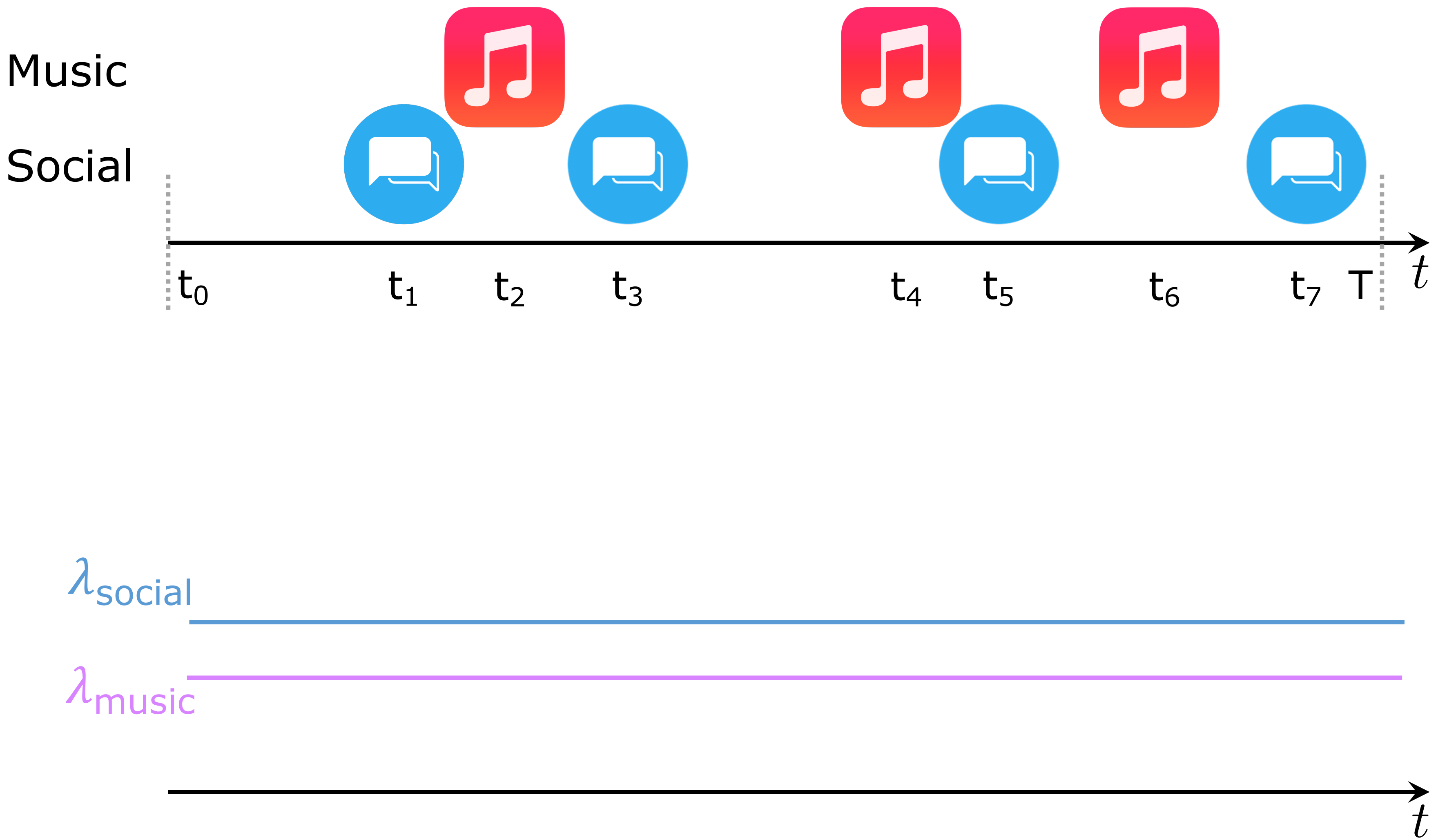
Multivariate Time Series



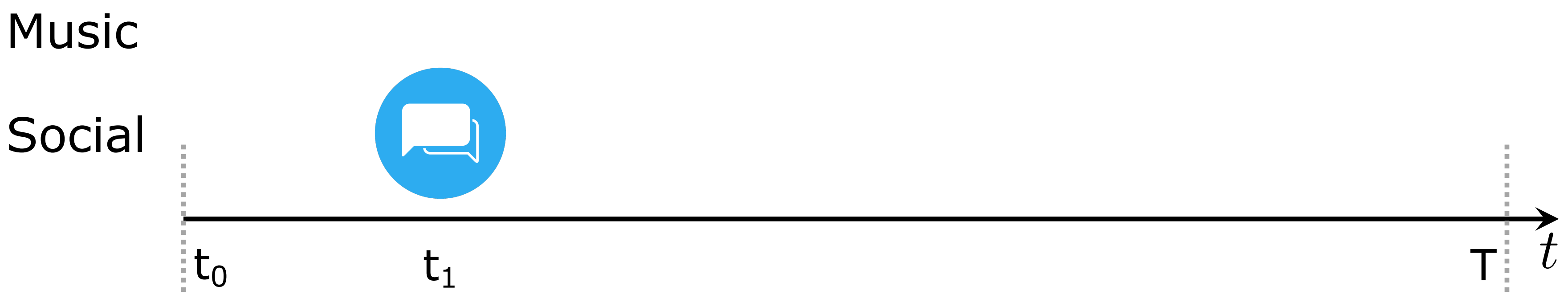
Multivariate Time Series

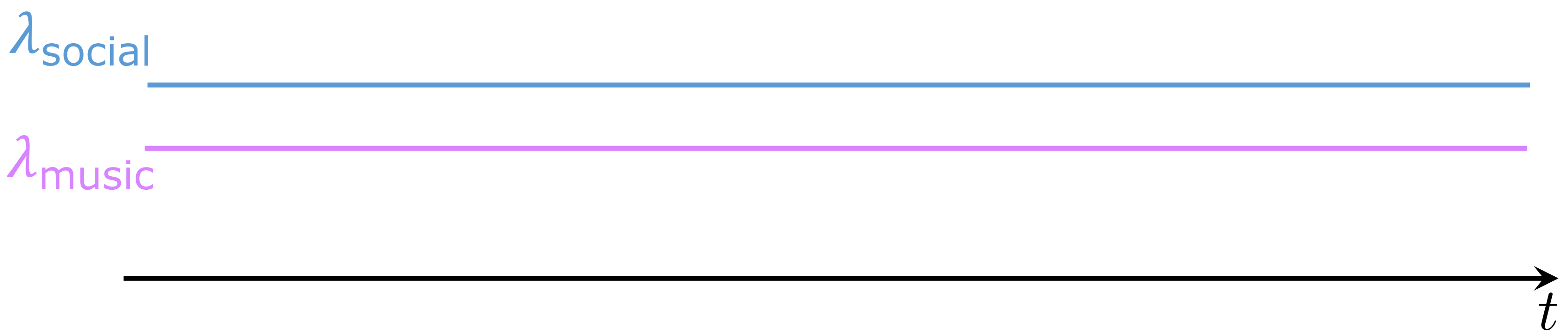


Multivariate Time Series



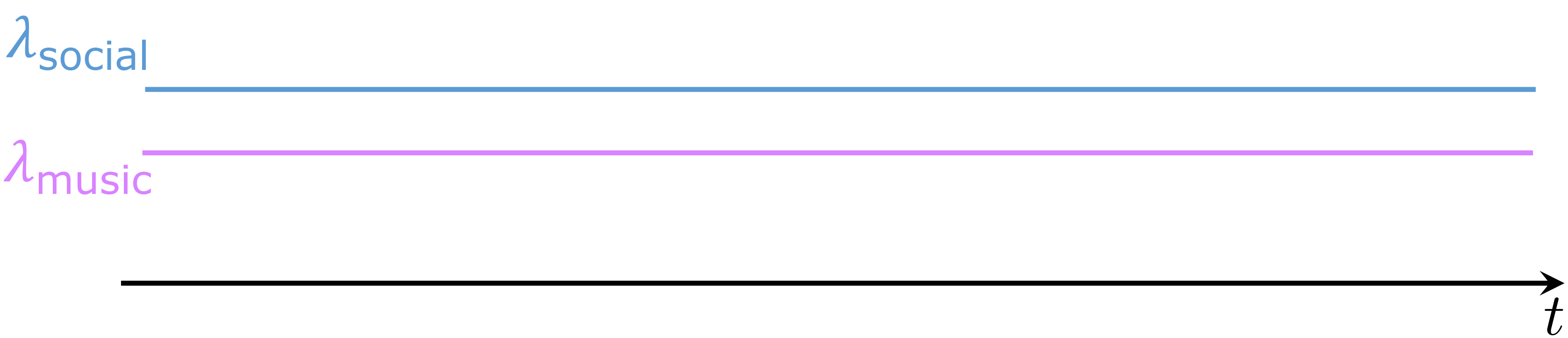
Multivariate Time Series





Multivariate Time Series

$(\lambda_{\text{music}} + \lambda_{\text{social}})dt$ = prob of *some* event at t



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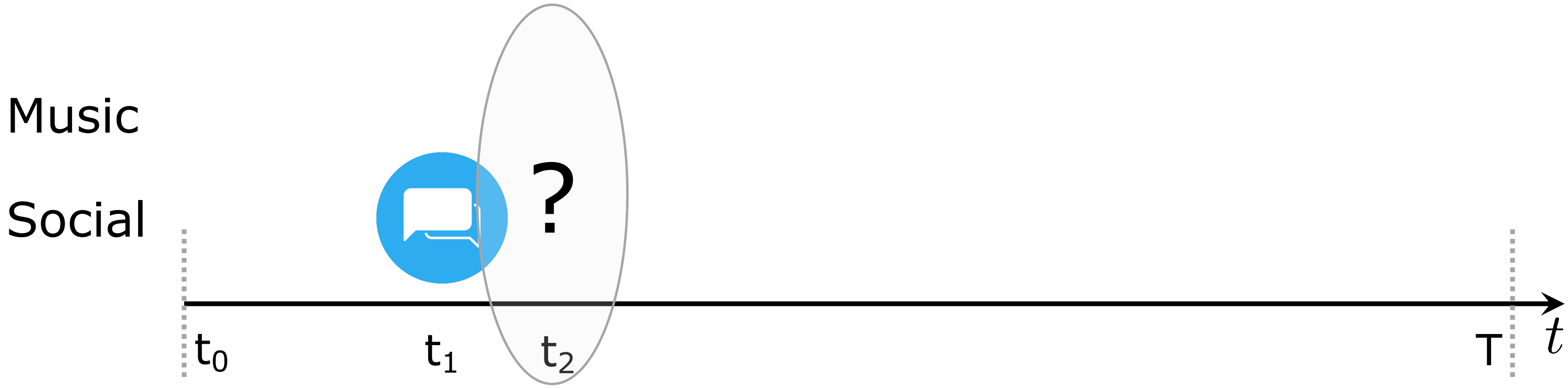
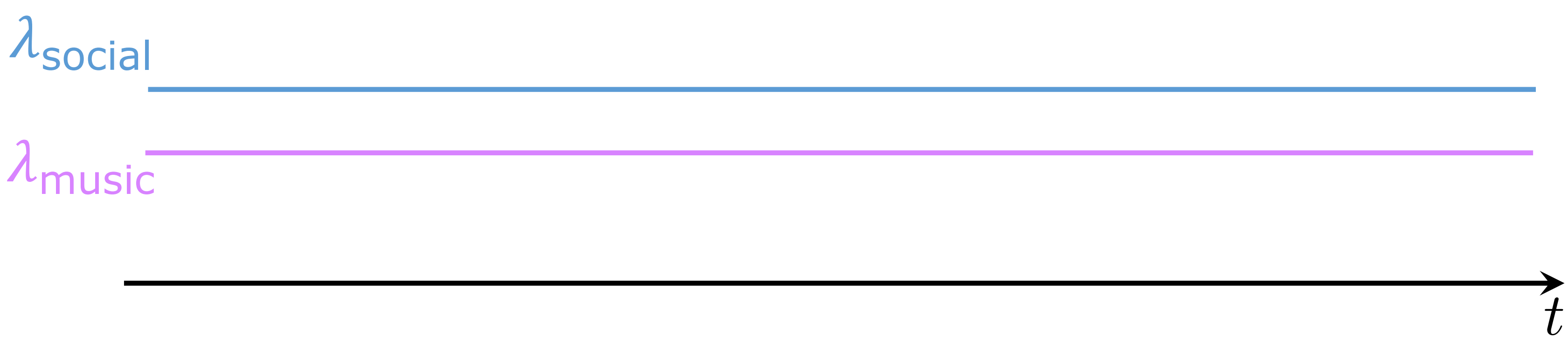
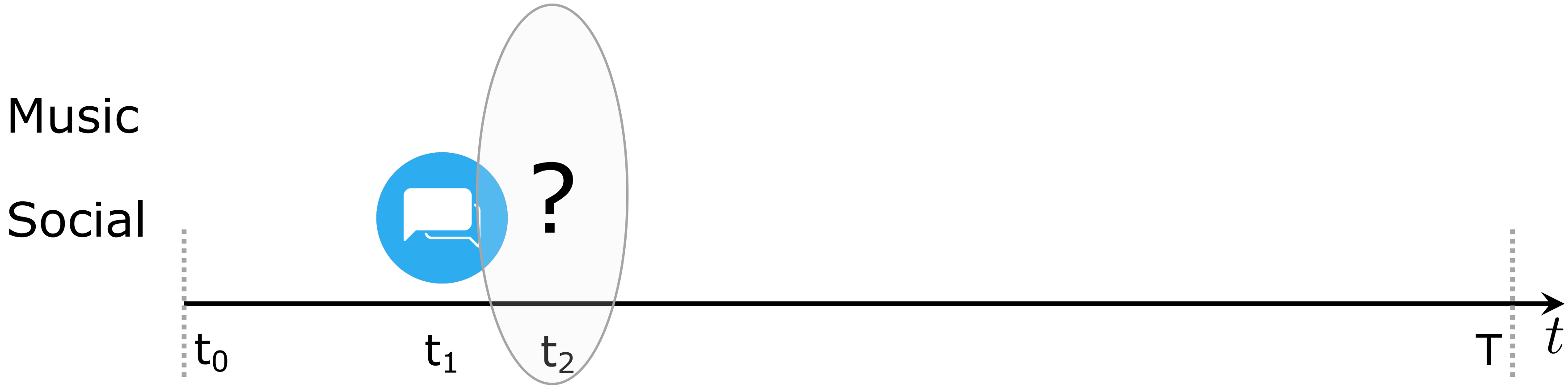
Multivariate Time Series



$(\lambda_{\text{music}} + \lambda_{\text{social}})dt$ = prob of *some* event at t

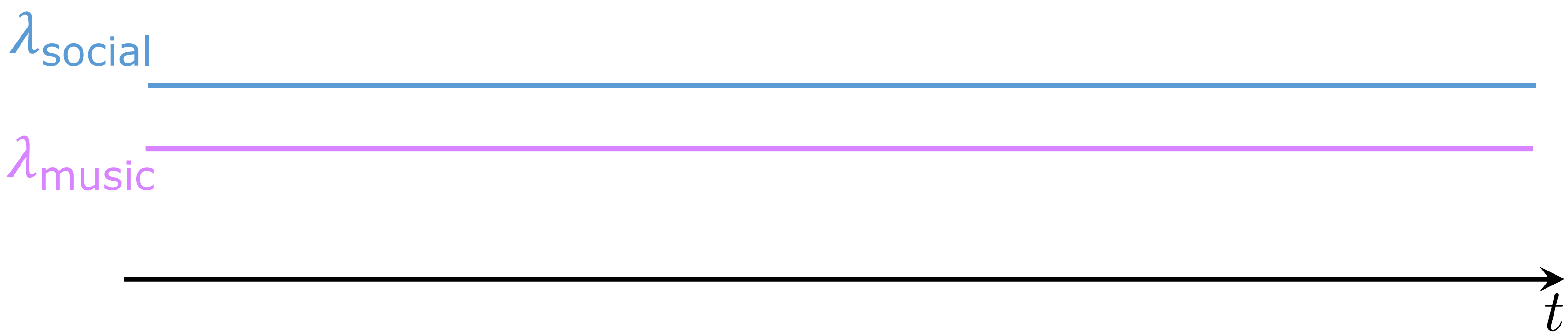


Multivariate Time Series



$\propto \lambda_{\text{music}}$ $\propto \lambda_{\text{social}}$

$(\lambda_{\text{music}} + \lambda_{\text{social}})dt$ = prob of *some* event at t



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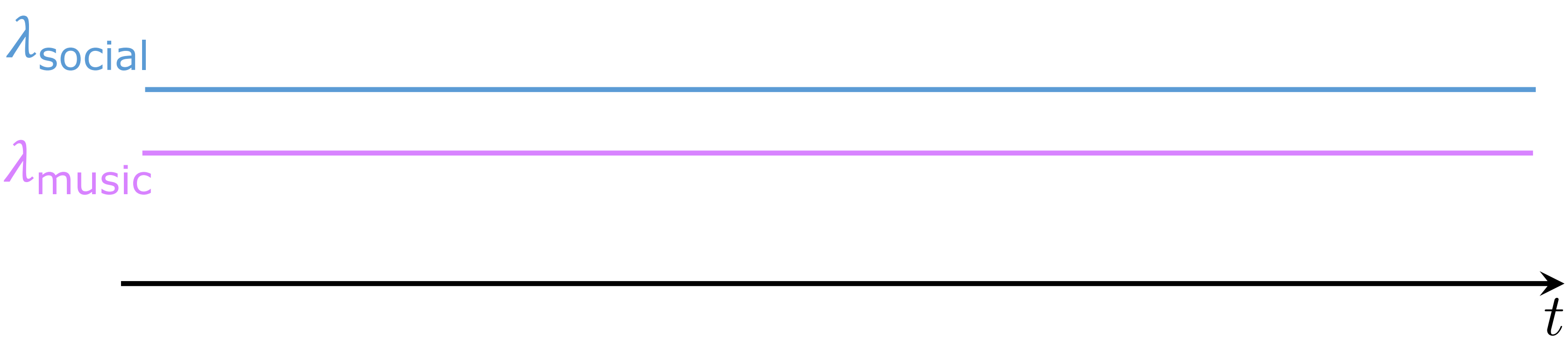
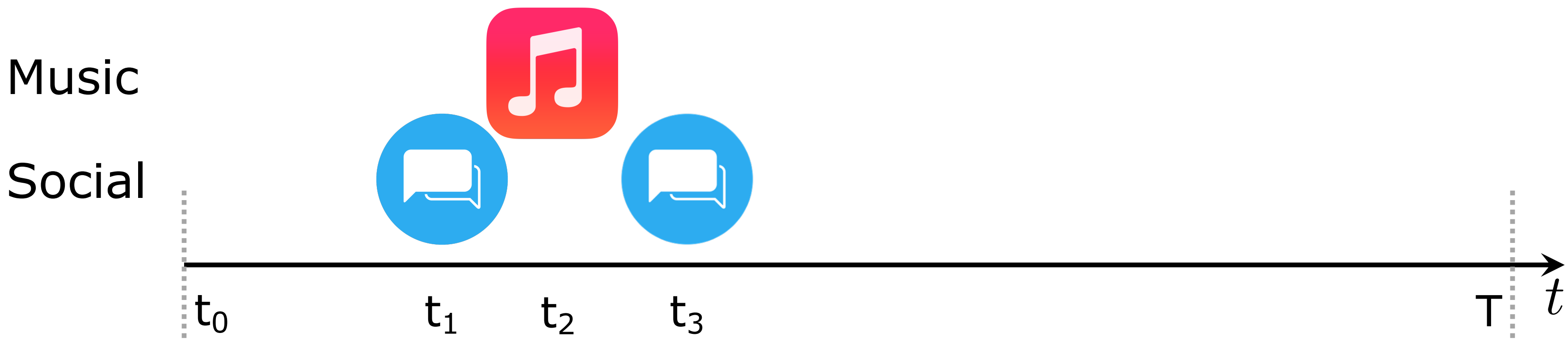
Multivariate **Time Series**

$\propto \lambda_{\text{music}}$ $\quad$ $\propto \lambda_{\text{social}}$

$(\lambda_{\text{music}} + \lambda_{\text{social}})dt$ = prob of *some* event at t

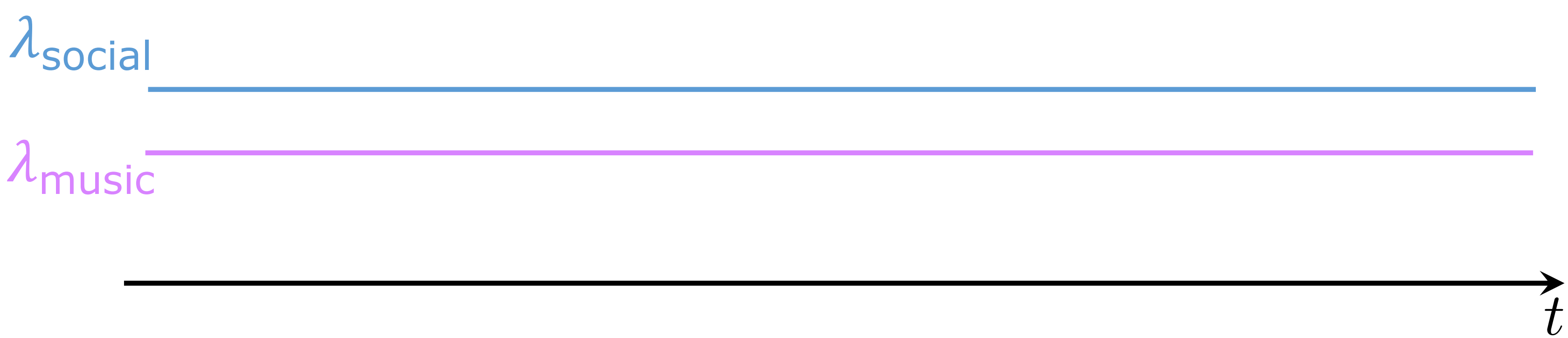


Multivariate Time Series

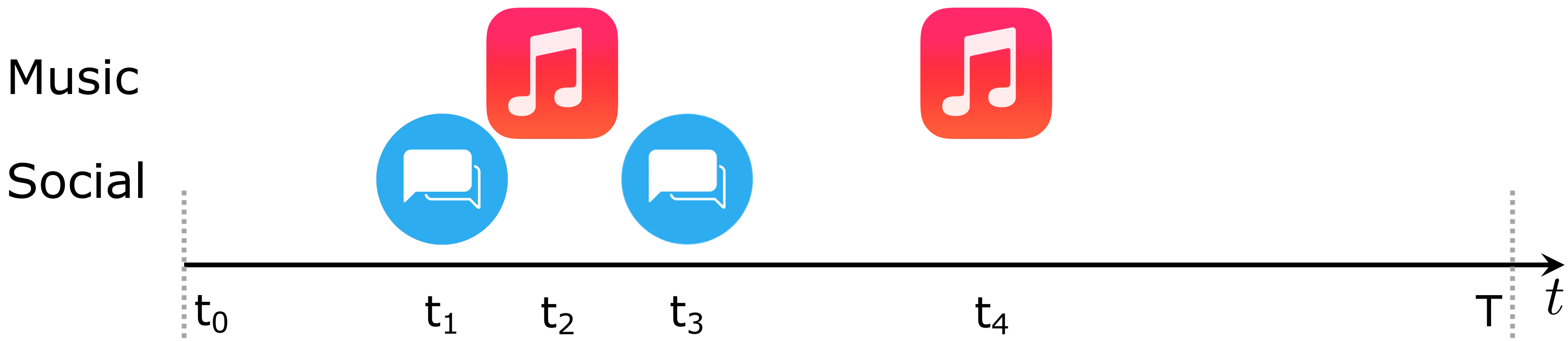


$\propto \lambda_{\text{music}}$ $\propto \lambda_{\text{social}}$

$(\lambda_{\text{music}} + \lambda_{\text{social}})dt$ = prob of *some* event at t

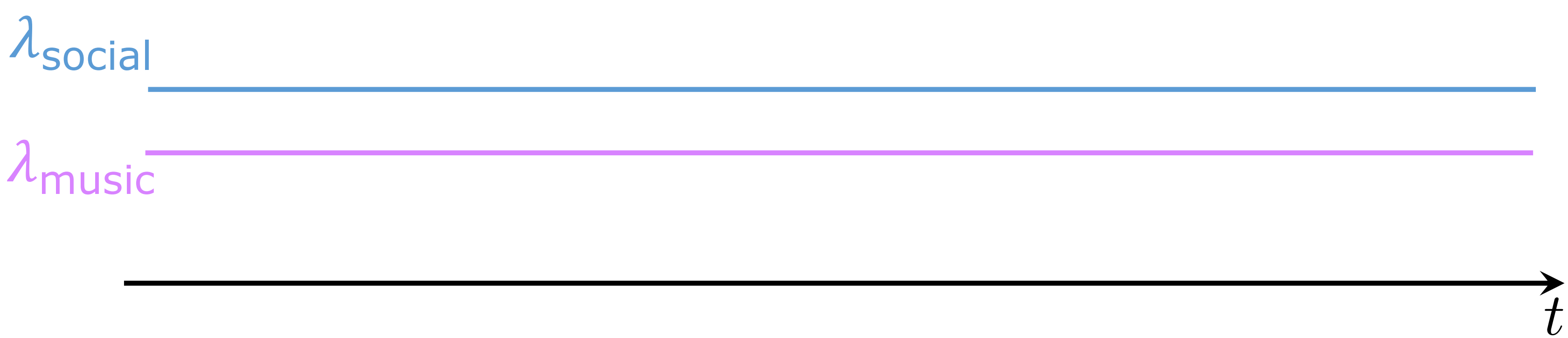


Multivariate Time Series

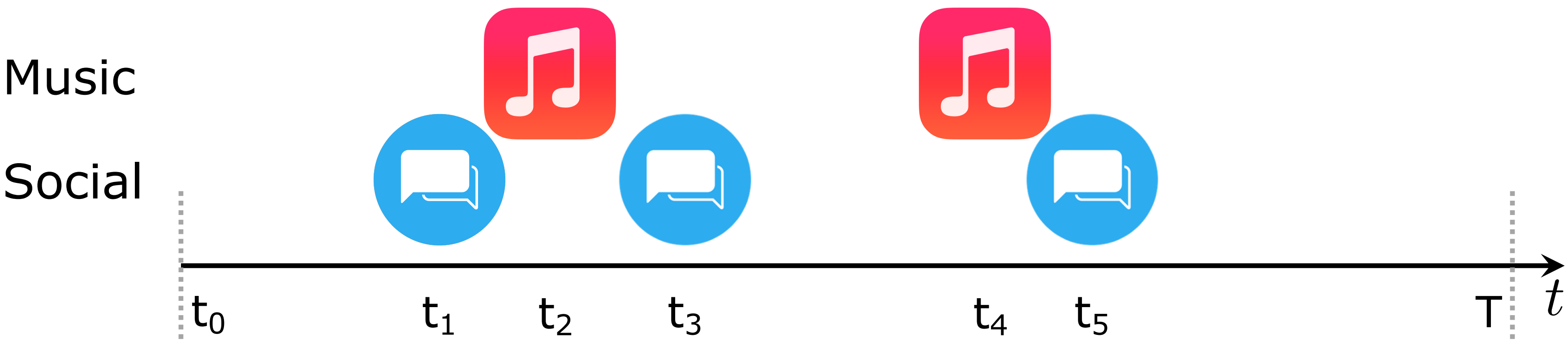


$\propto \lambda_{\text{music}}$ $\propto \lambda_{\text{social}}$

$(\lambda_{\text{music}} + \lambda_{\text{social}})dt$ = prob of *some* event at t



Multivariate Time Series



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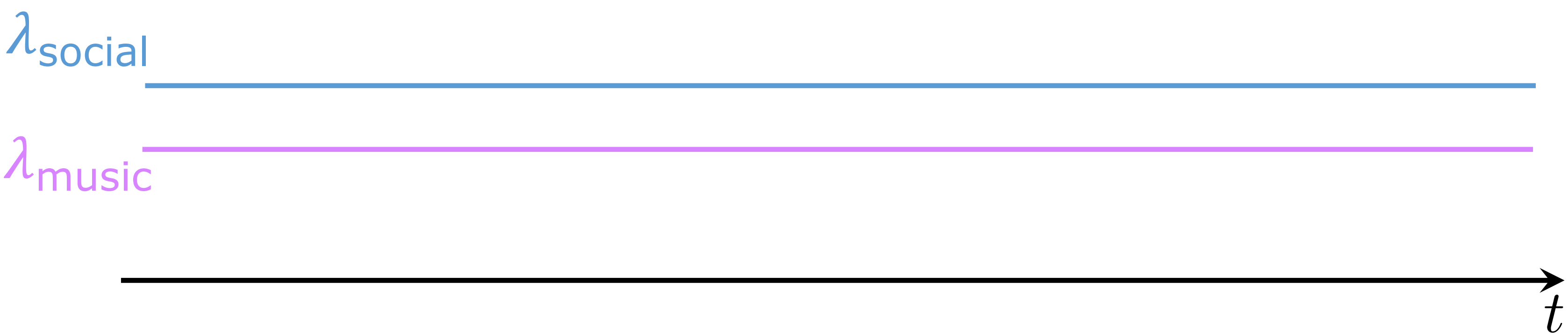
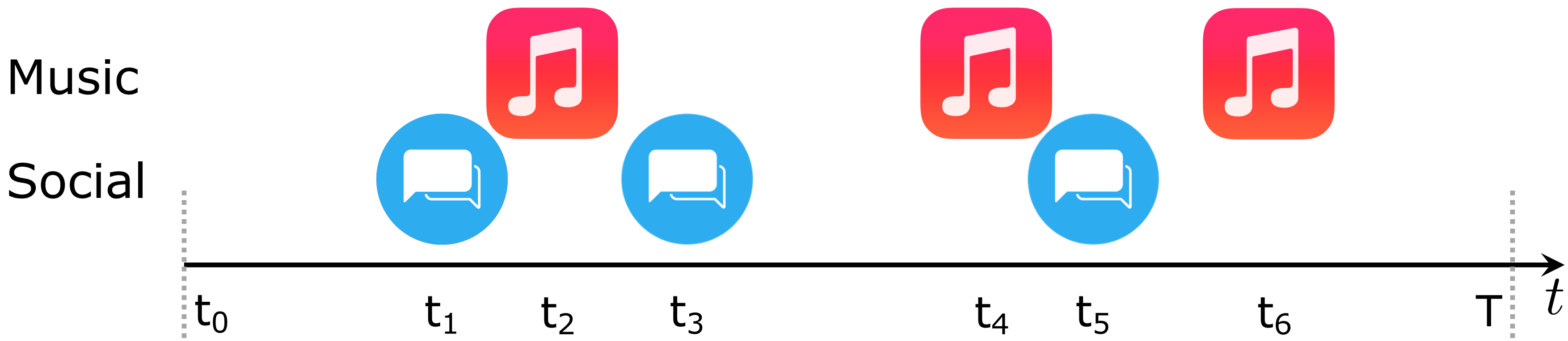
$(\lambda_{\text{music}} + \lambda_{\text{social}})dt$ = prob of *some* event at t



Multivariate Time Series



$\propto \lambda_{\text{music}}$ $\propto \lambda_{\text{social}}$

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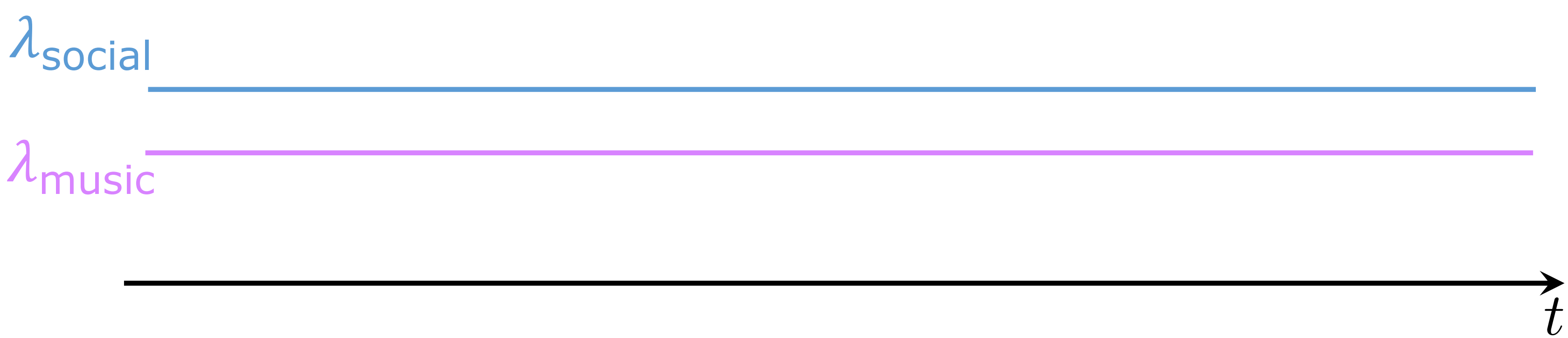
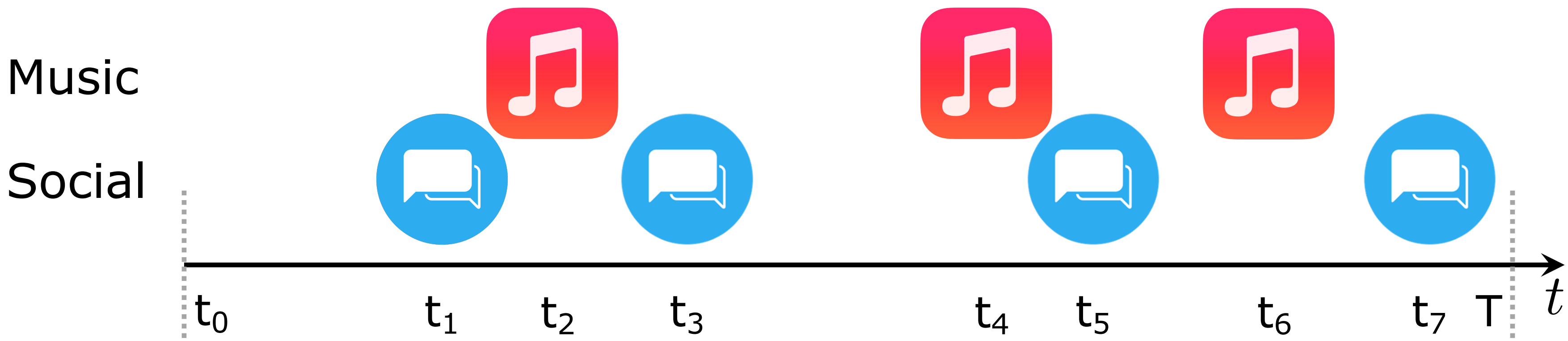


Multivariate Time Series



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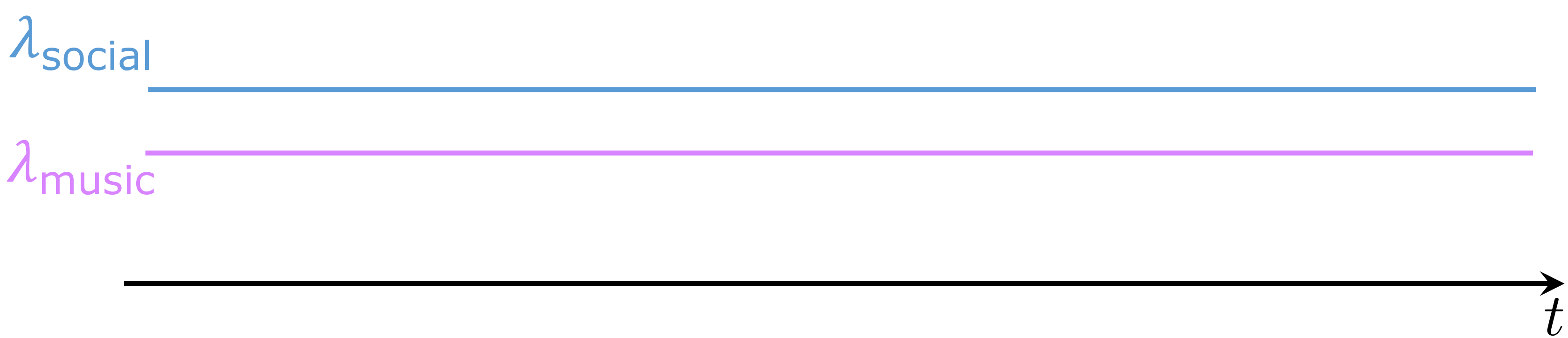
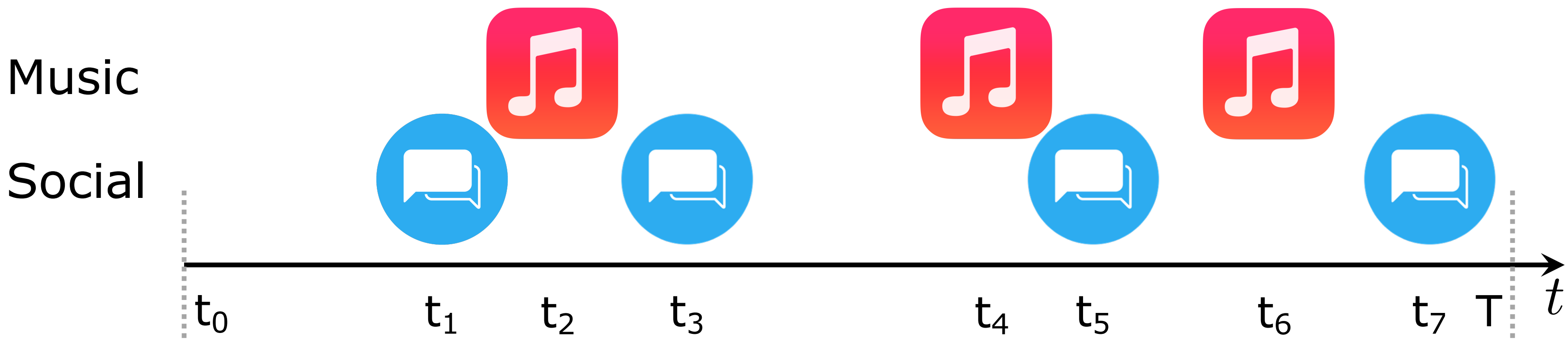
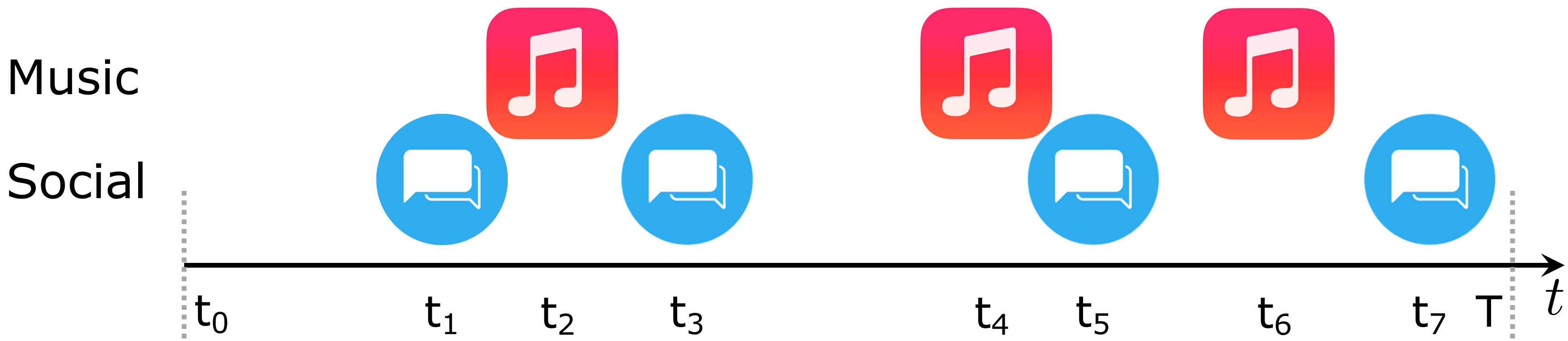


Multivariate Time Series: MLE



Multivariate Time Series: MLE



$$\lambda_{social} \exp(-\lambda\ (t_1 - t_0))$$

$$\times\ \lambda_{music} \exp(-\lambda\ (t_2 - t_1))$$

$$\times\ \lambda_{social} \exp(-\lambda\ (t_3 - t_2))$$

$$\dots\ \dots$$

$$= \lambda_{social}^4\ \lambda_{music}^3 \exp(-\lambda\ (T - t_0))$$

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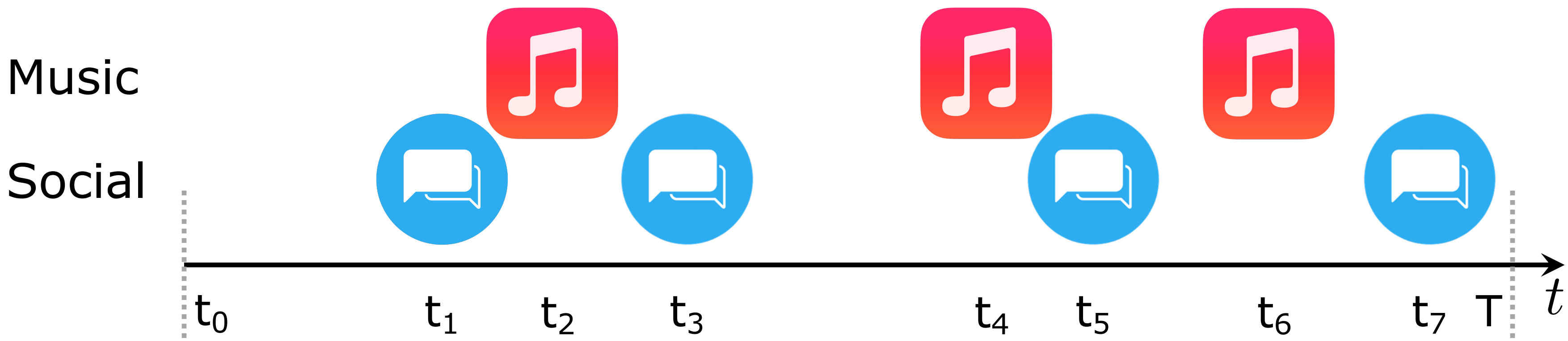
http://bburl/tpp-lab-p1

Multivariate Time Series: MLE



$$\lambda_{social}\, \exp(-\lambda\,(t_1 - t_0)) \rightarrow (1-\lambda dt) \ldots (1-\lambda dt)$$

$$\times\, \lambda_{music}\, \exp(-\lambda\,(t_2 - t_1))$$

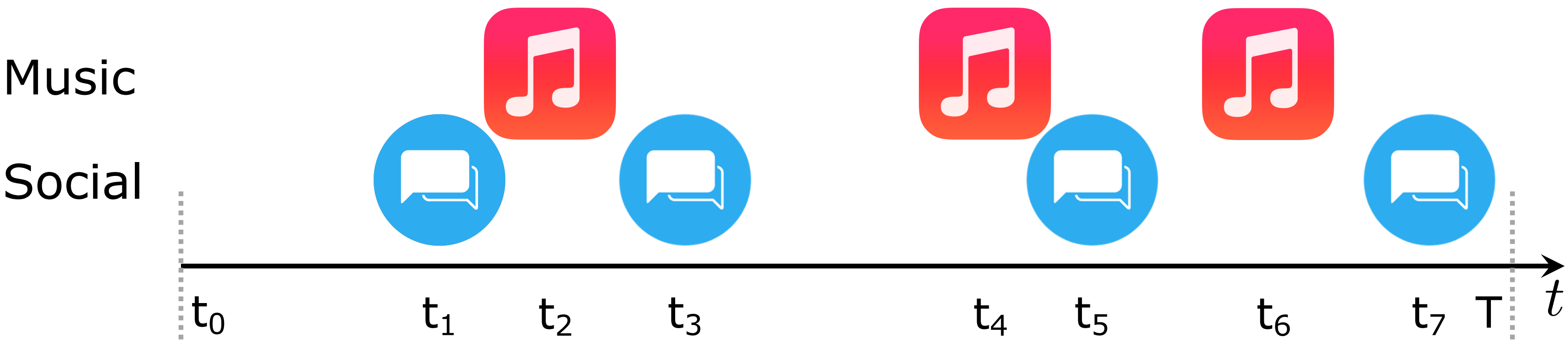
$$\lambda = \lambda_{music} + \lambda_{social}$$

$$\times\, \lambda_{social}\, \exp(-\lambda\,(t_3 - t_2))$$

… …

$$= \lambda_{social}^{4}\, \lambda_{music}^{3}\, \exp(-\lambda\,(T - t_0))$$

Multivariate Time Series: MLE



λ_{social} $\exp(-\lambda\ (t_1 - t_0))$

x λ_{music} $\exp(-\lambda\ (t_2 - t_1))$

x λ_{social} $\exp(-\lambda\ (t_3 - t_2))$

… …

$= \lambda_{social}^4\ \lambda_{music}^3\ \exp(-\lambda\ (T - t_0))$

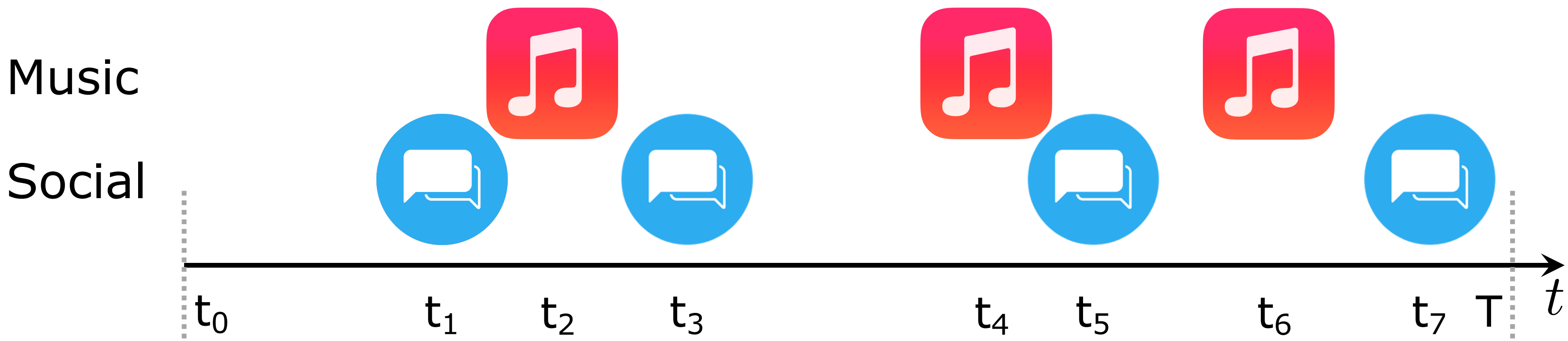
$(1-\lambda dt)\ \ldots\ (1-\lambda dt)$

$\lambda = \lambda_{music} + \lambda_{social}$

$\lambda_{social} = 4\ /\ (T - t_0)$

$\lambda_{music} = 3\ /\ (T - t_0)$

Multivariate Time Series: MLE



$$\lambda_{social} \exp(-\lambda\,(t_1 - t_0)) \rightarrow (1-\lambda dt) \ldots (1-\lambda dt)$$

$$\times\ \lambda_{music} \exp(-\lambda\,(t_2 - t_1))$$

$$\times\ \lambda_{social} \exp(-\lambda\,(t_3 - t_2))$$

$$\ldots\ \ldots$$

$$= \lambda_{social}^{4}\ \lambda_{music}^{3} \exp(-\lambda\,(T - t_0))$$

$$\lambda = \lambda_{music} + \lambda_{social}$$

$$\lambda_{social} = 4 / (T - t_0)$$

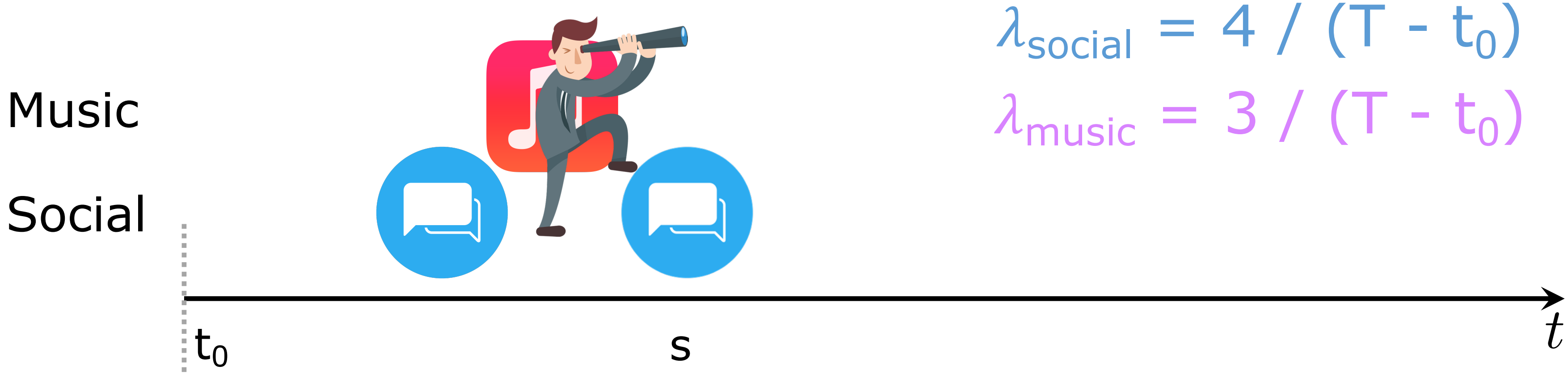
$$\lambda_{music} = 3 / (T - t_0)$$

\# of events
of *that* type

Multivariate Time Series: MBR

$\lambda_{social} = 4 / (T - t_0)$

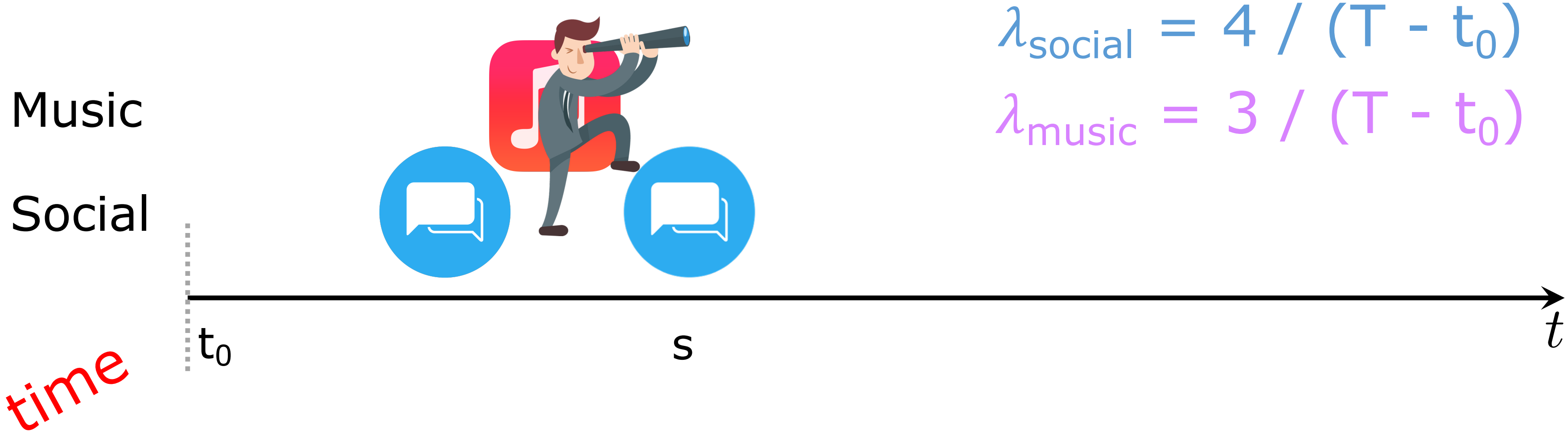
$\lambda_{music} = 3 / (T - t_0)$



Multivariate Time Series: MBR

$\lambda_{social} = 4 / (T - t_0)$

$\lambda_{music} = 3 / (T - t_0)$

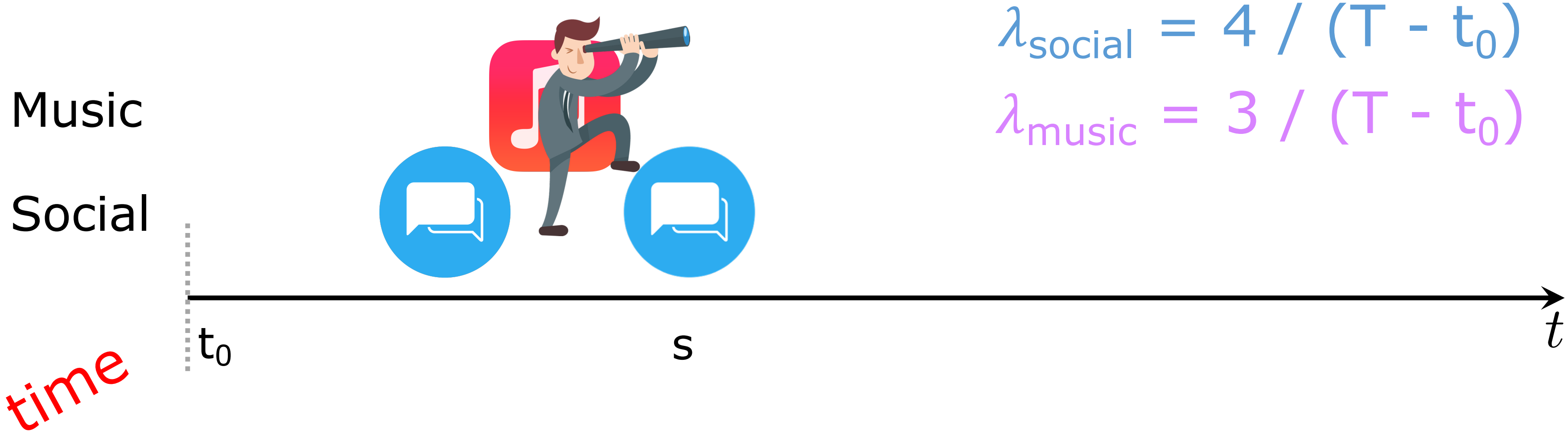


$t_{pred} = \int_s^{\infty} t\, p(t)\, dt \quad p(t) = \lambda \exp(-\lambda\, (t - s))$

Multivariate Time Series: MBR

$\lambda_{social} = 4 / (T - t_0)$

$\lambda_{music} = 3 / (T - t_0)$



$$t_{pred} = \int_s^\infty t \, p(t) \, dt \quad p(t) = \lambda \exp(-\lambda (t - s))$$

$$\lambda = \lambda_{music} + \lambda_{social} = 7 / (T - t_0)$$

Multivariate Time Series: MBR

$\lambda_{social} = 4 / (T - t_0)$

$\lambda_{music} = 3 / (T - t_0)$

Music

Social

t_0 s t

time

$t_{pred} = \int_s^\infty t\, p(t)\, dt \quad p(t) = \lambda \exp(-\lambda\, (t - s))$

$\lambda = \lambda_{music} + \lambda_{social} = 7 / (T - t_0)$

type?

Multivariate Time Series: MBR

$\lambda_{\text{social}} = 4 \,/\, (T - t_0)$

$\lambda_{\text{music}} = 3 \,/\, (T - t_0)$

Music

Social

t_0 s t

time

$$t_{\text{pred}} = \int_s^\infty t\, p(t)\, dt \quad p(t) = \lambda \exp(-\lambda\, (t - s))$$

$$\lambda = \lambda_{\text{music}} + \lambda_{\text{social}} = 7 \,/\, (T - t_0)$$

type?

$$k = \operatorname{argmax}_k\, p(k \mid t) = \operatorname{argmax}_k\, \lambda_k$$

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http://bburl/tpp-lab-p1

Multivariate Time Series: MBR

$\lambda_{social} = 4 / (T - t_0)$

$\lambda_{music} = 3 / (T - t_0)$

Music

Social

t_0 s t

time

$t_{pred} = \int_s^\infty t\, p(t)\, dt \quad p(t) = \lambda \exp(-\lambda\, (t - s))$

$\lambda = \lambda_{music} + \lambda_{social} = 7 / (T - t_0)$

type?

$k = \mathrm{argmax}_k\, p(k \mid t) = \mathrm{argmax}_k\, \lambda_k$

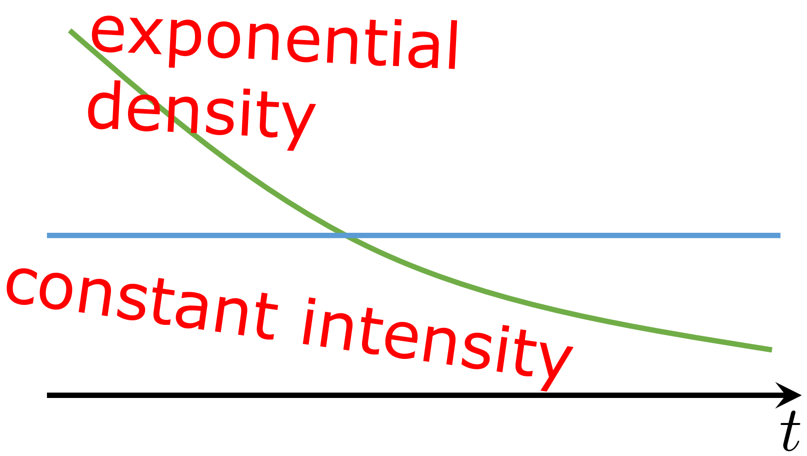
$\mathrm{argmin}_k \sum_j p(j \mid t)\, \mathbb{I}(k \neq j) \quad j, k \in \{music, social\}$

choose type k that makes $\sum_{j \neq k} p(j \mid t)$ small

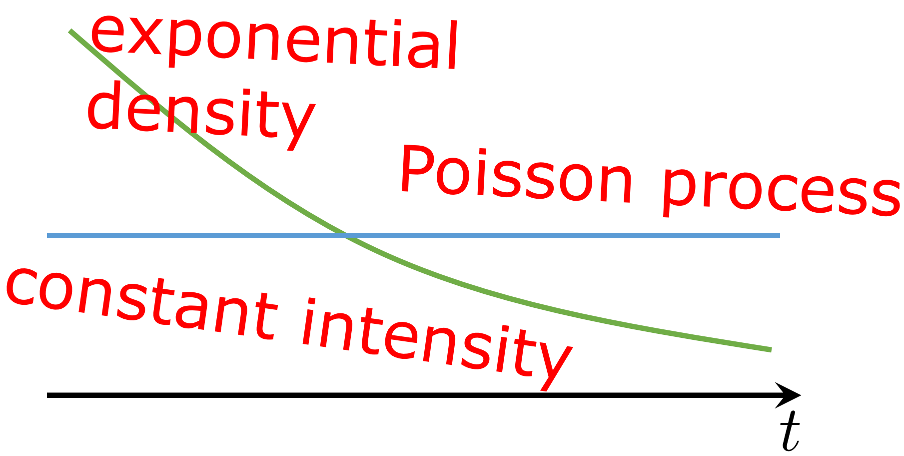
Other Modeling Designs

Other Modeling Designs

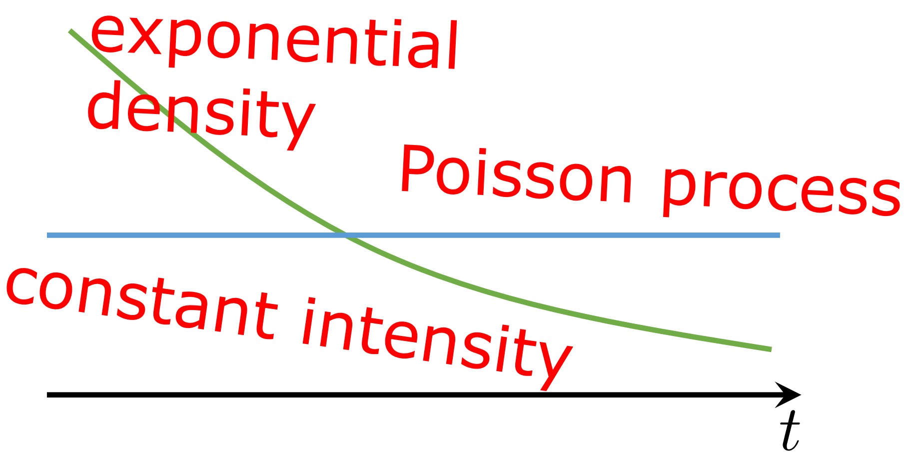
Other Modeling Designs

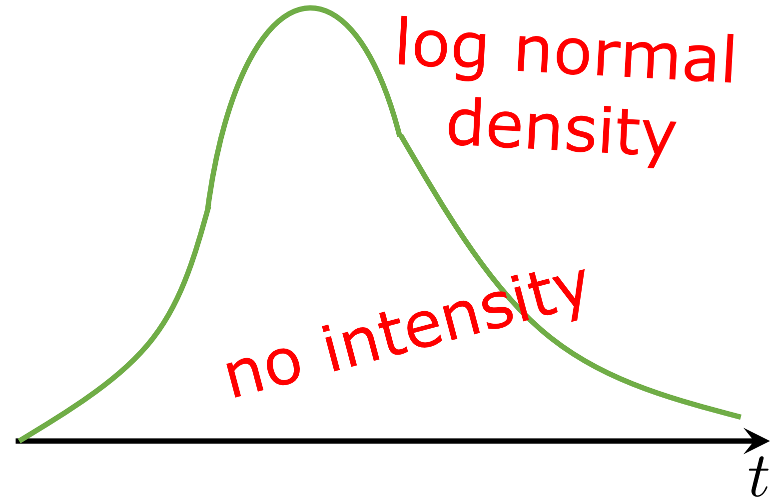


Other Modeling Designs



Other Modeling Designs





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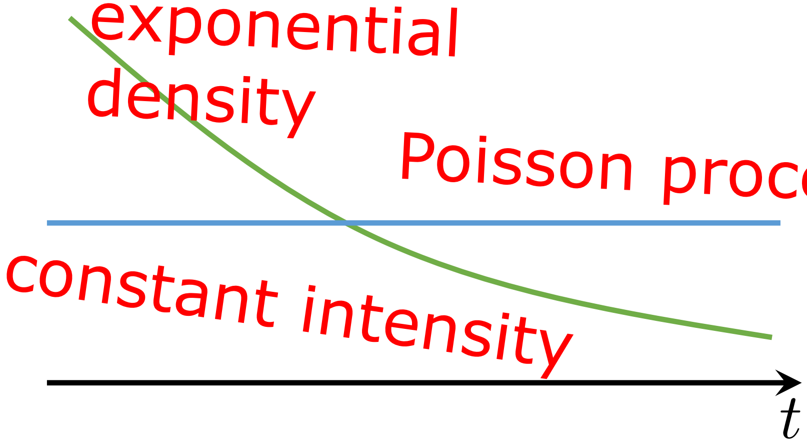
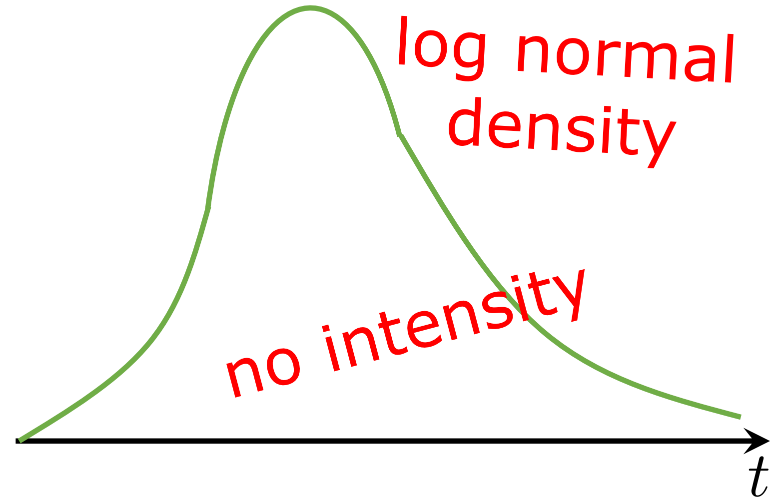
Other Modeling Designs





stochastic intensity
e.g., Cox process

Any Questions?

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