

Mathematics problems

1 Elementary algebra

Problem 1.1. Simplify

$$\frac{y^{58}}{y^4 \cdot y^{12}}$$

Problem 1.2. Solve for x :

$$8^2 \cdot 2^x = 2^9$$

Problem 1.3. Calculate the missing value. If $\frac{x}{y}$ is 3, then $x^{-2}y^2 = \dots$

Problem 1.4. Calculate

$$\frac{\sqrt{2^{13}}}{\sqrt{8^3}}$$

Problem 1.5. True or False (x and y and z are real numbers):

- (a) $x + y = y + x$
- (b) $x(y + z) = xy + xz$
- (c) $x^{y+z} = x^y x^z$
- (d) $\frac{x^y}{x^z} = x^{y-z}$

Problem 1.6. Find the solution for the equality below:

$$\frac{x^2 - 25}{x - 5} = 3$$

2 Functions of one variable

Problem 2.1 (Based on SYD 2.5.6). The relationship between temperatures measured in Kelvin and Fahrenheit is linear. 0 K is equivalent to -460°F and 1000 K is the same as 1340°F . Which temperature is measured by the same number on both scales?

Problem 2.2. Take the following function $f(x) = 2x + 3$. Find y if $f(y) = 17$.

Problem 2.3. Find all values of x that satisfy:

$$3^{2x^2 - 4x + 3} = 27$$

Problem 2.4. Solve the following problem. If the annual GDP growth of a country is 1%, how long does it take the economy to double its GDP?

Problem 2.5. Calculate the following value

$$\ln\left(\frac{e^2}{e^3}\right)$$

3 Calculus

Problem 3.1. Calculate the following sum

$$\sum_{i=0}^{\infty} \left(\frac{1}{6^i} + 0.25^i \right)$$

Problem 3.2. Find the following limit

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

Problem 3.3. Find the slope of the function $f(x) = x^3 - 4$ at $(-1, -5)$.

Problem 3.4. Find the following derivative

$$\left(\frac{x^2 + 3}{x + 2} \right)'$$

Problem 3.5. Find the second derivative of

$$f(x) = x^7 + 4x^2$$

Problem 3.6. Find the derivative of

$$f(x) = \frac{x^4 + 4^x}{\ln(x)}$$

Problem 3.7. Consider the following function. Find all of its stationary points and classify them as local minima, local maxima or inflection points.

$$f(x) = 3x^3 - 9x$$

Problem 3.8. Let $f(x, y) = x^2 + 2y^3$. Calculate $f(2, 3)$

Problem 3.9. Consider the following function: $f(x, y) = \ln(2x - y)$. For what combinations of x and y is this function defined?

Problem 3.10. Find all partial derivatives of the following function:

$$f(x, y) = x^5 e^y + x^2 y^3$$

Problem 3.11. Find the local maxima or minima of the following function:

$$f(x, y) = \sqrt{xy} - 0.7x - 0.7y$$

Problem 3.12. Solve the following constrained optimization problem using Lagrange's method: $\max x^2 y^2$
s.t. $x + y = 10$

4 Linear algebra

Problem 4.1. Take the following matrices:

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

What is $A \cdot B$?

Problem 4.2. Take the following matrices:

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

What is $B \cdot A$?

Problem 4.3. What is the transpose of the following matrix?

$$\begin{bmatrix} 3.3 & 5.1 \\ 6.1 & 1.23 \\ 45.76 & 0 \end{bmatrix}$$

Problem 4.4. Calculate the determinant of

$$\begin{bmatrix} 2 & 3 & 0 \\ 4 & 5 & 2 \\ 2 & 5 & 3 \end{bmatrix}$$

5 Probability theory

Problem 5.1. You run an experiment where you flip a coin twice. Each time you get either heads (H) or tails (T). What is the sample space of your experiment?

Problem 5.2. You are observing a race with 30 competitors. How many possible outcomes exist for the 1st, 2nd and 3rd place?

Problem 5.3. You run an experiment in which you toss a dice twice and record the results. What is the probability that at least one of the tosses end up being odd?