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Saathi

Linear Algebra Assignment - 5

1) Given:-

$$y = A + Bx + Cx^2$$

at (1, 1)

$$1 = A + B + C \quad \text{--- (1)}$$

at (2, -1)

$$-1 = A + 2B + 4C \quad \text{--- (2)}$$

at (3, 1)

$$1 = A + 3B + 9C \quad \text{--- (3)}$$

$$Ax = b$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 2 & 8 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$$

$$2C = 4 \Rightarrow C = 2$$

$$B + 3C = -2 \Rightarrow B = -8$$

$$A + B + C = 1$$

$$A - 8 + 2 = 1$$

$$A - 6 = 1$$

$$A = 7$$

$$\therefore y = 7 + (-8)x + 2x^2$$

$$y = 7 - 8x + 2x^2$$

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2) LU decomposition of a matrix i.e $A = LU$

Given $A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$

Sol^n:- $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 + 5R_1$
$R_4 \to R_4 - 5R_1$ $\sim \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix}$

$R_3 \to R_3 + 2R_2$
$R_4 \to R_4 + 2 \cdot R_2$ $\sim \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$

$R_4 \to R_4 - 3R_3$ $\sim \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix} = U$

$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$

3) ~~I(x, y, z)~~ $\neq$ $A = LU$

$$\begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & -14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$$

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3) $T(x,y,z) = (x+2y-z\ ,\ y+z\ ,\ x+y-2z)$

i) find T wrt standard basis of R_3

Basis for $R^3 = \{(1,0,0), (0,1,0), (0,0,1)$

$T(1,0,0) = (1,0,1)$
$T(0,1,0) = (2,1,1)$
$T(0,0,1) = (-1,1,-2)$

$\therefore$ column wise transform gives us

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

column 1, 2 in T produce pivots hence they are linearly independent

$C(A) = \{(1,0,1), (2,1,1)\}$

$\underline{\dim(C(A)) = 2}$

row space $C(A^T) = \{(1,2,-1), (0,1,1)\}$

$\underline{\dim(C(A^T)) = 2}$

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Finding $N(A)$ & $N(A^T)$

Convert T to row-reduced form

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \to R_1 - 2R_2} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

z is free variable

$x - 3z = 0$ $\quad x = 3z$

$y + z = 0$ $\quad y = -z$

$0x + 0y + 0z = 0$ $\quad$ Let $z = 1$

$$N(A) = z \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

$N(A) = \{(3, -1, 1)\}$

$\dim(N(A)) = 1$

finding $N(A^T)$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 1 & 1 & 2 & b_3 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & -1 & -1 & b_3 - b_1 \end{array}\right]$$

$R_3 \to R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & b_1 \\ 0 & 1 & 1 & b_2 \\ 0 & 0 & 0 & b_3 - b_1 + b_2 \end{array}\right]$$

for consistency $(-b_1 + b_2 + b_3 = 0)$

$\therefore$ $N(A^T) = \{(-1, 1, 1)\}$

$\dim(N(A^T)) = 1$

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4) Eigen value and Eigen Vectors

From given

$$\begin{bmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{bmatrix} = (A - \lambda I)$$

$$|A - \lambda I| = 0$$

$$(1-\lambda)\left[(1-\lambda)(-2-\lambda) - 1\right] + 2 + 1(1-\lambda) = 0$$

$$\cancel{(1-\lambda)^2}$$

$$(1-\lambda)\left[(-2-\lambda+2\lambda+\lambda^2-1)\right. + 2 + 1 - \lambda = 0$$

$$(1-\lambda)\left[\lambda^2+2\lambda-\lambda-3\right] + 3 - \lambda = 0$$

$$(1-\lambda)\left[\lambda^2+\lambda-3\right] - \lambda + 3 = 0$$

$$\cancel{\lambda^2} + \cancel{\lambda} \cancel{-3} - \lambda^3 - \cancel{\lambda^2} + 3\lambda - \cancel{\lambda} + \cancel{3} = 0$$

$$\lambda^3 = 3\lambda$$

$$\boxed{\lambda = \sqrt{3}, -\sqrt{3}, 0}$$

Eigen vector for $\lambda = \sqrt{3}$

$$\begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 1 & 1 & -2-\sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3 \to R_3 - \frac{1}{1-\sqrt{3}} R_1$$

$$\sim \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 1-\frac{2}{1-\sqrt{3}} & -(2+\sqrt{3})+\frac{1}{1-\sqrt{3}} \end{bmatrix} = \begin{bmatrix} -0.732 & 2 & -1 \\ 0 & -0.732 & 1 \\ 0 & 3.732 & -5.098 \end{bmatrix}$$

$$\sim \begin{bmatrix} -0.732 & 2 & -1 \\ 0 & -0.732 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$y = 1.366 z$

$\sqrt{}\ x = 0$

$-0.732x + 2y - z = 0$

$-0.732y + z = 0 \quad y = -z/0.732$

$-0.732x = -1.73202$

$\boxed{x = 2.36642}$

Eigen vector $= 2\begin{bmatrix} 2.3664 \\ 1.366 \\ 1 \end{bmatrix}$

for $\lambda = -\sqrt{3}$

$$\begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 1 & 1 & -2+\sqrt{3} \end{bmatrix} = \begin{bmatrix} 2.732 & 2 & -1 \\ 0 & 2.732 & 1 \\ 1 & 1 & -0.2679 \end{bmatrix}$$

$R_3 \to R_3 - \frac{1}{2.732}R_1 \sim \begin{bmatrix} 2.732 & 2 & -1 \\ 0 & 2.732 & 1 \\ 0 & 0.2679 & 0.0981 \end{bmatrix}$

$$\sim \begin{bmatrix} 2.732 & 2 & -1 \\ 0 & 2.732 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Let $z = 1$

$2.732y + z = 0$

$2.732y = -z$

$y = -0.3660$

$2.732x + 2y = z$

$x = z - 2y/2.732$

$x = 0.63396$

$x_2 = \begin{bmatrix} 0.63396 \\ -0.3660 \\ 1 \end{bmatrix}$

if $\lambda = 0$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$ from N(A) solution obtained previously

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5) Factorize $T = QR$

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$q_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad q_2 = \frac{l_2}{\|l_2\|}$$

$$l_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \left[\frac{2}{\sqrt{2}} + 0 + \frac{1}{\sqrt{2}}\right]\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\frac{1}{\sqrt{2}}$$

$$= \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 3/2 \\ 0 \\ 3/2 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \\ -0.5 \end{bmatrix}$$

$$q_2 = \frac{1}{\sqrt{1.5}}\begin{bmatrix} 0.5 \\ 0 \\ -0.5 \end{bmatrix}$$

$$q_3 = \frac{l_2}{\|l_3\|} \qquad l_3 = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} - \frac{1}{1.5}\begin{pmatrix} 0.5 \\ 0 \\ -0.5 \end{pmatrix}\left(-\tfrac{1}{2} + 1 + 1\right) + \frac{3}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$q_3 = \begin{bmatrix} -2/7 \\ 3/7 \\ -6/7 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1/\sqrt{2} & 0.5/\sqrt{1.5} & -2/7 \\ 0 & 1/\sqrt{1.5} & 3/7 \\ 1/\sqrt{2} & -0.5/\sqrt{1.5} & -6/7 \end{bmatrix}$$

$A = QR$

$Q^T A = R$

$$R = \begin{bmatrix} 1.4142 & 2.1213 & -2.121 \\ 0 & 1.2247 & 1.2247 \\ 0 & 0 & 0 \end{bmatrix}$$

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5) Find a best fit line $y = mx + c$

let $y = \begin{bmatrix} \\ \\ \end{bmatrix}_{n\times 1}$ $\quad A = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ \vdots & x_n \end{bmatrix}_{n\times 2}$

$$\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} c \\ m \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$A \quad x = B$$

wkt. $\hat{x} = (A^T A)^{-1} . A^T . B$

$$\hat{x} = \left[\begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}^T \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \right]^{-1} A^T . B$$

$$A^T A = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$A^T B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix} = \begin{bmatrix} 28 \\ 28+6 \end{bmatrix}_{2\times 1} = \begin{bmatrix} 28 \\ 34 \end{bmatrix}$$

$$(A^T A)^{-1} = \begin{bmatrix} 15/58 & -1/58 \\ -1/58 & 1/29 \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} 193/29 \\ 20/29 \end{bmatrix}$$

$m = 20/29$

$c = 193/29$

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i) Consider the Equation of plane

let $v = [1 \quad 1 \quad 3 \quad 0 \quad 4]$

$\underset{\text{pivot}}{1}$

$x_1 = -x_2 + (-3)x_3 - 4x_5$

let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = k_1 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + k_3 \begin{bmatrix} -4 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{5\times 3}$

WKT $P = A(A^TA)^{-1}.A^T$

using calculation

$P = \begin{bmatrix} 26/27 & -1/27 & -1/9 & 0 & 4/27 \\ -1/27 & 26/27 & -1/9 & 0 & 4/27 \\ -1/9 & -1/9 & 2/3 & 0 & 4/9 \\ 0 & 0 & 0 & \phi & 0 \\ -4/27 & -4/27 & -4/9 & 0 & 16/27 \end{bmatrix}$

7) For what values of a is the matrix +ve definite

Sol^n - w.k.t. Determinant of principle submatrices ≥ 0

$A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$

$|a| > 0$ so $a > 0$

$\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix} > 0$ so $a^2 - 4 > 0$ $\quad (a-2)(a+2) > 0$

$a^2 > 4$ $\quad (a-2) > 0$

$a > \pm 2$ $\quad a > 2 \quad a+2 >$

$a > -2$

so $a \neq 0, -1, 1, -2, 2$

$a \notin [-2, 2]$

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$|A| > 0$

$a(a^2-4) - 2(2a-4) + 2(4-2a) > 0$

$a(a-2)(a+2) - 4(a-2) + 4(2-a) > 0$

$a(a-2)(a+2) + 4(2-a) + 4(2-a) > 0$

$a(a-2)(a+2) + 8(2-a) > 0$

$(2-a)\left(a(a+2)(-1) + 8\right) > 0$

$a^3 - 12a + 16 > 0$

$2 - a > 0 \quad \Rightarrow a < 2$

$a > 2$

$a > -4$

$a > 1$

$\therefore a \in (2, \infty)$

$f = x^T A x$

$f = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 + (-2)(x_2x_3)$

$\downarrow \quad \downarrow \quad \downarrow \quad -1 \quad -1 \quad -1 \quad -1$

$a_{11} \quad a_{22} \quad a_{33} \quad a_{12} \quad a_{21} \quad a_{23} \quad a_{32}$

$\therefore a_{31} = a_{13} = 0$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

8) SVD of $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}$

Find A^TA

$$A^TA = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}$$

λ_1, λ_2 of A^TA?

$$\begin{bmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{bmatrix} = 0$$

$A - \lambda I = 0$

$(81-\lambda)(9-\lambda) - 27^2 = 0$

$729 - 81\lambda - 9\lambda + \lambda^2 - 27^2 = 0$

$\lambda_1 = 90 \quad \sqrt{\lambda_1} = 3\sqrt{10}$

$\lambda_2 = 0$

$$\Sigma = \begin{bmatrix} 3\sqrt{10} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

λ_1 vector x_1

x_1

$$\begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$9x + 27y = 0$

$27x + 81y = 0$

$$y\begin{bmatrix} 27/9 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} \quad y\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

if $\lambda = 0$

$$x_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

for $v = \lambda_1 = 90$

$$v_1 = \frac{x_1}{\|x_1\|} = \frac{1}{\sqrt{10}}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$\lambda_2 = 0$

$$V_2 = \frac{x_2}{\|x_2\|} = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$$

$$V = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix} = V^T$$

Finding U eigen values are 90, 0, 0

$$U_i = \frac{A V_i}{\sigma_1} \qquad U_1 = \frac{A V_1}{\sigma_1} = \frac{1}{3\sqrt{10}} A \cdot V = \begin{bmatrix} -0.266 \\ 0.533 \\ 0.533 \end{bmatrix}$$

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$U_2 = ?$ $\quad \sigma_2 = 0$ here we do $(AA^T - 0.I)x = 0$

so

$AA^T . x = 0$

$$\begin{bmatrix} 10 & -20 & -20 \\ -20 & 40 & 40 \\ -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Finding null space

$R_2 \rightarrow R_2 + 2R_1$

$$\begin{bmatrix} 10 & -20 & -20 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$10x - 20y - 20z = 0$

$x - 2y - 2z = 0$

$x = 2y + 2z$

$= y \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

for $\lambda = 0$ we get 2 vector eigen

choosing $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$

$$v = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

$$U = \begin{bmatrix} -0.266 & 0.8944 & a \\ 0.533 & 0.44721 & b \\ 0.533 & 0 & c \end{bmatrix}$$

U is orthogonal so $U_3 \perp U_2$ & $U_3 \perp U_1$

$-0.266a + 0.533b + 0.533c = 0$

$0.8944a + 0.44721b + 0c = 0$

Let $c = 1$

$-0.266a + 0.533b + 0.533c = 0$ —— (1)

$0.8944a + 0.44721 = 0$ —— (2)

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Eq ① × 3.3624

$-0.8944a + 1.792162b = 1.79216$

$+0.89449 + 0.44721b = 0$

$2.239b = 1.79216$

$\underline{b = 0.8}$

$$V_3 = \frac{1}{\sqrt{1.8}} \begin{bmatrix} 0.4 \\ 0.8 \\ 1 \end{bmatrix}$$

$a = 0.4$

$c = 1$

$$A = \begin{bmatrix} 1/3 & 2/\sqrt{5} & 0.4/\sqrt{1.8} \\ -2/3 & 1/\sqrt{5} & 0.8/\sqrt{1.8} \\ -2/3 & 0 & 1/\sqrt{1.8} \end{bmatrix} \begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$$

$A = \quad U \quad \Sigma \quad V^T$