Ising Model Solver for Combinatorial Optimization Problem

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I. Introduction

The Ising model is a mathematical model of a ferromagnetic material under the presence of a magnetic field, which allows for the analysis of the thermodynamics of a system of ferromagnetic particles [1]. When applied with the spin glass magnetic state, the Ising spin-glass model can be utilized to analyze the thermodynamics of unordered or chaotic systems. Particularly, the spin-glass Ising model aligns each ferromagnetic particle in a lattice with a random spin-interactions depicted as: $\sigma \in \{-1, +1\}$. For the purpose of this document, we will consider the spin-glass Ising model to compute NP-Hard problems.

Formally, the N-spin Ising problem aims to find the configuration of spins that minimizes the energy Hamiltonian,

$$H = -\sum_{i,j<N} J_{ij}\sigma_i\sigma_j - \sum_i h_i\sigma_i \tag{1}$$

where J_{ij} represents the coupling coefficient, negligible for non-neighboring spins, and h_i represents the magnetic field acting on spin i. This document will explore the solving of the NP-Hard problem MaxCut using the Ising spin glass model. As such, the energy of MaxCut can be modeled by the following equation:

$$Cut(s) = \frac{1}{2}\sum_{(i,j)\in E}(1 - \sigma_i\sigma_j) \tag{2}$$

To map this to the Hamiltonian for Ising spin glass and maintain the objective of the problem, we aim to solve for the ground state of $H(s) = -Cut(s)$. In other words, we have the following equation representing the Hamiltonian we will strive to minimize:

$$H(s) = \sum_{(i,j)\in E}\sigma_i\sigma_j \tag{3}$$

Due to the difficulty of finding the minimum Hamiltonian of a chaotic systems, the Ising spin glass model is a NP-Hard problem for classical computers. Naturally, we are able to correlate this property to all NP-Hard problems, and can justifiably state that Ising spin glasses can be polynomially mapped to all other NP-Hard problems [2].

II. Existing Methods

We will be comparing our results to algorithms such as BLS, CPLEX, Gurobi, and MCPG. A table is shown below documenting the results the BLS and MCPG algorithms achieved on Gset datasets according to their referenced papers. Whilst the results the Gurobi and CPLEX algorithms were achieved on Syn datasets. Both algorithms used the same problem formulation of an Binary Integer approximation [5] and was implemented using the PuLP python library.

TABLE I
Gset Dataset Results

Graph	Nodes	Edges	MCPG	BLS
G14	800	4,694	3,064	3064
G15	800	4,661	3,050	3,050
G22	2,000	19,990	13,359	13,359
G49	3,000	6,000	6,000	6,000
G50	3,000	6,000	5,880	5,880
G55	5,000	12,468	10,294	10,294
G70	10,000	9,999	9,595	9,541

TABLE II
Syn Dataset Results

Graph	Nodes	Edges	Gurobi	CPLEX
P_20_ID0	20	63	46	46
P_40_ID0	40	144	109	109
P_100_ID0	100	384	282	282
P_200_ID0	200	784	581	*
P_300_ID0	300	1182	*	*

The most impressive algorithm shown is the Monte Carlo Policy Gradient (MCPG) method. This algorithm has specific features that contribute to its success, including: a filter function that acts to reduce the probability the algorithm will fall into local minima; a sampling procedure with filter function that starts from the best solution found previously and aims to maintain diversity; a modified policy gradient algorithm to update the probabilistic model; and a probabilistic model that guides the sampling procedure towards potentially good solutions.

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