

# Ising Model Solver for Combinatorial Optimization Problem

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## I. INTRODUCTION

The Ising model is a mathematical model of a ferromagnetic material under the presence of a magnetic field, which allows for the analysis of the thermodynamics of a system of ferromagnetic particles [1]. When applied with the spin glass magnetic state, the Ising spin-glass model can be utilized to analyze the thermodynamics of unordered or chaotic systems. Particularly, the spin-glass Ising model aligns each ferromagnetic particle in a lattice with a random spin-interactions depicted as:  $\sigma \in \{-1, +1\}$ . For the purpose of this document, we will consider the spin-glass Ising model to compute NP-Hard problems.

Formally, the  $N$ -spin Ising problem aims to find the configuration of spins that minimizes the energy Hamiltonian,

$$H = - \sum_{i,j < N} J_{ij} \sigma_i \sigma_j - \sum_i h_i \sigma_i \quad (1)$$

where  $J_{ij}$  represents the coupling coefficient, negligible for non-neighboring spins, and  $h_i$  represents the magnetic field acting on spin  $i$ . This document will explore the solving of the NP-Hard problem MaxCut using the Ising spin glass model. As such, the energy of MaxCut can be modeled by the following equation:

$$Cut(s) = \frac{1}{2} \sum_{(i,j) \in E} (1 - \sigma_i \sigma_j) \quad (2)$$

To map this to the Hamiltonian for Ising spin glass and maintain the objective of the problem, we aim to solve for the ground state of  $H(s) = -Cut(s)$ . In other words, we have the following equation representing the Hamiltonian we will strive to minimize:

$$H(s) = \sum_{(i,j) \in E} \sigma_i \sigma_j \quad (3)$$

Due to the difficulty of finding the minimum Hamiltonian of a chaotic systems, the Ising spin glass model is a NP-Hard problem for classical computers. Naturally, we are able to correlate this property to all NP-Hard problems, and can justifiably state that Ising spin glasses can be polynomially mapped to all other NP-Hard problems [2].

## II. EXISTING METHODS

### REFERENCES

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