Ising Model Solver for Combinatorial Optimization Problem

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The Ising model is a mathematical model of a ferromagnetic material under the presence of a magnetic field, which allows for the analysis of the thermodynamics of a system of ferromagnetic particles [1]. When applied with the spin glass magnetic state, the Ising spin-glass model can be utilized to analyze the thermodynamics of unordered or chaotic systems. Particularly, the spin-glass Ising model aligns each ferromagnetic particle in a lattice with a random spin-interactions depicted as: $\sigma \in \{-1, +1\}$. For the purpose of this document, we will consider the spin-glass Ising model to solve for popular and known NP-Hard problems.

Formally, the N-spin Ising problem aims to find the configuration of spins that minimizes the energy Hamiltonian,

$$H = -\sum_{i,j<N} J_{ij}\sigma_i\sigma_j - \sum_i h_i\sigma_i \tag{1}$$

where J_{ij} represents the coupling coeffcient, negligible for non-neighboring spins. This document will explore the solving of the NP-Hard problem MaxCut using the Ising spin glass model. As such, the energy of MaxCut can be modeled by the following equation:

$$Cut(s) = \frac{1}{2}\sum_{(i,j)\in E}(1 - \sigma_i\sigma_j) \tag{2}$$

To map this to the Hamiltonian for Ising spin glass and maintain the objective of the problem, we aim to solve for the ground state of $H(s) = -Cut(s)$. In other words, we have the following equation representing the Hamiltonian we will strive to minimize:

$$H(s) = \sum_{(i,j)\in E}\sigma_i\sigma_j \tag{3}$$

Due to the difficulty of finding the minimum Hamiltonian of a chaotic systems, the Ising spin glass model is a NP-Hard problem for classical computers. Naturally, we are able to correlate this property to all NP-Hard problems, and can be justifiably stated that Ising spin glasses are able to be polynomially mapped to all other NP-Hard problems [2].

References

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