LBMP USING GUROBI

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PROBLEM DEFINITION

- Given a graph that has generators, loads, and transmission limits, find the lowest cost possible to satisfy all loads.
  - A generator can never supply more MW than it dictates.
  - All loads must be filled for the problem to reach a complete state.
  - Energy transfer between zones happens through a transmission bus. The two-way energy across this bus must never be exceeded.
- Locational Based Marginal Price (LBMP) can be calculated based on the maximum price required to meet all demand.
  - Congestion may occur where a transmission bus does not allow for more energy transfer, and a more expensive generator is required to fulfill the demand of a load.

LBMP PROBLEM:

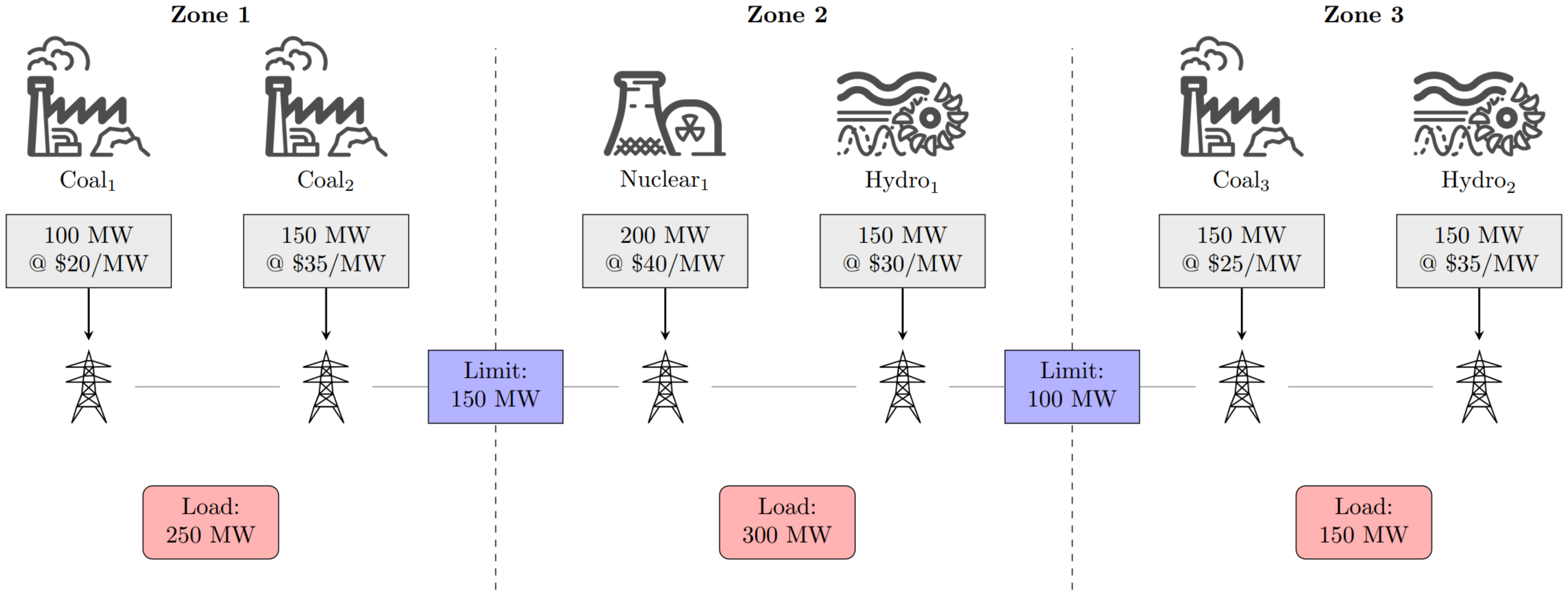

Figure 1: LBMP problem introduction

OBJECTIVE AND VARIABLES

- Objective Function:
  - Minimize: $\sum_{g \in \mathrm{G}} \sum_{l \in \mathrm{L}} c_g \times x_{g,l}$
  - Where G is the set of all generators; L is the set of all loads; c_g represents the USD\MW at generator g.
- Variables:
  - $x_{g,l} \geq 0 \ \forall \ G \cup L$

LINEAR CONSTRAINTS

- Constraints:
  - $\exists g \in G; \sum_{l \in L} x_{g,l} \leq S_g$
    - All generator-load pairs for each generator must remain under the generator supply limit.
    - Where S_g represents the maximum supply that generator g has (MW).
  - $\exists l \in L; \sum_{g \in G} x_{g,l} = D_l$
    - The demand for all loads must be met by the generator-load pairs for each load.
    - Where D_l represents the demand at load l (MW).
  - $\sum_{g \in G(T_{A,\mathrm{B}})} \sum_{l \in L(\overline{T_{A,B}})} x_{g,l} + \sum_{g \in G(\overline{T_{A,B}})} \sum_{l \in L(T_{A,B})} x_{g,l} \leq C_{A,B}$
    - The generator-load pairs across transmission bus (A, B) must remain under the capacity limit for bus (A, B).
    - Where $T_{A,B}$ represents the zones reachable from A when (A, B) is removed; $\overline{T_{A,B}}$ represents the remaining zones; $C_{A,B}$ represents the transmission cap across edge (A, B).

PROBLEM ENVIRONMENT SETUP

- Figure 2 defines the problem variables:
  - Set of generators, loads, and zones.
  - The supply (MW) for each generator.
  - The cost (USD/MW) for each generator.
  - The demand (MW) for each load.
  - Transmission limit between zoneA and zoneB for all transmission busses.
  - Separate the generators based on their corresponding zones.
  - Separate the loads based on their corresponding zones.

```
generators = ["coal1", "coal2", "nuclear1", "hydro1", "hydro2", "coal3"]
loads = ["load1", "load2", "load3"]
zones = ["zone1", "zone2", "zone3"]

# Maximum MW each generator can supply
supply = {
    "coal1":100,
    "coal2":150,
    "nuclear1":200,
    "hydro1":150,
    "hydro2":150,
    "coal3":150
}

# USD/MW for each generator
cost = {
    "coal1":20,
    "coal2":35,
    "nuclear1":40,
    "hydro1":30,
    "hydro2":35,
    "coal3":25
}

# Demand in MW of each load
demand = {
    "load1":250,
    "load2":300,
    "load3":150
}

# Transmission limit across zones
transLimit = {
    ("zone1", "zone2"): 150,
    ("zone2", "zone3"): 100,
}

# Define the corresponding zone-generator relationships
generatorByZone = {
    "zone1": ["coal1", "coal2"],
    "zone2": ["nuclear1", "hydro1"],
    "zone3": ["hydro2", "coal3"]
}

# Define the corresponding zone-load relationships
loadByZone = {
    "zone1": ["load1"],
    "zone2": ["load2"],
    "zone3": ["load3"]
}

# Define the corresponding load-zone relationsips
zoneByLoad = {
    "load1": "zone1",
    "load2": "zone2",
    "load3": "zone3"
}
```

Figure 2: Problem Environment setup

GUROBI MODEL CONFIGURATION PART I

- Figure 3 defines the Gurobi model.
- Create a 2D array relating each generator to each load, $x_{g,l}$.
- Setting the objective to minimize the product of cost and generator-load pairs.
- Add constraint defining the maximum supply for each generator.
- Add constraint defining the necessary demand for each load.

```python
m = Model("LBMP")

x = m.addVars(generators, loads, name="x", lb=0)

m.setObjective( sum(cost[g] * x[g,l] for g in generators for l in loads), GRB.MINIMIZE )

# Generator supply constraints
for g in generators:
    m.addConstr(sum(x[g,l] for l in loads) <= supply[g], name=f"supply_{g}")

# Load demand constraints
for l in loads:
    m.addConstr(sum(x[g,l] for g in generators) == demand[l], name=f"demand_{l}")
```

Figure 3: Gurobi Objective and Constraint setup

GUROBI MODEL CONFIGURATION PART II

- Figure 4 details the steps required for the transmission bus constraint
- Associate zones with neighboring zones.
- Partition the graph along each transmission bus.
  - The sum of generator-load pairs from Partition A to Partition B and Partition B to partition A must remain below the transmission cap between zones A and B.

```python
# define transmission bus constraints
for (zoneA,zoneB), cap in transLimit.items():
    # Side A contains all zones on one side of the partition
    T = reachable_without_edge(zoneA, zoneA, zoneB, adj)
    # Side B contains the remaining zones
    Tbar = set(zones) - T

    # Find all generators in each partition
    gensA = [g for z in T for g in generatorByZone[z]]
    gensB = [g for z in Tbar for g in generatorByZone[z]]

    # Find all loads in each partition
    loadsA = [l for z in T for l in loadByZone[z]]
    loadsB = [l for z in Tbar for l in loadByZone[z]]

    # Compute the sums of energy flowing across the partitions
    flow_A_to_B = sum(x[g,l] for g in gensA for l in loadsB)
    flow_B_to_A = sum(x[g,l] for g in gensB for l in loadsA)

    # Combine directional energy sums and ensure the total is <= the
    # transmission cap for the corresponding bus
    m.addConstr(
        flow_A_to_B + flow_B_to_A <= cap,
        name=f"cut_{zoneA}_{zoneB}"
    )
```

Figure 4: Transmission bus limit constraint configuration

GUROBI RESULTS

```
Optimal total cost: 21000.0
coal1 -> load1: 100.0 MW
coal2 -> load1: 150.0 MW
nuclear1 -> load2: 50.0 MW
hydro1 -> load2: 150.0 MW
hydro2 -> load3: 100.0 MW
coal3 -> load2: 100.0 MW
coal3 -> load3: 50.0 MW
```

Figure 5: Optimized cost and generator-load pairs

```
Price per Zone:
zone1:
        Marginal Price:         $35.00
        Congestion Cost:        $0.00
        Total Cost:             $35.00
zone2:
        Marginal Price:         $35.00
        Congestion Cost:        $5.00
        Total Cost:             $40.00
zone3:
        Marginal Price:         $35.00
        Congestion Cost:        $0.00
        Total Cost:             $35.00
```

Figure 6: Computed prices for each zone

- Figure 5 shows the total cost, neglecting marginal price, necessary to satisfy all loads is $21,000.
- Figure 5 displays the generator-load pairs in an optimal solution, along with the power purchased from each generator.
- Figure 6 shows the breakdown of zone charges based on marginal price rules and congestion considerations.

LBMP PROBLEM WITH CONGESTION:

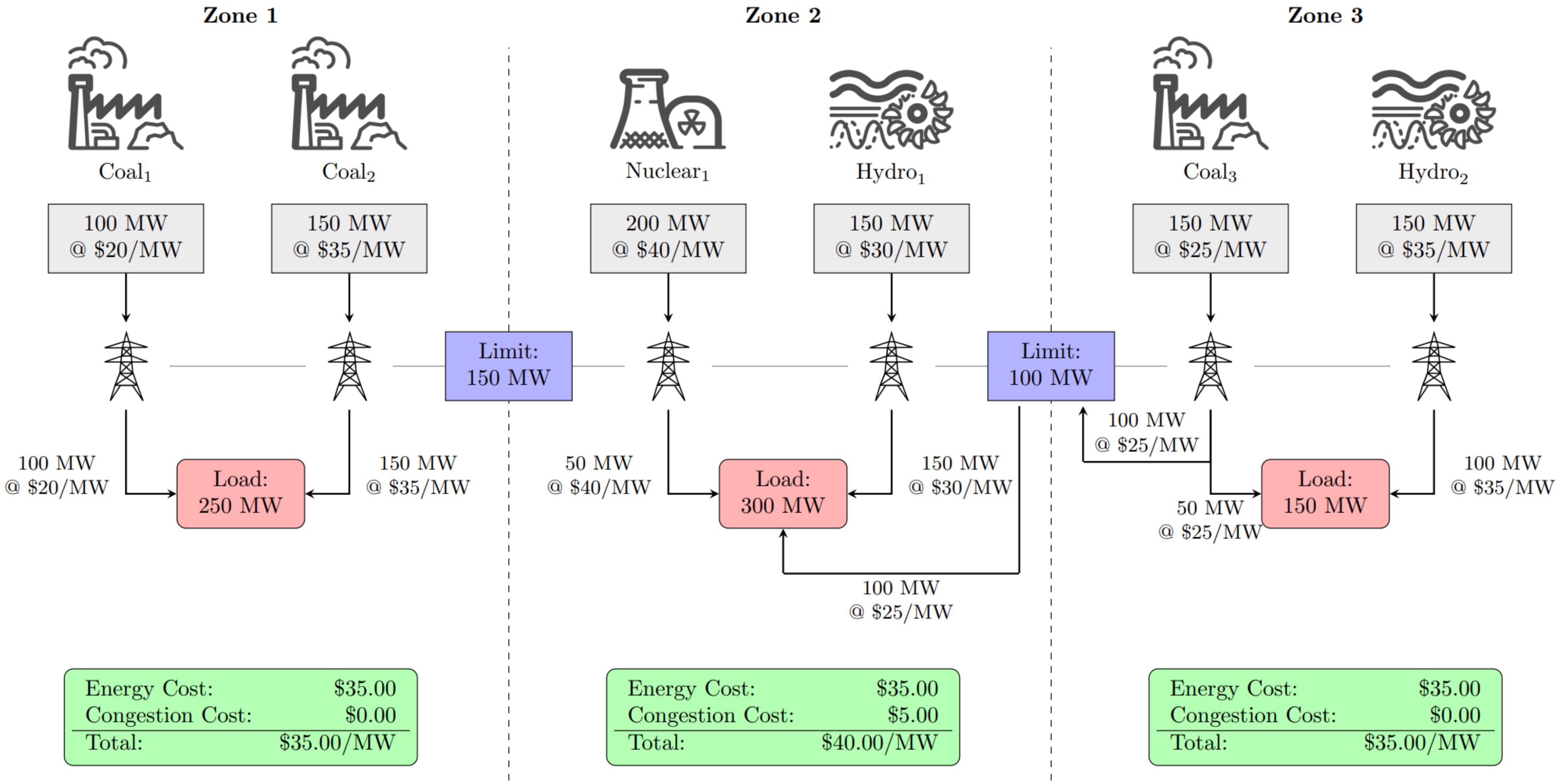

Figure 7: LBMP solution found by Gurobi