MSSC 6040 - Homework 2

Henri Medeiros Dos Reis

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Problem 1 (5 pts)**.** The discrete Fourier transform (DFT) of a vector $x \in \mathbb{C}^n$ is another vector $\hat{x} \in \mathbb{C}^n$ whose kth entry is given by

$$\hat{x}_k = \frac{1}{\sqrt{n}} \sum_{\ell=1}^{n} x_\ell \, \omega^{(k-1)(\ell-1)} \quad \text{where} \quad \omega := e^{-i2\pi/n}$$

for all $k = 1, ..., n$. Observe that the DFT can be expressed as $\hat{x} = Fx$ where $F \in \mathbb{C}^{n \times n}$ is the matrix with entries

$$F_{k\ell} = \frac{1}{\sqrt{n}} \omega^{(k-1)(\ell-1)} \quad \text{for all} \quad k, \ell \in \{1, ..., n\}.$$

(a) In the case of length $n = 3$ vectors, write down the corresponding DFT matrix F as well as its adjoint F^*, and verify that F is unitary, i.e., show $FF^* = I$ where I is the 3×3 identity matrix. Below are some tips to simplify calculations:

- Keep everything in terms of powers of ω as long as possible.
- Use the fact that $\overline{\omega^m} = \omega^{-m}$ for any integer m.
- Since FF^* is Hermitian, you only need to compute its entries on the upper diagonal and verify these match the upper diagonal entries of the 3×3 identity.
- When it comes time to compute sums of powers of ω, use the identity $(e^{i\theta})^m = e^{im\theta} = \cos(m\theta) + i\sin(m\theta)$ which holds for all $\theta \in \mathbb{R}$ and all integers m.

(b) For general length n vectors, prove that the corresponding DFT matrix F is unitary. (Hint: The identity $\sum_{\ell=1}^{n} z^{\ell-1} = \frac{1-z^n}{1-z}$, which holds for any $z \in \mathbb{C}$, $z \neq 1$, may be helpful.)

(c) Using the result of part (b), give a short proof of *Parseval's theorem*: for all $x \in \mathbb{C}^n$,

$$\sum_{k=1}^{n} |x_k|^2 = \sum_{k=1}^{n} |\hat{x}_k|^2.$$

Solution 1. (a)

$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$F = \begin{bmatrix} \frac{1}{\sqrt{3}}\omega^{(1-1)(1-1)} & \frac{1}{\sqrt{3}}\omega^{(1-1)(2-1)} & \frac{1}{\sqrt{3}}\omega^{(1-1)(3-1)} \\ \frac{1}{\sqrt{3}}\omega^{(2-1)(1-1)} & \frac{1}{\sqrt{3}}\omega^{(2-1)(2-1)} & \frac{1}{\sqrt{3}}\omega^{(2-1)(3-1)} \\ \frac{1}{\sqrt{3}}\omega^{(3-1)(1-1)} & \frac{1}{\sqrt{3}}\omega^{(3-1)(2-1)} & \frac{1}{\sqrt{3}}\omega^{(3-1)(3-1)} \end{bmatrix}$$

$$F^* = \begin{bmatrix} \frac{1}{\sqrt{3}}\overline{\omega}^{(1-1)(1-1)} & \frac{1}{\sqrt{3}}\overline{\omega}^{(1-1)(2-1)} & \frac{1}{\sqrt{3}}\overline{\omega}^{(1-1)(3-1)} \\ \frac{1}{\sqrt{3}}\overline{\omega}^{(2-1)(1-1)} & \frac{1}{\sqrt{3}}\overline{\omega}^{(2-1)(2-1)} & \frac{1}{\sqrt{3}}\overline{\omega}^{(2-1)(3-1)} \\ \frac{1}{\sqrt{3}}\overline{\omega}^{(3-1)(1-1)} & \frac{1}{\sqrt{3}}\overline{\omega}^{(3-1)(2-1)} & \frac{1}{\sqrt{3}}\overline{\omega}^{(3-1)(3-1)} \end{bmatrix}$$

$$F = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\omega^1 & \frac{1}{\sqrt{3}}\omega^2 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\omega^2 & \frac{1}{\sqrt{3}}\omega^4 \end{bmatrix}$$

$$F^* = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\overline{\omega}^1 & \frac{1}{\sqrt{3}}\overline{\omega}^2 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\overline{\omega}^2 & \frac{1}{\sqrt{3}}\overline{\omega}^4 \end{bmatrix}$$

$$FF^* = \begin{bmatrix} 1 & \frac{1}{3}+\frac{1}{3}\omega^1+\frac{1}{3}\omega^2 & \frac{1}{3}+\frac{1}{3}\omega^2+\frac{1}{3}\omega^4 \\ \frac{1}{3}+\frac{1}{3}\overline{\omega}^1+\frac{1}{3}\overline{\omega}^2 & \frac{1}{3}+\frac{1}{3}\omega^1\overline{\omega}^1+\frac{1}{3}\omega^2\overline{\omega}^2 & \frac{1}{3}+\frac{1}{3}\omega^2\overline{\omega}^1+\frac{1}{3}\omega^4\overline{\omega}^2 \\ \frac{1}{3}+\frac{1}{3}\overline{\omega}^2+\frac{1}{3}\overline{\omega}^4 & \frac{1}{3}+\frac{1}{3}\omega^1\overline{\omega}^2+\frac{1}{3}\omega^2\overline{\omega}^4 & \frac{1}{3}+\frac{1}{3}\omega^2\overline{\omega}^2+\frac{1}{3}\omega^4\overline{\omega}^4 \end{bmatrix}$$

$$FF^* = \begin{bmatrix} 1 & \frac{1}{3}+\frac{1}{3}\omega^1+\frac{1}{3}\omega^2 & \frac{1}{3}+\frac{1}{3}\omega^2+\frac{1}{3}\omega^4 \\ \frac{1}{3}+\frac{1}{3}\overline{\omega}^1+\frac{1}{3}\overline{\omega}^2 & 1 & \frac{1}{3}+\frac{1}{3}\omega^2\overline{\omega}^1+\frac{1}{3}\omega^4\overline{\omega}^2 \\ \frac{1}{3}+\frac{1}{3}\overline{\omega}^2+\frac{1}{3}\overline{\omega}^4 & \frac{1}{3}+\frac{1}{3}\omega^1\overline{\omega}^2+\frac{1}{3}\omega^2\overline{\omega}^4 & 1 \end{bmatrix}$$

$$FF^* = \begin{bmatrix} 1 & \frac{1}{3}(e^{\frac{i2\pi}{3}}+e^{\frac{i4\pi}{3}}) & \frac{1}{3}(e^{\frac{i4\pi}{3}}+e^{\frac{i8\pi}{3}}) \\ \frac{1}{3}(e^{\frac{-i2\pi}{3}}+e^{\frac{-i4\pi}{3}}) & 1 & \frac{1}{3}(e^{\frac{i2\pi}{3}}+e^{\frac{i4\pi}{3}}) \\ \frac{1}{3}(e^{\frac{-i4\pi}{3}}+e^{\frac{-i8\pi}{3}}) & \frac{1}{3}(e^{\frac{-i2\pi}{3}}+e^{\frac{-i4\pi}{3}}) & 1 \end{bmatrix}$$

$$FF^*_{col1} = \begin{bmatrix} 1 \\ \frac{1}{3}(1+cos(\frac{-2\pi}{3})+isin(\frac{-2\pi}{3})+cos(\frac{-4\pi}{3})+isin(\frac{-4\pi}{3})) \\ \frac{1}{3}(1+cos(\frac{-4\pi}{3})+isin(\frac{-4\pi}{3})+cos(\frac{-8\pi}{3})+isin(\frac{-8\pi}{3})) \end{bmatrix}$$

$$FF^*_{col2} = \begin{bmatrix} \frac{1}{3}(1+cos(\frac{2\pi}{3})+isin(\frac{2\pi}{3})+cos(\frac{4\pi}{3})+isin(\frac{4\pi}{3})) \\ 1 \\ \frac{1}{3}(1+cos(\frac{-2\pi}{3})+isin(\frac{-2\pi}{3})+cos(\frac{-4\pi}{3})+isin(\frac{-4\pi}{3})) \end{bmatrix}$$

$$FF^*_{col3} = \begin{bmatrix} \frac{1}{3}(1+cos(\frac{4\pi}{3})+isin(\frac{4\pi}{3})+cos(\frac{8\pi}{3})+isin(\frac{8\pi}{3})) \\ \frac{1}{3}(1+cos(\frac{2\pi}{3})+isin(\frac{2\pi}{3})+cos(\frac{4\pi}{3})+isin(\frac{4\pi}{3})) \\ 1 \end{bmatrix}$$

$$FF^* = \begin{bmatrix} 1 & \frac{1}{3}(1 - \frac{1}{2} + i\frac{\sqrt{3}}{2} - \frac{1}{2} + i\frac{-\sqrt{3}}{2}) & \frac{1}{3}(1 - \frac{1}{2} + i\frac{-\sqrt{3}}{2} - \frac{1}{2} + i\frac{\sqrt{3}}{2}) \\ \frac{1}{3}(1 - \frac{1}{2} + i\frac{-\sqrt{3}}{2} + \frac{1}{2} + i\frac{\sqrt{3}}{2}) & 1 & \frac{1}{3}(1 - \frac{1}{2} + i\frac{\sqrt{3}}{2} - \frac{1}{2} + i\frac{-\sqrt{3}}{2}) \\ \frac{1}{3}(1 + \frac{1}{2} + i\frac{\sqrt{3}}{2} - \frac{1}{2} + i\frac{-\sqrt{3}}{2}) & \frac{1}{3}(1 - \frac{1}{2} + i\frac{-\sqrt{3}}{2} + \frac{1}{2} + i\frac{\sqrt{3}}{2} & 1 \end{bmatrix}$$

$$FF^* = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & 1 \end{bmatrix}$$

Then performing $R_2 = R_2 - \frac{1}{3}R_1$ and $R_3 = R_3 - \frac{1}{3}R_1 - \frac{1}{3}R_2$

$$FF^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b)

$$F = \frac{1}{\sqrt{n}} \begin{bmatrix} 1 & 1 & \cdots & \cdots & 1 \\ 1 & \omega & \omega^2 & \cdots & \omega^{n-1} \\ \vdots & \omega^2 & \omega^4 & \cdots & \omega^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{n-1} & \omega^{2(n-1)} & \cdots & \omega^{(n-1)(n-1)} \end{bmatrix}$$

$$F^* = \frac{1}{\sqrt{n}} \begin{bmatrix} 1 & 1 & \cdots & \cdots & 1 \\ 1 & \overline{\omega} & \overline{\omega}^2 & \cdots & \overline{\omega}^{n-1} \\ \vdots & \overline{\omega}^2 & \overline{\omega}^4 & \cdots & \overline{\omega}^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \overline{\omega}^{n-1} & \overline{\omega}^{2(n-1)} & \cdots & \overline{\omega}^{(n-1)(n-1)} \end{bmatrix}$$

$$FF^* = \frac{1}{\sqrt{n}} \begin{bmatrix} 1 & 1 & \cdots & \cdots & 1 \\ 1 & \omega & \omega^2 & \cdots & \omega^{n-1} \\ \vdots & \omega^2 & \omega^4 & \cdots & \omega^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{n-1} & \omega^{2(n-1)} & \cdots & \omega^{(n-1)(n-1)} \end{bmatrix} \frac{1}{\sqrt{n}} \begin{bmatrix} 1 & 1 & \cdots & \cdots & 1 \\ 1 & \overline{\omega} & \overline{\omega}^2 & \cdots & \overline{\omega}^{n-1} \\ \vdots & \overline{\omega}^2 & \overline{\omega}^4 & \cdots & \overline{\omega}^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \overline{\omega}^{n-1} & \overline{\omega}^{2(n-1)} & \cdots & \overline{\omega}^{(n-1)(n-1)} \end{bmatrix}$$

Since $\omega\overline{\omega} = \omega^0 = 1$ for all powers, then:

$$FF^* = \frac{1}{n} \begin{bmatrix} n & \sum_{m=0}^{n-1} \omega^{-m} & \sum_{m=0}^{n-1} \omega^{-2m} & \cdots & \sum_{m=0}^{n-1} \omega^{-m(n-1)} \\ \sum_{m=0}^{n-1} \omega^{m} & n & \sum_{m=0}^{n-1} \omega^{-m} & \cdots & \sum_{m=0}^{n-1} \omega^{-m(n-2)} \\ \sum_{m=0}^{n-1} \omega^{2m} & \sum_{m=0}^{n-1} \omega^{m} & n & \cdots & \sum_{m=0}^{n-1} \omega^{-m(n-3)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{m=0}^{n-1} \omega^{m(n-1)} & \sum_{m=0}^{n-1} \omega^{m(n-2)} & \sum_{m=0}^{n-1} \omega^{m(n-3)} & \cdots & n \end{bmatrix}$$

If we take any general sum of the elements that are not in the diagonal. And, since $\omega^{\ell m} = (\omega^m)^\ell$, then all sums $\sum_{m=0}^{n-1} \omega^{\ell m} = \sum_{m=0}^{n-1} (\omega^m)^\ell$. Hence, we can use the hint

$\sum_{\ell=1}^{n} z^{\ell-1} = \frac{1-z^n}{1-z}$

$$\sum_{m=0}^{n-1} \omega^{\ell m} = \sum_{m=1}^{n} (\omega^{m-1})^{\ell} = \frac{1-(\omega^{m-1})^n}{1-\omega^{m-1}} = \frac{1-((e^{\frac{-i2\pi}{n}})^{m-1})^n}{1-(e^{\frac{-i2\pi}{n}})^{m-1}} \Rightarrow$$

$$\frac{1-(e^{-i2\pi(m-1)})}{1-(e^{\frac{-i2\pi}{n}})^{m-1}} = \frac{1-(cos((m-1)2\pi)+isin((m-1)2\pi)}{1-(cos((m-1)\frac{2\pi}{n})+isin((m-1)\frac{2\pi}{n}))} \Rightarrow$$

$$\frac{1-(1+0)}{1-1-(cos((m-1)\frac{2\pi}{n})+isin((m-1)\frac{2\pi}{n}))} = 0$$

So, all the entries that have indexes $i \neq j$ are 0, and the matrix

$$FF^* = \frac{1}{n}\begin{bmatrix} n & 0 & 0 & \cdots & 0 \\ 0 & n & 0 & \cdots & 0 \\ 0 & 0 & n & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & n \end{bmatrix} = I$$

Therefore, for general length n vectors, the corresponding DFT matrix F is unitary.

(c) Let $\hat{x} = Fx$. And since $FF^* = I$ Then by using $F^*\hat{x} = FF^*x = x$. Then

$$\sum_{k=1}^{n} |x_k|^2 = \sum_{k=1}^{n} |x_k||x_k| = \sum_{k=1}^{n} |F^*\hat{x_k}||F^*\hat{x_k}| \Rightarrow$$

$$\sum_{k=1}^{n} |\hat{x_k}F^*F\hat{x_k}| = \sum_{k=1}^{n} |\hat{x_k}\hat{x_k}| = \sum_{k=1}^{n} |\hat{x}|^2$$

Problem 2 (5 pts)**.** T-B 2.3 & 2.4

Solution 2.
T-B 2.3:

a-) To prove that all λ values are real, $\lambda = \lambda^*$. Then let's look at the 2-norm of x times λ:

$$\lambda||x||_2 = \lambda(x^*x) = x^*(\lambda x)$$

Since $Ax = \lambda x$, $x^*(Ax) = x^*(\lambda x)$, and A is hermitian, so:

$$x^*A^*x = (Ax)^*x = (\lambda x)^*x = \lambda^*(x^*x) \Rightarrow$$

$$\lambda^*(x^*x) = \lambda(x^*x)$$

Therefore, $\lambda^* = \lambda$

b-) Since x and y are eigenvectors, then we have $Ax = \lambda_x x$ and $Ay = \lambda_y y$, for any values λ_x and λ_y such that $\lambda_y \neq \lambda_x$. In order to show that x and y are orthogonal, their inner product has to be equal to 0, $x^* y = 0$. Then we can write:

$$\lambda_x(y^* x) = y^*(Ax) = (y^* A)x = (y^* A*)x = (Ay)^* x$$

Since hermitian and since $A_y = \lambda_y y$, then:

$$(Ay)^* x = (\lambda_y y)^* x = \lambda_y y^* x$$

Since $\lambda^* = \lambda$, then $\lambda_x(y^* x) = \lambda_y(y^* x)$. Finally, since $\lambda_x \neq \lambda_y$, the only way that this equation is true is for $y^* x = 0$, which proves that these vectors are orthogonal to each other.

T-B 2.4: For any unitary matrix U, $U^* = U^{-1}$. Then let λ be an eigenvalue of U, and x be the corresponding eigenvector. Then:

$$x^* x = x^*(U^* U)x = x^* U^* U x = (Ux)^* U x$$

Since $Ux = \lambda x$ and $x \neq 0$, then

$$x^* x = (\lambda x)^* \lambda x = x^* \lambda^* \lambda x$$

$$x^* x = ||\lambda|| x^* x \Rightarrow ||\lambda|| \frac{x^* x}{x^* x} = 1 = ||\lambda||$$

Problem 3 (5 pts)**.** Recall that the *Frobenius norm* of a matrix $A \in \mathbb{C}^{m \times n}$, denoted by $\|A\|_F$, is defined as

$$\|A\|_F := \left(\sum_{i=1}^{m} \sum_{j=1}^{n} |A_{ij}|^2 \right)^{\frac{1}{2}}.$$

(a) Prove that

$$\|A\|_F = \sqrt{\operatorname{Tr}(A^* A)} = \sqrt{\operatorname{Tr}(AA^*)}.$$

where $\operatorname{Tr}(\cdot)$ denotes the trace of a matrix, i.e., the sum of all entries along the main diagonal.

(b) Prove the Frobenius norm is unitarily invariant, i.e., for any matrix $A \in \mathbb{C}^{m \times n}$ and any unitary matrices $U \in \mathbb{C}^{m \times m}$ and $V \in \mathbb{C}^{n \times n}$ we have

$$\|UA\|_F = \|A\|_F \quad \text{and} \quad \|AV\|_F = \|A\|_F.$$

(c) Can the Frobenius norm be expressed as an induced matrix norm? Why or why not? (*Hint: What is the norm of the identity matrix for any induced matrix norm?*)

Solution 3. a-)

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}, A^* = \begin{bmatrix} a^*_{11} & a^*_{12} & \cdots & a^*_{1n} \\ a^*_{21} & a^*_{22} & \cdots & a^*_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a^*_{m1} & a^*_{m2} & \cdots & a^*_{mn} \end{bmatrix}$$

$$A^*A = \begin{bmatrix} \sum_{i=1}^m (a_{1i})^2 & \sum_{i=1}^m a^*_{1i}a_{2i} & \cdots & \sum_{i=1}^m a^*_{1i}a_{in} \\ \sum_{i=1}^m a^*_{2i}a_{i1} & \sum_{i=1}^m (a_{2i})^2 & \cdots & \sum_{i=1}^m a^*_{2i}a_{in} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^m a^*_{ni}a_{i1} & \sum_{i=1}^m a^*_{in}a_{2i} & \cdots & \sum_{i=1}^m (a_{ni})^2 \end{bmatrix}$$

$$AA^* = \begin{bmatrix} \sum_{i=1}^m (a_{1i})^2 & \sum_{i=1}^m a_{1i}a^*_{2i} & \cdots & \sum_{i=1}^m a_{1i}a^*_{in} \\ \sum_{i=1}^m a_{2i}a^*_{i1} & \sum_{i=1}^m (a_{2i})^2 & \cdots & \sum_{i=1}^m a_{2i}a^*_{in} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^m a_{ni}a^*_{i1} & \sum_{i=1}^m a_{in}a^*_{2i} & \cdots & \sum_{i=1}^m (a_{ni})^2 \end{bmatrix}$$

Then $Tr(A^*A) = \sum_{i=1}^m \sum_{j=1}^n (a_{ij})^2 = Tr(AA^*)$. Finally:

$$\sqrt{Tr(A^*A)} = \sum_{i=1}^m \sum_{j=1}^n (a_{ij})^2 = \sum_{i=1}^m \sum_{j=1}^n |A_{ij}|^2 = \sqrt{Tr(AA^*)}$$

b-) Since just proved that the Frobenius norm is $\sqrt{Tr(A^*A)}$, then:

Let A be a $m \times n$ matrix, and U be a $m \times m$ unitary matrix.

$$||UA||_F = \sqrt{Tr((UA)^*(UA))} = \sqrt{Tr(A^*U^*UA)}$$

Since U is unitary, $U^*U = I$

$$\sqrt{Tr(A^*U^*UA)} = \sqrt{Tr(A^*A)} = ||A||_F$$

Also, let V be a $n \times n$ matrix.

$$||AV||_F = \sqrt{Tr((AV)^*(AV))} = \sqrt{Tr(AV(AV)^*)} = \sqrt{Tr(AVV^*A} \Rightarrow$$

Since $VV^* = I$

$$\sqrt{Tr(AA^*)} = ||A||_F$$

c-) Let's consider the identity matrix $I_{n\times n}$. The Frobenius norm then is:

$$||I||_F = \sqrt{n}$$

While the induced norm of the identity matrix is:

$$||I|| = max\left\{\frac{||I\vec{x}||}{||\vec{x}||} : \vec{x} \neq 0, \vec{x} \in \mathbb{C}^n\right\} = max\left\{\frac{||\vec{x}||}{||\vec{x}||} : \vec{x} \neq 0, \vec{x} \in \mathbb{C}^n\right\} = 1$$

Therefore, for all values $n \neq 1$, the Frobenius norm cannot be expressed as an induced matrix norm, since $||I||_F = \sqrt{n} \neq 1 = ||I||$

Problem 4 (5 pts, MATLAB)**.** Write a MATLAB function that computes the ∞-norm of a complex matrix (see Ex. 3.4 in T-B). Use your function to compute the ∞-norm of the matrix `A = fft((1:100)'*(1:50))`. Check your answer with MATLAB's built-in command `norm(A,Inf)`. Include in your write-up a printout/screenshot of your code and the output.

Solution 4.

```matlab
%Declaration of the A Matrix
A = fft((1:100)'*(1:50));

%use the function created to compute the infinity norm
AinfNorm = compInfNormA(A);

%store 2 strings with the formatted results.
s = strcat(The infinity norm of A using my funcition is:
    , num2str(AinfNorm,'%2d'));
s1 = strcat(The infinity norm of A using MATLAB's funcition is:
    , num2str(norm(A,Inf),'%2d'));

%print the results
disp(s)
disp(s1)

%Define a function that will compute the infinity norm, given a
    matrix
function infNorm = compInfNormA(A)
    %declare a vector that will hold all the 1-norm of the rows
    myVector = []
    %compute the 1-norm for all the rows
    for i = 1:size(A,1)
        result = norm(A(i,:),1);
        myVector = [myVector, result];
    end
    %the infinity norm is the max inside that vector
    infNorm = max(myVector);
end
```

```
%-------OUTPUT-----------
%The infinity norm of A using my funcition is: 6438750
%The infinity norm of A using MATLAB's funcition is: 6438750
```

Problem 5 (5 pts)**.** T-B 3.2

Solution 5. We need to show that $\rho(A) = max(\lambda)\{|\lambda| : \lambda \text{ is an eigenvalue}\} \leq ||A|| = \frac{||Ax||}{||x||}$. Then let λ be the eigenvalue of A, and x be the corresponding eigenvector, $Ax = \lambda x$. Then:

$$||Ax|| = ||\lambda x|| = |\lambda|||x|| \Rightarrow |\lambda| = \frac{||Ax||}{||x||}$$

Therefore, by definition of the induced-norm of a matrix, $|\lambda| = \frac{||Ax||}{||x||} \leq max\{\frac{||Ay||}{||y||} : y \neq 0\} = ||A||$