Math 4540/MSSC 5540 - Activity #7

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1. Consider the bungee jumper using a strong cord

   i) Write down the equation of motion for the bungee jumper

   $$\frac{dv}{dt} = g - \frac{C_{drag}}{m}v^2, v(0) = 0$$

   ii) Use the gravitational constant g = 9.81 m/sc^2. Assume a maximum weight of 350 lb, a rope length 30m, an initial height of 500m, and a coefficient of drag of 0.25 kg/m. Show that the units of the equation are balanced.
   Since the gravitational constant is in meters/seconds^2, $\frac{C_{drag}}{m}v^2$ is Kilograms/meters*(meters/second)^2/Mass, which when simplifying gives meters/seconds^2. Which is the same units we have in the left hand side of the equation, as long as we convert the weight from pounds to kilograms.

   iii) Determine the terminal velocity of the jumper.
   At terminal velocity $\frac{dv}{dt} = 0$

   $$0 = g - \frac{C_{drag}}{m}v^2$$
   $$v_{final} = \sqrt{\frac{gm}{C_{drag}}}$$
   $$v_{final} = \sqrt{\frac{9.81 * 350/2.2}{0.25}}$$
   $$v_{final} = 79.01$$

   iv) Solve the ODE in Matlab using ode45. Plot the velocity as a function of time. Include the graph here.

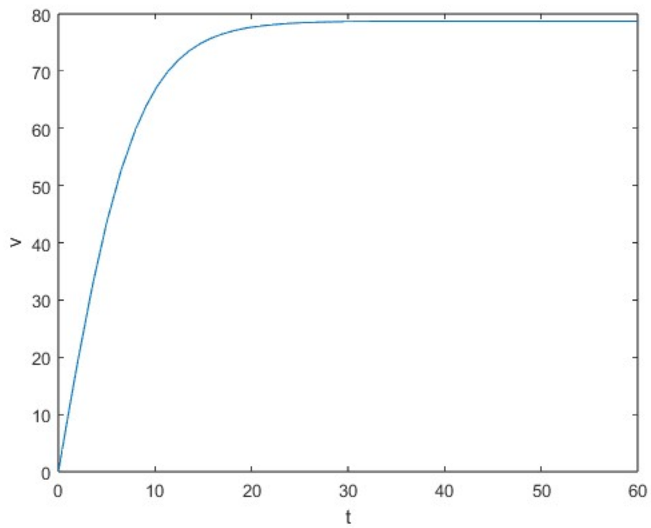
```
m = 158;
c_drag = 0.25;
g = 9.81;
```

```
f = @(t,v) g-(c_drag./m).*v.^2;

tspan = [0 60]; %time interval
v0 = 0; % Initial velocity

%%RK
[t,v] = ode45(f,tspan,v0);

%%
plot(t,v)
xlabel('t')
ylabel('v')
```



v) Is the terminal velocity correct?
   Yes, it is not exact, but very close. At the last v, we get 78.9279.

vi) How long does it take the jumper to reach a speed of 80 m/s?
   The jumper never reaches this speed if the jumper weights
   $350pounds = 350/2.20462kg = 158.757kg$

2. Consider the bungee jumper using a stretchable bungee cord

   i) Write down the equation of motion for the bungee jumper.

$$\frac{d^2y}{dt^2} = g - sign(v)a_{drag}v^2/mass - F_{cord}/mass$$

   Where

$$F_{cord} = \begin{cases} a_{spring}(y-L) + a_{damp}v & \text{if } y > L \\ 0 & \text{else} \end{cases}$$

ii) Assume a spring constant of k=40N/m and a coefficient of damping of 8 N s/m. Show that the units are balanced. Writing only the units for the equation gives

$$\begin{aligned}
\frac{d^2y}{dt^2} &= \frac{m}{s^2} - \frac{kg/m(m/s)^2}{kg} - \frac{N/m(m-m) + Ns/m(m/s)}{Kg} \\
&= \frac{m}{s^2} - \frac{m}{s^2} - \frac{N+N}{Kg} \\
&= \frac{m}{s^2} - \frac{m}{s^2} - \frac{Kgm/s^2 + Kgm/s^2}{Kg}
\end{aligned}$$

iii) Write the equation as a system of ODEs

Let

$$\begin{aligned}
y_1 &= y \\
y_2 &= v = \frac{dy}{dt} = y_1'
\end{aligned}$$

Then the system is

$$\begin{cases} y_1' & = y_2 \\ y_2' & = y'' \end{cases}$$

$$\begin{cases} y_1' & = y_2 \\ y_2' & = g - sign(v) a_{drag} y_2^2/mass - F_{cord}/mass \end{cases}$$

iv) Solve the ODEs is Matlab using ode 45. Plot the height above the ground as a function of time. How close to the ground does the jumper get? Include your code.

```
%% RK45 ODE solver
clear all; close all;
global a_damp a_spring a_drag g L mass
tspan = [0 100];
%parameters
a_damp = 8; a_spring=40; a_drag = 0.25; g = 9.81; L = 30;
mass = 158.757;init_height = 500;
xinit = init_height; yinit = 0;
ICs = [xinit yinit];
%% RK
[t,y] = ode45(@(t,y) bungee(t,y),tspan,ICs);

%% Graph
plot(t,y(:,1))
hold on
%%
```

```
function dydt = bungee(t,y)
global a_damp a_spring a_drag g L mass
q = y(2);
if y(1) > L      %Cord exerts force
    F_cord = sign(y(1))*a_spring.*(y(1) - L) + a_damp.*y
       (2);
else
    F_cord = 0;
end

TotalForce = g - y(2).*a_drag.*y(2).^2./mass - F_cord./
   mass;
r = TotalForce;
dydt = [q;r];
end
```

The jumper hits the ground. And goes below 345.7841. We can check using `min(y(:,1))`.

3. Now assume that we want to position the platform in the tree as close as possible to the ground while always maintaining at least an 8m height above the ground for the jumper.

   i) How high does the platform need to be? Explain or show how you obtained your solution.

   The minimum height it should be able to go is 8. So we have a total length of falling of $500 + 345.7841$, we need to add 8 to that, then the result is the height we should be the starting, so the platform should be 853.7841 high.

   ii) Now consider that we will also build a bungee jump for kids with maximum weight of 75 pounds. How high does the platform need to be? Explain or show how you obtained your solution.

   Change the 158.757 kg to 34.0194. Then we get that the kid would go below 83.3521. So it should have a platform of height 591.3521.