Math 4540/MSSC 5540 - Activity #11

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November 19, 2023

1. Write matlab code to solve our heat equation wit Neumann BCs using finite differences method

```
%% Finite Difference for BVPs
clear all; close all
%Solve the 1D Heat equation with Neumann BCs.
L = 10; Tair = 200; T0 = 40; TL = 400; w = 0.05;q0 = 0;
h = 2;
a1 = 2+w*h*h;
a2 = w*h*h*Tair;

A = [a1 -2 0 0 0;
    -1 a1 -1 0 0;
    0 -1 a1 -1 0;
    0 0 -1 a1 -1;
    0 0 0 -1 a1];
b = [w*h^2*Tair-2*h*q0 a2 a2 a2 TL+a2]';

uin = A\b;
u = [uin' 400];
x = 0:h:L;
plot(x,u)
```

2. An insulated heated rod with a uniform heat source can be modeled by Poisson's equation:

$$\frac{d^2u}{dx^2} = -f(x)$$

Given a heat source $f(x) = 25$ and boundary conditions $u(0) = 40$ and $u(10) = 200$

i) Write the finite differences system of equations with h=2

$$i = 1, \frac{u_0 - 2u_1 + u_2}{h^2} = -25 \Rightarrow 2u_1 - u_2 = 25h^2 + u_0$$

$$i = 2, -u_1 + 2u_2 - u_3 = 25h^2$$
$$i = 3, -u_2 + 2u_3 - u_4 = 25h^2$$
$$i = 4, -u_3 + 2u_4 = 25h^2 + u_f$$

Then
$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 25h^2 + u_0 \\ 25h^2 \\ 25h^2 \\ 25h^2 + u_f \end{bmatrix}$$

ii) Solve using matlab

```
%% Finite Difference for BVPs
clear all; close all
L=10;
u0 = 40;
uf = 200;
h = 2;
a1 = 2;
a2 = 25*h^2;

A = [a1 -1 0 0;
     -1 a1 -1 0;
     0 -1 a1 -1;
     0 0 -1 a1];
b = [a2+u0 a2 a2 a2+uf]';

uin = A\b;
u = [u0 uin' uf];
x = 0:h:L;
plot(x,u)
```

iii) Solve in matlab using h=0.2 (or larger if required by matlab)

```
%% iii
% solve the same with different h
clear all; close all
L=10;
u0 = 40;
uf = 200;
h = 0.02;
a1 = 2;
a2 = 25*h^2;
```

```
m_size = L/h-1;
A = diag(a1*ones(1,m_size)) + diag( ...
    -1*ones(1,m_size-1),1) + diag( ...
    -1*ones(1,m_size-1),-1);

b = a2+zeros(m_size,1);
b(1) = b(1)+u0;
b(end) = b(end)+uf;

uin = A\b;
u = [u0 uin' uf];
x = 0:h:L;
plot(x,u)
```

3. The following is a simple reaction-diffusion equation describing the stead-state concentration, c, of a substance that reacts in a long reactor and disperses axially:

$$D\frac{d^2c}{dx^2} - kc = 0$$

Where $D = 1.5$ the dispersion coefficient, $k - 5$ the reaction time, and $L = 100$

Boundary conditions are given by c(0)=0.1 and c(L) = 1.

(a) Write the system of equations

$$i = 1, D(\frac{u_0 - 2u_1 + u_2}{h^2} - Ku_1 = 0 \Rightarrow u_1\left(\frac{2D}{h^2} + k\right) - \frac{Du_2}{h^2} = \frac{Du_0}{h^2}$$

$$i = 2, \frac{-Du_1}{h^2} + u_2\left(\frac{2D}{h^2} + k\right) - \frac{Du_3}{h^2} = 0$$

$$i = 3, \frac{-Du_2}{h^2} + u_3\left(\frac{2D}{h^2} + k\right) - \frac{Du_4}{h^2} = 0$$

$$i = 3, \frac{-Du_3}{h^2} + u_4\left(\frac{2D}{h^2} + k\right) = \frac{Du_f}{h^2}$$

$$\begin{bmatrix} \frac{2D}{h^2} + k & -\frac{D}{h^2} & 0 & 0 \\ -\frac{D}{h^2} & \frac{2D}{h^2} + k & -\frac{D}{h^2} & 0 \\ 0 & -\frac{D}{h^2} & \frac{2D}{h^2} + k & -\frac{D}{h^2} \\ 0 & 0 & -\frac{D}{h^2} & \frac{2D}{h^2} + k \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \begin{bmatrix} u0\frac{D}{h^2} \\ 0 \\ 0 \\ uf\frac{D}{h^2} \end{bmatrix}$$

(b) Solve in matlab using finite differences with h=20

```matlab
%% Finite Difference for BVPs
clear all; close all
L=100;
k=5;
D=1.5;
u0 = 0.1;
uf = 1;
h = 20;

a1 = 2*D/h^2+k;
a2 = 0;
c = D/h^2;
A = [a1 -c 0 0;
     -c a1 -c 0;
     0 -c a1 -c;
     0 0 -c a1];
b = [a2+u0*D/h^2 a2 a2 a2+uf*D/h^2]';

uin = A\b;
u = [u0 uin' uf];
x = 0:h:L;
plot(x,u)
```

(c) Repeat with h=2 (or as small as matlab will permit)

```matlab
%% iii
% solve the same with different h
L=100;
k=5;
D=1.5;
u0 = 0.1;
uf = 1;
h = 2;

a1 = 2*D/h^2+k;
a2 = 0;
c = D/h^2;

m_size = L/h-1;
A = diag(a1*ones(1,m_size)) + diag( ...
    -c*ones(1,m_size-1),1) + diag( ...
    -c*ones(1,m_size-1),-1);
```

```
b = a2+zeros(m_size,1);
b(1) = b(1)+u0*D/h^2;
b(end) = b(end)+uf*D/h^2;

uin = A\b;
u = [u0 uin' uf];
x = 0:h:L;
plot(x,u)
```