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FOUR SPECIAL MATRICES + DIFFERENCES, DERIVATIVES + BCS

INSTRUCTIONS: You may work with your classmates but your write-up must be your own. Your write up should be clear and easy to follow with the full problem statement at the beginning of each problem (if not given). Be prepared to be present your work at the start of our next class period.

§1.1: PROBLEM 22: Use *LiveScript* in MATLAB for this problem. With $n = 1000$, $e =$ `ones`$(n, 1)$; $K =$ `spdiags`$([-e, 2 * e, -e], -1 : 1, n, n)$; to enter K_{1000} as a sparse matrix. Solve the sparse equation $Ku = e$ by using the backslash operator in MATLAB: $u = K \setminus e$. Plot the using using `plot`(u). Label the axes and give the plot a title using the `xlabel`, `ylabel`, and `title` commands.

At the end

§1.2: PROBLEM 3: The h^2 term in the error for a centered difference $\frac{u(x+h)-u(x-h)}{2h}$ is $\frac{1}{6}h^2u'''(x)$. Test by computing that difference for $u(x) = x^3$ and $u(x) = x^4$.

first, let's start with $u(x) = x^3$, then we have

$$u(x+h) = (x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$
$$u(x-h) = (x-h)^3 = x^3 - 3x^2h + 3xh^2 - h^3$$

and plugging to the centered difference quotient gives

$$\frac{u(x+h)-u(x-h)}{2h} = \frac{\cancel{x^3} + 3x^2h + \cancel{3xh^2} + h^3 - (\cancel{x^3} - 3x^2h + \cancel{3xh^2} - h^3)}{2h}$$

$$= \frac{6x^2h + 2h^3}{2h} = 3x^2 + h^2$$

which shows that the error is equal to h^2 since

$u'(x) = 3x^2$. Now computing $\frac{1}{6}h^2 u'''(x)$ gives

$u''(x) = 6x$, $u'''(x) = 6$, then $\frac{1}{6}h^2 \cdot 6 = h^2$

therefore $\frac{1}{6}h^2 u'''(x)$ is the error by computing the derivative with centered difference for $u(x) = x^3$

Now let's repeat with $u(x) = x^4$

$u(x+h) = (x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

$u(x-h) = (x-h)^4 = x^4 - 4x^3h + 6x^2h^2 - 4xh^3 + h^4$

and plugging to the centered difference quotient gives

$$\frac{u(x+h) - u(x-h)}{2h} = \frac{\cancel{x^4} + 4x^3h + \cancel{6x^2h^2} + 4xh^3 + h^4 - (\cancel{x^4} - 4x^3h + \cancel{6x^2h^2} - 4xh^3 + h^4)}{2h}$$

$$= \frac{8x^3h + 8xh^3}{2h} = 4x^3 + 4xh^2$$

which shows that the error is equal to $4xh^2$ since

$u'(x) = 4x^3$. Now computing $\frac{1}{6}h^2 u'''(x)$ gives

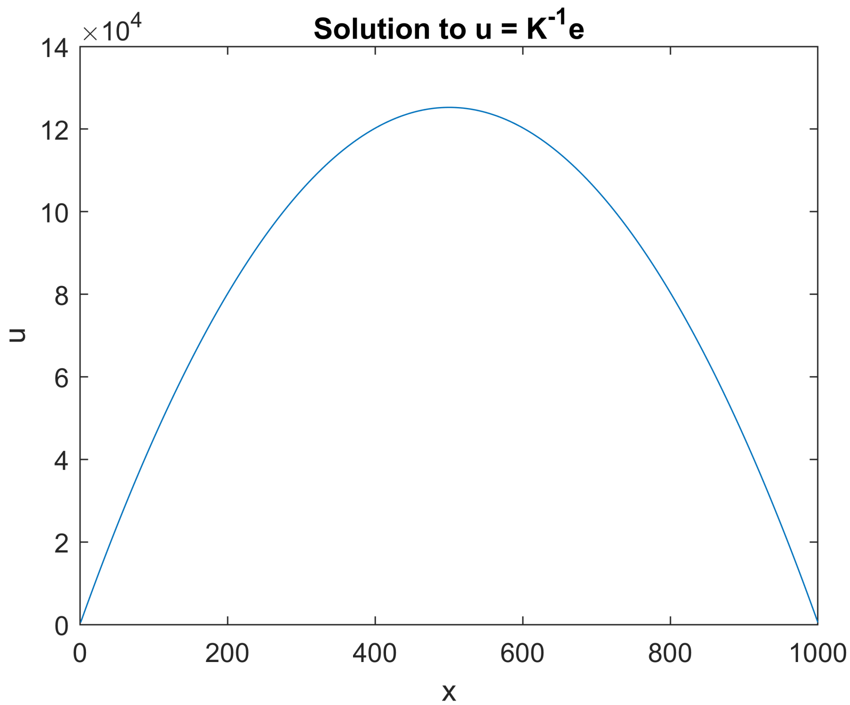
$u''(x) = 12x^2$, $u'''(x) = 24x$, then $\frac{1}{6}h^2(24x) = 4xh^2$

therefore $\frac{1}{6}h^2 u'''(x)$ is the error by computing the derivative with centered difference for $u(x) = x^4$

1.1 Problem 22:

Using the commands given in the problem and labeling the axes gives

```
% 1.1 Problem 22
n = 1000;
e = ones(n,1);
K = spdiags([-e,2*e,-e],-1:1,n,n);
u = K\e;
plot(u)
xlabel("x")
ylabel("u")
title("Solution to u = K^{-1}e")
```



As we can see, the graph shows up with the solution and alterations made.