Homework 3

MSSC 6010- Computational Probability

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Question 1. (3.6.39) From book. Let the random variable X be the sum of the numbers on two fair dice. Find an upper bound on $\mathbb{P}(|X-7| \geq 4|)$ using Chebyshev's Inequality as well as the exact probability for $\mathbb{P}(|X-7| \geq 4|)$.

```r
X = c(2,3,4,5,6,7,8,9,10,11,12)
X = rbind(X,c(1/36,2/36,3/36,4/36,5/36,6/36,5/36,4/36,3/36,2/36,1/36))
table(X)
```

```
X
0.0277777777777778 0.0555555555555556 0.0833333333333333  0.111111111111111
                 2                  2                  2                  2
 0.138888888888889  0.166666666666667                  2                  3
                 2                  1                  1                  1
                 4                  5                  6                  7
                 1                  1                  1                  1
                 8                  9                 10                 11
                 1                  1                  1                  1
                12
                 1
```

Question 2. (3.6.48) From book. Consider the random variable X, which takes the values $1, 2, 3,$ and 4 with probabilities $0.2, 0.3, 0.1,$ and 0.4, respectively. Calculate $E[X]$, $\frac{1}{E[X]}$,$E\left[\frac{1}{X}\right]$,$E[X^2]$, and $E[X]^2$, and check empirically that $E[X]^2 \neq E[X^2]$ and $\frac{1}{E[X]} \neq E\left[\frac{1}{X}\right]$

Question 3. (3.6.50) From book. Find the values of k such that the following functions are probability density functions.

- *(a)* $f(x) = \frac{kx^4}{5}, 0 < x < 1$

Question 4. (3.6.52) From book. Consider an experiment where two dice are rolled. Let the random variable X equal the sum of the two dice and the random variable Y be the difference of the two dice.

- *(a)* Find the mean of X
- *(b)* Find the variance of X
- *(c)* Find the skewness of X
- *(d)* Find the mean of Y
- *(e)* Find the variance of Y
- *(f)* Find the skewness of Y

Question 5. (3.6.58) From book. Consider the probability density function

$$f(x) = \frac{1}{36} x e^{\frac{-x}{6}}, x > 0$$

Derive the moment generating function, and calculate the mean and the variance.

Question 6. (3.6.60) From book. Prove that if a and b are real-valued constants, then

- *(1)*$M_{X+a}(t) = E[e^{(X+a)t}] = e^{at} M_X(t)$.
- *(2)*$M_{bX}(t) = E[e^{bXt}] = M_X(bt)$.
- *(3)*$M_{\frac{X+a}{b}}(t) = E[e^{(\frac{X+a}{b}t)}] = e^{\frac{a}{b}t} M_X(\frac{t}{b})$.