Activity 1

The first 3 problems relate to the differential equation dx/dt = 1/(x^2) with the initial condition x(0) = 1.

1. i) Solve the differential equation using separation of variables. ii) Use the initial condition to find the constant of integration, c. iii) Check your solution by showing that the LHS of the differential equation equals the RHS and that the solution satisfies the initial condition.
2. Apply Euler's method with step size deltat = 0.5. i) Calculate x(0.5) and x(1) by hand using Euler's method with stepsize deltat = 0.5. ii.) Calculate x(0.2) ... x(1) by hand using Euler's method with stepsize deltat = 0.2.
3. Using the analytic solution from 1. and your approximations from 2. compute the error at each time point.
4. Consider the pred-prey system of ODEs presented in class. Modify the model development for the following situation. Stock a pond with two species of fish: trout and bass that compete for the same resources. Both populations increase in the absence of the other since we assume there are unlimited resources. However, they are now on the same level on the food chain so that interactions, reduced their viability. i) Construct an ODE model of their population growth. Define the variables, state the assumptions and determine the system of ODEs describing this scenario. ii.) Then determine if the system has an equilibrium points. Describe the meaning and significance of those points.

$$\frac{dx}{dt} = 1/x^2 \quad , \quad x(0) = 1$$

1-) i-) $x^2\,dx = dt$

$$\int x^2\,dx = \int dt + C$$

$$\frac{x^3}{3} = t + c$$

$$x = (3t + 3c)^{1/3}$$

ii-) $x(0) = 1 \Rightarrow 1 = \sqrt[3]{0 + 3c}$

$$c = 1/3 \qquad \Rightarrow \qquad x = (3t + 1)^{1/3}$$

iii-) LHS: $\frac{dx}{dt} = 1/3\,(3t+1)^{-2/3}(3)$

$$= \frac{1}{(3t+1)^{2/3}}$$

RHS: $\frac{1}{x^2} = \frac{1}{((3t+1)^{1/3})^2} = \frac{1}{(3t+1)^{2/3}}$

$$x(0) = \frac{1}{(3(0)+1)^{2/3}} = 1$$

2-) Euler's Method

$$x_{i+1} = x_i + f(t_i, x_i)\,\Delta t$$

i-) $x(0.5) = x(0) + f(0, x(0))\,0.5$

$$= 1 + 1/(1^2) \cdot 0.5$$

$$= 1.5$$

$x(1) = x(0.5) + f(0.5, x(0.5))\,0.5$

$$= 1.5 + \frac{1}{(1.5)^2} \cdot 0.5 = 1.7222$$

ii.) $x(0.2) = x(0) + \frac{1}{x(0)^2} \cdot 0.2$

$= 1 + 0.2 = 1.2$

$x(0.4) = 1.2 + \frac{1}{1.2^2} \cdot 0.2 = 1.3388$

$x(0.6) = 1.3388 + \frac{1}{1.3388^2} \cdot 0.2 = 1.4504$

$x(0.8) = 1.4504 + 1/(1.4504)^2 \cdot 0.2 = 1.5455$

$x(1) = 1.5455 + 1/(1.5455)^2 \cdot 0.2 = 1.6292$

3.) Analytic solutions $x(0.2) = 1.1696$ $x(0.5) = 1.3572$

$x(0.4) = 1.3006$

$x(0.6) = 1.4095$

$x(0.8) = 1.5037$

$x(1) = 1.5874$

errors

$x(0.2) = |1.2 - 1.1696| = 0.0304$

$x(0.4) = |1.3388 - 1.3006| = 0.0382$

$x(0.6) = |1.4504 - 1.4095| = 0.0409$

$x(0.8) = |1.5455 - 1.5037| = 0.0418$

$x(1) = |1.6292 - 1.5874| = 0.0418$

$x(0.5) = |1.5 - 1.3572| = 0.1428$

$x(1) = |1.7222 - 1.5874| = 0.1348$

4.)

trout - bass pond

let

$x(t)$ = trout population at time t

$y(t)$ = bass population at time t

Isolation:

$\frac{dt}{dt}$ = rate of change (Δ) of $x \propto x \Rightarrow \frac{dt}{dt} = ax$, $a > 0$

$\frac{dy}{dt}$ = rate of change (Δ) of $y \propto y \Rightarrow \frac{dy}{dt} = my$, $m > 0$

<u>Interaction</u>: $\frac{dx}{dt} = -bxy$ $\frac{dy}{dt} = -nxy$

$\Rightarrow$ $\frac{dx}{dt} = ax - bxy$ $\frac{dy}{dt} = my - nxy$

$x(0) = x_0$ $y(0) = y_0$

Equilibrium points at 0 or anywhere where the derivative is 0, such as
$y = a/b$, $x = m/n$

this means that at those equilibrium points, both species would coexist.