# Math 4540/MSSC 5540 - Activity #3

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1. 1.5.1b). Apply two steps of the Secant Method to the following equations with initial guesses $x_0 = 1$, and $x_1 = 2$. (b) $e^x + x = 7$

$$f(x) = e^x + x - 7 = 0$$
$$i = 1, x_0 = 1, x_1 = 2$$

$$\begin{aligned} x_2 &= x_0 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} \\ &= 2 - \frac{(e^2 + 2 - 7)(2 - 1)}{(e^2 + 2 - 7) - (e^1 + 1 - 7)} \\ &\approx 1.5787 \end{aligned}$$

$$i = 2, x_1 = 2, x_2 = 1.5787$$

$$\begin{aligned} x_3 &= 1.5787 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)} \\ &= 1.5787 - \frac{(e^{1.5787} + 1.5787 - 7)(1.5787 - 2)}{(e^{1.5787} + 1.5787 - 7) - (e^2 + 2 - 7)} \\ &\approx 1.6602 \end{aligned}$$

2. Computer Exercise 1.5.1b). Use the Secant Method to find the (single) solution to the equation in Exercise 1.

```
clear all;
close all;

f = @(x) exp(x)+x-7;
x(1) = 1;
x(2) = 2;
```

```
tol = 1e-8;
deltax = inf;
i = 2;
while deltax > tol
    x(i+1) = x(i)-(f(x(i))*(x(i)-x(i-1)))/(f(x(i))-f(x(i-1)
       ));
    deltax = abs(x(i+1)-x(i));
    i = i+1;
end

fprintf(['Algorithm converged after %d ' ...
    'iterations, and the root is %.4f. ' ...
    '\n f(root) = %.8f\n'] ,i,x(i),f(x(i)))

% Output:
% Algorithm converged after 8 iterations, and the root is
   1.6728.
% f(root) = 0.00000000
```

3. 1.5.2b) Apply two steps of the Method of False Position with initial bracket $[1, 2]$ to the equation of Exercise 1.

$$f(x) = e^x + x - 7 = 0$$
$$i = 1, a = 1, b = 2$$

$$\begin{aligned} c &= \frac{bf(a) - af(b)}{f(a) - f(b)} \\ &= \frac{2(e^1 + 1 - 7) - 1(e^2 + 2 - 7)}{(e^1 + 1 - 7) - (e^2 + 2 - 7)} \\ &\approx 1.5787 \\ \Rightarrow f(a)f(c) &> 0 \\ \Rightarrow a &= 1.5787 \end{aligned}$$

$$i = 1, a = 1.5787, b = 2$$

$$
\begin{aligned}
c &= \frac{bf(a) - af(b)}{f(a) - f(b)} \\
&= \frac{2(e^{1.5787} + 1.5787 - 7) - 1.5787(e^2 + 2 - 7)}{(e^{1.5787} + 1.5787 - 7) - (e^2 + 2 - 7)} \\
&\approx 1.6602 \\
\Rightarrow f(a)f(c) &> 0 \\
\Rightarrow a &= 1.6602
\end{aligned}
$$

4. Computer Exercise 1.5.2b) Use Method of False Position to find the solution of each equation in Exercise 1.

```
clear all;
close all;

f = @(x) exp(x)+x-7;

a = 1;
b = 2;
x(1) = b;
tol = 1e-8;
deltax = inf;
i = 1;
while deltax > tol
    c = (b*f(a)-a*f(b))/(f(a)-f(b));
    if c==0
        x(i+1) = c;
        break;
    end
    if f(a)*f(c) < 0
        b = c;
        x(i+1) = b;
    else
        a = c;
        x(i+1) = a;
    end
    deltax = abs(x(i+1)-x(i));
    i = i+1;
end

fprintf(['Algorithm converged after %d ' ...
    'iterations, and the root is %.4f. ' ...
```

```
    '\n f(root) = %.8f\n'] ,i,x(i),f(x(i)))

% Output:
% Algorithm converged after 11 iterations, and the root is
   1.6728.
% f(root) = -0.00000001
```

5. Newton's Method 1.5.1b) Two steps by hand.

$$f(x) = e^x + x - 7 = 0, f'(x) = e^x + 1$$
$$i = 0, x_0 = 1$$

$$\begin{aligned} x_{i+1} &= x_i - \frac{f(x_i)}{f'(x_i)} \\ &= 1 - (e^1 + 1 - 7)e^1 + 1 \\ &= 1.8826 \end{aligned}$$

$$i = 1, x_1 = 1.8826$$

$$\begin{aligned} x_{i+1} &= x_i - \frac{f(x_i)}{f'(x_i)} \\ &= 1.8826 - \frac{(e^{1.8826} + 1.8826 - 7)}{e^{1.8826} + 1} \\ &= 1.6907 \end{aligned}$$

6. Newton's Method 1.5.1b) MATLAB code.

```
clear all;
close all;

f = @(x) exp(x)+x-7;
df = @(x) exp(x)+1;

x(1) = 1;

tol = 1e-8;
deltax = inf;
i = 1;
while deltax > tol
```

```matlab
    x(i+1) = x(i)-f(x(i))/df(x(i));
    deltax = abs(x(i+1)-x(i));
    i = i+1;
end

fprintf(['Algorithm converged after %d ' ...
    'iterations, and the root is %.4f. ' ...
    '\n f(root) = %.8f\n'] ,i,x(i),f(x(i)))

% Output:
% Algorithm converged after 6 iterations, and the root is
   1.6728.
% f(root) = -0.00000000
```