Math 4540/MSSC 5540 - Activity #11

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1. A more realistic ODE describing heat transfer in the rod, that also accounts for radiative transfer, is

$$u'' + w(T_{air} - u) + \sigma(T_{air}^4 - u^4) = 0$$

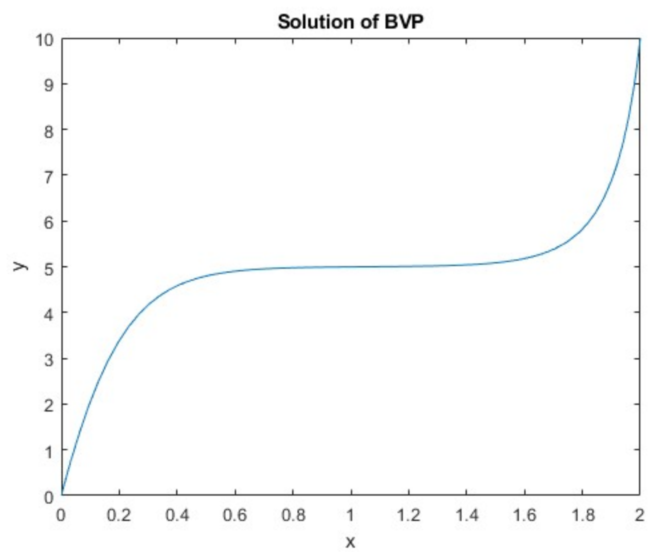
Note that this is a nonlinear ODE. Modify my matlab code to solve this BVP with the same parameters and BCs. Use σ= 0.1. Plot the resulting solution.

```
%%Shooting Method for Nonlinear BVP ODEs
clear all; close all
global y0 yL L Tair w sigma
y0 = 0; yL = 10; L = 2; Tair = 5; w = 4; sigma = 0.1;

%Before running, determine values of ydot that yield
%one + and one - residual (R= pred y(L) - y(L)).
%These values are the endpoints of the search interval.
%fzero will determine the root on that search interval
% u'_opt
ydotopt = fzero(@F, [1 25]) ;
%Value of y' that will hit the BC

%Solve the IVP using ydotopt
% u'
% Set up the system of ODEs
ydot = @(t,y) [y(2);
-w*(Tair - y(1))-sigma*(Tair.^4-y(1).^4)];
 %Set the BCs
tspan = [0 L];
ICs = [y0 ydotopt];
[t,y] = ode45(ydot, tspan, ICs);                %Solve the IVP
plot(t,y(:,1))
xlabel('x');
ylabel('y');title('Solution of BVP')
```

```
%% Residual function. fzero finds root of fn R
function R = F(s)
global y0 yL L Tair w sigma
% THIS GIVES RESIDUAL WITH DIFFERENT S VALUES
    ydot = @(t,y) [y(2);
    -w*(Tair - y(1))-sigma*(Tair.^4-y(1).^4)];
    tspan = [0 L]; ICs = [y0 s];
    [t,y] = ode45(ydot, tspan, ICs);
    R = y(end,1) - yL;
end
```



2. Consider constructing the analytic solution to the BVP

$$u'' = -wu, \; u(0) = 0, \; u(\pi/4) = 10.$$

We'll use the guessing method as done in class.

i.) Start with w = 1. What function(s) u satisfies the DE? How many solutions must there be?

$$w = 1 \Rightarrow u'' = -u \Rightarrow u = sin(x), cos(x)$$

So general solution is

$$u = c_1 sin(x) + c_2 cos(x)$$

ii.) Now solve the ODE with w = 4. Recall that the general solution is the linear combination of two linearly independent solutions.

$$w = 1 \Rightarrow u'' = -4u \Rightarrow u = sin(2x), cos(2x)$$

So general solution is

$$u = c_1 sin(2x) + c_2 cos(2x)$$

iii.) Impose the boundary conditions to determine the solutions to the BVP. for part i:

$$u(0) = 0 \Rightarrow c_2 = 0$$

$$u(\pi/4) = \sqrt{2}/2c_1 = 10 \Rightarrow c_1 = 20/\sqrt{2}$$

Therefore, the solution is

$$u = 20/\sqrt{2}sin(x)$$

for part ii:

$$u(0) = 0 \Rightarrow c_2 = 0$$

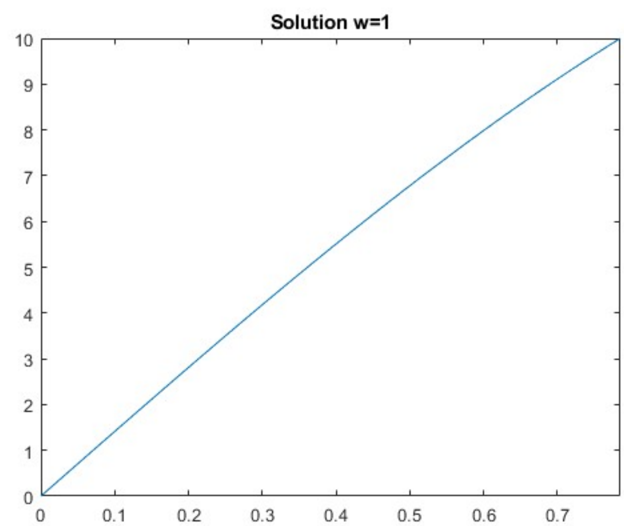
$$u(\pi/4) = c_1 = 10$$

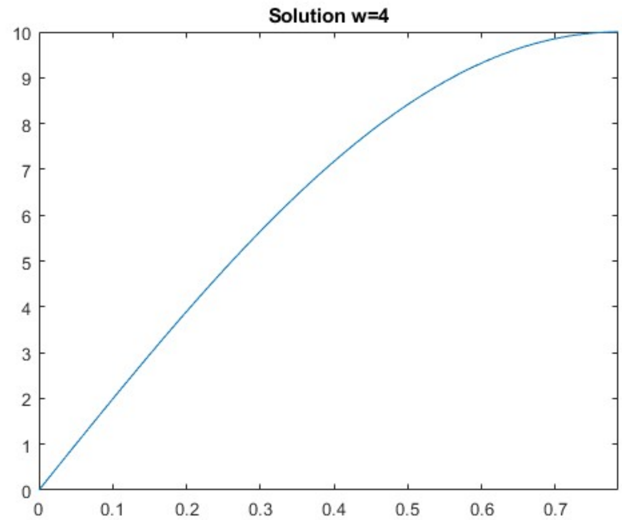
Therefore, the solution is

$$u = 10sin(2x)$$

iv.) Use matlab to plot the solutions.

```
figure
fplot(@(x) 10*sin(2*x), [0 pi/4])
figure
fplot(@(x) 20/sqrt(2)*sin(x),[0,pi/4])
```



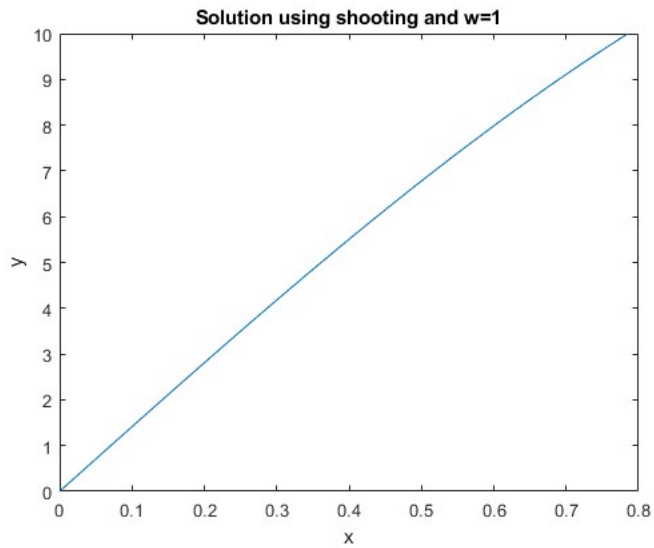


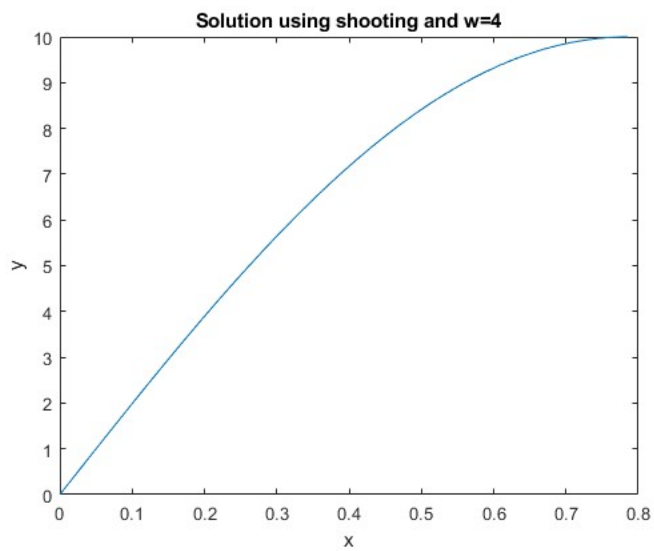
v.) Now solve the problem numerically using the shooting method. Plot the solutions and compare to iv).

```
%% Shooting Method for Nonlinear BVP ODEs
clear all; close all
global y0 yL L Tair w
y0 = 0; yL = 10; L = pi/4; Tair = 0; w = 1;

ydotopt = fzero(@F, [10 21]);
ydot = @(t, y) [y(2); -w *y(1)];
tspan = [0 L]; ICs = [y0 ydotopt];
[t, y] = ode45(ydot, tspan, ICs);
plot(t, y(:, 1))
xlabel('x'); ylabel('y');
title('Solution using shooting and w=1')

%% Residual function. fzero finds the root of fn R
function R = F(s)
global y0 yL L Tair w
ydot = @(t, y) [y(2); -w *y(1)];
tspan = [0 L]; ICs = [y0 s];
[t, y] = ode45(ydot, tspan, ICs);
R = y(end, 1) - yL;
end
```





Solutions look the same graphic wise, analysing the differences between those two numerically shows a little bit of a difference, but basically the same.

3. Students taking the course for graduate credit only:

   i) Consider the original heat equation (σ=0) with fixed temperature at x = L (Dirichlet condition) but with a "free" end at x = 0. This means that since we are considering steady-state (long time equilibrium), convection must equal conduction at the x = 0 end of the rod. Set up the heat balance there to determine the boundary condition at x = 0.

$$u'' + w(t_{air} - u) = 0, u''(L) = 0$$

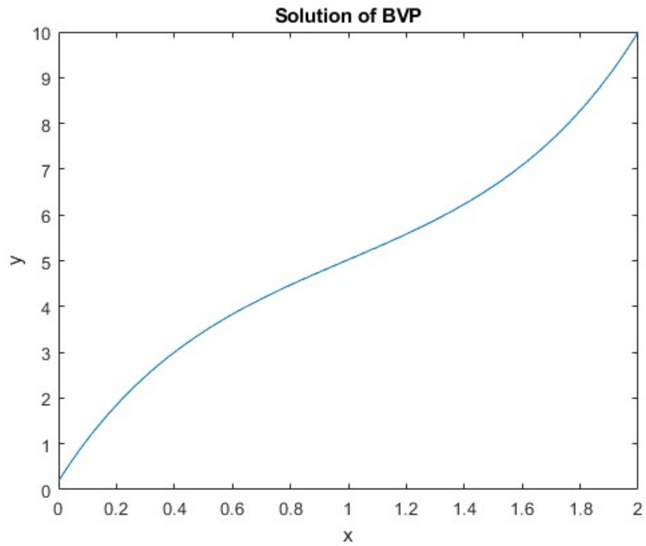
This steady state means a linear function of the form $u'(0) = w(t_{air} - u(0))$ at the end.

ii) Solve this new BVP using the shooting method. Plot the solution

```
%%Shooting Method for Nonlinear BVP ODEs
clear all; close all
global y0 yL L Tair w sigma
y0 = 0; yL = 10; L = 2; Tair = 5; w = 4; sigma = 0;

ydotopt = fzero(@F, [-1 1]) ;                    %Value of y
   ' that will hit the BC

%Solve the IVP using ydotopt
% u'
ydot = @(t,y) [y(2);
    -w*(Tair - y(1))-sigma*(Tair.^4-y(1).^4)];
tspan = [0 L]; ICs = [ydotopt yL];
[t,y] = ode45(ydot, tspan, ICs);
plot(t,y(:,1))
xlabel('x'); ylabel('y');title('Solution of BVP')
%% Residual function. fzero finds root of fn R
function R = F(s)
global y0 yL L Tair w sigma%%
%y0 = 0; yL = 10; L = 2; Tair = 5; w = 4;
% THIS GIVES RESIDUAL WITH DIFFERENT S VALUES
ydot = @(t,y) [y(2);
    -w*(Tair - y(1))-sigma*(Tair.^4-y(1).^4)];
tspan = [0 L]; ICs = [s yL];
[t,y] = ode45(ydot, tspan, ICs);
R = y(end,1) - yL;
end
```



iii) Interpret the solution. Does it make sense?

It makes sense, it is not starting exactly at 0, like it was previously. And it is ending exactly at 10.