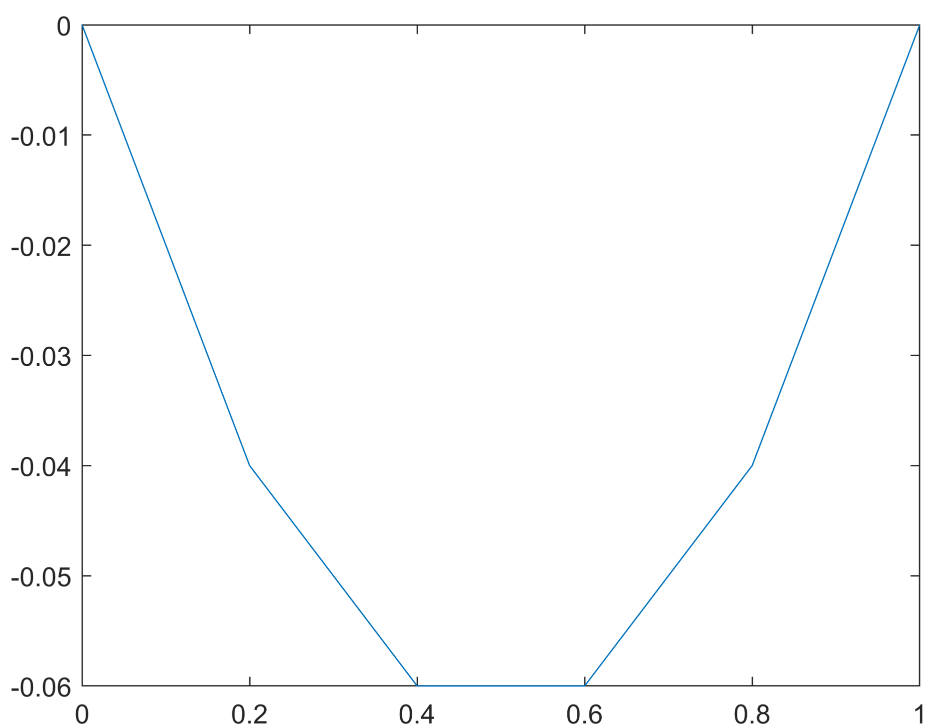
Strang 1.2 - Problem 18: Use *Livescript* in Matlab for this problem. Write a fininte difference approximation with $n = 4$ unknowns to

$$\frac{d^2u}{dx^2} = x, \text{with boundary conditions } u(0) = 0 \text{ and } u(1) = 0$$

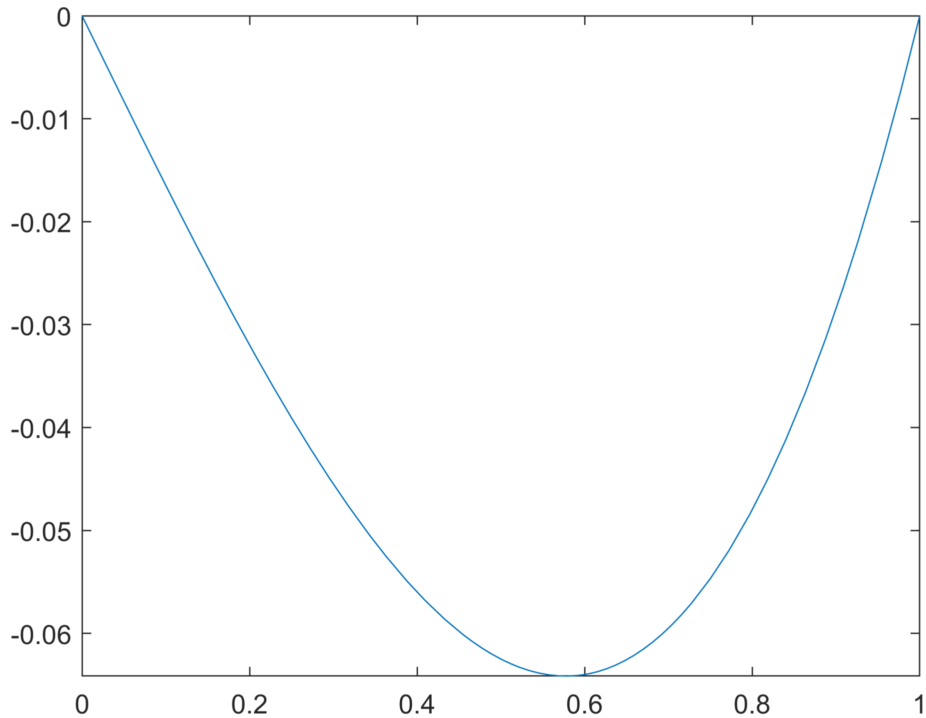
Solve for $u_1, u_2, u_3,$ and u_4. Compare them to the true solution (use Calculus). Decrease your stepsize $h = \Delta x$ to now have $n = 8$ unkowns. How does your solution change? How small need Δx be?

```
n = 4;
e = ones(n,1);
A = spdiags([e -2*e e],-1:1,n,n);
b = 1;
a = 0;
h = (b-a)/(n+1);

u0 = 0; uf = 0;
b_vec = 1/2*h.^2*ones(n,1);
b_vec(1) = b_vec(1)+u0;
b_vec(end) = b_vec(end)+uf;

u = A\b_vec;
u = [u0 u' uf];
x = 0:h:1;
plot(x,u)
```




```
f = @(x) 1/6*(x.^3-x);
```

```
fplot(f,[0 1])
```



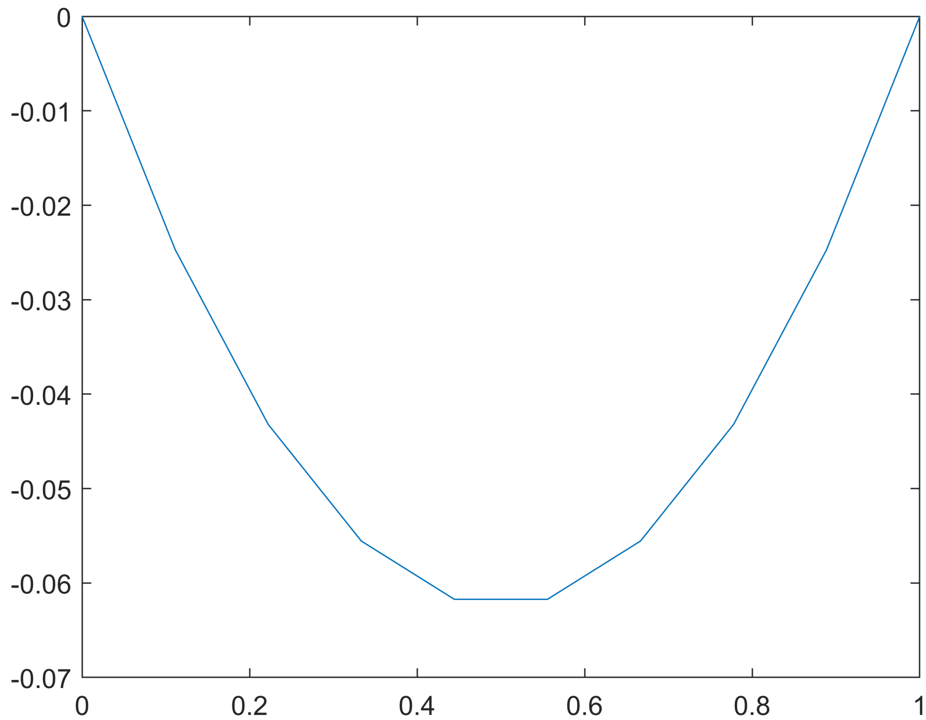
Besides comparing the graphs, we can look at the errors:

```
err = abs(f(u) - u)
```

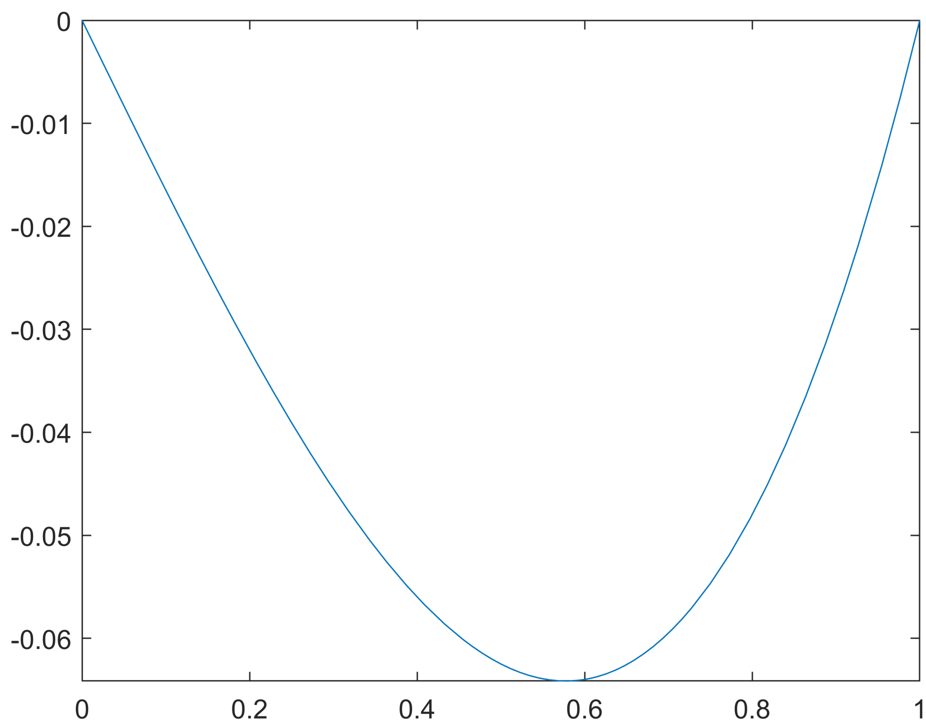
```
err = 1×6
         0    0.0467    0.0700    0.0700    0.0467         0
```

Then running everything again with $n = 8$ gives:

```
n = 8;
e = ones(n,1);
A = spdiags([e -2*e e],-1:1,n,n);
b = 1;
a = 0;
h = (b-a)/(n+1);

u0 = 0; uf = 0;
b_vec = 1/2*h.^2*ones(n,1); % not so sure why its 1/2h^2
b_vec(1) = b_vec(1)+u0;
b_vec(end) = b_vec(end)+uf;

u = A\b_vec;
u = [u0 u' uf];
x = 0:h:1;
plot(x,u)
```




```
f = @(x) 1/6*(x.^3-x);
fplot(f,[0 1])
```



Besides comparing the graphs, we can look at the errors:

```
err = abs(f(u) - u);

err(1:5)
```

```
ans = 1×5
         0    0.0288    0.0504    0.0648    0.0720
```

```
err(6:end)
```

```
ans = 1×5
    0.0720    0.0648    0.0504    0.0288         0
```