Math 4540/MSSC 5540 - Activity #6

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1. Exercise 5.5.4 c). Change variables using Substitution (5.46) to rewrite an integral over $[-1, 1]$

$$(c) \int_0^1 xe^x dx$$

From 5.46

$$\int_a^b f(x)dx = \int_{-1}^1 f\left(\frac{(b-a)t+b+a}{2}\right)(\frac{b-a}{2})dt$$

Then

$$\begin{aligned}\int_0^1 xe^x dx &= \int_{-1}^1 f\left(\frac{(1-0)t+1+0}{2}\right)(\frac{1-0}{2})dt \\ &= \int_{-1}^1 f\left(\frac{t+1}{2}\right)\frac{1}{2}dt \\ &= \int_{-1}^1 \frac{t+1}{2}e^{\frac{t+1}{2}}\frac{1}{2}dt\end{aligned}$$

2. Exercise 5.4.5 c). Approximate the integrals in Exercise 4, using $n = 3$ Gaussian Quadrature. Do this by writing a short matlab code. Include your code.

```matlab
clear all;
close all;
%%
f = @(x) (x+1)./2.*exp((x+1)./2).*(1/2);
c_i_n3 = [5/9 8/9 5/9];
x_i_n3 = [-sqrt(3/5) 0 sqrt(3/5)];
approx = sum(c_i_n3.*f(x_i_n3));

fprintf('Integral is approximatelly = %.4f\n',approx)

%Output:
% Integral is approximatelly = 1.0000
```

3. Exercise 5.5.7. Show the Legendre polynomials $p_1(x) = x$ and $p_2(x) = x^2 - 1/3$ are orthogonal on $[-1, 1]$.

   In order to be orthogonal, we need

$$\int_a^b p_j(x)p_k(x)dx = \begin{cases} 0 & j \neq k \\ \neq 0 & j = k \end{cases}$$

   Then

$$\begin{aligned}
\int_{-1}^{1} p_1(x)p_2(x)dx &= \int_{-1}^{1} x(x^2 - \frac{1}{3})dx \\
&= \int_{-1}^{1} x^3 - \frac{x}{3}dx \\
&= (\frac{x^4}{4} - \frac{x^2}{6})\Big|_{-1}^{1} \\
&= (\frac{1}{4} - \frac{1}{6}) - (\frac{1}{4} - \frac{1}{6}) \\
&= 0
\end{aligned}$$