Math 4540/MSSC 5540 - Activity #8

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Consider the Lorenz system of differential equations. Use s=10, r=28, and b = 8/3

1. Determine whether the system of equations has any equilibrium points.

   In order to find the equilibrium points, we need to set all left hand sides of the equations to 0 and solve for what the variables should be.

$$\frac{dx}{dt} = -sx + sy, \frac{dy}{dt} = -xz + rx, \frac{dz}{dt} = xy - bz$$

   Need $x = y$

$$\frac{dz}{dt} = 0 = x^2 - bz \Rightarrow z = x^2/b$$

$$\frac{dy}{dt} = 0 = -\frac{x^3}{b} + rx - x$$

$$x = +- \sqrt{b(r-1)} = y$$

$$z = \frac{+- b(r-1)}{b}$$

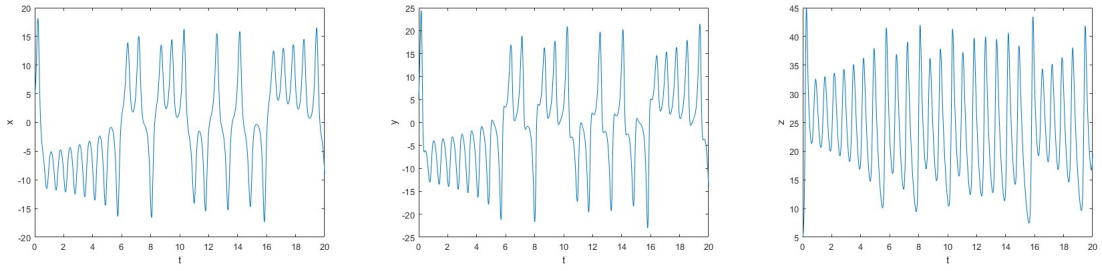
2. Construct matlab code to solve the system on $t = [0, 20]$ using ode45. Check that your code is working correctly by using initial conditions = equilibrium points.

```
%% RK45 ODE solver
clear all; close all;
global s r b
tspan = [0 20];
%parameters
s = 10;r=28;b=8/3;
x_init = -sqrt(b*(r-1));
y_init = x_init;
z_init = r-1;
vars = [x_init,y_init,z_init];
```

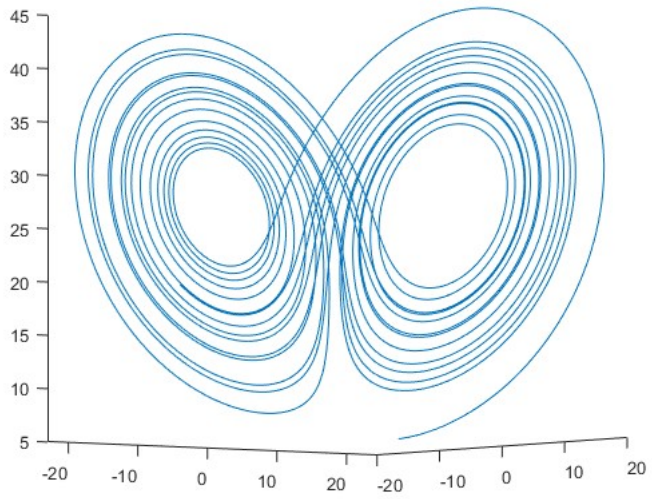
```
options = odeset('RelTol',1e-8,'AbsTol',1e-8);
[t,Y] = ode45(@(t,y) lorenz(t,y),tspan,vars,options);

function ddt = lorenz(t,Y)
global s r b
    x = Y(1);
    y = Y(2);
    z = Y(3);
    dxdt = -s*x+s*y;
    dydt = -x*z+r*x-y;
    dzdy = x*y-b*z;
    ddt = [dxdt,;dydt;dzdy];
end
```

3. Now use x(0) = y(0) = z(0) = 5. Generate plots of (t,x),(t,y), (t,z). Label the axes.

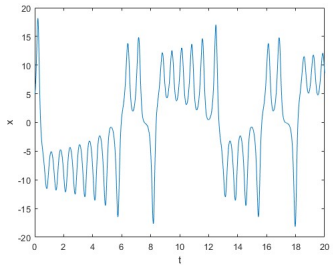


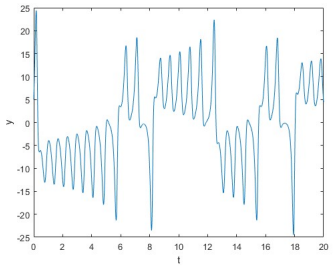
4. Plot the phase portrait of the system. Suppose [t,Y] is the output from ode45. Use plot3(Y(:,1),Y(:,2),Y(:,3)) to generate the plot.

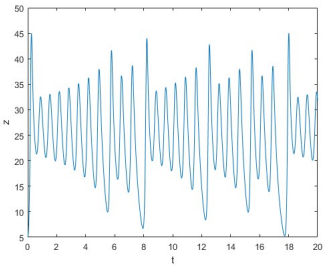


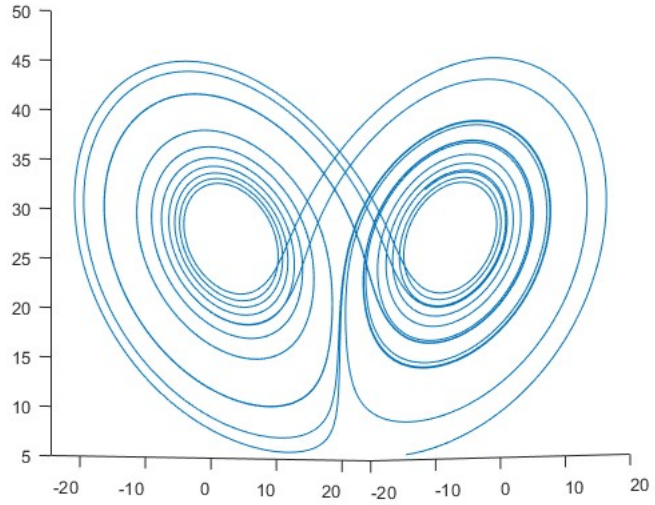
5. Repeat 3. and 4. using an initial value of 5.01,5.001,5.0001,5.00001.

5.01:

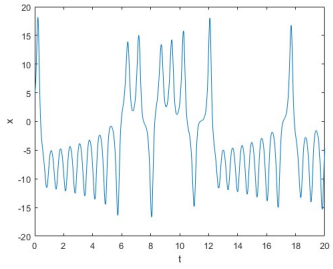


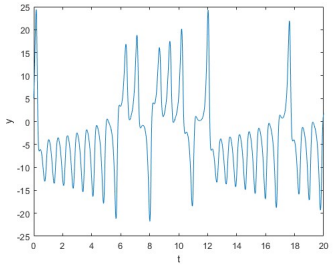


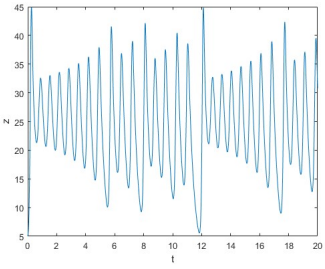


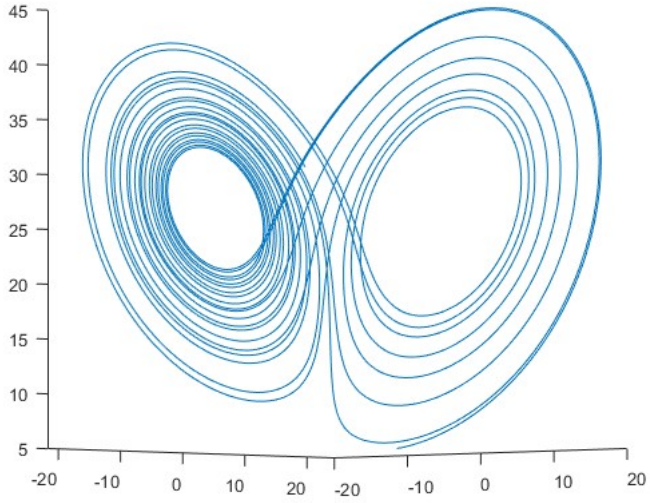


5.001:

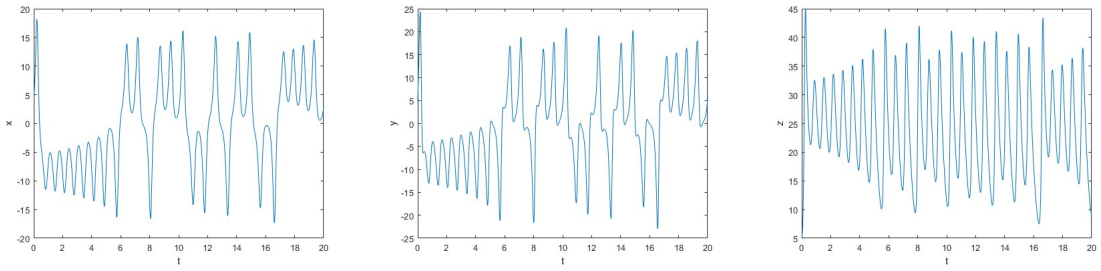


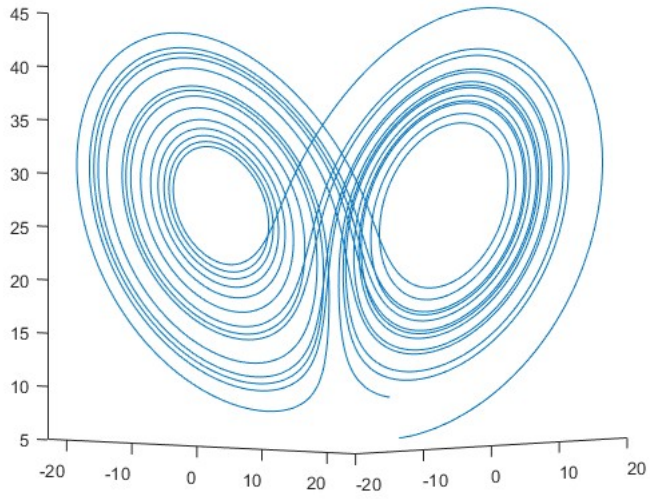




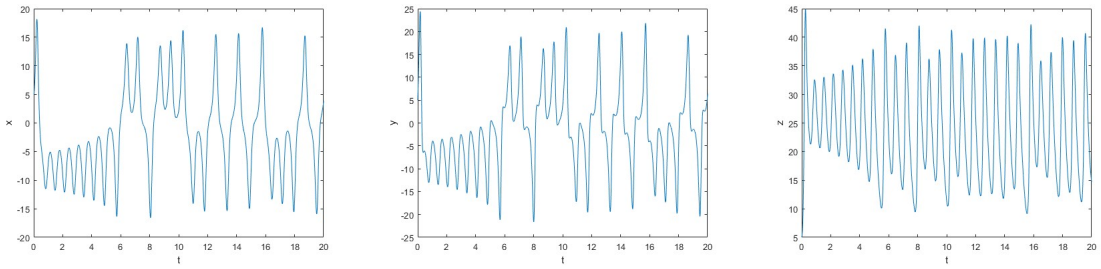


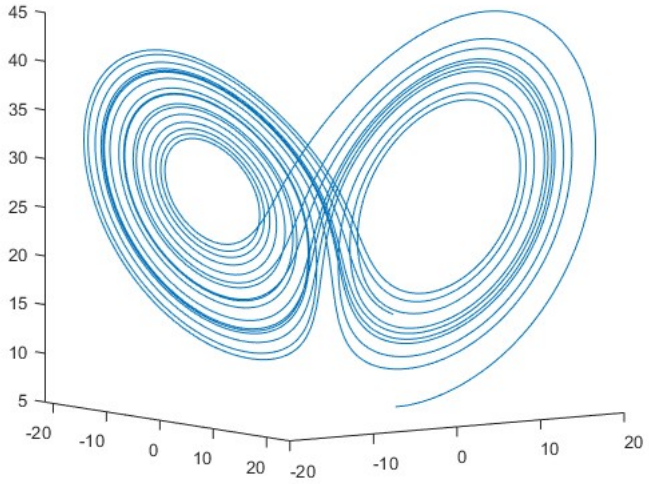
5.0001:





5.00001:





6. Describe the behavior you are seeing. Do you think the behavior is because of numerical error or dynamics of the system?What conclusions can you draw about weather prediction?

   Any small change to the initial values changes the output by a lot, solutions are unstable. There is not a smooth transition between input and output. I believe this is due to the system, and it implies that weather prediction is only reliable for small values of t. Since we can see the system getting chaotic after t around 12