



# Inductive Representation Learning on Large Graphs

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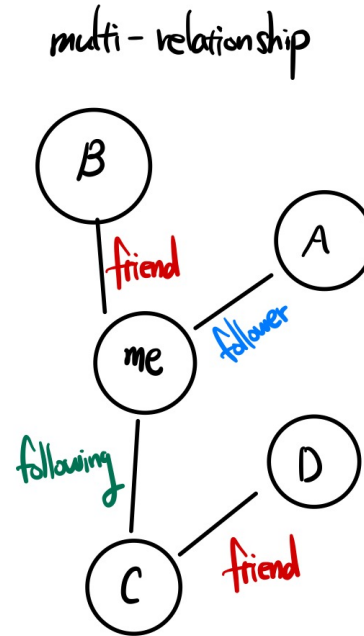
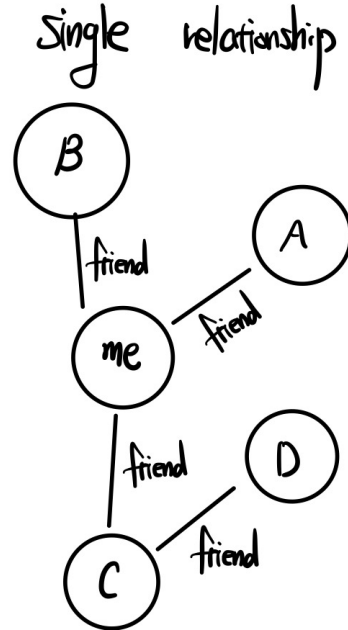
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# Previous work

*In Social Network*



## Previous work

표현력, 보편성 증가를 목표

*stochastic blockmodel*

*models based on tensor factorization*

*collective matrix factorization*



모델 복잡성 증가

높은 계산 비용

해석의 어려움

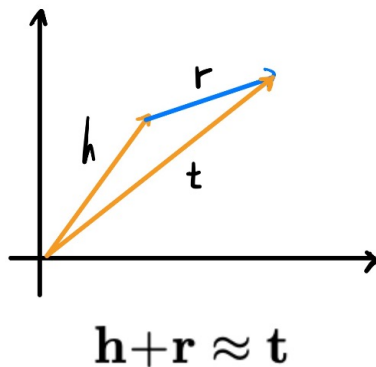
정규화 어려움

과적합 or 과소적합 문제 발생



성능은 비슷하게, 그러나 간단하게

# TransE



$$\mathcal{L} = \sum_{(h, \ell, t) \in S} \sum_{(h', \ell, t') \in S'_{(h, \ell, t)}} [\gamma + d(\mathbf{h} + \ell, \mathbf{t}) - d(\mathbf{h}' + \ell, \mathbf{t}')] ]$$

$$S'_{(h, \ell, t)} = \{(h', \ell, t) | h' \in E\} \cup \{(h, \ell, t') | t' \in E\}$$

Head와 Relation을 더했을 때, 최대한 Tail과 가깝도록

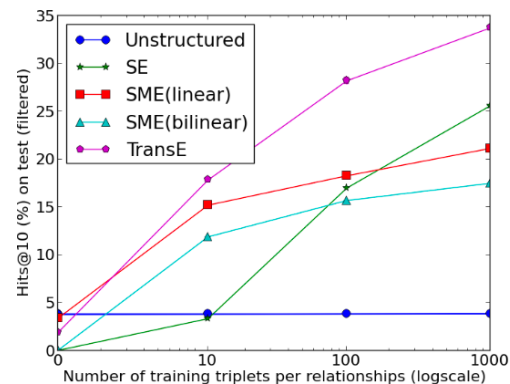
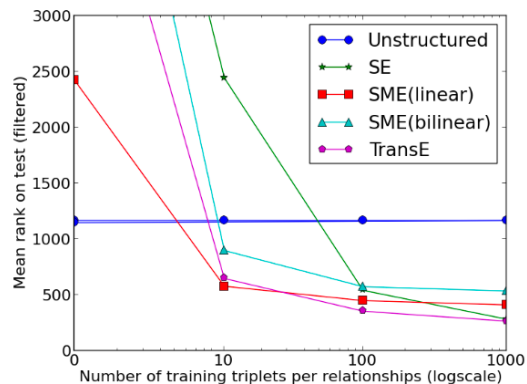
# TransE

Table 1: **Numbers of parameters** and their values for FB15k (in millions).  $n_e$  and  $n_r$  are the nb. of entities and relationships;  $k$  the embeddings dimension.

METHOD	NB. OF PARAMETERS	ON FB15K
Unstructured [2]	$O(n_e k)$	0.75
RESCAL [11]	$O(n_e k + n_r k^2)$	87.80
SE [3]	$O(n_e k + 2n_r k^2)$	7.47
SME(LINEAR) [2]	$O(n_e k + n_r k + 4k^2)$	0.82
SME(BILINEAR) [2]	$O(n_e k + n_r k + 2k^3)$	1.06
LFM [6]	$O(n_e k + n_r k + 10k^2)$	0.84
TransE	$O(n_e k + n_r k)$	0.81

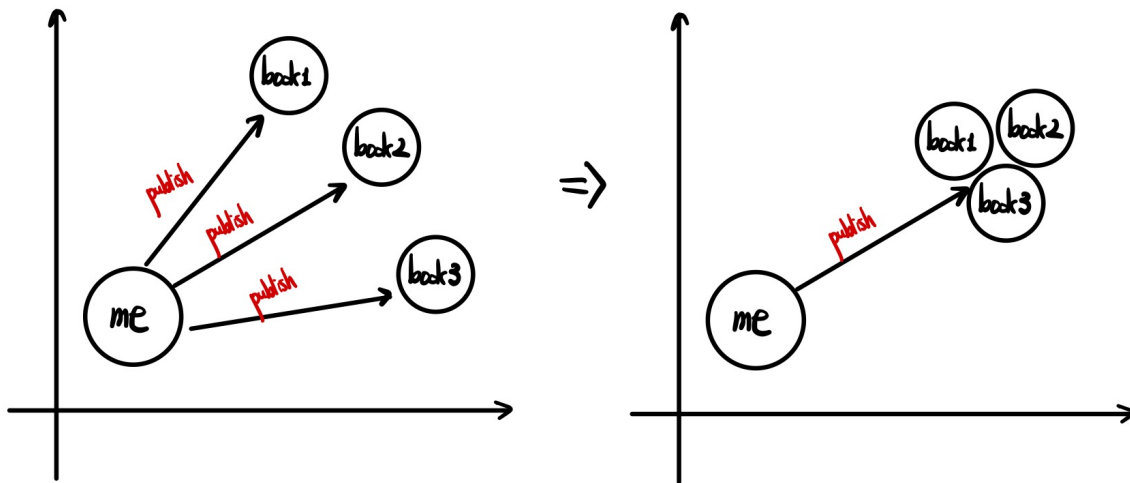
Mean rank: 모델이 제시한 정답 entity의 rank의 평균

Hits@10: 모델이 제시한 상위 10개의 정답 중 실제 정답이 있는 비율



# TransE

한계점: 1 - to - N



“모든 사람은 동물이다”

수량자

주어

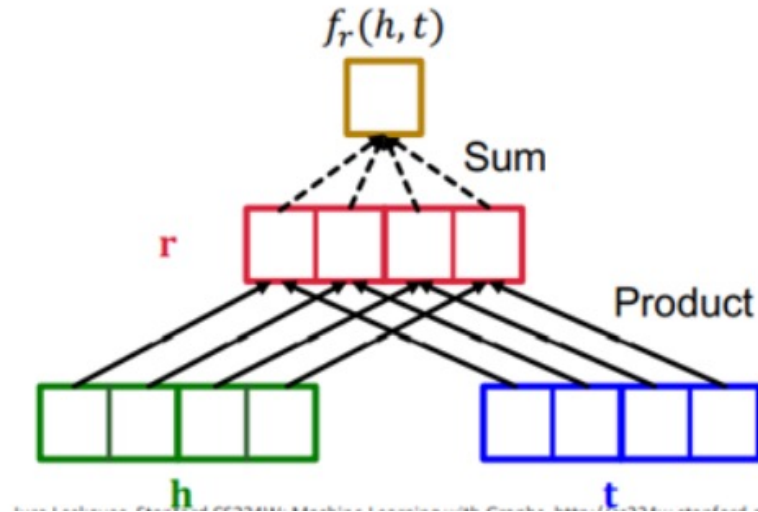
서술어



자연어 문장에서의 Rule Extraction을  
Knowledge Graph에서 시도



# DistMult



# DistMult

$$\mathbf{y}_{e_1} = f(\mathbf{W}\mathbf{x}_{e_1}), \mathbf{y}_{e_2} = f(\mathbf{W}\mathbf{x}_{e_2})$$

$$g_r^a(\mathbf{y}_{e_1}, \mathbf{y}_{e_2}) = \mathbf{A}_r^T \begin{pmatrix} \mathbf{y}_{e_1} \\ \mathbf{y}_{e_2} \end{pmatrix}$$

Linear transformation

$$g_r^b(\mathbf{y}_{e_1}, \mathbf{y}_{e_2}) = \mathbf{y}_{e_1}^T \mathbf{B}_r \mathbf{y}_{e_2}$$

Bilinear transformation

Models	$\mathbf{B}_r$	$\mathbf{A}_r^T$	Scoring Function
Distance (Bordes et al., 2011)	-	$(\mathbf{Q}_{r1}^T - \mathbf{Q}_{r2}^T)$	$-  g_r^a(\mathbf{y}_{e_1}, \mathbf{y}_{e_2})  _1$
Single Layer (Socher et al., 2013)	-	$(\mathbf{Q}_{r1}^T \quad \mathbf{Q}_{r2}^T)$	$\mathbf{u}_r^T \tanh(g_r^a(\mathbf{y}_{e_1}, \mathbf{y}_{e_2}))$
TransE (Bordes et al., 2013b)	$\mathbf{I}$	$(\mathbf{V}_r^T - \mathbf{V}_r^T)$	$-(2g_r^a(\mathbf{y}_{e_1}, \mathbf{y}_{e_2}) - 2g_r^b(\mathbf{y}_{e_1}, \mathbf{y}_{e_2}) +   \mathbf{V}_r  _2^2)$
NTN (Socher et al., 2013)	$\mathbf{T}_r$	$(\mathbf{Q}_{r1}^T \quad \mathbf{Q}_{r2}^T)$	$\mathbf{u}_r^T \tanh(g_r^a(\mathbf{y}_{e_1}, \mathbf{y}_{e_2}) + g_r^b(\mathbf{y}_{e_1}, \mathbf{y}_{e_2}))$

# DistMult

$$g_r^b(\mathbf{y}_{e_1}, \mathbf{y}_{e_2}) = \mathbf{y}_{e_1}^T \mathbf{M}_r \mathbf{y}_{e_2}$$

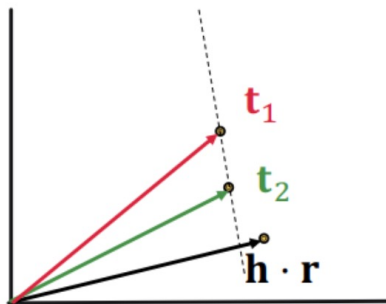
Diagonal matrix



$$r \quad [a, b, c] \Rightarrow M_r = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

TransE와 parameter 수 동일

$$\langle \mathbf{h}, \mathbf{r}, \mathbf{t}_1 \rangle = \langle \mathbf{h}, \mathbf{r}, \mathbf{t}_2 \rangle$$



# Experiment: Link Prediction

	FB15k		FB15k-401		WN	
	MRR	HITS@10	MRR	HITS@10	MRR	HITS@10
NTN	0.25	41.4	0.24	40.5	0.53	66.1
Bilinear+Linear	0.30	49.0	0.30	49.4	0.87	91.6
TransE (DISTADD)	0.32	53.9	0.32	54.7	0.38	90.9
Bilinear	0.31	51.9	0.32	52.2	<b>0.89</b>	92.8
Bilinear-diag (DISTMULT)	<b>0.35</b>	<b>57.7</b>	<b>0.36</b>	<b>58.5</b>	0.83	<b>94.2</b>

MRR: Rank 역수의 평균

Hits@10: 모델이 제시한 상위 10개의 정답 중 실제 정답이 있는 비율

# Experiment: Link Prediction

	Predicting subject entities				Predicting object entities			
	1-to-1	1-to-n	n-to-1	n-to-n	1-to-1	1-to-n	n-to-1	n-to-n
DISTADD	70.0	76.7	21.1	53.9	68.7	17.4	<b>83.2</b>	57.5
DISTMULT	<b>75.5</b>	<b>85.1</b>	<b>42.9</b>	<b>55.2</b>	<b>73.7</b>	<b>46.7</b>	81.0	<b>58.8</b>

Hits@10: 모델이 제시한 상위 10개의 정답 중 실제 정답이 있는 비율

# Experiment: Link Prediction

선형? 비선형?

$$\mathbf{y}_{e_1} = f(\mathbf{W}\mathbf{x}_{e_1}), \quad \mathbf{y}_{e_2} = f(\mathbf{W}\mathbf{x}_{e_2})$$

wv? Ev? default?

	MRR	HITS@10	MAP (w/ type checking)
DISTMULT	0.36	58.5	64.5
DISTMULT-tanh	0.39	63.3	76.0
DISTMULT-tanh-WV-init	0.28	52.5	65.5
DISTMULT-tanh-EV-init	<b>0.42</b>	<b>73.2</b>	<b>88.2</b>

MAP: 모델이 예측한 rank list에서 실제 정답이 높은 rank에 위치 할수록 높은 수치

w/ type checking: relation에 맞는 entity type을 확인 (BornInCity(Person, City))

# Experiment: Link Prediction

New / York / City

word vectors

New York City

entity vectors

# Experiment: Rule Extraction

$$B_1(a_1, a_2) \wedge B_2(a_2, a_3) \wedge \dots \wedge B_n(a_n, a_{n+1}) \implies H(a_1, a_{n+1})$$

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**Algorithm 1** EMBEDRULE
 

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```

1: Input:  $KB = \{(e_1, r, e_2)\}$ , relation set  $R$ 
2: Output: Candidate rules  $Q$ 
3: for each  $r$  in  $R$  do
4:   Select the set of start relations  $S = \{s : \mathcal{X}_s \cap \mathcal{X}_r \neq \emptyset\}$ 
5:   Select the set of end relations  $T = \{t : \mathcal{Y}_t \cap \mathcal{Y}_r \neq \emptyset\}$ 
6:   Find all possible relation sequences
7:   Select the  $K$ -NN sequences  $P' \subseteq P$  for  $r$  based on  $dist(\mathbf{M}_r, \mathbf{M}_{p_1} \circ \dots \circ \mathbf{M}_{p_n})$ 
8:   Form candidate rules using  $P'$  where  $r$  is the head relation and  $p \in P'$  is the body in a rule
9:   Add the candidate rules into  $Q$ 
10: end for
  
```

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B1: 사람 a가 도시 b에서 태어났다.

B2: 도시 b는 국가 c에 속한다.

H: 사람 a의 국적은 국가 c이다.



# Experiment: Rule Extraction

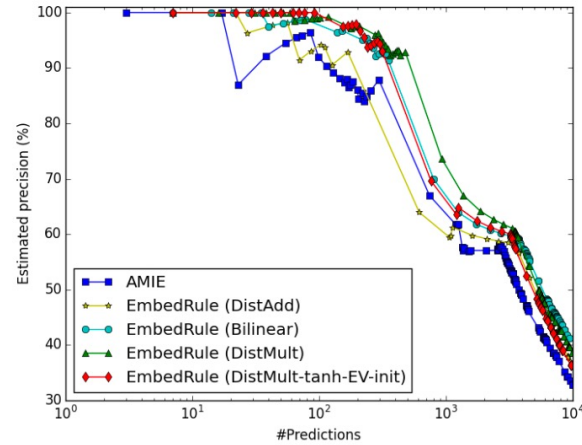


Figure 1: Aggregated precision of top length-2 rules extracted by different methods

# Experiment: Rule Extraction

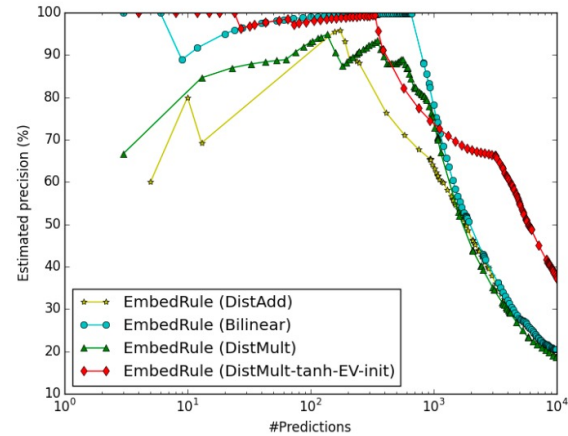
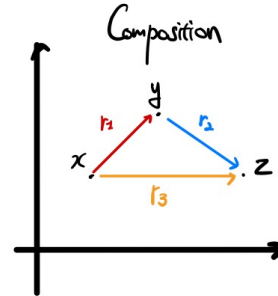
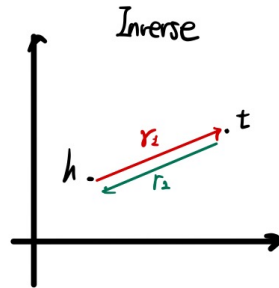
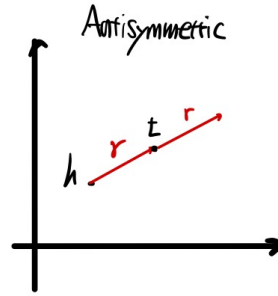
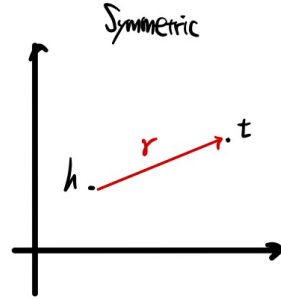


Figure 2: Aggregated precision of top length-3 rules extracted by different methods

# Limitations: Relation Patterns

TransE



DistMult

Symmetric

$$\langle h, r, t \rangle = \langle t, r, h \rangle$$

Antisymmetric

$$\langle h, r, t \rangle = \langle t, r, h \rangle$$

Inverse

$$\langle h, r_1, t \rangle = \langle t, r_2, h \rangle$$

Composition

$$(M_{r_1}, M_{r_2}) \neq M_{r_3}$$

# Conclusion

Multi-relationship을 기존 연구보다 간단하게 표현한 TransE

내적을 활용하여 Multi-relationship 표현과 Rule Extraction 모두 두각을 드러낸 DistMult

Experiment를 통해 성능을 시연하고 다양한 확장을 도모하였지만, 여전히 한계점 존재