



Link Prediction Based on Graph Neural Networks (SEAL)

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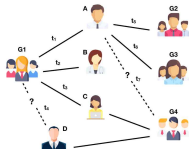
NeurIPS 2018



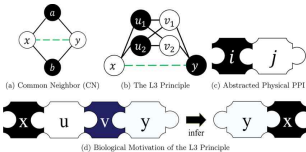
- Main interest
- Problem with previous models
- Theoretical justification
- Architecture
- Experiments
- Conclusion

■ Link prediction on graph networks

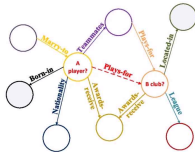
- Find whether there's a link between two nodes



▲ Social network recommendation



▲ PPI link prediction

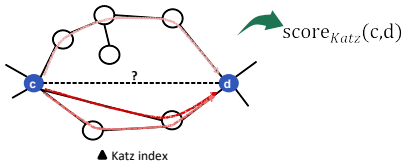
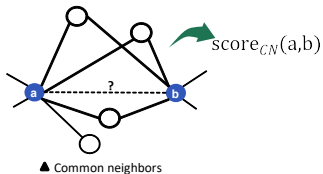


▲ Knowledge graph link prediction

■ Previous approaches

- Use hand made heuristic methods to determine links between nodes

Heuristic : A formula that defines the likelihood of a link between target nodes



Problem with previous models.

- Using hand made heuristic for link prediction is biased on data

- The assumptions of hand made heuristics can't be generalized to all data
(e.g., two proteins in PPI networks that share many common neighbors are **less likely to link**)



The heuristic needs to be flexible for all graph data



SEAL

*Subgraphs, **Embeddings**, **Attributes** for **Link prediction***

Problem with previous models.

- **SEAL wasn't the first approach for supervised(general) heuristic**
 - WLNK(KDD, 2017) also learns from local subgraphs
 - But WLNK has **several drawbacks**
 1. trains on fix sized input GNN, thus losing structural features
 2. doesn't utilize any latent/explicit node information
 3. theoretical justifications are missing

■ Theoretical background for general heuristic

- Does general heuristic for link prediction exist? **Yes!**
- Researchers proved that most high-order heuristics can be unified to **γ -decaying heuristic**

$$\mathcal{H}(x, y) = \eta \sum_{l=1}^{\infty} \gamma^l f(x, y, l)$$

$$\sum_{l=1}^{\infty} \beta^l |\text{walks}^{\langle l \rangle}(x, y)| \quad [\pi_x]_y + [\pi_y]_x \quad \gamma \frac{\sum_{a \in \Gamma(x)} \sum_{b \in \Gamma(y)} \text{score}(a, b)}{|\Gamma(x)| \cdot |\Gamma(y)|} \quad \dots$$

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■ Theoretical background for general heuristic

$$\mathcal{H}(x, y) = \eta \sum_{l=1}^{\infty} \gamma^l f(x, y, l)$$

γ : decaying factor between 0 and 1
 η : positive constant or function of γ that is upper bounded by a constant
 f : nonnegative function of x, y, l under the given network
 $G_{x,y}^h$: h - hop enclosing subgraph between target node x, y

As long as two properties are satisfied

- (property 1) $f(x, y, l) \leq \lambda^l$ where $\lambda < \frac{1}{\gamma}$; and
- (property 2) $f(x, y, l)$ is calculable from $G_{x,y}^h$ for $l = 1, 2, \dots, g(h)$, where $g(h) = ah + b$ with $a, b \in \mathbb{N}$ and $a > 0$,

then $\mathcal{H}(x, y)$ can be approximated from $G_{x,y}^h$ and the approximation error decreases at least exponentially with h .

- **What we achieved from the general heuristic analysis**

1. γ -decaying heuristic can be generalized to all graph networks
2. γ -decaying heuristic can be approximated via enclosing subgraph

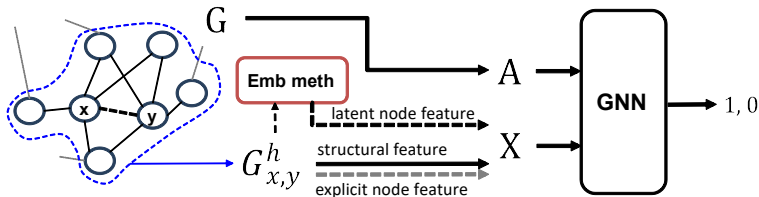
Thus our objective boils down to

“Construct a model that could accurately calculate $f(x,y,l)$ ”

- SEAL framework

*node2vec and DGCNN is selected for Emb meth, GNN respectively

* node2vec pre-trained on the network with negative injection(explained later)



SEAL framework(subgraph extraction and structural feature)

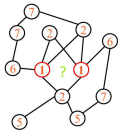
- Extract a h-hop subgraph surrounding the target nodes(h selected from {1, 2})
- Label the structural role of nodes in the subgraph via **Double-Radius Node Labeling(DRNL)**

$$f_l(i) = 1 + \min(d_x, d_y) + (d/2)[(d/2) + (d\%2) - 1]$$

$$d_x := d(i, x), d_y := d(i, y), d := d_x + d_y,$$

	1	2	3	4	5	...
1	2	3	4	6	8	11
2		5	7	9	12	15
3			10	13	16	20
4				17	21	25
5					26	...
...						...

4 Distance to x
5 Distance to y
21 Node label

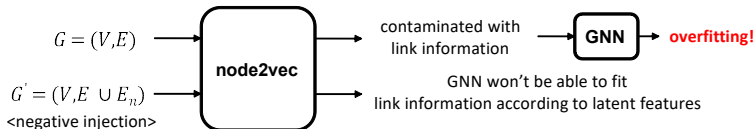


$d(i,j)$ = shortest distance between i,j

SEAL framework(latent features)

- Latent features capture global properties and long range effects of nodes
- Use negative injection in training node2vec to avoid overfitting in the GNN

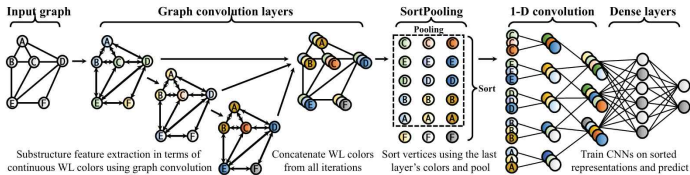
$$E_p \subseteq E, E_n \cap E = \emptyset$$



- **SEAL framework(explicit features)**
 - Not always available, but included if available
- **Example of explicit features**
 - citation networks : word distribution of document nodes
 - social networks : user's profile information

■ DGCNN for SEAL

- Adjacency matrix(A), node feature matrix(X) is given as input
- SortPooling allows two things
 1. nodes with similar structural roles are likely to go to the same input layer
 2. allows fixed size input



- **Evaluation metrics**

- AUC, average precision(AP)

- **Datasets**

- USAir, NS, PB, Yeast, C.ele, Power, Router, and E.coli

- **Compared methods**

- heuristics(CN, PA, AA, RA, Katz, PR, SR, ENS)
- latent feature methods(MF, SBM, node2vec, SPC, VGAE)
- heuristic learning methods(WLK, WLNLM)

■ Comparison to heuristic methods

- Restrict SEAL to use only graph structure features

Table 1: Comparison with heuristic methods (AUC).

Data	CN	Jaccard	PA	AA	RA	Katz	PR	SR	ENS	WLK	WLNLM	SEAL
USAir	93.80±1.22	89.79±1.61	88.84±1.45	95.06±1.03	95.77±0.92	92.88±1.42	94.67±1.08	78.89±2.31	88.96±1.44	96.63 ±0.73	95.95±1.10	96.62 ±0.72
NS	94.42±0.95	94.43±0.93	68.65±2.03	94.45±0.93	94.45±0.93	94.85±1.10	94.89±1.08	94.79±1.08	97.64±0.25	98.57±0.51	98.61±0.49	98.85 ±0.47
PB	92.04±0.35	87.41±0.39	90.14±0.45	92.36±0.34	92.46±0.37	92.92±0.35	93.54±0.41	77.08±0.80	90.15±0.45	93.83±0.59	93.49±0.47	94.72 ±0.46
Yeast	89.37±0.61	89.32±0.60	82.20±1.02	89.43±0.62	89.45±0.62	92.24±0.61	92.76±0.55	91.49±0.57	82.36±1.02	95.86±0.54	95.62±0.52	97.91 ±0.52
C.ele	85.13±1.61	80.19±1.64	74.79±2.04	86.95±1.40	87.49±1.41	86.34±1.89	90.32 ±1.49	77.07±2.00	74.94±2.04	89.72±1.67	86.18±1.72	90.30 ±1.35
Power	58.80±0.88	58.79±0.88	44.33±1.02	58.79±0.88	58.79±0.88	65.39±1.59	66.00±1.59	76.15±1.06	79.52±1.78	82.41±3.43	84.76±0.98	87.61 ±1.57
Router	56.43±0.52	56.40±0.52	47.58±1.47	56.43±0.51	56.43±0.51	38.62±1.35	38.76±1.39	37.40±1.27	47.58±1.48	87.42±2.08	94.41±0.88	96.38 ±1.45
E.coli	93.71±0.39	81.31±0.61	91.82±0.58	95.36±0.34	95.95±0.35	93.50±0.44	95.57±0.44	62.49±1.43	91.89±0.58	96.94±0.29	97.21±0.27	97.64 ±0.22

■ Comparison to latent feature methods

- Structural feature + latent feature incorporated to the node information matrix
- Interestingly, joint learning is not always good(e.g., Power)

Table 2: Comparison with latent feature methods (AUC).

Data	MF	SBM	N2V	LINE	SPC	VGAE	SEAL
USAir	94.08 \pm 0.80	94.85 \pm 1.14	91.44 \pm 1.78	81.47 \pm 10.71	74.22 \pm 3.11	89.28 \pm 1.99	97.09\pm0.70
NS	74.55 \pm 4.34	92.30 \pm 2.26	91.52 \pm 1.28	80.63 \pm 1.90	89.94 \pm 2.39	94.04 \pm 1.64	97.71\pm0.93
PB	94.30 \pm 0.53	93.90 \pm 0.42	85.79 \pm 0.78	76.95 \pm 2.76	83.96 \pm 0.86	90.70 \pm 0.53	95.01\pm0.34
Yeast	90.28 \pm 0.69	91.41 \pm 0.60	93.67 \pm 0.46	87.45 \pm 3.33	93.25 \pm 0.40	93.88 \pm 0.21	97.20\pm0.64
C.ele	85.90 \pm 1.74	86.48 \pm 2.60	84.11 \pm 1.27	69.21 \pm 3.14	51.90 \pm 2.57	81.80 \pm 2.18	89.54\pm2.04
Power	50.63 \pm 1.10	66.57 \pm 2.05	76.22 \pm 0.92	55.63 \pm 1.47	91.78\pm0.61	71.20 \pm 1.65	84.18 \pm 1.82
Router	78.03 \pm 1.63	85.65 \pm 1.93	65.46 \pm 0.86	67.15 \pm 2.10	68.79 \pm 2.42	61.51 \pm 1.22	95.68\pm1.22
E.coli	93.76 \pm 0.56	93.82 \pm 0.41	90.82 \pm 1.49	82.38 \pm 2.19	94.92 \pm 0.32	90.81 \pm 0.63	97.22\pm0.28

■ Incorporating explicit features

- VGAE and SEAL are the only methods that incorporate explicit features
- Structural feature + latent feature + explicit feature

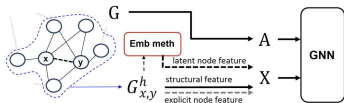
Table 12: Comparison with network embedding methods (AUC and standard deviation, OOM: out of memory).

	N2V	LINE	SPC	VGAE	WLNLM	SEAL
arXiv	96.18 \pm 0.40	84.64 \pm 0.03	87.00 \pm 0.14	OOM	99.19 \pm 0.03	99.40\pm0.14
Facebook	99.05 \pm 0.07	89.63 \pm 0.06	98.59 \pm 0.11	98.21 \pm 0.22	99.24 \pm 0.03	99.40\pm0.08
BlogCatalog	85.97 \pm 1.56	90.92 \pm 2.05	96.74 \pm 0.31	OOM	96.55 \pm 0.08	98.10\pm0.60
Wikipedia	76.59 \pm 2.06	74.44 \pm 0.66	99.54 \pm 0.04	89.74 \pm 0.18	99.05 \pm 0.03	99.63\pm0.05
PPI	70.31 \pm 0.79	72.82 \pm 1.53	92.27 \pm 0.22	85.86 \pm 0.43	88.79 \pm 0.38	93.52\pm0.37

Conclusion

■ Supervised learning link prediction model SEAL

- Use Subgraphs, Embeddings, Attributes for Link prediction
- Provided theoretical justification
- Proposed DRNL to label structural roles of nodes
- Utilized DGCNN to preserve the structural information



■ Inspiration to other researchs

- Opened new directions for knowledge graph completion and recommender systems etc.

Thank You!



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