



# **RNNLogic- Learning Logic Rules for Reasoning on Knowledge Graphs**

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# Previous work

End-to-End differentiable model

이산적 공간의 structure와 연속적 공간의 score를  
**Gradient based optimization**을 통해 학습시키자



*NeuralLP, DRUM*

End-to-End differentiable model한 RNN 기반 모델  
large search space -> High-quality의 logic rule을 찾기 어려움

# RNNLogic

Rule Generator  $p_{\theta}$

다양한 rule body의 rule 생성

Reasoning Predictor  $p_w$

생성된 rule에 대한 scoring

+ EM algorithm



Rule Generator, Reasoning Predictor의  
parameter를 단계적으로 학습

# RNNLogic

Objective function

$$\max_{\theta, w} \mathcal{O}(\theta, w) = \mathbb{E}_{(\mathcal{G}, \mathbf{q}, \mathbf{a}) \sim p_{\text{data}}} [\log p_{w, \theta}(\mathbf{a} | \mathcal{G}, \mathbf{q})] = \mathbb{E}_{(\mathcal{G}, \mathbf{q}, \mathbf{a}) \sim p_{\text{data}}} [\log \mathbb{E}_{p_{\theta}(\mathbf{z} | \mathbf{q})} [p_w(\mathbf{a} | \mathcal{G}, \mathbf{q}, \mathbf{z})]].$$

**G**: background knowledge graph (triplets)

**q**: query (h, r, ?)

**a**: answer (a = t)

**z**: rule generator가 정의한 latent rules

주어진  $G, q$ 에 대하여  $z$ 로 얻은 answer가  
실제 answer  $\mathbf{a}$ 를 예측할 확률을 최대화

# RNNLogic

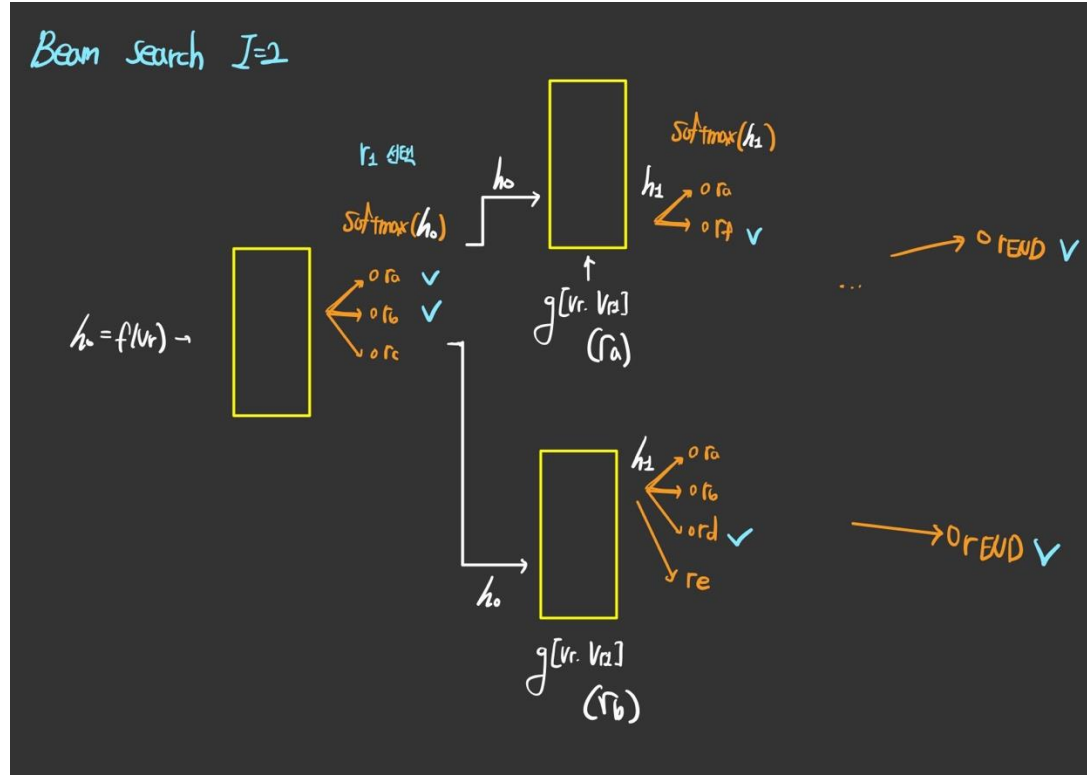
Rule generator

$$p_{\theta}(\mathbf{z}|\mathbf{q}) = \text{Mu}(\mathbf{z}|N, \text{RNN}_{\theta}(\cdot|\mathbf{r}))$$

Rule Generator는 LSTM 기반 RNN 모델 사용  
(Hyperparameter)  $N$ 개의 rule에 대한 multinomial distribution 정의

=> 규칙들이 각기 다른 확률을 가지고 sampling 됨

# RNNLogic



$$h_0 = f(v_r).$$

$$\text{softmax}(o(h_{t+1}))$$

$$h_t = \text{LSTM}(h_{t-1}, g([v_r, v_{r_t}]));$$

# RNNLogic

top 1.  $z_1 [r. r_{11} r_{z11} \dots, r_{END}]$   
top 2.  $z_2 [r. r_{z21} r_{z22} \dots, r_{END}]$  }  $\hat{z}$



# RNNLogic

## Reasoning Predictor

$$\text{score}_w(e) = \sum_{rule \in \mathbf{z}} \text{score}_w(e|rule) = \sum_{rule \in \mathbf{z}} \sum_{path \in \mathcal{P}(h, rule, e)} \psi_w(rule) \cdot \phi_w(path).$$

Rule에 대한 learnable parameter와 Path에 대한 score 존재

### Path score

RotatE를 사용하여 h부터 rule body에 대한 embedding을 움직여

Candidate answer e의 embedding과 거리로 점수 계산

$$\phi_w(path) = \sigma(\delta - d(\mathbf{x}_{e_0} \circ \mathbf{x}_{r_1} \circ \mathbf{x}_{r_2} \circ \cdots \circ \mathbf{x}_{r_l}, \mathbf{x}_{e_l}))$$

# RNNLogic

## Reasoning Predictor

$$p_w(\mathbf{a} = e | \mathcal{G}, \mathbf{q}, \mathbf{z}) = \frac{\exp(\text{score}_w(e))}{\sum_{e' \in \mathcal{A}} \exp(\text{score}_w(e'))}$$

Softmax function을 통해

다른 candidate answer들 간 score에 대한 확률 계산

# RNNLogic

## EM algorithm

E-step: 주어진 **현재 parameter**를 바탕으로 latent 변수의 기댓값 계산

M-step: E-step에서 계산된 latent 변수의 기댓값을 이용해 **parameter**를 최대화

# RNNLogic

EM algorithm

**E-step** : 현재 step에서 rule generator가 생성한  $\hat{z}$ 에서 high quality의 rule 식별

$$p_{\theta, w}(\mathbf{z}_I | \mathcal{G}, \mathbf{q}, \mathbf{a}) \propto p_w(\mathbf{a} | \mathcal{G}, \mathbf{q}, \mathbf{z}_I) \bar{p}_{\theta}(\mathbf{z}_I | \mathbf{q})$$

$\mathcal{G}, \mathbf{q}, \mathbf{a}$ 가 주어졌을 때, 좋은 rule  $\mathbf{z}$ 를 찾는 과정

by posterior probability

# RNNLogic

EM algorithm

but, Rule에 대한 multinomial distribution 형식으로 나타내기 어려움



$$H(rule) = \left\{ \text{score}_w(t|rule) - \frac{1}{|\mathcal{A}|} \sum_{e \in \mathcal{A}} \text{score}_w(e|rule) \right\} + \log \text{RNN}_\theta(rule|r),$$

$$\left| \log p_{\theta,w}(\mathbf{z}_I | \mathcal{G}, \mathbf{q}, \mathbf{a}) - \left( \sum_{rule \in \mathbf{z}_I} H(rule) + \gamma(\mathbf{z}_I) + \text{const} \right) \right| \leq s^2 + O(s^4)$$



$$q(\mathbf{z}_I) \propto \exp(\sum_{rule \in \mathbf{z}_I} H(rule) + \gamma(\mathbf{z}_I))$$

# RNNLogic

EM algorithm

$$\exp(H(rule)) / (\sum_{rule' \in \hat{z}} \exp(H(rule')))$$



Top - K 개의 rule이 sampling되어  $\hat{z}_I$  구성

# RNNLogic

EM algorithm

**M-step** : high quality의 rule을 통해 Rule Generator를 update

$$\mathcal{O}_{(\mathcal{G}, \mathbf{q}, \mathbf{a})}(\theta) = \log p_{\theta}(\hat{\mathbf{z}}_I | \mathbf{q}) = \sum_{rule \in \hat{\mathbf{z}}_I} \log \text{RNN}_{\theta}(rule | r) + \text{const.}$$

**Objective function:** Rule generator가 high quality rule을 생성할 확률을 최대화

# RNNLogic





# Experiment

Evaluation Metrics: (h, r, ?), (t, r<sup>-1</sup>, ?)

RNNLogic+ : Aggregator + MLP + KGE 활용한 scoring 선택

$$\text{score}_w(e) = \text{MLP}(\text{AGG}(\{\mathbf{v}_{rule}, |\mathcal{P}(h, rule, e)|\}_{rule \in \hat{z}_I}) + \eta \text{KGE}(h, r, e).$$

Inverse relation 제거된 dataset

Category	Algorithm	FB15k-237					WN18RR				
		MR	MRR	H@1	H@3	H@10	MR	MRR	H@1	H@3	H@10
No Rule Learning	TransE*	357	0.294	-	-	46.5	3384	0.226	-	-	50.1
	DistMult*	254	0.241	15.5	26.3	41.9	5110	0.43	39	44	49
	ComplEx*	339	0.247	15.8	27.5	42.8	5261	0.44	41	46	51
	ComplEx-N3*	-	<b>0.37</b>	-	-	<b>56</b>	-	0.48	-	-	57
	ConvE*	244	0.325	23.7	35.6	50.1	4187	0.43	40	44	52
	TuckER*	-	0.358	<b>26.6</b>	<b>39.4</b>	54.4	-	0.470	44.3	48.2	52.6
	RotatE*	<b>177</b>	0.338	24.1	37.5	53.3	<b>3340</b>	0.476	42.8	49.2	57.1
Rule Learning	PathRank	-	0.087	7.4	9.2	11.2	-	0.189	17.1	20.0	22.5
	NeuralLP <sup>†</sup>	-	0.237	17.3	25.9	36.1	-	0.381	36.8	38.6	40.8
	DRUM <sup>†</sup>	-	0.238	17.4	26.1	36.4	-	0.382	36.9	38.8	41.0
	NLIL*	-	0.25	-	-	32.4	-	-	-	-	-
	M-Walk*	-	0.232	16.5	24.3	-	-	0.437	41.4	44.5	-
RNNLogic	w/o emb.	538	0.288	20.8	31.5	44.5	7527	0.455	41.4	47.5	53.1
	with emb.	232	0.344	25.2	38.0	53.0	4615	0.483	44.6	49.7	55.8
RNNLogic+	w/o emb.	480	0.299	21.5	32.8	46.4	7204	0.489	45.3	50.6	56.3
	with emb.	178	0.349	25.8	38.5	53.3	4624	<b>0.513</b>	<b>47.1</b>	<b>53.2</b>	<b>59.7</b>

$$\phi_w(path) = 1$$

$$\phi_w(path) = \sigma(\delta - d(x_{e_0} \circ x_{e_1} \circ x_{e_2} \circ \dots \circ x_{e_i}, x_{e_i}))$$

relation / inverse relation 에 대한 평가

# Experiment

## Evaluation Metrics

Category	Algorithm	FB15k-237					WN18RR				
		MR	MRR	H@1	H@3	H@10	MR	MRR	H@1	H@3	H@10
Rule Learning	MINERVA*	-	0.293	21.7	32.9	45.6	-	0.448	41.3	45.6	51.3
	MultiHopKG*	-	0.407	32.7	-	56.4	-	0.472	43.7	-	54.2
RNNLogic	w/o emb.	459.0	0.377	28.9	41.2	54.9	7662.8	0.478	43.8	50.3	55.3
	with emb.	<b>146.1</b>	<b>0.443</b>	<b>34.4</b>	<b>48.9</b>	<b>64.0</b>	<b>3767.0</b>	<b>0.506</b>	<b>46.3</b>	<b>52.3</b>	<b>59.2</b>

(h, r, ?)만 사용하는 모델들과도 비교

# Experiment

## Evaluation Metrics

Category	Algorithm	Kinship					UMLS				
		MR	MRR	H@1	H@3	H@10	MR	MRR	H@1	H@3	H@10
<b>No Rule Learning</b>	DistMult	8.5	0.354	18.9	40.0	75.5	14.6	0.391	25.6	44.5	66.9
	ComplEx	7.8	0.418	24.2	49.9	81.2	13.6	0.411	27.3	46.8	70.0
	ComplEx-N3	-	0.605	43.7	71.0	92.1	-	0.791	68.9	87.3	95.7
	TuckER	6.2	0.603	46.2	69.8	86.3	5.7	0.732	62.5	81.2	90.9
	RotatE	3.7	0.651	50.4	75.5	93.2	4.0	0.744	63.6	82.2	93.9
<b>Rule Learning</b>	MLN	10.0	0.351	18.9	40.8	70.7	7.6	0.688	58.7	75.5	86.9
	Boosted RDN	25.2	0.469	39.5	52.0	56.7	54.8	0.227	14.7	25.6	37.6
	PathRank	-	0.369	27.2	41.6	67.3	-	0.197	14.8	21.4	25.2
	NeuralLP	16.9	0.302	16.7	33.9	59.6	10.3	0.483	33.2	56.3	77.5
	DRUM	11.6	0.334	18.3	37.8	67.5	8.4	0.548	35.8	69.9	85.4
	MINERVA	-	0.401	23.5	46.7	76.6	-	0.564	42.6	65.8	81.4
	CTP	-	0.335	17.7	37.6	70.3	-	0.404	28.8	43.0	67.4
<b>RNNLogic</b>	w/o emb.	3.9	0.639	49.5	73.1	92.4	5.3	0.745	63.0	83.3	92.4
	with emb.	<b>3.1</b>	<b>0.722</b>	<b>59.8</b>	<b>81.4</b>	<b>94.9</b>	<b>3.1</b>	<b>0.842</b>	<b>77.2</b>	<b>89.1</b>	<b>96.5</b>

# Experiment

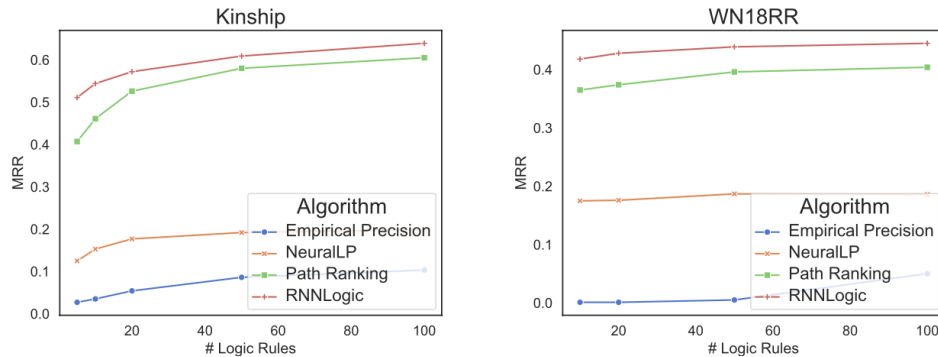


Figure 2: Performance w.r.t. # logic rules. RNNLogic achieves competitive results even with 10 rules per query relation.

주어진 query relation  $r$ 에 대해 10 개의 rule 생성하여 추론한 결과

# Experiment

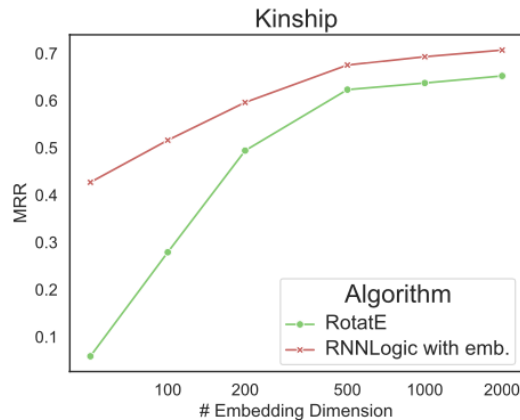


Figure 3: Performance w.r.t. embedding dimension.

모든 embedding 차원에서 RotatE 능가

# Experiment

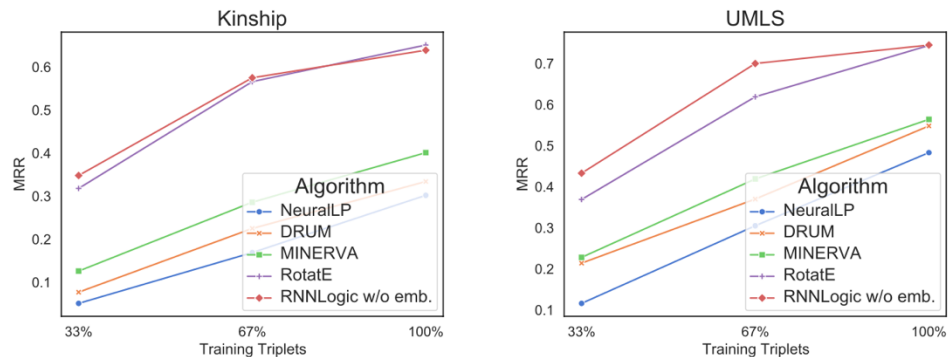


Figure 4: Performance w.r.t. # training triplets. RNNLogic is more robust to data sparsity even without using embeddings.

훈련시킨 triplet 개수에 대한 효율성

# Conclusion

NeuralLP, DRUM에서 채택한 end-to-end differentiable은 search space가 커지는 문제 발생

RNNLogic은 Rule Generator, Reasoning Predictor를 통해 rule 생성과 scoring을 분리

EM algorithm을 활용하여 parameter 최적화

Experiment를 통해 high-quality의 rule을 통한 answer 추론이 효과적임