EXAMPLE E.1A W-SHAPE COLUMN DESIGN WITH PINNED ENDS

Given:

Select a W-shape column to carry the loading as shown in Figure E.1A. The column is pinned top and bottom in both axes. Limit the column size to a nominal 14-in. shape. A column is selected for both ASTM A992 and ASTM A913 Grade 65 material.

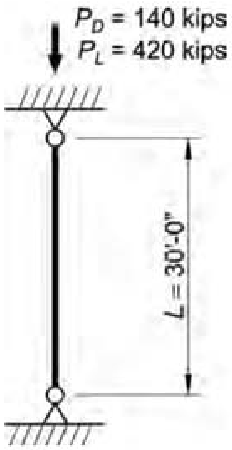

Fig. E.1A. Column loading and bracing.

$$1.2 \cdot 140\ kip + 1.6 \cdot 420\ kip = \left(3.737 \cdot 10^{3}\right)\ kN$$

W14x132

$$h := 14.7\ in = 373.38\ mm$$
$$b_f := 14.7\ in = 373.38\ mm$$
$$t_f := 1.03\ in = 26.162\ mm$$
$$t_w := 0.645\ in = 16.383\ mm$$
$$k := 1.63\ in = 41.402\ mm$$

$$A_g := 2 \cdot t_f \cdot b_f + \left(h - 2 \cdot t_f\right) \cdot t_w = \left(2.48 \cdot 10^{4}\right)\ mm^2$$

$$I_x := \left(2 \cdot \left(b_f \cdot \frac{t_f^{\ 3}}{12} + b_f \cdot t_f \cdot \left(\frac{\left(h - 2 \cdot t_f\right)}{2} + \frac{t_f}{2}\right)^2\right) + t_w \cdot \frac{\left(h - 2 \cdot t_f\right)^3}{12}\right) = \left(6.351 \cdot 10^{8}\right)\ mm^4$$

$$S_x := \frac{I_x}{\left(\frac{h}{2}\right)} = \left(3.402 \cdot 10^{6}\right)\ mm^3$$

$$Z_x := b_f \cdot t_f \cdot \left(h - t_f\right) + \frac{1}{4} \cdot \left(h - 2\ t_f\right)^2 \cdot t_w = \left(3.814 \cdot 10^{6}\right)\ mm^3$$

$$r_x := \sqrt{\frac{I_x}{A_g}} = 160.043\ mm$$

$$I_y := 2 \cdot \left(t_f \cdot \frac{b_f^{\ 3}}{12}\right) + \left(h - 2 \cdot t_f\right) \cdot \frac{t_w^{\ 3}}{12} = \left(2.271 \cdot 10^{8}\right)\ mm^4$$

$$S_y := \frac{I_y}{\frac{b_f}{2}} = \left(1.216 \cdot 10^{6}\right)\ mm^3$$

$$Z_y := \frac{1}{2} \cdot b_f^{\,2} \cdot t_f + \frac{1}{4} \cdot \left(h - 2 \cdot t_f\right) \cdot t_w^{\,2} = \left(1.845 \cdot 10^{6}\right)\ mm^{3}$$

$$r_y := \sqrt{\frac{I_y}{A_g}} = 95.698\ mm$$

$$c_w := \frac{\left(h - t_f\right)^{2} \cdot b_f^{\,3} \cdot t_f}{24} = \left(6.841 \cdot 10^{12}\right)\ mm^{6}$$

$$J := \frac{2 \cdot b_f \cdot t_f^{\,3} + \left(h - t_f\right) \cdot t_w^{\,3}}{3} = \left(4.966 \cdot 10^{6}\right)\ mm^{4}$$

$$r_{ts} := \sqrt{\frac{\sqrt{I_y \cdot c_w}}{S_x}} = 107.636\ mm$$

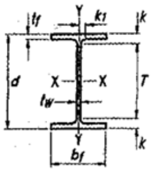

Table 1-1 (continued)
W-Shapes
Dimensions

Shape	Area, A	Depth, d		Web Thickness, t_w		Web $\frac{t_w}{2}$	Flange Width, b_f		Flange Thickness, t_f		Distance k k_{des}	Distance k k_{det}	Distance k_1	Distance T	Workable Gage
	in.^{2}	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W14×132	38.8	14.7	14 5/8	0.645	5/8	5/16	14.7	14 3/4	1.03	1	1.63	2 5/16	1 9/16	10	5 1/2
×120	35.3	14.5	14 1/2	0.590	9/16	5/16	14.7	14 5/8	0.940	15/16	1.54	2 1/4	1 1/2	↓	↓
×109	32.0	14.3	14 3/8	0.525	1/2	1/4	14.6	14 5/8	0.860	7/8	1.46	2 3/16	1 1/2		
×99[f]	29.1	14.2	14 1/8	0.485	1/2	1/4	14.6	14 5/8	0.780	3/4	1.38	2 1/16	1 7/16		
×90[f]	26.5	14.0	14	0.440	7/16	1/4	14.5	14 1/2	0.710	11/16	1.31	2	1 7/16		

Table 1-1 (continued)
W-Shapes
Properties
W14-W12

Nominal Wt.	Compact Section Criteria $\frac{b_f}{2t_f}$	Compact Section Criteria $\frac{h}{t_w}$	Axis X-X I	Axis X-X S	Axis X-X r	Axis X-X Z	Axis Y-Y I	Axis Y-Y S	Axis Y-Y r	Axis Y-Y Z	r_{ts}	h_o	Torsional Properties J	Torsional Properties C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
132	7.15	17.7	1530	209	6.28	234	548	74.5	3.76	113	4.23	13.7	12.3	25500
120	7.80	19.3	1380	190	6.24	212	495	67.5	3.74	102	4.20	13.6	9.37	22700
109	8.49	21.7	1240	173	6.22	192	447	61.2	3.73	92.7	4.17	13.4	7.12	20200
99	9.34	23.5	1110	157	6.17	173	402	55.2	3.71	83.6	4.14	13.4	5.37	18000
90	10.2	25.9	999	143	6.14	157	362	49.9	3.70	75.6	4.10	13.3	4.06	16000

$$E := 29000\ ksi = \left(1.999 \cdot 10^{5}\right)\ MPa$$

$$F_y := 50\ ksi = 344.738\ MPa$$

$\frac{b_f}{2 \cdot t_f} = 7.136$ $\quad 0.56 \cdot \sqrt{\frac{E}{F_y}} = 13.487$ $\quad$ Non-Slender Flange

$\frac{\left(h - 2 \cdot (k)\right)}{t_w} = 17.736$ $\quad 1.49 \cdot \sqrt{\frac{E}{F_y}} = 35.884$ $\quad$ Non-Slender Web

$$L_c := 30\ ft = \left(9.144 \cdot 10^{3}\right)\ mm$$

$$4.71 \cdot \sqrt{\frac{E}{F_y}} = 113.432$$

$$\frac{L_c}{r_x} = 57.135$$

$$\frac{L_c}{r_y} = 95.551$$

$$F_e := \frac{\pi^2 \cdot E}{\left(\frac{L_c}{r_x}\right)^2} = 604.53 \ MPa$$

$$\mathbf{F_{cr}} := \mathrm{if}\left(\frac{L_c}{r_x} > 4.71 \cdot \sqrt{\frac{E}{F_y}}, 0.877 \cdot F_e, 0.658^{\frac{F_y}{F_e}} \cdot F_y\right) = 271.538 \ MPa$$

$$F_e := \frac{\pi^2 \cdot E}{\left(\frac{L_c}{r_y}\right)^2} = 216.147 \ MPa$$

$$\mathbf{F_{cr}} := \mathrm{if}\left(\frac{L_c}{r_y} > 4.71 \cdot \sqrt{\frac{E}{F_y}}, 0.877 \cdot F_e, 0.658^{\frac{F_y}{F_e}} \cdot F_y\right) = 176.837 \ MPa$$

$$F_e := \frac{\pi^2 \cdot E}{\left(\max\left(\frac{L_c}{r_x}, \frac{L_c}{r_y}\right)\right)^2} = 216.147 \ MPa$$

$$\mathbf{F_{cr}} := \mathrm{if}\left(\max\left(\frac{L_c}{r_x}, \frac{L_c}{r_y}\right) > 4.71 \cdot \sqrt{\frac{E}{F_y}}, 0.877 \cdot F_e, 0.658^{\frac{F_y}{F_e}} \cdot F_y\right) = 176.837 \ MPa$$

$$\phi := 0.9$$

$$\phi P_n := \phi \cdot \mathbf{F_{cr}} \cdot A_g = \left(3.946 \cdot 10^3\right) \ kN$$

$$1.2 \cdot 140 \ kip + 1.6 \cdot 420 \ kip = \left(3.737 \cdot 10^3\right) \ kN$$

LRFD	ASD
$\phi_c P_n = 893 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\frac{P_n}{\Omega_c} = 594 \text{ kips} > 560 \text{ kips}$ **o.k.**

Given:

Verify a W14×90 is adequate to carry the loading as shown in Figure E.1B. The column is pinned top and bottom in both axes and braced at the midpoint about the *y-y* axis and torsionally. The column is verified for both ASTM A992 and ASTM A913 Grade 65 material.

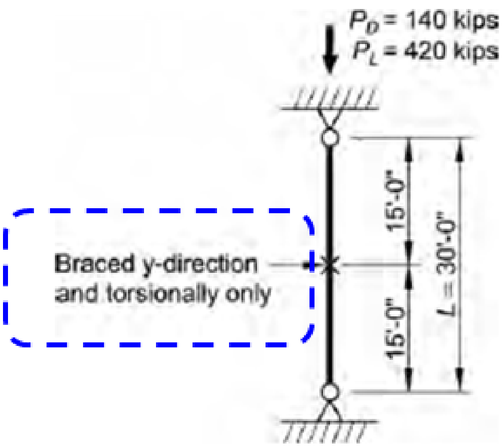

Fig. E.1B. Column loading and bracing.

W14x132

$h := 14.7\ in = 373.38\ mm$
$b_f := 14.7\ in = 373.38\ mm$
$t_f := 1.03\ in = 26.162\ mm$
$t_w := 0.645\ in = 16.383\ mm$
$k := 1.63\ in = 41.402\ mm$

$$A_g := 2 \cdot t_f \cdot b_f + \left(h - 2 \cdot t_f\right) \cdot t_w = \left(2.48 \cdot 10^4\right)\ mm^2$$

$$I_x := \left(2 \cdot \left(b_f \cdot \frac{t_f^{\ 3}}{12} + b_f \cdot t_f \cdot \left(\frac{\left(h - 2 \cdot t_f\right)}{2} + \frac{t_f}{2}\right)^2\right) + t_w \cdot \frac{\left(h - 2 \cdot t_f\right)^3}{12}\right) = \left(6.351 \cdot 10^8\right)\ mm^4$$

$$S_x := \frac{I_x}{\left(\frac{h}{2}\right)} = \left(3.402 \cdot 10^6\right)\ mm^3$$

$$Z_x := b_f \cdot t_f \cdot \left(h - t_f\right) + \frac{1}{4} \cdot \left(h - 2\ t_f\right)^2 \cdot t_w = \left(3.814 \cdot 10^6\right)\ mm^3$$

$$r_x := \sqrt{\frac{I_x}{A_g}} = 160.043\ mm$$

$$I_y := 2 \cdot \left(t_f \cdot \frac{b_f^{\ 3}}{12}\right) + \left(h - 2 \cdot t_f\right) \cdot \frac{t_w^{\ 3}}{12} = \left(2.271 \cdot 10^8\right)\ mm^4$$

$$S_y := \frac{I_y}{\frac{b_f}{2}} = \left(1.216 \cdot 10^6\right)\ mm^3$$

$$Z_y := \frac{1}{2} \cdot b_f^{\ 2} \cdot t_f + \frac{1}{4} \cdot \left(h - 2 \cdot t_f\right) \cdot t_w^{\ 2} = \left(1.845 \cdot 10^6\right)\ mm^3$$

$$r_y := \sqrt{\frac{I_y}{A_g}} = 95.698\ mm$$

$$c_w := \frac{(h-t_f)^2 \cdot b_f^3 \cdot t_f}{24} = \left(6.841 \cdot 10^{12}\right)\ mm^6$$

$$J := \frac{2 \cdot b_f \cdot t_f^3 + (h-t_f) \cdot t_w^3}{3} = \left(4.966 \cdot 10^6\right)\ mm^4$$

$$r_{ts} := \sqrt{\frac{\sqrt{I_y \cdot c_w}}{S_x}} = 107.636\ mm$$

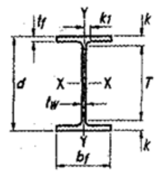

Table 1-1 (continued)
W-Shapes
Dimensions

Shape	Area, A	Depth, d		Web Thickness, t_w		Web $\frac{t_w}{2}$	Flange Width, b_f		Flange Thickness, t_f		Distance k k_{des}	Distance k k_{det}	k_1	T	Workable Gage
	in.^{2}	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W14×132	38.8	14.7	14 5/8	0.645	5/8	5/16	14.7	14 3/4	1.03	1	1.63	2 5/16	1 9/16	10	5 1/2
×120	35.3	14.5	14 1/2	0.590	9/16	5/16	14.7	14 5/8	0.940	15/16	1.54	2 1/4	1 1/2	↓	↓
×109	32.0	14.3	14 3/8	0.525	1/2	1/4	14.6	14 5/8	0.860	7/8	1.46	2 3/16	1 1/2		
×99^f	29.1	14.2	14 1/8	0.485	1/2	1/4	14.6	14 5/8	0.780	3/4	1.38	2 1/16	1 7/16		
×90^f	26.5	14.0	14	0.440	7/16	1/4	14.5	14 1/2	0.710	11/16	1.31	2	1 7/16		

Table 1-1 (continued)
W-Shapes
Properties

W14-W12

Nominal Wt.	Compact Section Criteria $\frac{b_f}{2t_f}$	Compact Section Criteria $\frac{h}{t_w}$	Axis X-X I	Axis X-X S	Axis X-X r	Axis X-X Z	Axis Y-Y I	Axis Y-Y S	Axis Y-Y r	Axis Y-Y Z	r_{ts}	h_o	Torsional Properties J	Torsional Properties C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
132	7.15	17.7	1530	209	6.28	234	548	74.5	3.76	113	4.23	13.7	12.3	25500
120	7.80	19.3	1380	190	6.24	212	495	67.5	3.74	102	4.20	13.6	9.37	22700
109	8.49	21.7	1240	173	6.22	192	447	61.2	3.73	92.7	4.17	13.4	7.12	20200
99	9.34	23.5	1110	157	6.17	173	402	55.2	3.71	83.6	4.14	13.4	5.37	18000
90	10.2	25.9	999	143	6.14	157	362	49.9	3.70	75.6	4.10	13.3	4.06	16000

$$E := 29000\ ksi = \left(1.999 \cdot 10^5\right)\ MPa$$

$$F_y := 50\ ksi = 344.738\ MPa$$

$$\frac{b_f}{2 \cdot t_f} = 7.136 \qquad 0.56 \cdot \sqrt{\frac{E}{F_y}} = 13.487$$ Non-Slender Flange

$$\frac{(h - 2 \cdot (k))}{t_w} = 17.736 \qquad 1.49 \cdot \sqrt{\frac{E}{F_y}} = 35.884$$ Non-Slender Web

$$L_{cx} := 30\ ft = \left(9.144 \cdot 10^3\right)\ mm$$

$$L_{cy} := 15\ ft = \left(4.572 \cdot 10^3\right)\ mm$$

$$4.71 \cdot \sqrt{\frac{E}{F_y}} = 113.432$$

$$\frac{L_{cx}}{r_x} = 57.135$$

$$\frac{L_{cy}}{r_y} = 47.775$$

$$F_e := \frac{\pi^2 \cdot E}{\left(\frac{L_{cx}}{r_x}\right)^2} = 604.53\ MPa$$

$$\mathbf{F_{cr}} := \mathbf{if}\left(\frac{L_{cx}}{r_x} > 4.71 \cdot \sqrt{\frac{E}{F_y}}, 0.877 \cdot F_e, 0.658^{\frac{F_y}{F_e}} \cdot F_y\right) = 271.538\ MPa$$

$$F_e := \frac{\pi^2 \cdot E}{\left(\frac{L_{cy}}{r_y}\right)^2} = 864.59\ MPa$$

$$\mathbf{F_{cr}} := \mathbf{if}\left(\frac{L_{cy}}{r_y} > 4.71 \cdot \sqrt{\frac{E}{F_y}}, 0.877 \cdot F_e, 0.658^{\frac{F_y}{F_e}} \cdot F_y\right) = 291.75\ MPa$$

$$F_e := \frac{\pi^2 \cdot E}{\left(\max\left(\frac{L_{cx}}{r_x}, \frac{L_{cy}}{r_y}\right)\right)^2} = 604.53\ MPa$$

$$\mathbf{F_{cr}} := \mathbf{if}\left(\max\left(\frac{L_{cx}}{r_x}, \frac{L_{cy}}{r_y}\right) > 4.71 \cdot \sqrt{\frac{E}{F_y}}, 0.877 \cdot F_e, 0.658^{\frac{F_y}{F_e}} \cdot F_y\right) = 271.538\ MPa$$

$$\phi := 0.9$$

$$\phi P_n := \phi \cdot \mathbf{F_{cr}} \cdot A_g = \left(6.06 \cdot 10^3\right)\ kN$$

$$1.2 \cdot 140\ kip + 1.6 \cdot 420\ kip = \left(3.737 \cdot 10^3\right)\ kN$$

W14x90

$$h := 14\ in = 355.6\ mm$$
$$b_f := 14.5\ in = 368.3\ mm$$
$$t_f := 0.71\ in = 18.034\ mm$$
$$t_w := 0.44\ in = 11.176\ mm$$
$$k := 1.31\ in = 33.274\ mm$$

$$A_g := 2 \cdot t_f \cdot b_f + \left(h - 2 \cdot t_f\right) \cdot t_w = \left(1.685 \cdot 10^{4}\right)\ mm^2$$

$$I_x := \left(2 \cdot \left(b_f \cdot \frac{t_f^{\,3}}{12} + b_f \cdot t_f \cdot \left(\frac{\left(h - 2 \cdot t_f\right)}{2} + \frac{t_f}{2}\right)^2\right) + t_w \cdot \frac{\left(h - 2 \cdot t_f\right)^3}{12}\right) = \left(4.092 \cdot 10^{8}\right)\ mm^4$$

$$S_x := \frac{I_x}{\left(\frac{h}{2}\right)} = \left(2.301 \cdot 10^{6}\right)\ mm^3$$

$$Z_x := b_f \cdot t_f \cdot \left(h - t_f\right) + \frac{1}{4} \cdot \left(h - 2 \cdot t_f\right)^2 \cdot t_w = \left(2.527 \cdot 10^{6}\right)\ mm^3$$

$$r_x := \sqrt{\frac{I_x}{A_g}} = 155.808\ mm$$

$$I_y := 2 \cdot \left(t_f \cdot \frac{b_f^{\,3}}{12}\right) + \left(h - 2 \cdot t_f\right) \cdot \frac{t_w^{\,3}}{12} = \left(1.502 \cdot 10^{8}\right)\ mm^4$$

$$S_y := \frac{I_y}{\frac{b_f}{2}} = \left(8.156 \cdot 10^{5}\right)\ mm^3$$

$$Z_y := \frac{1}{2} \cdot b_f^{\,2} \cdot t_f + \frac{1}{4} \cdot \left(h - 2 \cdot t_f\right) \cdot t_w^{\,2} = \left(1.233 \cdot 10^{6}\right)\ mm^3$$

$$r_y := \sqrt{\frac{I_y}{A_g}} = 94.398\ mm$$

$$c_w := \frac{\left(h - t_f\right)^2 \cdot b_f^{\,3} \cdot t_f}{24} = \left(4.278 \cdot 10^{12}\right)\ mm^6$$

$$J := \frac{2 \cdot b_f \cdot t_f^{\,3} + \left(h - t_f\right) \cdot t_w^{\,3}}{3} = \left(1.597 \cdot 10^{6}\right)\ mm^4$$

$$r_{ts} := \sqrt{\frac{\sqrt{I_y \cdot c_w}}{S_x}} = 104.949\ mm$$

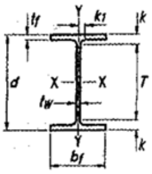

Table 1-1 (continued)
W-Shapes
Dimensions

Shape	Area, A	Depth, d		Web Thickness, t_w		$\frac{t_w}{2}$	Flange Width, b_f		Flange Thickness, t_f		Distance k k_{des}	k k_{det}	k_1	T	Workable Gage
	in.^{2}	in.		in.		in.	in.		in.		in.	in.	in.	in.	in.
W14×132	38.8	14.7	14 5/8	0.645	5/8	5/16	14.7	14 3/4	1.03	1	1.63	2 5/16	1 9/16	10	5 1/2
×120	35.3	14.5	14 1/2	0.590	9/16	5/16	14.7	14 5/8	0.940	15/16	1.54	2 1/4	1 1/2	↓	↓
×109	32.0	14.3	14 3/8	0.525	1/2	1/4	14.6	14 5/8	0.860	7/8	1.46	2 3/16	1 1/2		
×99[f]	29.1	14.2	14 1/8	0.485	1/2	1/4	14.6	14 5/8	0.780	3/4	1.38	2 1/16	1 7/16		
×90[f]	26.5	14.0	14	0.440	7/16	1/4	14.5	14 1/2	0.710	11/16	1.31	2	1 7/16		

Table 1-1 (continued)
W-Shapes
Properties

W14-W12

Nominal Wt.	Compact Section Criteria $\frac{b_f}{2t_f}$	$\frac{h}{t_w}$	Axis X-X I	S	r	Z	Axis Y-Y I	S	r	Z	r_{ts}	h_o	Torsional Properties J	C_w
lb/ft			in.^{4}	in.^{3}	in.	in.^{3}	in.^{4}	in.^{3}	in.	in.^{3}	in.	in.	in.^{4}	in.^{6}
132	7.15	17.7	1530	209	6.28	234	548	74.5	3.76	113	4.23	13.7	12.3	25500
120	7.80	19.3	1380	190	6.24	212	495	67.5	3.74	102	4.20	13.6	9.37	22700
109	8.49	21.7	1240	173	6.22	192	447	61.2	3.73	92.7	4.17	13.4	7.12	20200
99	9.34	23.5	1110	157	6.17	173	402	55.2	3.71	83.6	4.14	13.4	5.37	18000
90	10.2	25.9	999	143	6.14	157	362	49.9	3.70	75.6	4.10	13.3	4.06	16000

$$E := 29000\ ksi = \left(1.999 \cdot 10^5\right)\ MPa$$

$$F_y := 50\ ksi = 344.738\ MPa$$

$$\frac{b_f}{2 \cdot t_f} = 10.211 \qquad 0.56 \cdot \sqrt{\frac{E}{F_y}} = 13.487$$

Non-Slender Flange

$$\frac{(h - 2 \cdot (k))}{t_w} = 25.864 \qquad 1.49 \cdot \sqrt{\frac{E}{F_y}} = 35.884$$

Non-Slender Web

$$L_{cx} := 30\ ft = \left(9.144 \cdot 10^3\right)\ mm$$

$$L_{cy} := 15\ ft = \left(4.572 \cdot 10^3\right)\ mm$$

$$4.71 \cdot \sqrt{\frac{E}{F_y}} = 113.432$$

$$\frac{L_{cx}}{r_x} = 58.688$$

$$\frac{L_{cy}}{r_y} = 48.433$$

$$F_e := \frac{\pi^2 \cdot E}{\left(\frac{L_{cx}}{r_x}\right)^2} = 572.956\ MPa$$

$$\mathrm{F_{cr}} := \mathrm{if}\left(\frac{L_{cx}}{r_x} > 4.71 \cdot \sqrt{\frac{E}{F_y}}, 0.877 \cdot F_e, 0.658^{\frac{F_y}{F_e}} \cdot F_y\right) = 267.99\ MPa$$

$$F_e := \frac{\pi^2 \cdot E}{\left(\frac{L_{cy}}{r_y}\right)^2} = 841.261\ MPa$$

$$\mathbf{F_{cr}} := \mathrm{if}\left(\frac{L_{cy}}{r_y} > 4.71 \cdot \sqrt{\frac{E}{F_y}}, 0.877 \cdot F_e, 0.658^{\frac{F_y}{F_e}} \cdot F_y\right) = 290.402\ MPa$$

$$F_e := \frac{\pi^2 \cdot E}{\left(\max\left(\frac{L_{cx}}{r_x}, \frac{L_{cy}}{r_y}\right)\right)^2} = 572.956\ MPa$$

$$\mathbf{F_{cr}} := \mathrm{if}\left(\max\left(\frac{L_{cx}}{r_x}, \frac{L_{cy}}{r_y}\right) > 4.71 \cdot \sqrt{\frac{E}{F_y}}, 0.877 \cdot F_e, 0.658^{\frac{F_y}{F_e}} \cdot F_y\right) = 267.99\ MPa$$

$$\phi := 0.9$$

$$\phi P_n := \phi \cdot \mathbf{F_{cr}} \cdot A_g = \left(4.065 \cdot 10^3\right)\ kN$$

$$1.2 \cdot 140\ kip + 1.6 \cdot 420\ kip = \left(3.737 \cdot 10^3\right)\ kN$$

LRFD	ASD
$\phi_c P_n = 903$ kips > 840 kips **o.k.**	$\frac{P_n}{\Omega_c} = 601$ kips > 560 kips **o.k.**