

Exercise Sheet #8

Submit by Friday 26-03-2021

Exercise 1. - Spiral arms

Consider the stars in the Milky Way to have a constant rotation velocity of 220 km s^{-1} and assume that, when the Sun was formed $4.5 \cdot 10^9$ yr ago, the Galaxy already showed a pattern of the spiral arms similar to the present and following the same differential rotation as the rest of the disc.

- (a) How many orbits around the Galactic center would a star located at 4 kpc from the Galactic center (the inner end of an arm), have completed since the formation of the Sun? (10 points)
- (b) How many orbits would a star located at 10 kpc from the Galactic center (the outer end of the arm), have completed during the same time interval? (10 points)
- (c) Assuming that the stars populating the spiral arms have been the same during the entire life of the Sun (material arms), what would be the appearance of the spiral arm nowadays? Does this correspond to the real appearance of the spiral arms? If not, comment on possible solutions. (10 points)

Exercise 2. - Sérsic profile

The radially decreasing intensity profile of galaxies can be reasonably well described with Sérsic's (1963, 1968) $R^{(1/n)}$ model:

$$I(R) = I_e \cdot \exp(-b_n[(R/R_e)^{(1/n)} - 1]) \quad (1)$$

where I_e is the intensity at the effective radius R_e that encloses half of the total light from the model. The profile shape is determined by n , which has become widely known as the "Sérsic index". The value of b_n can be approximated with $b_n = 1.9992 \cdot n - 0.3271$ (for $0.5 < n < 10$).

The above "Sérsic profile" can be expressed in terms of surface brightness:

$$\mu(R) = \mu_e + \frac{2.5 \cdot b_n}{\ln(10)} [(R/R_e)^{(1/n)} - 1] \quad (2)$$

Note that μ_e denotes the surface brightness at R_e , as opposed to the frequently used quantity $\langle \mu_e \rangle$, which is averaged within R_e . Further information: For this and other useful quantities related to the Sérsic profile, we recommend Graham & Driver (2005, PASA 22, 118).

While the light profile of galaxy disks can usually be described with a Sérsic index of 1 (an exponential profile), bright elliptical galaxies are better described with a Sérsic index of $n = 4$ (known as "de Vaucouleurs profile"). For the sake of simplicity, here you can approximate spiral galaxies with $n = 1$ regardless of their bulge.

- (a) In the near-infrared H -band, bright ellipticals and spirals roughly have $\mu_e = 18 \text{ mag/arcsec}^2$. Suppose that the limiting surface brightness of your images is $\mu = 23 \text{ mag/arcsec}^2$. Out to how many effective radii can you see the light of the elliptical and spiral galaxy? How does this compare to the spiral galaxy? Plot the $\mu(R)$ vs. R/R_e dependence. (15 points)
- (b) Similar to the part a), but now for surface brightness at $\mu_e = 20 \text{ mag/arcsec}^2$, for same galaxies with the same limiting surface brightness $\mu = 23 \text{ mag/arcsec}^2$. Plot the $\mu(R)$ vs. R/R_e dependence. Compare and comment on difference between part a) and b). (10 points)