ZAH - Astronomisches Rechen-Institut
Galactic and Extragalactic Astronomy (MVAstro3) - Winter semester 2021
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Exercise Sheet #9

Submit by Wednesday 31-03-2021

Exercise 1. - Black Holes in Galaxy Cores

The Schwarzschild radius R_s of a black hole is defined as $R_s = 2\mathrm{G}M_{BH}/\mathrm{c}^2$, where G is the gravitational constant, c the speed of light, and M_{BH} the mass of the black hole. It is the boundary beyond which events are not visible for an outside observer.

(a) Determine the Schwarzschild radius of a black hole at the center of a galaxy, assuming a black hole mass $M_{BH} = 10^9 M_\odot$. Convert it both into angular units (arcsec) assuming a distance of 16 Mpc, as well as into astronomical units (AU). (10 points)

(b) In the giant elliptical M87 (a cD galaxy in the Virgo cluster at $d = 16\,\mathrm{Mpc}$) a gaseous disk with an inclination of 42° has been detected around the center of the galaxy and the radius has been determined to be 7.76 pc. The orbital speeds of the gas have been measured to be $1000\,\mathrm{kms}^{-1}$. What is the enclosed mass within the gas orbit? (10 points)

(c) Determine the mean density inside of the orbit of the gas and compare with the typical density at the center of globular clusters ($10^4\,\mathrm{M}_\odot\,\mathrm{pc}^{-3}$). Is it likely that the central object is composed entirely of stars? (10 points)

(d) What is the spatial resolution (in arcsec) you would need to actually resolve the rotating disk described in part b)? Is that possible? Compare to the angular resolution you would need to resolve the Schwarzschild radius from part a). (10 points)

Exercise 2. - The Faber-Jackson relation

(a) Consider the kinetic (K) and potential (U) energy of an elliptical galaxy (described as a sphere of radius R_e) to be:

$$K = \frac{3}{2} M\sigma^2 \quad , \quad U = -\frac{3}{5}\frac{\mathrm{G}M^2}{R_e}, \tag{1}$$

where the σ is the velocity dispersion. Show by means of the virial theorem that $M \propto \sigma^2 R_e$. (10 points)

(b) Assuming the mass-to-light ratio M/L and the effective surface brightness μ_e to be constant and $L \propto \mu_e R_e^2$, show that $L \propto \sigma^4$ (the so-called Faber-Jackson relation) holds. (10 points)