Introduction to Algorithms, Fall, 2023 Programming Homework 5

Problem A Maximum Sum Path on DAG

Time limit: 1 second Memory limit: 2048 megabytes

Problem Description

Given a Directed Acyclic Graph (DAG) with n vertices and m directed edges, where the vertices are numbered from 1 to n. The i-th vertex has a weight a_i .

A path in a graph is a sequence of distinct verticies v_1, v_2, \ldots, v_k , where $k \geq 1$ and each pair of adjacent vertices share an edge. Find the maximum sum of weights among all paths in the given DAG.

Input Format

The first line of the input contains two integers n and m. The second line of input contains n space-separated integers a_1, a_2, \ldots, a_n . Each of the following m lines contains two integers u_i and v_i , where the i-th line denotes that there is a directed edge from vertex u_i to v_i in the graph.

Output Format

Output the maximum sum of weights in one line.

Technical Specification

- $1 \le n \le 2 \times 10^5$
- $0 \le m \le \min(\frac{n(n-1)}{2}, 2 \times 10^5)$
- $-10^9 \le a_i \le 10^9 \text{ for } i = 1, 2, \dots, n$
- $1 \le u_i, v_i \le n \text{ for } i = 1, 2, \dots, m$
- It is guaranteed that the input graph does not contain loops and multiple edges.
- It is guaranteed that the input graph is a DAG.

Scoring

- 1. (30 points) m = n 1 and $(u_i, v_i) = (i, i + 1)$ for i = 1, 2, ..., n 1.
- 2. (70 points) No additional constraints.

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Sample Input 1



Sample Output 1

6

Sample Input 2

```
8 6
-2 6 8 -1 3 0 -4 5
3 4
1 3
7 2
5 2
3 6
5 7
```

Sample Output 2

9

Sample Input 3

1 0 -100000000

Sample Output 3

-100000000

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Sample Input 4

Sample Output 4

399999000

Hint

Do a topological sort on the DAG, and try some dynamic programming algorithms with it.