Introduction to Algorithms, Fall, 2023 Programming Homework 3

Problem B2 Dynamic Order Statistic Queries (Hard Version)

Time limit: 1 second

Memory limit: 2048 megabytes

Problem Description

Given an empty array, a fixed positive integer k and q queries to be performed on the array. Each query belongs to one of the following two types:

- 1. 1 x_i —Append x_i to the end of the array.
- 2. 2 —Find the k-th order statistic of the numbers from the array and remove **one copy** of it. If there are fewer than k elements in the array, nothing is done to the array.

Can you answer all these queries?

Input Format

The first line of the input contains two integers q and k. The i-th of the following q lines contains the i-th query in the format as in the problem description.

Output Format

For each query of type 2, output the k-th order statistic in one line. If there are fewer than k elements in the array, output -1.

Technical Specification

- $1 \le q \le 3 \times 10^5$
- $1 \le x_i \le 10^9$ for $i = 1, 2, \dots, q$ of query type 1
- It is guaranteed that there is at least one query of type 2 in the input.

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Sample Input 1



Sample Output 1

```
-1
-1
7
3
5
9
12
-1
-1
```

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Sample Input 2

```
7 1
1 1000000000
1 1000000000
2
2
2
1 1000000000
```

Sample Output 2

```
100000000
100000000
-1
100000000
```

Hint

Two heaps are required in this problem. One is a max heap holding the k smallest values, and the other one is a min heap holding other values.