Introduction to Algorithms, Fall, 2023 Programming Homework 1

Problem B Range Order Statistic Queries

Time limit: 3 seconds

Memory limit: 2048 megabytes

Problem Description

Given an array of n integers a_1, a_2, \ldots, a_n and q queries to be performed on these integers. Each query belongs to one of the following two types:

- 1. 1 $idx_i \ v_i$ —Change a_{idx_i} to v_i . That is, $a_{idx_i} := v_i$.
- 2. 2 $l_i r_i k_i$ —Find the k_i -th order statistic of the numbers $a_{l_i}, a_{l_i+1}, \ldots, a_{r_i}$. In other words, sort the numbers $a_{l_i}, a_{l_i+1}, \ldots, a_{r_i}$ in ascending order and output the k_i -th element. **Note** that k_i is at most 5 in this problem.

Can you answer all these queries?

Input Format

The first line of the input contains two integers n and q. The second line of the input contains n space-separated integers a_1, a_2, \ldots, a_n . The i-th of the following q lines contains the i-th query in the format as in the problem description.

Output Format

For each query of type 2, output the k_i -th order statistic in one line.

Technical Specification

- $1 \le n, q \le 2 \times 10^5$
- $1 < a_i < 10^9 \text{ for } i = 1, 2, \dots, n$
- $1 \le idx_i \le n$ for i = 1, 2, ..., q of query type 1
- $1 \le v_i \le 10^9$ for i = 1, 2, ..., q of query type 1
- $1 \le l_i \le r_i \le n$ for $i = 1, 2, \dots, q$ of query type 2
- $1 \le k_i \le \min(5, r_i l_i + 1)$ for i = 1, 2, ..., q of query type 2
- It is guaranteed that there is at least one query of type 2 in the input.

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Sample Input 1

```
      5
      4

      3
      2
      1
      5
      4

      2
      2
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      1
      1</td
```

Sample Output 1

```
1
5
2
```

Sample Input 2

```
6 7
3 6 10 3 5 8
2 1 6 5
2 2 5 3
1 3 1
2 3 6 4
1 6 4
2 4 6 2
2 2 2 1
```

Sample Output 2

```
8
6
8
4
6
```

Hint

Use a segment tree to perform these queries. Since the value of k_i is at most 5, we can keep the 5 smallest values in each node of the segment tree.

When constructing the segment tree, each internal node has to fetch the 5 smallest values from its left child and 5 from its right child, and select the 5 smallest values among them. This can

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be done by merging two sorted arrays and select the first few elements of the merged array. In C++, you can use the 'std::merge' function from the standard library.

Updates and queries can also be performed in a similar way.