Introduction to Algorithms, Fall, 2023 Final Exam

Problem E Shortest Paths

Time limit: 2 seconds

Memory limit: 2048 megabytes

Problem Description

You are given an undirected graph containing n vertices and m undirected edges, where the vertices are numbered from 1 to n. Every edge in this graph is associated with a weight w_i .

A shortest path from vertex u to vertex v is a path between u and v where no other paths are shorter than it. It is important to note that there can be multiple shortest paths within a graph. For instance, consider a three-node undirected graph with weighted edges: (1,2) with a weight of 1, (2,3) with a weight of 1, and (1,3) with a weight of 2. In this case, there are two shortest paths from vertex 1 to vertex 3: $1 \to 2 \to 3$ and $1 \to 3$.

This problem comprises two types of questions falling into the following categories:

- 1. Determine the length of the shortest path from vertex 1 to vertex 2.
- 2. Calculate the count of shortest paths from vertex 1 to vertex 2. Output the answer modulo $10^9 + 7$.

Note that the edges weights are always positive in this problem.

Input Format

The first line of the input contains three integers n, m and type. type denotes the type of questions: type = 1 denotes the first type while type = 2 denotes the second type. Each of the following m lines contains three integers u_i , v_i and w_i , where the i-th line denotes that there is an undirected edge from vertex u_i to v_i in the graph.

Output Format

For type = 1, output the length of the shortest path from vertex 1 to vertex 2. For type = 2, output the count of shortest paths from vertex 1 to vertex 2 modulo $10^9 + 7$. In any type of the question, if vertex 2 is unreachable from vertex 1, output -1 instead.

Technical Specification

- $2 \le n \le 2 \times 10^5$
- $0 \le m \le 2 \times 10^5$
- $1 \le type \le 2$
- $1 \le u_i, v_i \le n$ and $u_i \ne v_i$ for $i = 1, 2, \dots, m$
- $1 \le w_i \le 10^8$ for $i = 1, 2, \dots, m$

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Scoring

- 1. (4 points) type = 1, $1 \le n \le 1000$ and $0 \le m \le 1000$.
- 2. (5 points) type = 1.
- 3. (4 points) type = 2, $1 \le n \le 1000$ and $0 \le m \le 1000$.
- 4. (2 points) type = 2.

Sample Input 1

```
7
8
1

1
6
1

6
3
2

1
4
2

4
3
1

3
5
3

5
2
4

3
7
1

7
2
6
```

Sample Output 1

10

Sample Input 2

```
7
8
2

1
6
1

6
3
2

1
4
2

4
3
1

3
5
3

5
2
4

3
7
1

7
2
6
```

Sample Output 2

4

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Sample Input 3

156156 0 2

Sample Output 3

-1

Sample Input 4

4 6 1

1 3 5

3 2 4

3 2 5

2 4 7

1 2 100

3 4 10

Sample Output 4

9