Figure 1: BIC Plots. This figure shows the plots of Bayesian Information Criterion (BIC) value against moving average window size (m) for all the specifications of the MF2-GARCH-in-mean model *(see Table 4 for parameter estimates)*. The optimal m value is chosen as the one that minimizes the BIC. The left column shows proportional variants and the right column shows the non-proportional ones. From top to bottom, the rows show the plots for the short-term component specification, the long-term component specification, the two component specification and the overall conditional variance specification.

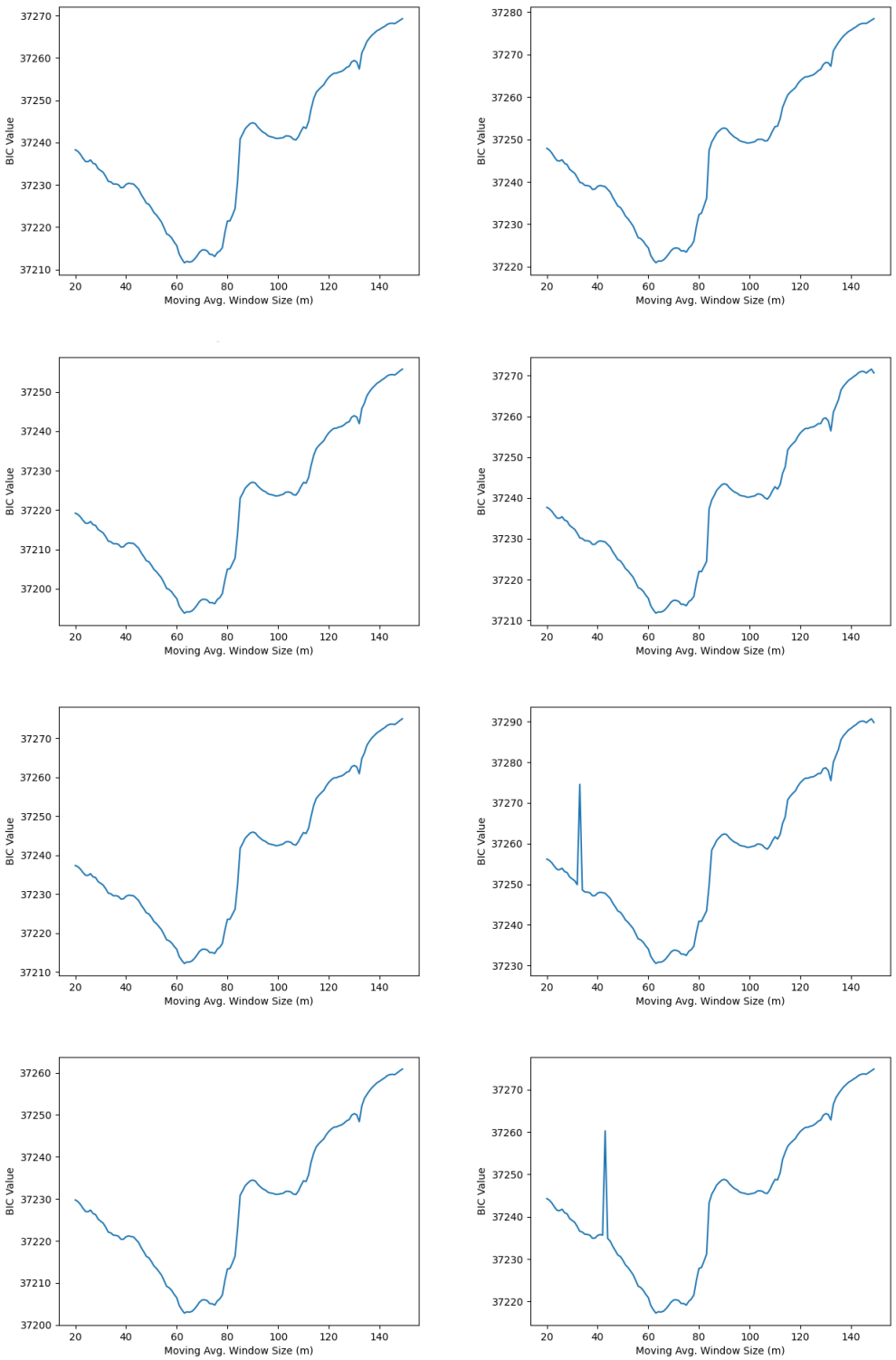

Table 1: Parameter Values for Monte Carlo Simulations

α	γ	β	λ_0	λ_1	λ_2	δ_0	$\delta_{1,s}$	$\delta_{1,l}$	δ_1
Short-term component									
Proportional									
0.006	0.160	0.842	0.011	0.085	0.902	-	0.027	-	-
Non-Proportional									
0.006	0.160	0.842	0.011	0.085	0.902	0.033	-0.003	-	-
Long-term component									
Proportional									
0.006	0.160	0.842	0.011	0.085	0.902	-	-	0.049	-
Non-Proportional									
0.006	0.160	0.842	0.011	0.085	0.902	0.003	-	0.045	-
Both components (additive)									
Proportional									
0.006	0.160	0.842	0.011	0.085	0.902	-	-0.005	0.054	-
Non-Proportional									
0.006	0.160	0.842	0.011	0.085	0.902	0.008	-0.008	0.046	-
Overall conditional variance (multiplicative)									
Proportional									
0.006	0.160	0.842	0.011	0.085	0.902	-	-	-	0.042
Non-Proportional									
0.006	0.160	0.842	0.011	0.085	0.902	0.020	-	-	0.023

Notes: This table presents the "true" parameter values I used in Monte Carlo simulations of daily market premium data. The MF2-GARCH-in-mean model is fitted $R = 1,000$ times on these simulated samples, each of size $T = 15,120$. This is repeated for the proportional and non-proportional variant of every specification. These values were chosen based on estimates from real data *(see Table 4 below).*

Table 2: Summary Statistics - Market Premia

	mean	sd	skew	kurtosis	min	max	AC(1)
r_t	0.028	1.028	-0.487	15.564	-17.440	11.360	0.016

Notes: This table shows summary statistics for the U.S. daily market premium data. The data runs from January 1964 to April 2025. The columns present the mean, standard deviation (sd), skewness, kurtosis, minimum (min), maximum (max) and the first-order autocorrelation coefficient (AC(1)).

Table 3: NBER Recession Periods

Start date	End date	Remarks
December 1969	November 1970	-
November 1973	March 1975	1973 oil crisis and stagflation
January 1980	July 1980	Volcker recession I
July 1981	November 1982	Volcker recession II
July 1990	March 1991	-
March 2001	November 2001	Dot-com bubble
December 2007	June 2009	Global financial crisis
February 2020	April 2020	COVID-19 pandemic

Notes: This table shows the start and end dates of recession periods defined by the U.S. National Bureau of Economic Research (NBER) which fall within the sample period on which the MF2-GARCH-in-mean model is estimated. The dummy variable used to control for periods of crisis is given the value 1 for the above periods and 0 otherwise.

Table 4: Combined Specification Parameter Estimates (Controlling for Crises)

α	γ	β	λ_0	λ_1	λ_2	δ_0	$\delta_{1,s}$	$\delta_{1,l}$	δ_1	θ_0	$\theta_{1,s}$	$\theta_{1,l}$	θ_1	LLF	BIC
Panel A: Short-term component															
Proportional															
0.007	0.158***	0.841***	0.012	0.086	0.900***	-	0.027**	-	-	-	-0.013	-	-	-18567	37212
(0.014)	(0.019)	(0.018)	(0.020)	(0.150)	(0.173)		(0.011)				(0.020)				
Non-Proportional															
0.005*	0.166***	0.845***	0.011***	0.081***	0.907***	0.033***	-0.003***	-	-	0.009	-0.009	-	-	-18562	37221
(0.003)	(0.015)	(0.012)	(0.002)	(0.018)	(0.018)	(0.007)	(0.001)			(0.008)	(0.009)				
LRT	9.928***														
Panel B: Long-term component															
Proportional															
0.006**	0.160***	0.842***	0.011***	0.085***	0.902***	-	-	0.049***	-	-	-	-0.002***	-	-18558	**37194**
(0.003)	(0.015)	(0.013)	(0.003)	(0.018)	(0.020)			(0.009)				(0.001)			
Non-Proportional															
0.006*	0.161***	0.843***	0.011*	0.080***	0.907***	0.003	-	0.045***	-	-0.054	-	0.049	-	-18558	37212
(0.004)	(0.015)	(0.014)	(0.006)	(0.030)	(0.036)	(0.003)		(0.017)		(0.060)		(0.049)			
LRT	1.358														
Panel C: Both components (additive)															
Proportional															
0.006	0.161***	0.844***	0.012*	0.085**	0.901***	-	-0.005	0.054**	-	-	-0.012**	0.009	-	-18558	37212
(0.013)	(0.015)	(0.016)	(0.006)	(0.035)	(0.042)		(0.021)	(0.023)			(0.006)	(0.012)			
Non-Proportional															
0.006	0.162***	0.844***	0.011	0.081**	0.906***	0.008	-0.008	0.046***	-	-0.050	-0.002	0.048	-	-18557	37230
(0.003)	(0.016)	(0.014)	(0.007)	(0.036)	(0.043)	(0.006)	(0.007)	(0.012)		(0.067)	(0.002)	(0.041)			
LRT	0.990														
Panel D: Overall conditional variance (multiplicative)															
Proportional															
0.007	0.158***	0.839***	0.011**	0.080***	0.907***	-	-	-	0.042***	-	-	-	-0.014*	-18563	37203
(0.009)	(0.016)	(0.014)	(0.005)	(0.031)	(0.035)				(0.010)				(0.008)		
Non-Proportional															
0.006	0.162***	0.841***	0.011	0.080	0.908***	0.020	-	-	0.023	-0.005	-	-	-0.004	-18560	37217
(0.012)	(0.025)	(0.018)	(0.016)	(0.084)	(0.101)	(0.059)			(0.083)	(0.016)			(0.022)		
LRT	4.850*														

Notes: This table shows the results of the QMLE parameter estimates for the MF2-GARCH-in-mean model. The numbers in parentheses are Bollerslev–Wooldridge robust standard errors. *** ,** and * indicate significance at the 1%, 5% and 10% level. Each panel shows the results for a different specification, depending on which components of volatility are included. Each panel shows the proportional (no intercept) and non-proportional variant. The likelihood ratio test (LRT) statistic for the proportional variant against the non-proportional variant is also shown at the bottom of each panel. All specifications are estimated using daily U.S. market premium data for the period January 1964 to April 2025 inclusive. The moving average window size (m) which minmizes the Bayesian Information Criterionfor all specifications is m=63

Table 5: Summary Statistics - Volatility
(Proportional Long-Term Component Specification)

	mean	min	max	AC(1)
σ_t^2	1.051	0.122	60.992	0.94736
h_t	1.186	0.474	69.731	0.98251
τ_t	0.830	0.236	3.810	0.99988

Notes: This table shows summary statistics for the conditional variance and its components as estimated by MF2-GARCH in the proportional (no intercept) long-term component specification.

Table 6: Monte Carlo Parameter Estimates & Standard Deviations

	α	γ	β	λ_0	λ_1	λ_2	δ_0	$\delta_{1,s}$	$\delta_{1,l}$	δ_1
Short-term component										
Proportional										
True	0.00600	0.16000	0.84200	0.01100	0.08500	0.90200	-	0.02700	-	-
Bias	-0.00116	-0.00128	0.00198	0.00082	0.00119	-0.00226	-	-0.00011	-	-
(%)	(-19.33%)	(-0.80%)	(0.24%)	(7.45%)	(1.40%)	(-0.25%)	-	(-0.41%)	-	-
S.d.	0.00353	0.00684	0.00675	0.00601	0.03777	0.04460	-	0.00468	-	-
Non-Proportional										
True	0.00600	0.16000	0.84200	0.01100	0.08500	0.90200	0.03300	-0.00300	-	-
Bias	-0.00115	-0.00072	0.00122	0.00086	0.00147	-0.00259	-0.00155	0.00150	-	-
(%)	(-19.17%)	(-0.45%)	(0.14%)	(7.82%)	(1.73%)	(-0.29%)	(-4.70%)	(-50.00%)	-	-
S.d.	0.00356	0.00691	0.00711	0.00603	0.03789	0.04476	0.01192	0.01303	-	-
Long-term component										
Proportional										
True	0.00600	0.16000	0.84200	0.01100	0.08500	0.90200	-	-	0.04900	-
Bias	-0.00116	-0.00070	0.00139	0.00071	0.00052	-0.00145	-	-	-0.00010	-
(%)	(-19.33%)	(-0.44%)	(0.17%)	(6.45%)	(0.61%)	(-0.16%)	-	-	(-0.20%)	-
S.d.	0.00353	0.00693	0.00678	0.00578	0.03593	0.04253	-	-	0.00636	-
Non-Proportional										
True	0.00600	0.16000	0.84200	0.01100	0.08500	0.90200	0.00300	-	0.04500	-
Bias	-0.00116	-0.00070	0.00140	0.00073	0.00075	-0.00170	-0.00035	-	0.00022	-
(%)	(-19.33%)	(-0.44%)	(0.17%)	(6.64%)	(0.88%)	(-0.19%)	(-11.67%)	-	(0.49%)	-
S.d.	0.00355	0.00692	0.00679	0.00577	0.03660	0.04314	0.01413	-	0.01833	-
Both components (additive)										
Proportional										
True	0.00600	0.16000	0.84200	0.01100	0.08500	0.90200	-	-0.00500	0.05400	-
Bias	-0.00116	-0.00064	0.00124	0.00066	0.00050	-0.00138	-	0.00057	-0.00079	-
(%)	(-19.33%)	(-0.40%)	(0.15%)	(6.00%)	(0.59%)	(-0.15%)	-	(-11.40%)	(-1.46%)	-
S.d.	0.00354	0.00692	0.00702	0.00563	0.03599	0.04239	-	0.01071	0.01272	-
Non-Proportional										
True	0.00600	0.16000	0.84200	0.01100	0.08500	0.90200	0.00800	-0.00800	0.04600	-
Bias	-0.00115	-0.00061	0.00108	0.00068	0.00054	-0.00145	-0.00166	0.00156	0.00010	-
(%)	(-19.17%)	(-0.38%)	(0.13%)	(6.18%)	(0.64%)	(-0.16%)	(-20.75%)	(-19.50%)	(0.22%)	-
S.d.	0.00356	0.00693	0.00710	0.00568	0.03638	0.04281	0.01735	0.01309	0.01863	-
Overall conditional variance (multiplicative)										
Proportional										
True	0.00600	0.16000	0.84200	0.01100	0.08500	0.90200	-	-	-	0.04200
Bias	-0.00119	-0.00144	0.00211	0.00077	0.00095	-0.00195	-	-	-	-0.00006
(%)	(-19.83%)	(-0.90%)	(0.25%)	(7.00%)	(1.12%)	(-0.22%)	-	-	-	(-0.14%)
S.d.	0.00350	0.00682	0.00672	0.00594	0.03701	0.04378	-	-	-	0.00610
Non-Proportional										
True	0.00600	0.16000	0.84200	0.01100	0.08500	0.90200	0.02000	-	-	0.02300
Bias	-0.00117	-0.00109	0.00176	0.00081	0.00116	-0.00221	-0.00018	-	-	0.00011
(%)	(-19.50%)	(-0.68%)	(0.21%)	(7.36%)	(1.36%)	(-0.25%)	(-0.90%)	-	-	(0.48%)
S.d.	0.00353	0.00686	0.00689	0.00600	0.03745	0.04429	0.00846	-	-	0.01140

Notes: This table presents the results of MF2-GARCH-in-mean QMLE parameter estimation on data generated by Monte Carlo simulations of daily market premia. Each specification was fitted on a simulated sample of size $T = 30,240$ and this was repeated $R = 1,000$ times. The table shows the true parameter values (True), the average bias of the parameter estimates (value and percent), and the standard deviation (S.d.) of the parameter estimates across the 1,000 simulations.