Table 1: Parameter Values for Monte Carlo Simulations

| $\alpha$ | $\gamma$ | $\beta$ | $\lambda_0$ | $\lambda_1$ | $\lambda_2$ | $\delta_0$ | $\delta_{1,s}$ | $\delta_{1,l}$ | $\delta_1$ |
|---|---|---|---|---|---|---|---|---|---|
| **Short-term component** | | | | | | | | | |
| Proportional | | | | | | | | | |
| 0.006 | 0.160 | 0.842 | 0.011 | 0.085 | 0.902 | - | 0.027 | - | - |
| Non-Proportional | | | | | | | | | |
| 0.006 | 0.160 | 0.842 | 0.011 | 0.085 | 0.902 | 0.033 | -0.003 | - | - |
| **Long-term component** | | | | | | | | | |
| Proportional | | | | | | | | | |
| 0.006 | 0.160 | 0.842 | 0.011 | 0.085 | 0.902 | - | - | 0.049 | - |
| Non-Proportional | | | | | | | | | |
| 0.006 | 0.160 | 0.842 | 0.011 | 0.085 | 0.902 | 0.003 | - | 0.045 | - |
| **Both components (additive)** | | | | | | | | | |
| Proportional | | | | | | | | | |
| 0.006 | 0.160 | 0.842 | 0.011 | 0.085 | 0.902 | - | -0.005 | 0.054 | - |
| Non-Proportional | | | | | | | | | |
| 0.006 | 0.160 | 0.842 | 0.011 | 0.085 | 0.902 | 0.008 | -0.008 | 0.046 | - |
| **Overall conditional variance (multiplicative)** | | | | | | | | | |
| Proportional | | | | | | | | | |
| 0.006 | 0.160 | 0.842 | 0.011 | 0.085 | 0.902 | - | - | - | 0.042 |
| Non-Proportional | | | | | | | | | |
| 0.006 | 0.160 | 0.842 | 0.011 | 0.085 | 0.902 | 0.020 | - | - | 0.023 |

**Notes:** This table presents the "true" parameter values I used in Monte Carlo simulations of daily market premium data. The MF2-GARCH-in-mean model is fitted $R = 1,000$ times on these simulated samples, each of size $T = 15,120$. This is repeated for the proportional and non-proportional variant of every specification. These values were chosen based on estimates from real data *(see Table 4 below).*

Table 2: Summary Statistics - Market Premia

| | mean | sd | skew | kurtosis | min | max | AC(1) |
|---|---|---|---|---|---|---|---|
| $r_t$ | 0.028 | 1.028 | -0.487 | 15.564 | -17.440 | 11.360 | 0.016 |

**Notes:** This table shows summary statistics for the U.S. daily market premium data. The data runs from January 1964 to April 2025. The columns present the mean, standard deviation (sd), skewness, kurtosis, minimum (min), maximum (max) and the first-order autocorrelation coefficient (AC(1)).

Table 3: NBER Recession Periods

| Start date | End date | Remarks |
|---|---|---|
| December 1969 | November 1970 | - |
| November 1973 | March 1975 | 1973 oil crisis and stagflation |
| January 1980 | July 1980 | Volcker recession I |
| July 1981 | November 1982 | Volcker recession II |
| July 1990 | March 1991 | - |
| March 2001 | November 2001 | Dot-com bubble |
| December 2007 | June 2009 | Global financial crisis |
| February 2020 | April 2020 | COVID-19 pandemic |

**Notes:** This table shows the start and end dates of recession periods defined by the U.S. National Bureau of Economic Research (NBER) which fall within the sample period on which the MF2-GARCH-in-mean model is estimated. The dummy variable used to control for periods of crisis is given the value 1 for the above periods and 0 otherwise.

Table 5: Summary Statistics - Volatility
(Proportional Long-Term Component Specification)

| | mean | min | max | AC(1) |
|---|---|---|---|---|
| $\sigma_t^2$ | 1.051 | 0.122 | 60.992 | 0.94736 |
| $h_t$ | 1.186 | 0.474 | 69.731 | 0.98251 |
| $\tau_t$ | 0.830 | 0.236 | 3.810 | 0.99988 |

**Notes:** This table shows summary statistics for the conditional variance and its components as estimated by MF2-GARCH in the proportional (no intercept) long-term component specification.

Table 6: Monte Carlo Parameter Estimates & Standard Deviations

| | $\alpha$ | $\gamma$ | $\beta$ | $\lambda_0$ | $\lambda_1$ | $\lambda_2$ | $\delta_0$ | $\delta_{1,s}$ | $\delta_{1,l}$ | $\delta_1$ |
|---|---|---|---|---|---|---|---|---|---|---|
| **Short-term component** | | | | | | | | | | |
| **Proportional** | | | | | | | | | | |
| True | 0.00600 | 0.16000 | 0.84200 | 0.01100 | 0.08500 | 0.90200 | - | 0.02700 | - | - |
| Bias | -0.00116 | -0.00128 | 0.00198 | 0.00082 | 0.00119 | -0.00226 | - | -0.00011 | - | - |
| (%) | (-19.33%) | (-0.80%) | (0.24%) | (7.45%) | (1.40%) | (-0.25%) | - | (-0.41%) | - | - |
| S.d. | 0.00353 | 0.00684 | 0.00675 | 0.00601 | 0.03777 | 0.04460 | - | 0.00468 | - | - |
| **Non-Proportional** | | | | | | | | | | |
| True | 0.00600 | 0.16000 | 0.84200 | 0.01100 | 0.08500 | 0.90200 | 0.03300 | -0.00300 | - | - |
| Bias | -0.00115 | -0.00072 | 0.00122 | 0.00086 | 0.00147 | -0.00259 | -0.00155 | 0.00150 | - | - |
| (%) | (-19.17%) | (-0.45%) | (0.14%) | (7.82%) | (1.73%) | (-0.29%) | (-4.70%) | (-50.00%) | - | - |
| S.d. | 0.00356 | 0.00691 | 0.00711 | 0.00603 | 0.03789 | 0.04476 | 0.01192 | 0.01303 | - | - |
| **Long-term component** | | | | | | | | | | |
| **Proportional** | | | | | | | | | | |
| True | 0.00600 | 0.16000 | 0.84200 | 0.01100 | 0.08500 | 0.90200 | - | - | 0.04900 | - |
| Bias | -0.00116 | -0.00070 | 0.00139 | 0.00071 | 0.00052 | -0.00145 | - | - | -0.00010 | - |
| (%) | (-19.33%) | (-0.44%) | (0.17%) | (6.45%) | (0.61%) | (-0.16%) | - | - | (-0.20%) | - |
| S.d. | 0.00353 | 0.00693 | 0.00678 | 0.00578 | 0.03593 | 0.04253 | - | - | 0.00636 | - |
| **Non-Proportional** | | | | | | | | | | |
| True | 0.00600 | 0.16000 | 0.84200 | 0.01100 | 0.08500 | 0.90200 | 0.00300 | - | 0.04500 | - |
| Bias | -0.00116 | -0.00070 | 0.00140 | 0.00073 | 0.00075 | -0.00170 | -0.00035 | - | 0.00022 | - |
| (%) | (-19.33%) | (-0.44%) | (0.17%) | (6.64%) | (0.88%) | (-0.19%) | (-11.67%) | - | (0.49%) | - |
| S.d. | 0.00355 | 0.00692 | 0.00679 | 0.00577 | 0.03660 | 0.04314 | 0.01413 | - | 0.01833 | - |
| **Both components (additive)** | | | | | | | | | | |
| **Proportional** | | | | | | | | | | |
| True | 0.00600 | 0.16000 | 0.84200 | 0.01100 | 0.08500 | 0.90200 | - | -0.00500 | 0.05400 | - |
| Bias | -0.00116 | -0.00064 | 0.00124 | 0.00066 | 0.00050 | -0.00138 | - | 0.00057 | -0.00079 | - |
| (%) | (-19.33%) | (-0.40%) | (0.15%) | (6.00%) | (0.59%) | (-0.15%) | - | (-11.40%) | (-1.46%) | - |
| S.d. | 0.00354 | 0.00692 | 0.00702 | 0.00563 | 0.03599 | 0.04239 | - | 0.01071 | 0.01272 | - |
| **Non-Proportional** | | | | | | | | | | |
| True | 0.00600 | 0.16000 | 0.84200 | 0.01100 | 0.08500 | 0.90200 | 0.00800 | -0.00800 | 0.04600 | - |
| Bias | -0.00115 | -0.00061 | 0.00108 | 0.00068 | 0.00054 | -0.00145 | -0.00166 | 0.00156 | 0.00010 | - |
| (%) | (-19.17%) | (-0.38%) | (0.13%) | (6.18%) | (0.64%) | (-0.16%) | (-20.75%) | (-19.50%) | (0.22%) | - |
| S.d. | 0.00356 | 0.00693 | 0.00710 | 0.00568 | 0.03638 | 0.04281 | 0.01735 | 0.01309 | 0.01863 | - |
| **Overall conditional variance (multiplicative)** | | | | | | | | | | |
| **Proportional** | | | | | | | | | | |
| True | 0.00600 | 0.16000 | 0.84200 | 0.01100 | 0.08500 | 0.90200 | - | - | - | 0.04200 |
| Bias | -0.00119 | -0.00144 | 0.00211 | 0.00077 | 0.00095 | -0.00195 | - | - | - | -0.00006 |
| (%) | (-19.83%) | (-0.90%) | (0.25%) | (7.00%) | (1.12%) | (-0.22%) | - | - | - | (-0.14%) |
| S.d. | 0.00350 | 0.00682 | 0.00672 | 0.00594 | 0.03701 | 0.04378 | - | - | - | 0.00610 |
| **Non-Proportional** | | | | | | | | | | |
| True | 0.00600 | 0.16000 | 0.84200 | 0.01100 | 0.08500 | 0.90200 | 0.02000 | - | - | 0.02300 |
| Bias | -0.00117 | -0.00109 | 0.00176 | 0.00081 | 0.00116 | -0.00221 | -0.00018 | - | - | 0.00011 |
| (%) | (-19.50%) | (-0.68%) | (0.21%) | (7.36%) | (1.36%) | (-0.25%) | (-0.90%) | - | - | (0.48%) |
| S.d. | 0.00353 | 0.00686 | 0.00689 | 0.00600 | 0.03745 | 0.04429 | 0.00846 | - | - | 0.01140 |

**Notes:** This table presents the results of MF2-GARCH-in-mean QMLE parameter estimation on data generated by Monte Carlo simulations of daily market premia. Each specification was fitted on a simulated sample of size $T = 30,240$ and this was repeated $R = 1,000$ times. The table shows the true parameter values (True), the average bias of the parameter estimates (value and percent), and the standard deviation (S.d.) of the parameter estimates across the 1,000 simulations.