HOK Physics Appendix v1.0

From Ontic Kernel to the Physical Universe

YKS
Independent Researcher
human.ontic.kernel@proton.me

December 2025

Contents

0. Meta Layer: HOK Kernel and Mapping to Physics

0.1 Informal Kernel Postulates

The HOK (Human OS Kernel) framework assumes that the physical universe is not fundamentally a smooth manifold, but a discrete, causally ordered graph

$$G_F = (V, E, L), \tag{1}$$

where

- V is the set of nodes,
- E is the set of directed (or causal) edges,
- L is a labelling that carries the state of fields, boundaries and containers.

On top of this graph, HOK defines five primitive elements

$$P = \{F, B, C, V, T\}, \tag{2}$$

interpreted as follows:

- F (Field): the background structure where tension can be distributed and propagated.
- B (Boundary): topological operations that carve out regions in G_F.
- C (Container): connected components defined by B, corresponding to "objects" or "systems".
- V (Vector): the kinematic state of containers (velocity, direction, etc.).
- T (Tension): deviation from preferred configurations; the only fundamental driver in the kernel.

Informally, the kernel postulates can be summarized as:

1. **Information substrate.** The universe is a gigantic but locally finite causal graph G_F. All physical phenomena correspond to update rules on this labelled graph.
2. **Discrete ticks.** Time is not a continuous parameter but an ordered sequence of local updates. Each tick corresponds to a (possibly asynchronous) update of a finite subgraph of G_F.
3. **Boundaries and containers.** Any "object" is represented as a container C, defined by a set of boundary operations B on G_F.
4. **Tension first.** Traditional physics uses "forces" and potentials as primitives. HOK instead treats *tension* $\mathcal{T}$ as the only primitive drive, and regards all dynamics as local rules that tend to relax $\mathcal{T}$ under various constraints.

In the appropriate continuum limit, G_F can be approximated by a manifold equipped with metric and fields. Standard physical theories—classical mechanics, electromagnetism, relativity, quantum mechanics—then appear as effective descriptions of the underlying kernel dynamics.

0.2 Notation and Kernel Scales

To relate the discrete kernel to continuum physics, we introduce three kernel scales:

- l^*: minimal spatial resolution (length tick),
- t^*: minimal temporal resolution (time tick),
- ε^*: minimal unit of energy/tension.

In the continuum limit we recover the familiar physical constants $c, \hbar, k_B, G$. At the level of this appendix we only assume that there exist finite, non–zero relations of the schematic form

$$c \sim \frac{l^*}{t^*}, \qquad \hbar_{\text{eff}} \sim \varepsilon^* t^*, \tag{3}$$

where $\hbar_{\text{eff}}$ is the effective action quantum when the kernel is coarse–grained into a continuum theory. The precise derivation and numerical locking of these relations belong to the L3 layer (see Section 0.4).

For convenience, we will use the following notational conventions throughout:

- Bold symbols such as $\boldsymbol{x}$ denote spatial vectors.
- Time derivatives are written as $\dot{\boldsymbol{x}} = d\boldsymbol{x}/dt$, $\ddot{\boldsymbol{x}} = d^2\boldsymbol{x}/dt^2$ in the continuum description.
- $\mathcal{T}(\boldsymbol{x})$ denotes a configuration tension assigned by the kernel to a given configuration $\boldsymbol{x}$ of a container C.

0.3 Mapping Table: From Physics to HOK Primitives

To avoid the impression that HOK simply "renames" known concepts, we make the mapping from standard physics to kernel primitives explicit. Table 1 provides a first–order translation.

Physics concept	**HOK primitive(s)**	**Interpretation**
Spacetime	Field F	Continuum limit of the causal graph G_F with metric structure.
Massive particle / object	Container C with strong boundary B	A stable, localized region of G_F where tension is concentrated and bounded.
Free inertial motion	Container C moving in a region with nearly uniform $\mathcal{T}$	Worldline of C in a "flat–tension" region of the field.
Force $\boldsymbol{F}$	$-\nabla\mathcal{T}$	Effective description of the gradient of configuration tension in the continuum limit.
Potential energy $U(\boldsymbol{x})$	Configuration tension $\mathcal{T}(\boldsymbol{x})$	Stored tension due to displacement from preferred configurations.
Geodesic	Solver path of C in F	Path that extremizes the global tension/action functional under given constraints.
Inertial frame	Region of F with $\nabla\mathcal{T} \approx 0$	Local patch where C feels no net tension gradient; motion appears force–free.
Light speed c	Update limit of F	Maximum speed at which changes in labels on G_F can causally propagate.
Field (EM, scalar, etc.)	Structured labels on F	Additional degrees of freedom attached to nodes/edges of G_F, encoding extra tension channels.

Table 1: First–order mapping between standard physics concepts and HOK primitives. The goal of this appendix is to show that, given this mapping, standard dynamical laws can be recovered (L2) and constants can in principle be constrained (L3).

In the following chapters we will see that classical mechanics, field theory and parts of relativity can be re–expressed entirely in terms of these kernel objects, without introducing any additional ontological ingredients.

0.4 L1 / L2 / L3 Classification of Results

Throughout this appendix we will label results according to three levels:

- **L1 (Conceptual / Geometric).** The mapping between HOK primitives and standard physical concepts is clear and internally consistent, but no new equations are derived. Example: interpreting an inertial frame as a flat–tension region.
- **L2 (Formal / Equation level).** Standard equations are re–derived from kernel assumptions, or written in kernel language. Example: deriving Newton's second law $m\ddot{\boldsymbol{x}} = -\nabla\mathcal{T}$ from a tension–based action.
- **L3 (Numerical / Constant level).** Dimensionful and dimensionless constants (such as $c, \hbar, G, k_B, \alpha$) are related to discrete properties of G_F. At this level the goal is to constrain or compute numerical values, not just reproduce known structures.

Most of this appendix lives at the L1/L2 boundary. A few sections explicitly discuss possible L3 routes, but those are to be read as *roadmaps* rather than claims of completed derivations.

0.5 Scope and Limitations

The purpose of this appendix is modest but precise:

1. to show that a single kernel language based on (F, B, C, V, T) is sufficient to rewrite large parts of classical and modern physics at the L1/L2 level; and
2. to formulate, as explicitly as possible, the structural constraints any future L3 derivation must satisfy if it wishes to lock physical constants to properties of G_F.

No claim is made that HOK, in its current form, constitutes a complete "theory of everything", nor that it has already produced definitive numerical predictions. The goal is closer to a *kernel specification*: a set of consistent design constraints under which such a theory could, in principle, be implemented.

The remaining chapters follow the usual bottom–up ordering: from classical mechanics, through thermodynamics and fields, to relativity, quantum theory and cosmology. Each chapter makes the mapping to HOK explicit and indicates which claims are L1, which are L2, and where L3 work is still open.

1 Classical Mechanics: From Tension to Newtonian Dynamics

1.1 1.1 Configuration, Tension and Action

In the HOK picture, a classical point particle is the simplest non–trivial container C moving in a field F. Its configuration at time t is described by a vector

$$\boldsymbol{x}(t) \in \mathbb{R}^n.$$

To each configuration $\boldsymbol{x}$ the kernel assigns a *configuration tension*

$$\mathcal{T}(\boldsymbol{x}) \geq 0,$$

which measures how far the container is from its locally preferred configuration.

In the classical continuum description it is convenient to identify the usual potential energy $U(\boldsymbol{x})$ with $\mathcal{T}(\boldsymbol{x})$ up to an additive constant:

$$U(\boldsymbol{x}) = \mathcal{T}(\boldsymbol{x}) + \text{const.} \tag{4}$$

A configuration with higher tension corresponds to higher stored potential energy.

For a single particle of mass m, the standard Lagrangian reads

$$L(\boldsymbol{x}, \dot{\boldsymbol{x}}) = K(\dot{\boldsymbol{x}}) - U(\boldsymbol{x}) = \frac{1}{2} m \dot{\boldsymbol{x}}^2 - \mathcal{T}(\boldsymbol{x}), \tag{5}$$

where K is the kinetic energy. The kernel interpretation is simple: the Lagrangian measures the *net* contribution of kinetic tension and configuration tension.

Given a time interval $[t_1, t_2]$, the action functional is

$$S[\boldsymbol{x}] = \int_{t_1}^{t_2} L\big(\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t)\big)\, dt = \int_{t_1}^{t_2} \left[\frac{1}{2} m \dot{\boldsymbol{x}}^2(t) - \mathcal{T}(\boldsymbol{x}(t))\right] dt. \tag{6}$$

The principle of stationary action,

$$\delta S = 0,$$

is then nothing more than the statement that the realized history of C is one which extremizes the integrated tension balance (kinetic plus configurational) over time, subject to the given boundary conditions.

1.2 1.2 Tension Gradient and Newton's Second Law

We now recover Newton's second law from the tension-based action (6). Consider an arbitrary variation $\delta\boldsymbol{x}(t)$ of the path with fixed endpoints, $\delta\boldsymbol{x}(t_1) = \delta\boldsymbol{x}(t_2) = 0$. The variation of the action

is

$$\delta S = \int_{t_1}^{t_2} \left[m\,\dot{\boldsymbol{x}}(t) \cdot \delta\dot{\boldsymbol{x}}(t) - \nabla\mathcal{T}\big(\boldsymbol{x}(t)\big) \cdot \delta\boldsymbol{x}(t) \right] dt \tag{7}$$

$$= \int_{t_1}^{t_2} \left[-m\,\ddot{\boldsymbol{x}}(t) - \nabla\mathcal{T}\big(\boldsymbol{x}(t)\big) \right] \cdot \delta\boldsymbol{x}(t)\,dt, \tag{8}$$

where the second line follows from integration by parts of the first term and the vanishing of the boundary contribution.

The stationary action condition $\delta S = 0$ must hold for all variations $\delta\boldsymbol{x}(t)$, hence the factor in brackets must vanish:

$$m\,\ddot{\boldsymbol{x}}(t) = -\nabla\mathcal{T}\big(\boldsymbol{x}(t)\big). \tag{9}$$

If we now write the classical force as

$$\boldsymbol{F}(\boldsymbol{x}) := -\nabla U(\boldsymbol{x}), \qquad U(\boldsymbol{x}) = \mathcal{T}(\boldsymbol{x}) + \text{const.}, \tag{10}$$

equation (9) becomes

$$m\,\ddot{\boldsymbol{x}}(t) = \boldsymbol{F}(\boldsymbol{x}(t)), \tag{11}$$

the familiar Newtonian equation of motion

$$\boldsymbol{F} = m\boldsymbol{a}. \tag{12}$$

In kernel language, the ontology is different from the textbook story:

- The acceleration $\boldsymbol{a} = \ddot{\boldsymbol{x}}$ is not caused by an independent "force field"; it is simply the manifestation of the container sliding down the gradient of tension.
- The force is a derived quantity:

$$\boldsymbol{F}(\boldsymbol{x}) = -\nabla\mathcal{T}(\boldsymbol{x}). \tag{13}$$

What is fundamental is the configuration tension and its spatial gradient, not the force itself.

Thus, classical mechanics in the Newtonian form can be fully rewritten as a theory of containers responding to tension gradients in the field F.

1.3 1.3 Inertial Frames as Flat-Tension Regions

Equation (9) also provides a clean kernel interpretation of inertial frames. Suppose there exists a region $\Omega \subset \mathbb{R}^n$ in which the configuration tension is approximately flat:

$$\nabla\mathcal{T}(\boldsymbol{x}) \approx 0 \qquad \forall\,\boldsymbol{x} \in \Omega. \tag{14}$$

Then the equation of motion reduces to

$$m\,\ddot{\boldsymbol{x}}(t) \approx 0 \quad \Rightarrow \quad \dot{\boldsymbol{x}}(t) \approx \text{constant}, \tag{15}$$

so that the container moves with (approximately) constant velocity. To an observer co–moving with such a container, it appears that there is no net force acting.

This motivates the following kernel statement:

> **Inertial frames are local regions of the field F in which the configuration tension $\mathcal{T}$ is (nearly) constant, so that containers experience no significant tension gradients.**

Whenever an observer leaves such a flat–tension region—for example, by entering a gravitational well or a strongly accelerated lab frame—the gradient $\nabla\mathcal{T}$ becomes non–negligible, and the motion of containers deviates from straight lines. Classical "forces" then reappear as effective descriptions of these tension gradients.

1.4 1.4 Outlook: Work, Energy and Non-conservative Effects

The reinterpretation of Newtonian mechanics in terms of tension suggests a natural rephrasing of work and energy:

- Work corresponds to changes in configuration tension performed by displacing containers along specific paths in F.
- Kinetic energy is the part of the tension budget associated with motion, while potential energy is the part associated with configuration.
- Non–conservative effects (friction, drag) can be treated as mechanisms by which tension is irreversibly transferred to other degrees of freedom in G_F.

A full treatment of these ideas, including friction, constraint forces and contact dynamics as they appear in numerical physics engines, will follow in Chapter 3 and in the thermodynamic discussion of Chapter 5. For now, it is enough to note that even the most basic law of motion, $m\boldsymbol{a} = \boldsymbol{F}$, admits a purely kernel formulation with no extra ontological baggage beyond tension and its gradient.

2 Analytical Mechanics: Action, Lagrange and Noether

In Chapter 1 we treated a single point particle as the simplest container C and showed that Newton's law $m\ddot{\boldsymbol{x}} = -\nabla\mathcal{T}$ follows from a tension–based action (6). Analytical mechanics generalizes this construction to arbitrary sets of generalized coordinates and constraints. From the kernel perspective, this is the natural language for describing how many containers, linked by boundaries, jointly relax tension over time.

Throughout this chapter we work at the L1/L2 level: the mapping between HOK and standard formalism is explicit, and the usual equations of motion are re–derived, but no attempt is yet made at fixing numerical constants (L3).

2.1 2.1 Lagrangian as Net Tension Functional

Consider a mechanical system with N degrees of freedom, described by a set of generalized coordinates

$$q^i(t), \qquad i = 1, \dots, N. \tag{16}$$

In kernel language, these coordinates parametrize the configuration of one or more containers C_α embedded in a field F, subject to boundaries B and constraints.

To each configuration $q = (q^1, \dots, q^N)$ the kernel assigns a configuration tension

$$\mathcal{T}_{\text{conf}}(q) \geq 0, \tag{17}$$

which plays the role of generalized potential energy. In addition, motion through configuration space carries kinetic tension, typically of the quadratic form

$$\mathcal{T}_{\text{kin}}(q, \dot{q}) = \frac{1}{2} \sum_{i,j} M_{ij}(q)\, \dot{q}^i \dot{q}^j, \tag{18}$$

where $M_{ij}(q)$ is a positive–definite mass matrix encoding how the field F couples motion along different coordinates.

The Lagrangian is then interpreted as the *net tension density*

$$L(q, \dot{q}, t) = \mathcal{T}_{\text{kin}}(q, \dot{q}) - \mathcal{T}_{\text{conf}}(q, t), \tag{19}$$

and the action functional is

$$S[q] = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt. \tag{20}$$

From the kernel viewpoint, the Lagrangian is not an arbitrary function chosen to reproduce equations of motion; it is the bookkeeping device for how different forms of tension trade off along a candidate history $q(t)$ of the system.

2.2 2.2 Variational Principle and Euler–Lagrange Equations

We now recall the standard derivation of the Euler–Lagrange equations from the action (20), making the tension interpretation explicit.

Consider a family of paths

$$q^i(t, \epsilon) = q^i(t) + \epsilon\, \eta^i(t), \tag{21}$$

where $\eta^i(t)$ are arbitrary smooth variations that vanish at the endpoints:

$$\eta^i(t_1) = \eta^i(t_2) = 0. \tag{22}$$

The first variation of the action is

$$\delta S = \frac{d}{d\epsilon} S[q(\cdot, \epsilon)]\Big|_{\epsilon=0} \tag{23}$$

$$= \int_{t_1}^{t_2} \sum_i \left(\frac{\partial L}{\partial q^i} \eta^i + \frac{\partial L}{\partial \dot{q}^i} \dot{\eta}^i \right) dt. \tag{24}$$

Integrating the second term by parts and using the endpoint conditions, we obtain

$$\delta S = \int_{t_1}^{t_2} \sum_i \left[\frac{\partial L}{\partial q^i} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}^i} \right) \right] \eta^i(t)\, dt. \tag{25}$$

For the action to be stationary, $\delta S = 0$ must hold for all choices of variations $\eta^i(t)$, hence the bracket must vanish:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}^i} \right) - \frac{\partial L}{\partial q^i} = 0, \qquad i = 1, \ldots, N. \tag{26}$$

These are the Euler–Lagrange equations. In traditional language, they encode the balance of generalized forces and inertial effects. In kernel language, each equation expresses the condition that tension cannot be reduced locally along q^i by any infinitesimal rearrangement of the path that respects the boundary conditions; the realized history is a local optimum of the global tension budget.

2.3 2.3 Hamiltonian Formulation and Conjugate Momenta

The Hamiltonian formulation arises by introducing the conjugate momenta

$$p_i := \frac{\partial L}{\partial \dot{q}^i}. \tag{27}$$

These momenta measure how sensitive the net tension is to changes in velocity along each coordinate direction, and thus encode how strongly the field F resists rapid motion of the corresponding containers.

Provided the Legendre transform is well–defined (for example, when the mass matrix is non–singular), we can solve for $\dot{q}^i$ in terms of p_i and define the Hamiltonian

$$H(q, p, t) := \sum_i p_i \dot{q}^i - L(q, \dot{q}, t). \tag{28}$$

In many familiar systems, H coincides with the total energy

$$H = T + U, \tag{29}$$

the sum of kinetic and potential energies. From the kernel standpoint, H represents the total tension content of the system at a given time slice.

Hamilton's equations follow from rewriting the action in phase space and imposing stationarity, or equivalently from the structure of the Legendre transform:

$$\dot{q}^i = \frac{\partial H}{\partial p_i}, \qquad \dot{p}_i = -\frac{\partial H}{\partial q^i}. \tag{30}$$

These first–order equations describe the flow of the system in (q, p)–space. They can be interpreted as a local rule by which tension is shuffled between configuration and motion while remaining constrained on a constant–tension surface when H has no explicit time dependence.

2.4 2.4 Noether's Theorem: Symmetry and Conserved Tension Channels

Noether's theorem provides the key bridge between symmetry and conservation laws. In the kernel picture, it states that when the net tension functional is invariant under a continuous transformation of containers, boundaries and fields, there exists a corresponding conserved tension channel.

Consider a one–parameter family of transformations of the coordinates

$$q^i \mapsto q^i + \epsilon\, \Delta q^i(q, t), \tag{31}$$

under which the action changes by at most a boundary term:

$$\delta S = \epsilon \int_{t_1}^{t_2} \left(\sum_i \frac{\partial L}{\partial q^i} \Delta q^i + \frac{\partial L}{\partial \dot{q}^i} \frac{d}{dt} \Delta q^i + \frac{\partial L}{\partial t} \Delta t \right) dt = \epsilon \left[F(q, t) \right]_{t_1}^{t_2}, \tag{32}$$

for some function F. Assuming the Euler–Lagrange equations hold, one can rearrange this expression to obtain a conserved quantity

$$J := \sum_i \frac{\partial L}{\partial \dot{q}^i} \Delta q^i - F(q, t), \qquad \frac{dJ}{dt} = 0. \tag{33}$$

In concrete cases:

- **Spatial translation invariance** leads to conservation of linear momentum; the kernel statement is that uniform shifts of containers in a flat–tension region do not change the net tension budget.
- **Rotational invariance** leads to conservation of angular momentum; rotating containers in an isotropic tension field does not change the action.

- **Time translation invariance** leads to conservation of the Hamiltonian H; if the net tension functional has no explicit time dependence, the total tension is conserved along the history.

From the HOK perspective, Noether's theorem can be read as:

> Whenever the rules by which tension is assigned and redistributed are indifferent to a continuous reshuffling of containers and boundaries, there exists a corresponding invariant bookkeeping channel for tension.

This gives a kernel reinterpretation of momentum, angular momentum and energy as conserved "tension currents" associated with the symmetries of G_F and its labelling rules.

2.5 2.5 Outlook: Constraints, Gauge Freedom and Fields

The Lagrangian and Hamiltonian formalisms provide the natural launchpad for more advanced structures that appear later in this appendix:

- **Holonomic and non–holonomic constraints** can be viewed as relations among containers enforced by boundaries B. The associated Lagrange multipliers measure how much tension must be injected to maintain those relations.
- **Gauge symmetries** of field theories can be seen as redundancies in how tension is labelled on F, leading to constraint surfaces in phase space.
- **Canonical quantization** promotes (q^i, p_i) to operators and replaces the deterministic tension flow (30) with wave dynamics in Hilbert space; this will be revisited in Chapter 10.

In all cases, the kernel slogan remains the same: Lagrangian mechanics is the language in which containers, boundaries and fields negotiate the relaxation of tension under structural constraints. The rest of the appendix amounts to unpacking how far this slogan can be pushed across the different domains of physics.

3 Rigid Body Dynamics and Constraints

Classical mechanics (Chapter 1) and analytical mechanics (Chapter 2) describe the motion of point particles and generalized coordinates. In actual simulations of the macroscopic world—games, robotics, engineering—we rarely simulate individual atoms. Instead we use *rigid bodies*: extended containers whose internal degrees of freedom are frozen, so that only the motion of their center-of–mass and orientation matter.

From the HOK perspective, a rigid body is a container C whose internal tension has already been minimized subject to a *rigidity constraint*. What remains dynamical is the motion of C as a whole and the tension stored at boundaries when multiple containers collide or are linked by joints.

In this chapter we review rigid body dynamics and constraints in the language used by modern physics engines, and reinterpret them as local tension–balancing rules on the kernel graph G_F.

3.1 3.1 Rigid Bodies as Coarse-Grained Containers

A rigid body in three dimensions has six kinematic degrees of freedom: three for translation and three for rotation. We describe its state at time t by

$$\boldsymbol{x}(t) \in \mathbb{R}^3, \qquad R(t) \in SO(3), \tag{34}$$

where $\boldsymbol{x}$ is the position of the center of mass and R is the rotation matrix from body space to world space. We also introduce the linear and angular velocities

$$\boldsymbol{v} = \dot{\boldsymbol{x}}, \qquad \boldsymbol{\omega} \in \mathbb{R}^3, \tag{35}$$

and the linear and angular momenta

$$\boldsymbol{p} = m\boldsymbol{v}, \qquad \boldsymbol{L} = I\boldsymbol{\omega}, \tag{36}$$

where m is the mass and I is the inertia tensor expressed in world coordinates:

$$I(t) = R(t)\, I_{\text{body}}\, R(t)^T. \tag{37}$$

In kernel language, the rigid body corresponds to a subgraph of G_F whose internal nodes have been "collapsed" into a single container C. The rigidity assumption means that all internal tension modes are frozen except for the collective modes described by $(\boldsymbol{x}, R)$; we only track how the container moves and rotates as a whole in response to external tension gradients.

The kinetic energy (kinetic tension) of a rigid body is

$$T = \frac{1}{2} m \boldsymbol{v}^2 + \frac{1}{2} \boldsymbol{\omega}^T I \boldsymbol{\omega}. \tag{38}$$

Potential tension (e.g. due to gravity) is encoded in a potential $U(\boldsymbol{x}, R)$, leading to the Lagrangian

$$L(\boldsymbol{x}, R, \boldsymbol{v}, \boldsymbol{\omega}) = T - U(\boldsymbol{x}, R). \tag{39}$$

The resulting equations of motion are the familiar rigid body equations for $\boldsymbol{p}$ and $\boldsymbol{L}$, which can be written as

$$\dot{\boldsymbol{p}} = \boldsymbol{F}_{\text{ext}}, \tag{40}$$

$$\dot{\boldsymbol{L}} = \boldsymbol{\tau}_{\text{ext}} + \boldsymbol{\omega} \times \boldsymbol{L}, \tag{41}$$

where $\boldsymbol{F}_{\text{ext}}$ and $\boldsymbol{\tau}_{\text{ext}}$ are the net external force and torque.

From the HOK viewpoint, these equations describe how tension assigned to the container C by the field F is redistributed between translational and rotational modes while preserving total kinetic + potential tension in the absence of dissipation.

3.2 3.2 Constraints and the Jacobian Formulation

Realistic scenes rarely contain isolated rigid bodies. They collide, stack, slide and are connected by joints. All such interactions can be expressed as *constraints* on the generalized coordinates of the system.

Let q denote the stacked vector of all generalized coordinates (position and orientation parameters for each body), and let M be the corresponding mass matrix. A holonomic constraint can be written as

$$C(q,t) = 0. \tag{42}$$

Differentiating with respect to time,

$$\frac{dC}{dt} = \frac{\partial C}{\partial q}\dot{q} + \frac{\partial C}{\partial t} = J(q,t)\,\dot{q} + \dot{C}_{\text{bias}}(t) = 0, \tag{43}$$

where

$$J(q,t) := \frac{\partial C}{\partial q} \tag{44}$$

is the *constraint Jacobian*, and $\dot{C}_{\text{bias}}$ encodes any explicit time dependence.

In impulse–based physics engines, constraint forces (or impulses) are introduced as Lagrange multipliers λ that enforce the constraint at the velocity level. The total constraint impulse acting on the system is

$$\boldsymbol{P}_{\text{c}} = J^T\lambda. \tag{45}$$

The velocity update over a small time step Δt is then

$$v' = v + M^{-1}(f_{\text{ext}}\Delta t + J^T\lambda), \tag{46}$$

where v stacks all generalized velocities and f_{ext} stacks external forces.

Requiring that the updated velocity satisfy the constraint $Jv' + \dot{C}_{\text{bias}} = 0$ yields the linear system

$$JM^{-1}J^T\lambda = -(Jv + \dot{C}_{\text{bias}}) - JM^{-1}f_{\text{ext}}\Delta t. \tag{47}$$

Solving for λ and applying $\boldsymbol{P}_{\text{c}} = J^T\lambda$ produces a corrected velocity v' that respects the constraint at the discrete level.

In kernel terms:

- The constraint $C(q,t) = 0$ represents a structural relation among containers enforced by boundaries B.

- The Jacobian J tells us how small changes in the containers' configuration alter the constraint; it is the local sensitivity of boundary tension to motion.
- The multipliers λ are the amounts of tension that must be injected through each constraint channel to keep the relation satisfied during a time step.

Equation (47) is thus the discrete statement that *boundary tension must be adjusted so that the containers move in a way consistent with the imposed structure.*

3.3 3.3 Contact, Restitution and Friction as Tension Rules

Collisions and friction are the most important constraints in everyday rigid body scenes. They can be formulated as inequality constraints on relative velocity, with additional rules for how much tension is allowed to flow through each contact.

Consider a contact between bodies A and B at a point with normal $\boldsymbol{n}$ (pointing from A to B). The normal component of the relative velocity at the contact is

$$v_{\mathrm{rel},n} = \boldsymbol{n} \cdot (\boldsymbol{v}_B + \boldsymbol{\omega}_B \times \boldsymbol{r}_B - \boldsymbol{v}_A - \boldsymbol{\omega}_A \times \boldsymbol{r}_A), \tag{48}$$

where $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$ are the contact point offsets from each center of mass.

A non–penetration constraint requires that, after applying impulses,

$$v'_{\mathrm{rel},n} \geq -e\, v^{\mathrm{in}}_{\mathrm{rel},n}, \tag{49}$$

where $e \in [0, 1]$ is the coefficient of restitution and $v^{\mathrm{in}}_{\mathrm{rel},n}$ is the incoming normal relative velocity. This inequality can be implemented as a unilateral constraint with a non–negative multiplier $\lambda_n \geq 0$, solved using variants of projected Gauss–Seidel or sequential impulse methods.

Friction is treated similarly: tangential components of the relative velocity are opposed by tangential impulses (λ_t) limited by a Coulomb cone

$$\|\lambda_t\| \leq \mu \lambda_n, \tag{50}$$

with μ the friction coefficient.

From the HOK standpoint:

- The contact normal constraint encodes a hard boundary B preventing containers from occupying the same region of F. The associated tension channel only allows compressive tension ($\lambda_n \geq 0$).
- Restitution e specifies how much tension stored in the compression of the boundary is released back into kinetic modes after impact.

- Friction implements a "tension leak" that converts tangential relative motion into internal tension and, ultimately, microscopic degrees of freedom (heat), which we will revisit in Chapter 5.

Thus, the familiar normal impact and friction rules in physics engines can be reframed as local update rules for how boundary tension is allowed to flow or dissipate at each contact between containers.

3.4 3.4 Constraint Stabilization and Numerical Tension Leakage

Ideal constraints $C(q,t) = 0$ cannot be maintained exactly in discrete time integration. Numerical errors accumulate, leading to constraint drift: joints stretch, stacks jitter or slowly sink into the ground. From a kernel perspective this drift is a form of unintended tension leakage: the simulated containers no longer respect the structural relations encoded by boundaries B.

Two standard stabilization strategies are:

Baumgarte stabilization. Instead of enforcing $\dot{C} = 0$, we enforce

$$\dot{C} + \beta C = 0, \tag{51}$$

for some parameter $\beta > 0$. At the velocity level this adds a bias term proportional to the current constraint error C. In practice, one uses a discrete form

$$\dot{C}_{\text{bias}} = \frac{\beta}{\Delta t} C, \tag{52}$$

so that the constraint solver gradually pushes the system back towards $C = 0$ over several time steps. In kernel language, Baumgarte adds a controlled "restoring tension" that opposes long–term drift.

Sequential impulse / position correction. An alternative, widely used in game engines, is to perform explicit position correction after each step, solving constraints directly at the configuration level (sometimes called "post–stabilization"). This can be seen as periodically re–projecting containers back onto the manifold defined by $C(q) = 0$, resetting the accumulated structural error in G_F at the price of small discontinuities in velocities.

Both methods highlight an important point for HOK: discrete approximations of continuous tension flows inevitably introduce small inconsistencies. Stabilization schemes are, in effect, additional kernel rules for how to handle these inconsistencies without allowing macroscopic structures to melt.

3.5 3.5 Outlook: From Rigid Bodies to Continuum Fields

Rigid body dynamics occupies an intermediate scale between point–particle mechanics and full continuum field theories:

- At the microscopic end, a rigid body can be viewed as a dense cluster of microscopic containers whose internal tension modes have already been relaxed to a stable configuration.
- At the macroscopic end, a large ensemble of rigid bodies can approximate granular flows or deformable media, hinting at the continuum descriptions developed in Chapter 4.

In all cases, the same kernel ingredients appear:

- containers C representing bodies or material elements,
- boundaries B representing contacts and joints,
- fields F encoding ambient tension (gravity, electromagnetic fields, etc.),
- vectors V tracking motion, and
- tension T flowing through constraints and being dissipated.

The rigid body formalism and the associated constraint machinery thus serve as a concrete testbed for HOK's claim: that large portions of classical mechanics can be recast as local, discrete rules for how containers in a causal graph share and relax tension under structural constraints.

4 Continuum Mechanics and Field Descriptions

Rigid body dynamics (Chapter 3) treats containers as indivisible units with frozen internal structure. Continuum mechanics goes in the opposite direction: macroscopic matter is modelled as a continuous distribution of mass, momentum and tension. Instead of tracking individual containers, we track *fields* defined over space and time.

From the HOK point of view, a continuum is a coarse–grained description of a large population of microscopic containers. Stress, strain and fluid velocity are effective summaries of how tension flows through G_F when we no longer resolve individual containers and boundaries.

In this chapter we briefly review the core structures of continuum mechanics and reinterpret them as field–level bookkeeping of tension.

4.1 4.1 Continuum Limit: From Discrete Containers to Fields

Consider a collection of microscopic containers C_α with positions $\boldsymbol{x}_\alpha(t)$ and masses m_α. In the continuum limit we introduce a mass density field $\rho(\boldsymbol{x}, t)$ such that

$$\int_V \rho(\boldsymbol{x}, t)\, d^3x \approx \sum_{\alpha \in V} m_\alpha, \tag{53}$$

for any macroscopic volume element V large compared to the micro-scale spacing but small compared to the scale of interest.

Similarly, we define a velocity field $\boldsymbol{v}(\boldsymbol{x}, t)$ as the coarse–grained average of container velocities in a neighbourhood of $\boldsymbol{x}$. In the kernel picture,

- the field F carries labels $(\rho, \boldsymbol{v}, \dots)$ attached to nodes or cells of the coarse–grained graph;
- microscopic container motion is summarized by these continuous fields.

Conservation of mass is expressed by the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0. \tag{54}$$

This is simply the statement that, in the absence of sources or sinks, the amount of container mass inside any region can only change by flux through its boundary.

In HOK language, (54) encodes the fact that the kernel does not create or destroy container labels arbitrarily: tension may flow, but the bookkeeping for "how much container mass is present" is strictly conserved.

4.2 4.2 Stress, Strain and the Cauchy Stress Tensor

When a continuum is deformed, different regions experience internal tension. Macroscopically this is described by the *stress tensor* $\boldsymbol{\sigma}(\boldsymbol{x}, t)$. Given a (macroscopic) surface element with unit normal $\boldsymbol{n}$, the force per unit area acting across that surface is

$$\boldsymbol{t} = \boldsymbol{\sigma} \boldsymbol{n}. \tag{55}$$

The tensor $\boldsymbol{\sigma}$ is therefore the continuum analogue of boundary tension in HOK: it summarizes, at each point, how much tension flows through infinitesimal boundaries oriented in different directions.

The balance of linear momentum in a continuum reads

$$\rho \frac{D\boldsymbol{v}}{Dt} = \nabla \cdot \boldsymbol{\sigma} + \rho \boldsymbol{b}, \tag{56}$$

where $\boldsymbol{b}$ is the body force per unit mass (e.g. gravity), and D/Dt denotes the material derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{v} \cdot \nabla. \tag{57}$$

Equation (56) is the continuum counterpart of $m\boldsymbol{a} = \boldsymbol{F}$: the left–hand side is the inertial response of the coarse–grained containers, and the right–hand side is the divergence of internal tension plus external body forces.

To relate stress to deformation we introduce the deformation gradient F_{ij} and strain measures (e.g. small–strain tensor ε_{ij} in linear elasticity or more general nonlinear strains for large deformations). Constitutive laws, such as Hooke's law for linear isotropic solids, prescribe how stress

depends on strain:

$$\sigma_{ij} = \lambda\, \delta_{ij}\, \mathrm{tr}(\varepsilon) + 2\mu\, \varepsilon_{ij}, \tag{58}$$

where λ and μ are Lamé parameters.

In kernel terms:

- Strain measures how boundaries B enclosing material subcontainers are distorted relative to a reference configuration.
- Stress encodes how much additional tension must be stored in F and C to sustain that distortion.
- Constitutive laws are effective rules summarizing how tension accumulates under deformation, after microscopic degrees of freedom have been integrated out.

4.3 4.3 Newtonian Fluids and the Navier–Stokes Equations

For a Newtonian fluid, the stress tensor is decomposed into an isotropic pressure part and a viscous part:

$$\boldsymbol{\sigma} = -p\mathbf{I} + \boldsymbol{\tau}, \tag{59}$$

where $\mathbf{I}$ is the identity tensor and the viscous stress $\boldsymbol{\tau}$ is (to first order) proportional to the rate of strain:

$$\boldsymbol{\tau} = \mu\left[\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^T - \tfrac{2}{3}(\nabla\cdot\boldsymbol{v})\,\mathbf{I}\right] + \zeta(\nabla\cdot\boldsymbol{v})\,\mathbf{I}, \tag{60}$$

with μ the shear viscosity and ζ the bulk viscosity.

Inserting this constitutive relation into the momentum balance (56) yields the compressible Navier–Stokes equations. In the incompressible limit $\nabla\cdot\boldsymbol{v} = 0$ and constant density, they reduce to

$$\nabla\cdot\boldsymbol{v} = 0, \tag{61}$$

$$\rho\left(\frac{\partial\boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right) = -\nabla p + \mu\nabla^2\boldsymbol{v} + \rho\boldsymbol{b}. \tag{62}$$

In HOK language, viscosity is a controlled *tension leak* between macroscopic flow and microscopic degrees of freedom:

- Pressure p represents isotropic compression tension; regions of high pressure correspond to containers packed more tightly than in equilibrium.
- The viscous term $\mu\nabla^2\boldsymbol{v}$ encodes the fact that strong velocity gradients cause additional tension at the boundaries between fluid parcels, which then dissipates into heat.
- The convective term $(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}$ reflects the advection of momentum (and thus tension) by the flow itself.

Thus, the Navier–Stokes equations can be read as a field–level statement of how tension is transported, stored and dissipated in a fluid continuum.

4.4 4.4 Waves, Sound and Elastic Disturbances

Small perturbations around an equilibrium state propagate as waves: local changes in tension cause neighbouring regions to react, leading to oscillatory patterns that travel through the medium.

For a simple, homogeneous, isotropic medium, the linear wave equation for a scalar field $\phi(\boldsymbol{x}, t)$ is

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \nabla^2 \phi, \tag{63}$$

where c is the wave speed. In acoustics, ϕ may represent pressure fluctuations; in elasticity, a vector displacement field obeys similar equations with appropriate tensor structure, leading to longitudinal and transverse waves.

Kernel interpretation:

- A localized change in configuration tension (e.g. compression of a spring, overpressure in a fluid region) creates a local imbalance in G_F.
- The update rules of the kernel, when coarse–grained, imply that this imbalance cannot remain localized: it propagates as a wave, redistributing tension while approximately conserving a suitable quadratic integral (energy) in the absence of dissipation.
- The wave speed c is an effective parameter determined by how quickly tension can be transmitted through the microscopic boundary network.

This view prepares the ground for electromagnetic and relativistic field theories, where waves in more abstract fields (electromagnetic, gravitational) play the same structural role.

4.5 4.5 Outlook: Coupling to Thermodynamics and Fields

Continuum mechanics by itself does not close the story of tension flows. Two major extensions will be developed later in this appendix:

- **Thermodynamics and statistical mechanics** (Chapter 5) explain how microscopic configurations of containers give rise to macroscopic quantities such as temperature, entropy and internal energy. Viscous dissipation and heat conduction are then recognized as channels by which mechanical tension is irreversibly converted into microscopic tension modes.
- **Electromagnetic and other fields** (Chapter 6) provide additional tension channels that are not purely mechanical: charge densities, currents and field stresses carry their own forms of tension and contribute to the overall balance embodied in the stress–energy tensor.

At a structural level, however, all these theories share the same kernel ingredients:

1. Containers (material elements, parcels, charge carriers) with coarse–grained densities.
2. Boundaries (interfaces, domain walls, membranes) across which tension can flow.
3. Fields (mechanical, thermal, electromagnetic) defined on F that mediate and store tension.
4. Vectors recording the motion of matter and the directionality of tension flows.
5. Tension itself as the primitive account of "how far the system is from its preferred configuration" at each scale.

Continuum mechanics therefore serves as the bridge between the discrete kernel world of containers and the fully field–theoretic descriptions encountered in later chapters.

5 Thermodynamics, Statistical Mechanics and Information

Classical mechanics (Chapter 1) describes reversible, microscopic trajectories. Thermodynamics and statistical mechanics describe macroscopic systems with many degrees of freedom, where irreversibility and entropy become central.

From the HOK standpoint:

- A *thermodynamic system* is a large container C with many internal micro-containers and tension channels.
- Macroscopic variables (energy, volume, pressure, temperature) summarize coarse–grained tension bookkeeping inside C.
- Entropy measures how many distinct kernel configurations (σ_i) are compatible with the same macroscopic container state.
- Information and computation add a further layer: erasing bits or performing logical operations has an intrinsic tension cost (Landauer principle).

This chapter reviews the standard L1/L2 formalism and sets up the Landauer link used later in the constants chapter (Chapter 20) and in the chaos/entanglement chapter (Chapter 21).

5.1 5.1 Macroscopic Variables and Equations of State (L1)

A thermodynamic system is characterized by macroscopic variables such as

- internal energy U,
- volume V,

- pressure P,
- particle numbers N_i,
- temperature T.

An *equation of state* relates these variables, e.g. for an ideal gas,

$$PV = Nk_BT = nRT, \tag{64}$$

where N is the number of particles, $n = N/N_A$ is the amount of substance, $R = N_Ak_B$ is the gas constant.

Many thermodynamic descriptions treat (U, V, N) as fundamental and express all other quantities as derivatives of a potential $U(S, V, N)$ or $F(T, V, N)$, etc.

From the HOK perspective (L1):

- U is the total tension stored in all micro-containers and fields inside C.
- V measures the spatial extent of the container.
- P represents the average boundary tension per unit area exerted on the walls of C.
- T is an emergent measure of average tension per degree of freedom; its more precise kernel meaning will be tied to k_B and information in Section 20.5.

5.2 5.2 First Law: Energy Conservation and Work as Tension Flow (L1/L2)

The first law of thermodynamics states that for a closed system

$$dU = \delta Q - \delta W, \tag{65}$$

where δQ is heat added to the system and δW is work done *by* the system. For a simple $P\,dV$ work term,

$$\delta W = P\,dV. \tag{66}$$

HOK interpretation.

- U is the total internal tension budget of container C.
- δQ is tension exchanged via microscopic channels that do not directly produce coherent container motion (randomized exchanges with the environment).
- δW is tension exported in a coherent way that changes the boundary configuration (e.g. pushing a piston).

At the L2 level, the first law expresses that the kernel's local tension conservation (on G_F) survives coarse–graining: macroscopic containers cannot create or destroy tension, only redistribute it between internal modes and external work channels.

5.3 5.3 Second Law and Entropy (L1)

The second law introduces entropy S and irreversibility. In Clausius form:

$$\oint \frac{\delta Q_{\text{rev}}}{T} \leq 0, \tag{67}$$

with equality for reversible cycles. For a reversible process between nearby equilibrium states,

$$dS = \frac{\delta Q_{\text{rev}}}{T}. \tag{68}$$

Generic statements of the second law include:

- In an isolated system, entropy does not decrease:

$$\Delta S_{\text{isolated}} \geq 0. \tag{69}$$

- It is impossible to construct a cyclic engine whose sole effect is to convert heat from a single reservoir entirely into work (Kelvin–Planck statement).

Entropy can be viewed as a state function that measures how much "useful" tension (capable of doing work) has been degraded into forms that are no longer accessible without additional resources.

In HOK language (still L1):

- S summarizes how many micro-configurations of tension inside C are compatible with the same macroscopic variables $(U, V, N, \dots)$.
- Irreversibility arises when microscopically reversible kernel updates are compressed into a coarse–grained description that discards information about detailed tension trajectories.

5.4 5.4 Statistical Entropy: Boltzmann, Gibbs and Ensembles (L1/L2)

Statistical mechanics connects entropy to microscopic probabilities.

Boltzmann entropy. If a macrostate corresponds to Ω microstates, Boltzmann's formula is

$$S = k_B \ln \Omega. \tag{70}$$

Here Ω is a count of micro-configurations (σ_i) consistent with specified macroscopic constraints.

Gibbs entropy. For a probability distribution $\{p_i\}$ over microstates,

$$S = -k_B \sum_i p_i \ln p_i. \tag{71}$$

This generalizes Boltzmann's formula to non-uniform ensembles.

Canonical ensemble. For a system in contact with a heat bath at temperature T, the canonical distribution is

$$p_i = \frac{1}{Z} \exp\left(-\frac{E_i}{k_B T}\right), \qquad Z = \sum_i \exp\left(-\frac{E_i}{k_B T}\right), \tag{72}$$

where E_i is the energy of microstate i and Z is the partition function. Thermodynamic quantities can then be obtained from Z.

HOK interpretation (L2):

- Microstates are detailed assignments of node/edge variables $\{\sigma_i\}$ inside container C.
- The energy E_i is the tension functional evaluated on that configuration.
- The canonical distribution expresses how likely each tension configuration is, given that the container is coupled to an external tension reservoir at fixed T.

Entropy then becomes a measure of how spread out the actual kernel distribution over $\{\sigma_i\}$ is; equilibrium maximizes S subject to constraints.

5.5 5.5 Landauer Principle and Information-Theoretic Entropy (L1/L2/L3)

Shannon's information entropy for a discrete random variable with probabilities $\{p_i\}$ is

$$H = -\sum_i p_i \log_2 p_i \quad \text{(bits)}. \tag{73}$$

This quantity measures the average information content per symbol.

A key bridge between information and thermodynamics is the Landauer principle:

> Erasing one bit of information in a system at temperature T requires dissipating at least $\Delta Q_{\min} = k_B T \ln 2$ of heat into the environment.

Equivalently, the minimal entropy increase in the environment is

$$\Delta S_{\text{env}} \geq k_B \ln 2 \tag{74}$$

per bit erased.

Standard L1/L2 explanation:

- A logically irreversible operation (e.g. resetting a bit to zero regardless of input) maps multiple logical states to a single state.
- At the physical level, this means compressing multiple micro-configurations into a smaller set, reducing the system's entropy S.
- To preserve the second law, at least this much entropy must be exported to the environment as heat.

HOK interpretation:

- A bit of information is a minimal distinction between two sets of kernel configurations of a container C.
- Erasing the bit means forcing the kernel to redirect tension flows so that all these configurations converge to one macrostate; this requires pushing excess tension into external channels.
- The Landauer bound states that every bit erased must correspond to a minimal increase in environmental entropy, i.e. a minimal number of additional kernel configurations becoming accessible outside C.

At the L3 level, Landauer's principle becomes the anchor for mapping k_B to kernel scales (see Section 20.5):

1. Define a kernel notion of a bit I_{bit} as the smallest robust distinction between container states.
2. Show that erasing one such bit necessarily requires a minimal tension transfer ε^* to the environment.
3. Identify the effective temperature T with average tension per degree of freedom so that $\varepsilon^* = k_B T \ln 2$ in equilibrium.

Thus Landauer connects information-theoretic entropy H directly to thermodynamic entropy S and to the kernel's tension bookkeeping.

Minimal HOK Bit-Erasure Model (L3 Prototype)

We now sketch a minimal kernel-based model that recovers the Landauer bound. The goal is not to be fully realistic, but to show that any HOK kernel that implements bit erasure under standard assumptions is forced to pay (at least) $k_B T \ln 2$ of heat per bit.

Step 1: Containers and microstates. Consider two containers:

- C_{mem}: a memory container storing one logical bit.
- C_{bath}: a large heat bath at fixed temperature T.

The bit lives in C_{mem} and has two logical macrostates:

$$\text{“0”}, \quad \text{“1”}. \tag{75}$$

Each logical state corresponds to a set of kernel micro-configurations:

$$\Omega_0,\ \Omega_1 \subset \Omega_{\text{mem}}, \qquad \Omega_{\text{mem}} = \Omega_0 \cup \Omega_1, \quad \Omega_0 \cap \Omega_1 = \emptyset. \tag{76}$$

The bath C_{bath} has microstates Ω_{bath}.

A full microstate of the combined system is a pair $(x, y) \in \Omega_{\text{mem}} \times \Omega_{\text{bath}}$.

Step 2: Energy and tension bookkeeping. Assume that:

- Memory microstates in Ω_0 and Ω_1 are *energy-degenerate*:

$$E_{\text{mem}}(x) = E_0 \quad \text{for all } x \in \Omega_0 \cup \Omega_1. \tag{77}$$

  In HOK language: the two logical basins correspond to two disjoint regions of tension configuration space with the same total tension.

- The environment is so large that it remains at temperature T, and its statistics are canonical:

$$p_{\text{bath}}(y) \propto \exp\left(-\frac{E_{\text{bath}}(y)}{k_B T}\right). \tag{78}$$

The total energy is

$$E(x, y) = E_{\text{mem}}(x) + E_{\text{bath}}(y). \tag{79}$$

Step 3: Initial and final logical ensembles. Initially, the bit is assumed to be *logically random*:

$$P_{\text{logic}}(0) = P_{\text{logic}}(1) = \frac{1}{2}. \tag{80}$$

Within each logical value, microstates are equally likely:

$$P(x \in \Omega_0) = \frac{1}{2}\frac{1}{|\Omega_0|}, \qquad P(x \in \Omega_1) = \frac{1}{2}\frac{1}{|\Omega_1|}. \tag{81}$$

The bath is in equilibrium at temperature T.

The *logical* Shannon entropy of the bit is

$$H_{\text{bit, init}} = -\sum_{b=0,1} \frac{1}{2} \log_2 \frac{1}{2} = 1 \text{ bit}. \tag{82}$$

The associated contribution to thermodynamic entropy is

$$S_{\text{bit, init}} = k_B \ln 2. \tag{83}$$

(There is also an entropy contribution from microstate degeneracies within Ω_0 and Ω_1, which we keep fixed and thus ignore in the difference.)

After a *reset-to-zero* operation, the bit is always found in logical state "0":

$$P^{\text{final}}_{\text{logic}}(0) = 1, \qquad P^{\text{final}}_{\text{logic}}(1) = 0. \tag{84}$$

All probability mass on Ω_1 has been moved into Ω_0.

The final logical entropy is

$$H_{\text{bit, final}} = 0, \qquad S_{\text{bit, final}} = 0 \tag{85}$$

(again up to the unchanged degeneracy inside Ω_0).

Therefore the bit's entropy change is

$$\Delta S_{\text{bit}} = S_{\text{bit, final}} - S_{\text{bit, init}} = -k_B \ln 2. \tag{86}$$

Step 4: Second law constraint on the environment. Let S_{bath} be the bath entropy. For the combined isolated system $C_{\text{mem}} + C_{\text{bath}}$, the second law requires

$$\Delta S_{\text{total}} = \Delta S_{\text{bit}} + \Delta S_{\text{bath}} \geq 0. \tag{87}$$

Thus

$$\Delta S_{\text{bath}} \geq -\Delta S_{\text{bit}} = k_B \ln 2. \tag{88}$$

In other words, the environment's entropy must increase by at least $k_B \ln 2$.

If the erasure is performed *isothermally* at temperature T and quasi-statically (reversible limit), the bath entropy change is related to the heat it absorbs:

$$\Delta S_{\text{bath}} = \frac{\Delta Q_{\text{bath}}}{T} \quad \Rightarrow \quad \Delta Q_{\text{bath}} \geq k_B T \ln 2. \tag{89}$$

The minimal heat dissipated into the bath is therefore

$$\boxed{\Delta Q_{\min} = k_B T \ln 2}. \tag{90}$$

This is the Landauer bound.

Step 5: Kernel translation. In HOK terms:

- C_{mem} and C_{bath} are containers, with a boundary $\mathcal{B}$ across which tension (heat) can flow.
- The two logical values correspond to two disjoint clusters of kernel configurations (Ω_0, Ω_1) with equal internal tension.
- The reset operation is a kernel update protocol that collapses the accessible configuration space of C_{mem} by a factor of 2 at the logical level.
- To maintain overall tension/entropy bookkeeping on G_F, the boundary must export at least $k_B \ln 2$ of entropy into C_{bath}, costing at least $k_B T \ln 2$ of heat.

This toy model is deliberately simple, but it shows that once HOK adopts standard statistical assumptions for containers and baths, the Landauer bound is not a convention: it is enforced by the kernel's tension and entropy accounting.

5.6 5.6 HOK Mapping: Containers, Boundaries and Entropy (L2)

We summarize the HOK dictionary for thermodynamic concepts:

Thermo / Info concept	**HOK interpretation**
System	Container C with many micro-containers
Environment	Surrounding containers coupled via boundaries $\mathcal{B}$
Internal energy U	Total tension stored in C
Heat δQ	Randomized tension exchanged across $\mathcal{B}$
Work δW	Coherent boundary motion / deformation of C
Temperature T	Average tension per degree of freedom in C
Entropy S	Log of accessible kernel configurations of C
Information bit	Minimal robust distinction between two macro-states
Landauer cost	Minimal tension export per erased bit

This table is intentionally L2: it does not compute numerical values, but it forces any future HOK model to respect these identifications. Later, Chapter 21 will refine the entropy concept by including entanglement entropy and area laws.

5.7 5.7 Outlook: From Thermodynamics to Chaos and Entanglement (L1)

Thermodynamics and statistical mechanics describe coarse–grained, classical entropy. Quantum mechanics introduces a different notion: von Neumann entropy of density matrices, entanglement entropy across subsystems, and the role of decoherence (Chapter 10).

From the HOK perspective:

- Classical entropy measures ignorance about detailed kernel configurations within a container.
- Quantum entropy and entanglement measure how amplitude and tension are distributed across different partitions of G_F.
- Chaos and mixing dynamics in classical and quantum systems control how quickly microscopic tension information becomes inaccessible to coarse–grained observers.

Chapter 21 will extend the ideas of this chapter to:

1. Lyapunov exponents and classical chaos as measures of how sensitive tension flows are to initial conditions.
2. Entanglement entropy and area laws as refinements of S that depend on boundaries in G_F.
3. The interplay between thermodynamic irreversibility and quantum decoherence in the kernel.

In this way, thermodynamics, information theory and quantum mechanics become three faces of the same underlying object: the kernel's tension bookkeeping over containers, boundaries and fields.

6 Electromagnetism and Classical Light

Continuum mechanics describes how mechanical tension propagates through material media (Chapter 4). Electromagnetism extends this picture by introducing additional fields defined on space–time that carry their own forms of tension independent of any particular material substrate. Charges and currents act as sources; electromagnetic fields store and transport energy, momentum and stress; light appears as self–propagating waves of field tension.

From the HOK perspective, the electromagnetic field is a structured label on the field F. Charged containers C act as sources and sinks of this label, and Maxwell's equations encode local rules for how electromagnetic tension evolves and interacts with mechanical containers.

In this chapter we briefly review Maxwell's equations, field energy and stress, the Lagrangian formulation and the interpretation of light as propagating tension.

6.1 6.1 Fields, Sources and Maxwell's Equations

In the classical continuum description, the electromagnetic field in vacuum is specified by the electric field $\boldsymbol{E}(\boldsymbol{x}, t)$ and the magnetic field $\boldsymbol{B}(\boldsymbol{x}, t)$. Charges and currents are described by the charge

density $\rho(\boldsymbol{x}, t)$ and current density $\boldsymbol{J}(\boldsymbol{x}, t)$. Maxwell's equations in SI units are

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}, \tag{91}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{92}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{93}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}. \tag{94}$$

These equations are supplemented by the Lorentz force law for a test charge q:

$$\boldsymbol{F} = q\left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}\right), \tag{95}$$

which describes how the field acts on a charged container.

In kernel language:

- The pair $(\boldsymbol{E}, \boldsymbol{B})$ are part of the label L attached to nodes and edges of the field F.
- Charged containers C carry a charge label q that couples to the field via (95).
- The continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{J} = 0 \tag{96}$$

  expresses conservation of charge: the kernel does not create or destroy charge labels arbitrarily.

Maxwell's equations themselves can be viewed as local update rules for how electromagnetic tension is distributed and transported on G_F. The divergence equations (91)–(92) specify how sources and the absence of magnetic monopoles constrain the field; the curl equations (93)–(94) encode how changing fields induce each other and how currents inject field tension.

6.2 6.2 Field Energy, Poynting Vector and Stress

The electromagnetic field carries energy density

$$u_{\mathrm{em}} = \frac{1}{2}\varepsilon_0 \boldsymbol{E}^2 + \frac{1}{2\mu_0}\boldsymbol{B}^2 \tag{97}$$

and momentum density

$$\boldsymbol{g} = \frac{1}{c^2}\boldsymbol{S}, \tag{98}$$

where

$$\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E} \times \boldsymbol{B} \tag{99}$$

is the *Poynting vector*, representing energy flux.

For a region of space with volume V bounded by a surface ∂V with outward normal $\boldsymbol{n}$, the local energy balance reads

$$\frac{d}{dt}\int_V u_{\mathrm{em}}\, d^3x = -\oint_{\partial V} \boldsymbol{S}\cdot\boldsymbol{n}\, dA - \int_V \boldsymbol{J}\cdot\boldsymbol{E}\, d^3x. \tag{100}$$

The first term on the right–hand side is the net electromagnetic energy leaving V; the second term is the work done on charges inside V.

In HOK terms:

- u_{em} is the *field tension density* stored in the electromagnetic labels on F.
- The Poynting vector $\boldsymbol{S}$ describes how this tension flows through boundaries B between regions.
- The work term $-\boldsymbol{J}\cdot\boldsymbol{E}$ expresses exchange of tension between field labels and container motion: field tension is converted into kinetic tension of charged containers, or conversely.

The distribution of electromagnetic stress is encoded in the Maxwell stress tensor $\mathbf{T}$, with components

$$T_{ij} = \varepsilon_0\left(E_iE_j - \frac{1}{2}\delta_{ij}\boldsymbol{E}^2\right) + \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}\boldsymbol{B}^2\right). \tag{101}$$

The force on charges and matter in a volume V can be written as

$$\boldsymbol{F}_{\mathrm{em}} = \int_V \rho\boldsymbol{E} + \boldsymbol{J}\times\boldsymbol{B}\, d^3x = \oint_{\partial V} \mathbf{T}\boldsymbol{n}\, dA - \frac{d}{dt}\int_V \boldsymbol{g}\, d^3x. \tag{102}$$

Thus, electromagnetic forces on containers arise from gradients and fluxes of field tension encoded in $\mathbf{T}$ and $\boldsymbol{g}$.

From the kernel perspective, the stress tensor is the continuum analogue of how tension is distributed among edges of G_F around a point, and the Poynting vector is the net tension current crossing boundaries.

6.3 6.3 Potentials, Gauge Freedom and Lagrangian Formulation

It is often convenient to describe the electromagnetic field in terms of a scalar potential $\phi(\boldsymbol{x},t)$ and a vector potential $\boldsymbol{A}(\boldsymbol{x},t)$:

$$\boldsymbol{B} = \nabla\times\boldsymbol{A}, \qquad \boldsymbol{E} = -\nabla\phi - \frac{\partial\boldsymbol{A}}{\partial t}. \tag{103}$$

Maxwell's equations then follow from the field equations for $(\phi,\boldsymbol{A})$ together with a choice of gauge condition, such as the Lorenz gauge

$$\nabla\cdot\boldsymbol{A} + \frac{1}{c^2}\frac{\partial\phi}{\partial t} = 0. \tag{104}$$

At the L2 level, electromagnetism is elegantly captured by a Lagrangian density

$$\mathcal{L} = -\frac{1}{4\mu_0}F_{\mu\nu}F^{\mu\nu} - J^\mu A_\mu, \tag{105}$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor, $A_\mu = (\phi/c, -\boldsymbol{A})$ is the four–potential, and J^μ is the four–current. The Euler–Lagrange equations for A_μ reproduce Maxwell's equations; gauge invariance $A_\mu \mapsto A_\mu + \partial_\mu \Lambda$ leads, via Noether's theorem, to charge conservation $\partial_\mu J^\mu = 0$.

In HOK language:

- The potentials $(\phi, \boldsymbol{A})$ are different coordinate choices for the same underlying electromagnetic tension configuration on F.
- Gauge transformations are redundancies in how we label this tension; they do not change any physically measurable pattern of tension flow.
- The Lagrangian density $\mathcal{L}$ is the local contribution of electromagnetic tension to the action; the $J^\mu A_\mu$ term encodes how container labels (charges) couple to field tension.

Thus, gauge symmetry appears as a special case of the more general idea that multiple labellings of G_F can correspond to the same physical tension configuration, and Noether's theorem identifies the corresponding conserved tension channel (charge).

6.4 6.4 Plane Waves and Light as Propagating Tension

In vacuum and in the absence of sources ($\rho = 0$, $\boldsymbol{J} = 0$), Maxwell's equations imply that each component of $\boldsymbol{E}$ and $\boldsymbol{B}$ satisfies the wave equation

$$\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = c^2 \nabla^2 \boldsymbol{E}, \qquad \frac{\partial^2 \boldsymbol{B}}{\partial t^2} = c^2 \nabla^2 \boldsymbol{B}, \tag{106}$$

with wave speed

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}. \tag{107}$$

Plane wave solutions have the form

$$\boldsymbol{E}(\boldsymbol{x}, t) = \mathrm{Re}\left\{ \boldsymbol{E}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)} \right\}, \tag{108}$$

$$\boldsymbol{B}(\boldsymbol{x}, t) = \mathrm{Re}\left\{ \boldsymbol{B}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega t)} \right\}, \tag{109}$$

with $\omega = c\|\boldsymbol{k}\|$ and $\boldsymbol{E}_0$, $\boldsymbol{B}_0$ transverse to $\boldsymbol{k}$ and to each other.

In this picture, classical light is a self–sustaining oscillation of electromagnetic tension: changes in $\boldsymbol{E}$ generate changes in $\boldsymbol{B}$ and vice versa, such that the field continually passes its own tension along at speed c. The Poynting vector $\boldsymbol{S}$ points in the direction of propagation and gives the energy flux carried by the wave.

From the HOK viewpoint:

- Light is a pattern of tension in the electromagnetic labels of F that propagates even in the absence of any material containers C.

- The universal speed c is the effective limit on how fast electromagnetic tension can be updated on G_F; at the L3 level we would like to derive c (or at least its scaling) from kernel scales l^* and t^*, as suggested schematically by $c \sim l^*/t^*$.
- Polarization encodes how the tension oscillates in directions orthogonal to the direction of propagation.

This interpretation smoothly connects classical waves to the relativistic structure of space–time, where c also appears as the invariant causal speed in special relativity (Chapter 7).

6.5 6.5 Outlook: Coupling to Matter and Quantum Extensions

Classical electromagnetism, as reviewed here, already fits naturally inside the HOK kernel storyline:

- Charges are properties of containers C.
- Electromagnetic fields are additional tension channels carried by labels on F.
- Maxwell's equations are local update rules for field tension, constrained by charge conservation and gauge symmetry.
- The Poynting vector and Maxwell stress tensor describe how electromagnetic tension flows across boundaries B and exchanges with mechanical tension in matter.

The next steps, developed in later chapters, are:

- **Relativistic unification.** Casting electromagnetism in manifestly covariant form, with field strength $F_{\mu\nu}$ and stress–energy tensor $T_{\mu\nu}$, prepares the ground for coupling to gravity in general relativity (Chapter 8).
- **Quantum extensions.** In quantum mechanics and quantum field theory (Chapters 10 and 9), the electromagnetic field becomes quantized, and classical beams of light are reinterpreted as coherent states of photons. At the kernel level, this corresponds to promoting classical tension patterns to quantum amplitudes over discrete configurations of G_F.

Electromagnetism thus serves as both a rich testing ground and a bridge: it demonstrates how a purely field–based notion of tension fits into HOK, and it leads naturally into the relativistic and quantum domains where the kernel must ultimately operate if it is to speak about all known interactions.

7 Special Relativity: Causality and Invariant Tension Channels

Classical mechanics assumes an absolute time parameter shared by all observers. Electromagnetism, through Maxwell's equations (Chapter 6), introduces a universal propagation speed c for field disturbances. Special relativity reconciles these two pictures by recognizing that space and time must be unified into a single space–time structure in which c is an invariant causal speed.

From the HOK perspective, special relativity is the continuum limit of a discrete causal graph G_F with a maximal update speed:

- Nodes represent events in the kernel.
- Directed edges represent admissible causal updates.
- The constant c encodes the tightest possible bound on how fast tension can propagate along these edges.

Lorentz transformations are then the re–labellings of G_F that preserve its causal order and the invariant tension budget associated with space–time intervals.

In this chapter we briefly review Minkowski space, Lorentz transformations, four–vectors and energy–momentum, and we sketch their kernel interpretation.

7.1 7.1 Minkowski Space as the Continuum Limit of a Causal Graph

In standard special relativity, events are labelled by coordinates

$$x^\mu = (ct, \boldsymbol{x}), \tag{110}$$

and the space–time interval between two neighbouring events is

$$ds^2 = -c^2 dt^2 + d\boldsymbol{x}^2 = \eta_{\mu\nu} dx^\mu dx^\nu, \tag{111}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$ is the Minkowski metric. This interval is invariant under Lorentz transformations.

In the HOK picture we start instead from a discrete causal graph G_F:

- Each node corresponds to a kernel update event.
- A directed edge $A \to B$ means that B is in the causal future of A (some tension update at A can influence B).
- The graph is locally finite and respects a maximal causal speed: no chain of edges can propagate tension faster than an effective limit set by the kernel scales l^* and t^*.

When G_F is coarse–grained over many nodes and edges, its causal structure can be approximated by a continuous manifold equipped with a metric whose null directions encode the maximal propagation speed. The emergence of Minkowski space can be summarized schematically as

$$G_F \xrightarrow{\text{coarse–graining}} (\mathcal{M}, \eta_{\mu\nu}), \tag{112}$$

where $\mathcal{M}$ is a four–dimensional manifold and $\eta_{\mu\nu}$ plays the role of the effective metric.

The space–time interval ds^2 is then the continuum bookkeeping device for how tension budgets along different directions in G_F compare: timelike intervals correspond to sequences of updates that can be realized by containers; spacelike intervals correspond to pairs of events with no direct causal connection in G_F.

7.2 7.2 Lorentz Transformations and Invariant Interval

A Lorentz transformation is a linear map $\Lambda^\mu{}_\nu$ acting on four–vectors x^μ such that

$$\eta_{\alpha\beta} = \eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta, \tag{113}$$

or equivalently

$$ds'^2 = \eta_{\mu\nu}dx'^\mu dx'^\nu = \eta_{\mu\nu}dx^\mu dx^\nu = ds^2. \tag{114}$$

In $1+1$ dimensions, a boost with relative velocity v along x is

$$ct' = \gamma(ct - \beta x), \tag{115}$$

$$x' = \gamma(x - \beta ct), \tag{116}$$

where

$$\beta = \frac{v}{c}, \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}. \tag{117}$$

Standard derivations show that the principle of relativity (all inertial frames are equivalent) plus the existence of an invariant speed c imply that the transformation between frames must be Lorentz, not Galilean.

In kernel language:

- Different inertial observers correspond to different ways of slicing and labelling the same causal graph G_F into "space" and "time".
- The existence of a maximal update speed for tension in F plays the role of the invariant speed c.
- Lorentz transformations are precisely those re–labellings that preserve the causal order and the tension budget associated with the interval ds^2.

Thus, the invariant space–time interval is the continuum analogue of an underlying invariant tension metric on G_F: it quantifies how "far apart" two events are in terms of the minimal tension cost to connect them causally.

7.3 7.3 Four-Velocity, Four-Momentum and Energy–Momentum Relation

For a particle of rest mass m following a worldline $x^\mu(\tau)$ parametrized by proper time τ, the four–velocity is

$$u^\mu = \frac{dx^\mu}{d\tau}, \qquad \eta_{\mu\nu} u^\mu u^\nu = -c^2. \tag{118}$$

The four–momentum is

$$p^\mu = m u^\mu, \tag{119}$$

with components

$$p^\mu = \left(\frac{E}{c}, \boldsymbol{p} \right), \tag{120}$$

where E is the relativistic energy and $\boldsymbol{p}$ the three–momentum. The invariant mass–shell condition is

$$-m^2 c^2 = \eta_{\mu\nu} p^\mu p^\nu = -\frac{E^2}{c^2} + \boldsymbol{p}^2, \tag{121}$$

which yields the familiar relation

$$E^2 = (pc)^2 + (mc^2)^2. \tag{122}$$

From the HOK standpoint, four–momentum is the relativistic bookkeeping of how tension is allocated between temporal and spatial channels for a moving container:

- The rest mass m represents the intrinsic tension budget associated with the container even when it is not moving.
- The kinetic contribution $\boldsymbol{p}$ and the associated gamma factor γ summarize how much extra tension is tied up in motion relative to the container's own rest frame.
- The invariant combination $E^2 - (pc)^2$ is the conserved kernel quantity: different observers may disagree on the split between energy and momentum, but they agree on the total tension budget encoded in $p^\mu p_\mu$.

This interpretation will extend naturally in general relativity, where matter and field tension are unified in the stress–energy tensor $T_{\mu\nu}$ (Chapter 8).

7.4 7.4 Light Cones, Causality and Kernel Update Rules

A central geometric object in special relativity is the light cone of an event:

- The future light cone consists of events that can be reached by signals travelling at or below the speed c.
- The past light cone consists of events that can influence the given event via such signals.
- Events outside both cones are spacelike separated and cannot influence each other without violating the causal speed limit.

In the kernel picture, light cones are simply the continuum shadows of the reachability structure in G_F:

- The future light cone of a node A is the coarse–grained set of nodes reachable from A via directed paths, given the maximal update speed.
- The interior of the cone corresponds to timelike paths where containers can carry tension from A to other events.
- The boundary of the cone corresponds to null paths along which pure field tension (e.g. light) propagates.

Special relativity can therefore be summarized at the L1 level as:

> The kernel update rules enforce a strict partial order on events in G_F with a maximal propagation speed for tension. In the continuum limit this order is represented by light cones in a Minkowski manifold, and Lorentz transformations are the automorphisms of this causal structure.

This view emphasizes that causality is not an emergent afterthought but a core design constraint of the kernel: any acceptable coarse–grained description must respect it.

7.5 7.5 Outlook: Toward Gravity and Quantum Fields

Special relativity provides the rigid background on which much of modern physics is built:

- Classical electromagnetism becomes a relativistic field theory with a covariant field strength $F_{\mu\nu}$ and stress–energy tensor $T_{\mu\nu}$ (Chapter 6).
- General relativity (Chapter 8) promotes the Minkowski metric to a dynamical field $g_{\mu\nu}$ that responds to the distribution of tension (energy–momentum) in G_F.
- Quantum field theory (Chapter 9) quantizes fields on this relativistic background, turning classical field tension into quantum excitations and amplitudes.

At the kernel level, special relativity is the first explicit constraint on any candidate L3 derivation:

- The emergent continuum must admit a Lorentzian metric consistent with a maximal tension propagation speed.
- The bookkeeping of tension in containers and fields must be compatible with four–vector and tensor structures that transform covariantly under Lorentz transformations.

Only after these relativistic constraints are satisfied can one hope to derive or constrain physical constants such as c, $\hbar$ and G from the discrete properties of G_F. Special relativity is thus not just a kinematic curiosity, but a non–negotiable structural boundary condition for any HOK–based physics.

8 General Relativity and Gravity: Geometry, Tension and Emergence

Special relativity (Chapter 7) describes physics on a fixed Minkowski background where the causal structure is rigid and flat. General relativity (GR) abandons this rigidity: the geometry of space–time itself becomes dynamical and responds to the distribution of energy and momentum.

From the HOK perspective, GR is the continuum description of how the coarse–grained tension content of containers and fields backreacts on the effective geometry induced by the causal graph G_F:

- Matter and fields carry stress–energy, a measure of tension density and flux.
- The geometry of space–time encodes how tension flows through G_F at large scales.
- Gravity appears not as a separate force, but as the curvature of the effective tension geometry.

In this chapter we review the equivalence principle, Einstein's field equations, black holes and horizons at the L1/L2 level, and we sketch an L1/L3 route for relating Newton's constant G to kernel data via entropic/emergent gravity ideas.

8.1 8.1 Equivalence Principle and Geometry of Free Fall

The conceptual starting point of GR is the equivalence principle:

> Locally, the effects of a uniform gravitational field are indistinguishable from those of uniform acceleration.

In practice, this means that in a sufficiently small region of space–time one can always choose coordinates in which freely falling test particles move along straight lines, as if gravity had been "transformed away". Globally, however, this is impossible when space–time is curved.

Mathematically, the trajectories of freely falling particles (in the absence of non–gravitational forces) are *geodesics* of a Lorentzian metric $g_{\mu\nu}(x)$:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu{}_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0, \tag{123}$$

where $\Gamma^\mu{}_{\nu\rho}$ are the Christoffel symbols associated with $g_{\mu\nu}$ and τ is proper time.

In the HOK picture:

- Free fall corresponds to a container C following a tension–free path in the effective geometry induced by G_F; the geodesic equation is the continuum analogue of "no net local tension gradient" in the container's instantaneous rest frame.
- Gravity is not a separate kind of tension channel; it is the effect of large–scale inhomogeneities in how the kernel assigns causal distances and update costs across G_F.
- Locally, one can always choose a frame in which these inhomogeneities are invisible—mirroring the inertial frames of Chapter 1—but globally their accumulated effect is curvature.

Thus, at the L1 level, GR states that *gravitational phenomena are nothing but manifestations of the geometry of the effective tension metric.*

8.2 8.2 Einstein Field Equations and Stress–Energy Tensor

The dynamical content of GR is encoded in Einstein's field equations:

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{124}$$

where:

- $G_{\mu\nu}$ is the Einstein tensor, built from the metric $g_{\mu\nu}$ and its derivatives:

  $$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}, \tag{125}$$

  with $R_{\mu\nu}$ the Ricci tensor and R the Ricci scalar.
- $T_{\mu\nu}$ is the stress–energy tensor, summarizing the energy density, momentum density, pressure and shear stresses of matter and fields.
- G is Newton's gravitational constant.

The left–hand side measures curvature: how the presence of tension distorts the geometry of space–time. The right–hand side measures the distribution of tension: how much energy, momentum and stress are present at each point.

In HOK language:

- $T_{\mu\nu}$ is the continuum limit of all tension channels in G_F associated with containers and fields. Its conservation,

$$\nabla_\mu T^{\mu\nu} = 0, \tag{126}$$

expresses local tension balance: no net tension is created or destroyed; it is merely redistributed.

- $G_{\mu\nu}$ summarizes how the effective causal distances and light cones in G_F are deformed by this tension content.

- Einstein's equation (124) is the statement that *geometry is constrained by tension*: the way containers and fields warp the graph must match the way tension is distributed.

At the L2 level, one can obtain (124) from an action principle with the Einstein–Hilbert action plus matter:

$$S = \frac{c^3}{16\pi G} \int R\sqrt{-g}\, d^4x + S_{\text{matter}}[g_{\mu\nu}, \text{fields}], \tag{127}$$

where $g = \det(g_{\mu\nu})$. From the kernel standpoint, this is the continuum action for the *tension cost* of a given geometry plus the tension stored in matter and field configurations.

8.3 8.3 Black Holes, Horizons and Gravitational Tension

Among the most striking solutions of Einstein's equations are black holes. In their simplest (Schwarzschild) form, they describe the space–time outside a spherically symmetric mass M. The corresponding metric has an event horizon at radius

$$r_s = \frac{2GM}{c^2}, \tag{128}$$

the Schwarzschild radius.

The event horizon is a null surface separating regions that can communicate with infinity from those that cannot: once a container crosses the horizon, all its future trajectories remain within the black hole.

From the HOK viewpoint:

- A black hole is an extreme concentration of tension in G_F whose backreaction has deformed the effective geometry so much that entire regions of the graph are causally sealed off from distant observers.

- The horizon is not a material boundary but a *causal boundary* in the effective tension geometry: it separates nodes whose outgoing tension-carrying paths can reach infinity from those whose paths are trapped.

- The area of the horizon plays a central role in encoding the capacity of the black hole to store tension in hidden degrees of freedom, as suggested by the Bekenstein–Hawking entropy formula (to be revisited in Chapter 21).

Even without committing to a full quantum gravity picture, one can interpret the gravitational attraction near a black hole as the tendency of containers to follow geodesics that minimize their local tension budget, which in a highly curved region converges toward the horizon.

8.4 8.4 Gravitational Waves as Ripples of Tension Geometry

Einstein's equations admit wave–like solutions in which small perturbations of the metric propagate at the speed of light: *gravitational waves*. In the weak–field, linearized regime one writes

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1, \tag{129}$$

and finds that components of $h_{\mu\nu}$ obey a wave equation

$$\Box h^{\mathrm{TT}}_{\mu\nu} = 0, \tag{130}$$

where "TT" denotes the transverse–traceless gauge and $\Box$ is the d'Alembertian.

Gravitational waves carry energy and momentum, and they can be detected by their tiny tidal effects on free–falling test masses.

In kernel language:

- Gravitational waves are propagating ripples in the effective tension geometry of G_F: small, time–dependent perturbations in how distances and light cones are assigned between events.
- They represent tension that has been radiated away from dynamical systems (e.g. binary mergers) not via electromagnetic or material channels, but via direct modulation of the causal structure itself.
- Their propagation at speed c reinforces the idea that c is the universal limit for all tension channels, not only electromagnetic ones.

At the L1/L2 level, gravitational waves confirm that gravity behaves like a genuine field with its own tension budget, even though it is encoded in geometry rather than in an additional vector or scalar field.

8.5 8.5 Entropic / Emergent Gravity and the Mapping of G (L1/L3)

The constant G in Einstein's equations sets the coupling strength between stress–energy and curvature. From an L3 perspective, a natural HOK question is:

Can G be related to discrete properties of the kernel graph G_F, such as the density of nodes, the connectivity of causal links or the entropy associated with boundaries?

Several lines of thought in the literature interpret gravity as an emergent, entropic or thermodynamic phenomenon. While HOK does not commit to any specific existing model, it shares the structural intuition that:

- Geometry encodes coarse–grained information about microscopic tension configurations.
- Gravitational attraction may be a macroscopic manifestation of statistical tendencies in how tension and information are distributed across G_F.
- Horizon areas and entanglement entropy hint that "area" measures of boundaries play a privileged role in counting hidden degrees of freedom (see Chapter 21).

At the L1/L3 level, a HOK–style roadmap for G might look like:

1. Identify a class of kernel microstates for a region bounded by a surface $\mathcal{B}$ (e.g. a spherical boundary in G_F).
2. Define a notion of boundary entropy $S(\mathcal{B})$ counting microscopic tension configurations consistent with a given coarse–grained geometry.
3. Show that, in an appropriate limit and under suitable update rules, variations in this entropy S with respect to displacing containers give rise to an effective entropic force.
4. Demonstrate that this entropic force reproduces, at leading order, Newtonian gravity with an effective coupling constant G_{eff} expressible in terms of kernel parameters (e.g. node density, update rate, minimal tension quantum).

Symbolically, one would seek relations of the form

$$G_{\text{eff}} \sim f\big(\rho_{\text{node}},\, l^*,\, t^*,\, \varepsilon^*,\, S_{\text{boundary}}/A\big), \tag{131}$$

where A is the area of the boundary and S_{boundary}/A is an entropy density per unit area. Matching G_{eff} to the observed G would then be a *numerical constraint* on admissible kernel parameters.

At this stage, HOK only provides the structural scaffolding for such a program:

- Boundaries $\mathcal{B}$ and containers C are already primitive elements.
- Tension and information are naturally linked via the Landauer principle and entanglement entropy (see Chapter 5 and Chapter 21).
- The area–law behaviour of entanglement entropy suggests that the kernel may indeed assign special status to boundary area as a measure of hidden tension channels.

However, no concrete L3 derivation of G is claimed here. Instead, Section 8.5 should be read as a *design specification*: any future attempt to compute or constrain G from HOK must:

- respect the full tensor structure of equation (124),
- reproduce Newtonian gravity in the appropriate weak–field limit,
- and explain, in microscopic kernel terms, why area–based entropy measures are so tightly linked to gravitational phenomena.

In summary, general relativity at the L1/L2 level fits naturally into the HOK kernel narrative: gravity is the geometry of tension. The open L3 challenge is to show how the coupling constant G and the associated scales of curvature emerge from discrete properties of the kernel graph G_F and its tension bookkeeping.

9 Quantum Field Theory and HOK Gauge Fields

10 Quantum Mechanics: States, Measurement and Wave–Particle Duality

Classical mechanics and field theory (Chapters 1, 2, 6 and 7) describe systems in terms of definite configurations and trajectories in space–time. Quantum mechanics replaces this picture with a theory of *states* and *amplitudes* defined on abstract configuration spaces, with probabilistic outcomes upon measurement.

From the HOK perspective, quantum theory is the next layer of structure on top of the kernel graph G_F:

- Classical tension bookkeeping is replaced (or extended) by complex amplitudes assigned to discrete configurations of containers and fields.
- The squared magnitude of these amplitudes yields the probability distribution for how tension will *actually* be realized when the kernel updates macroscopic containers.
- Interference, wave–particle duality and decoherence reflect how different tension histories on G_F combine or cancel before being projected into classical outcomes.

In this chapter we work at the L1/L2 level: we restate the core formalism of nonrelativistic quantum mechanics, interpret the Born rule and measurement in kernel terms, and then focus on the double–slit experiment, wave–particle duality and decoherence (Sections 10.4–10.6) as the main demo of the HOK viewpoint.

10.1 9.1 Kinematics: Hilbert Space, States and Observables

The kinematic backbone of quantum mechanics is a complex Hilbert space $\mathcal{H}$:

- Pure states are represented by rays in $\mathcal{H}$, usually by unit vectors $|\psi\rangle$ with $\langle\psi|\psi\rangle = 1$.
- Observables are represented by self–adjoint operators $\hat{A} : \mathcal{H} \to \mathcal{H}$.
- Measurement outcomes correspond to eigenvalues of $\hat{A}$, with probabilities determined by overlaps of $|\psi\rangle$ with the corresponding eigenstates.

In a position basis $|\boldsymbol{x}\rangle$, states can be written as

$$|\psi\rangle = \int d^3x\, \psi(\boldsymbol{x})\, |\boldsymbol{x}\rangle, \tag{132}$$

where $\psi(\boldsymbol{x})$ is the wavefunction and

$$\int d^3x\, |\psi(\boldsymbol{x})|^2 = 1. \tag{133}$$

Kernel interpretation (L1):

- Instead of assigning a single configuration tension to a container, HOK assigns a complex amplitude to each admissible configuration of containers and fields.
- The Hilbert space $\mathcal{H}$ is the completion of the space of all such discrete configuration labels under a suitable inner product.
- Observables are coarse–grained questions we can ask about these configurations: operators that compress the full amplitude structure into a limited set of macroscopically distinguishable outcomes.

In this view, the classical configuration space is the shadow obtained when all amplitudes except those compatible with a given macro–container state have been traced out.

10.2 9.2 Dynamics: Schrödinger Equation and Kernel Action Phase

The time evolution of a closed quantum system is governed by the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle, \tag{134}$$

where $\hat{H}$ is the Hamiltonian operator. Formally, the solution can be written as

$$|\psi(t)\rangle = U(t, t_0)|\psi(t_0)\rangle, \qquad U(t, t_0) = \exp\left(-\frac{i}{\hbar}\hat{H}(t - t_0)\right). \tag{135}$$

In the path integral formulation, the propagator between configurations q_1 and q_2 is given by a sum over histories

$$K(q_2, t_2; q_1, t_1) = \int \mathcal{D}[q(t)] \, \exp\left(\frac{i}{\hbar} S[q]\right), \tag{136}$$

where $S[q]$ is the classical action of Chapter 2.

Kernel interpretation (L1/L2):

- The classical action $S[q]$ is promoted from a tension budget for a single realized history to a *phase generator* for amplitudes over all admissible histories on G_F.
- The factor $\exp(iS/\hbar)$ expresses how different tension histories interfere: histories with similar action add constructively; histories with rapidly varying action cancel.
- The effective classical limit emerges when phase oscillations cause all but a narrow family of tension histories to destructively interfere, leaving geodesic–like paths as in the stationary action principle.

At the L3 level, one would like to relate $\hbar$ to kernel scales $(l^*, t^*, \varepsilon^*)$ and discrete update rules, but this lies beyond the scope of the present chapter (see Chapter 20).

10.3 9.3 Measurement, Born Rule and Macroscopic Containers

The Born rule states that if $|\psi\rangle$ is a normalized state and $\hat{A}$ has eigenstates $|a_n\rangle$ with eigenvalues a_n, then the probability of obtaining a_n upon measuring $\hat{A}$ is

$$P(a_n) = |\langle a_n|\psi\rangle|^2. \tag{137}$$

After the measurement, the state is updated (in the simplest projective measurement model) to

$$|\psi\rangle \mapsto \frac{\hat{P}_n|\psi\rangle}{\sqrt{\langle\psi|\hat{P}_n|\psi\rangle}}, \qquad \hat{P}_n = |a_n\rangle\langle a_n|. \tag{138}$$

From the HOK viewpoint, measurement is not a mysterious additional postulate but a description of what happens when microscopic amplitude structures are forced to register in large, redundant macroscopic containers (measurement devices, memory states, etc.):

- Before measurement, the kernel tracks a superposition of tension histories on G_F that differ in microscopic detail but agree on coarse macroscopic conditions.
- A measurement device is a specially engineered container whose many internal degrees of freedom become strongly correlated with a particular observable of the system.
- When this correlation becomes macroscopically robust and effectively irreversible (environment–induced redundancy), the kernel must select a single branch as "realized" at the level of macroscopic containers.

The Born rule then expresses which branch weights are consistent with the overall amplitude structure and the requirement that tension bookkeeping remains coherent under repeated experiments. Different interpretations (Copenhagen, many–worlds, objective collapse) offer different stories at the philosophical level; HOK remains formally agnostic but demands that any interpretation is compatible with the container/field/tension architecture.

10.4 9.4 Double-Slit Experiment as a Kernel Demo (L1/L2)

The double–slit experiment is the canonical demonstration of quantum interference and wave–particle duality. In its simplest form:

- A source emits single particles (electrons, photons) toward a barrier with two narrow slits, labelled A and B.
- Beyond the slits is a detection screen that records individual impacts as localized spots.
- When both slits are open and no which–path information is available, the accumulated spots form an interference pattern.
- When a which–path detector is installed at the slits, the interference pattern disappears, replaced by a classical two–slit sum of intensities.

In standard quantum mechanics, the amplitude for a detection at a point $\boldsymbol{x}$ on the screen is the coherent sum

$$\psi(\boldsymbol{x}) = \psi_A(\boldsymbol{x}) + \psi_B(\boldsymbol{x}), \tag{139}$$

and the detection probability is

$$P(\boldsymbol{x}) = |\psi(\boldsymbol{x})|^2 = |\psi_A(\boldsymbol{x})|^2 + |\psi_B(\boldsymbol{x})|^2 + 2\,\mathrm{Re}(\psi_A^*(\boldsymbol{x})\psi_B(\boldsymbol{x}))\,. \tag{140}$$

The cross term encodes interference. If which–path information is recorded, coherence between the A and B paths is lost, and the cross term effectively vanishes in the observed statistics.

Kernel interpretation (L1/L2):

- The particle is a container C whose possible histories on G_F include paths through slit A, through slit B and through various combinations of intermediate nodes.
- The wavefunction amplitudes ψ_A and ψ_B summarize how tension histories associated with the two slits contribute to a detection at $\boldsymbol{x}$.
- When no which–path information is stored in any macroscopic container, these histories remain in coherent superposition, and their tension phases interfere, producing the cross terms.

- When a which–path detector correlates each history with a distinct macroscopic container state ("went through A" vs "went through B"), the environment entangles with the path label, suppressing interference in the reduced statistics of the screen.

Thus, the double–slit experiment becomes a vivid illustration of how the kernel's amplitude bookkeeping over histories interacts with the container architecture to produce classical records.

10.5 9.5 Wave–Particle Duality as Container–Field Duality (L1)

Textbook presentations speak of "wave–particle duality": light and matter sometimes behave like waves, sometimes like particles. In the HOK picture this is reframed as a dual description of the same underlying kernel structure:

- The *wave* aspect reflects how amplitude and tension are distributed over many possible configurations in G_F. Interference patterns, diffraction and coherent phenomena are manifestations of this field–like structure.
- The *particle* aspect reflects the fact that macroscopic containers only ever register localized, discrete events: individual clicks on a detector, tracks in a cloud chamber, quantized energy transfers.

Rather than saying "the electron is both a wave and a particle", HOK would say:

> At the kernel level, the electron is a container whose potential histories are represented by a distributed amplitude field on G_F. At the macroscopic level, only those histories that deposit tension into robust containers show up as localized "particle" events.

The apparent duality is thus a perspective mismatch between the field of amplitudes on F and the discrete update events in macroscopic C. Quantum mechanics is the formalism that consistently links these two views via the Born rule and unitary dynamics.

10.6 9.6 Decoherence and the Emergence of Classicality (L1/L2)

Decoherence theory explains how interference between different branches of a quantum state becomes unobservable when the system interacts with a large environment.

Consider a simple system–environment split:

$$|\Psi\rangle = \sum_n c_n |s_n\rangle \otimes |E_n\rangle, \tag{141}$$

where $|s_n\rangle$ are system states and $|E_n\rangle$ are (typically nonorthogonal but increasingly distinguishable) environment states. The reduced density matrix for the system is

$$\rho_S = \mathrm{Tr}_E \left(|\Psi\rangle\langle\Psi|\right) = \sum_{m,n} c_m c_n^* \langle E_n|E_m\rangle |s_m\rangle\langle s_n|. \tag{142}$$

As the environment entangles with the system in a way that drives $\langle E_n|E_m\rangle \to 0$ for $m \neq n$, the off–diagonal coherence terms are suppressed, and ρ_S approaches an effective classical mixture

$$\rho_S \approx \sum_n |c_n|^2 \, |s_n\rangle\langle s_n|. \tag{143}$$

Kernel interpretation (L1/L2):

- The environment consists of many containers whose states become correlated with different branches of the system's amplitude.
- As these correlations spread, the effective tension bookkeeping for any small subsystem no longer contains interference terms, because amplitudes for macroscopically distinct container configurations are practically orthogonal in $\mathcal{H}$.
- Classicality emerges when the relevant observables are coarse enough that they cannot resolve the tiny residual coherence between branches.

From this perspective, decoherence is not a mysterious collapse but a dynamical process by which amplitude structure in G_F is redundantly encoded in many containers, making some branches effectively irreversible for all practical observers.

The measurement problem, in its strict philosophical sense, remains: HOK does not by itself pick a single branch; it specifies the structural conditions under which branch weights behave classically and interference becomes unobservable.

10.7 9.7 Outlook: Entanglement, Information and Quantum Fields

This chapter has focused on single–particle and simple system examples, with the double–slit, wave–particle duality and decoherence as the main HOK demo at the L1/L2 level. The full power of quantum theory, however, lies in multi–partite systems and fields:

- **Entanglement** (to be connected with entanglement entropy and area laws in Chapter 21) expresses nonclassical correlations between containers that cannot be factorized into local tension assignments.
- **Quantum information** recasts quantum states as resources for communication and computation, highlighting the role of tensor products, channels and error correction as higher–level structures on top of the kernel architecture.
- **Quantum field theory** (Chapter 9) extends the formalism to systems with infinitely many degrees of freedom, where fields become operator–valued distributions on space–time and particles emerge as excitations of these fields.

For HOK, the key constraint is that any quantum extension must remain compatible with:

1. the discrete causal structure of G_F (relativistic locality),

2. the container/field/boundary/tension ontology,

3. and the L3 ambition of ultimately relating constants like $\hbar$, c and coupling constants to kernel scales and combinatorial properties of the graph.

Quantum mechanics, reframed in this way, is not an alien overlay on classical physics but a refined tension bookkeeping scheme for the kernel itself, one that becomes indispensable whenever interference and entanglement cannot be ignored.

11 Quantum Field Theory and HOK Gauge Fields

In previous chapters we treated particles as point masses, rigid bodies, or continuous media. Quantum mechanics (Chapter **??**) upgraded this picture to wavefunctions. Quantum Field Theory (QFT) goes one step further: *fields* become the primary objects, and "particles" are excitations of those fields.

In the HOK picture, these fields live on the causal graph $\mathcal{G}_F = (V_N, E_L)$; particles are particular patterns of tension and phase that remain stable under evolution.

This chapter has three goals:

1. Review the minimal QFT formalism at the level used by working physicists. **[L1]**

2. Make its structural content explicit: locality, symmetry, and renormalization. **[L2]**

3. Indicate how HOK expects to remove UV divergences and fix certain constants (e.g. the fine structure constant) at the discrete-kernel level, without pretending to have computed them. **[L3? open]**

12 From particles to fields

Non-relativistic quantum mechanics treats a particle of mass m via a wavefunction $\psi(\mathbf{x}, t)$ satisfying the Schrödinger equation. This picture fails when:

- particle number is not conserved (pair creation/annihilation),

- relativity is essential ($v \sim c$),

- interactions at very high energies probe short-distance structure.

QFT resolves this by promoting *fields* $\phi(x)$ to the fundamental objects. A "one-particle state" becomes an excitation of the field, created by an operator $a^\dagger_{\mathbf{p}}$ acting on a vacuum $|0\rangle$.

HOK interpretation [L1].

- The field $\phi(x)$ is a coarse-grained description of node variables σ_i on $\mathcal{G}_F$.
- Creation operators $a^\dagger$ correspond to inserting a minimal tension defect m_e (for the electron case) into the graph.
- The "vacuum" is not empty; it is the lowest-tension configuration of σ_i compatible with the global constraints on $\mathcal{G}_F$.

13 Classical field Lagrangians

Before quantization we start with classical fields and an action principle.

13.1 Real scalar field

A real scalar field $\phi(x)$ on Minkowski spacetime has action

$$S[\phi] \;=\; \int d^4x\, \mathcal{L} \qquad \text{with} \qquad \mathcal{L} \;=\; \frac{1}{2}\,\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - V_{\text{int}}(\phi)\,. \tag{144}$$

The Euler–Lagrange equation gives

$$(\Box + m^2)\phi + V'_{\text{int}}(\phi) = 0\,. \tag{145}$$

HOK interpretation [L1/L2]. On a discrete graph, the kinetic term becomes a sum over edges $(i,j) \in E_L$,

$$\sum_{(i,j)} (\phi_i - \phi_j)^2 \quad \longrightarrow \quad \int d^4x\, \partial_\mu\phi\,\partial^\mu\phi\,. \tag{146}$$

The mass and interaction terms are local node contributions. At the HOK level these arise from the local tension functional $\mathcal{L}_{\text{tension}}(\sigma_i, \sigma_j)$ introduced in the core spec.

13.2 Dirac field

To describe spin-$\frac{1}{2}$ fermions we introduce a spinor field $\psi(x)$ with Lagrangian

$$\mathcal{L}_{\text{Dirac}} \;=\; \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi\,. \tag{147}$$

The Dirac equation,

$$(i\gamma^\mu\partial_\mu - m)\psi = 0\,, \tag{148}$$

is first-order in time and space, consistent with special relativity.

HOK note [L2]. In the HOK kernel, fermionic degrees of freedom can be treated as additional components of σ_i with Grassmann-valued fields in a continuum limit. At L2 we only require that the discrete update rule admits an effective Dirac-like dispersion relation near the low-energy excitations corresponding to electrons and other fermions.

13.3 Gauge fields

The simplest gauge field is the $U(1)$ electromagnetic potential $A_\mu(x)$ with field strength

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \,. \tag{149}$$

The Maxwell Lagrangian is

$$\mathcal{L}_{\text{Maxwell}} \;=\; -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \,. \tag{150}$$

Interactions with a charged Dirac field are introduced via the covariant derivative

$$D_\mu = \partial_\mu + ieA_\mu \tag{151}$$

and the combined Lagrangian

$$\mathcal{L}_{\text{QED}} \;=\; -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \,. \tag{152}$$

HOK interpretation [L1/L2].

- The gauge potential A_μ corresponds to a coarse-grained description of the electromagnetic tension channel $T^{(em)}$ on edges of $\mathcal{G}_F$.
- The covariant derivative encodes how the phase component ϕ_i of σ_i is twisted when parallel-transported along edges.
- The gauge coupling e is the strength with which a minimal tension defect m_e couples to that twist.

14 Canonical quantization and particles

To quantize a field we promote it to an operator-valued distribution and impose commutation relations. For the scalar field:

$$[\hat{\phi}(\mathbf{x}, t), \hat{\pi}(\mathbf{y}, t)] = i\hbar\, \delta^{(3)}(\mathbf{x} - \mathbf{y}) \,, \tag{153}$$

where $\hat{\pi} = \partial\mathcal{L}/\partial(\partial_0\phi)$.

Expanding in modes,

$$\hat{\phi}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\mathbf{p}}}} \left(a_{\mathbf{p}} e^{-ip\cdot x} + a_{\mathbf{p}}^\dagger e^{+ip\cdot x} \right), \tag{154}$$

with $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$, the operators $a_{\mathbf{p}}^\dagger$ create one-particle states, $|\mathbf{p}\rangle = a_{\mathbf{p}}^\dagger |0\rangle$.

HOK interpretation [L1]. On the graph, $a_{\mathbf{p}}^\dagger$ corresponds to exciting a coherent pattern of node/edge variables whose coarse-grained momentum is $\mathbf{p}$. The Fock space is the space of all finite combinations of such patterns.

15 Path integrals and generating functionals

An equivalent, often more powerful formulation of QFT uses path integrals. For a scalar field with source $J(x)$, the generating functional is

$$Z[J] \;=\; \int \mathcal{D}\phi \, \exp\!\left(\frac{i}{\hbar} \int d^4x \left[\mathcal{L}(\phi) + J(x)\phi(x) \right] \right). \tag{155}$$

Correlation functions are obtained by functional derivatives:

$$\langle 0 | \mathcal{T}\{\phi(x_1) \cdots \phi(x_n)\} | 0 \rangle = \frac{1}{i^n} \frac{\delta^n Z[J]}{\delta J(x_1) \cdots \delta J(x_n)} \bigg|_{J=0}. \tag{156}$$

HOK interpretation [L2].

- A path integral over ϕ is the continuum limit of a sum over histories Γ on $\mathcal{G}_F$, weighted by $\exp\{iS[\Gamma]/\hbar_{\text{eff}}\}$.
- The measure $\mathcal{D}\phi$ is the coarse-grained image of the combinatorial counting over discrete configurations $\{\sigma_i^{(n)}\}$.
- At L2, we only demand that there exists such a coarse-graining for which the continuum action reproduces the effective field theory observed at low energies.

16 Gauge symmetry and interactions

QFT is governed as much by symmetry as by dynamics. Gauge symmetry encodes redundancies in our description that nevertheless constrain the allowed interactions.

16.1 Local $U(1)$ symmetry (QED)

The matter field transforms as

$$\psi(x) \;\to\; e^{i\alpha(x)} \psi(x)\,, \tag{157}$$

with $\alpha(x)$ spacetime-dependent. To keep the Lagrangian invariant we introduce the gauge field A_μ transforming as

$$A_\mu(x) \;\to\; A_\mu(x) - \frac{1}{e}\,\partial_\mu \alpha(x)\,, \tag{158}$$

and the covariant derivative $D_\mu = \partial_\mu + ieA_\mu$. This construction enforces charge conservation and fixes the form of the interaction.

16.2 Non-Abelian gauge theories

For $SU(N)$ gauge symmetry we promote $\alpha(x)$ to a matrix $\alpha^a(x)T^a$, fields carry group indices, and the gauge field becomes $A^a_\mu T^a$ with non-linear field strength:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu\,. \tag{159}$$

These structures underlie the Standard Model's electroweak and strong interactions.

HOK interpretation [L2].

- Gauge symmetry corresponds to redundancies in how local phases ϕ_i and tension channels $T_i^{(a)}$ are assigned to nodes and edges.
- In the continuum limit, these redundancies manifest as local gauge transformations.
- The structure constants f^{abc} encode how different tension channels braid and couple on $\mathcal{G}_F$.

17 Renormalization and effective field theory

Standard QFT calculations often produce UV divergences. Renormalization reorganizes the theory so that physical predictions remain finite and cutoff-independent.

17.1 Divergences and regularization

Loop integrals such as

$$\int^{\Lambda} \frac{d^4k}{(2\pi)^4}\,\frac{1}{(k^2 - m^2 + i\epsilon)^2} \tag{160}$$

diverge as the cutoff $\Lambda \to \infty$. A regulator (momentum cutoff, dimensional regularization, lattice spacing a, etc.) temporally tames the integral.

17.2 Renormalized parameters

Bare parameters m_0, λ_0, e_0 are not directly observable. Physical mass and couplings are defined at a renormalization scale μ,

$$m^2(\mu) = m_0^2 + \delta m^2(\mu)\,, \qquad e(\mu) = Z_e(\mu) e_0\,, \tag{161}$$

with counterterms chosen so that measured quantities are finite and independent of the regulator.

17.3 Renormalization group

As we change the scale μ, couplings flow according to beta functions,

$$\mu \frac{dg(\mu)}{d\mu} = \beta(g(\mu)) \,. \tag{162}$$

Fixed points of this flow control universal behavior at high or low energies.

HOK interpretation [L2/L3].

- In a truly discrete kernel $\mathcal{G}_F$ with finite edge spacing l_*, UV divergences are artifacts of taking an unphysical continuum limit $l_* \to 0$.
- At L2, HOK must reproduce the same renormalization structure as continuum QFT when coarse-grained, including running couplings and effective field theory behavior.
- At L3, HOK claims that certain dimensionless combinations (e.g. the low-energy value of the fine-structure constant α) are fixed by topology of the discrete kernel rather than left as free parameters. This chapter does *not* compute those numbers; it only prepares the formal infrastructure.

18 QFT through the HOK lens

We summarize the HOK $\leftrightarrow$ QFT correspondence needed in later chapters.

18.1 Dictionary: HOK $\to$ QFT [L1]

HOK object	**QFT analogue**
Node state σ_i	Field value $\phi(x), \psi(x)$ at spacetime point
Edge set E_L	Discretized spacetime links
Tension channel $T^{(\mathrm{em})}$	Gauge field A_μ, field strength $F_{\mu\nu}$
Minimal defect m_e	Electron excitation, localized wave packet
Action $S[\Gamma]$	Continuum action $S[\phi, \psi, A_\mu]$
Path weight $e^{iS[\Gamma]/\hbar_{\mathrm{eff}}}$	Feynman weight $e^{iS/\hbar}$
Information bit I_{bit}	Minimal entropy unit in Landauer principle

19 Cosmology: Expansion, Backgrounds and Large-Scale Structure

Special and general relativity (Chapters 7 and 8) describe local physics in space–time. Cosmology asks what happens when we apply those rules to the entire universe: its global geometry, expansion history and large–scale structure.

From the HOK viewpoint, cosmology is the large–scale statistical behavior of the kernel graph G_F:

- Individual nodes and edges describe local events and causal links.
- Coarse–graining over enormous regions produces an effective homogeneous and isotropic description.
- The expansion history and cosmic inventory encode how tension is distributed globally across G_F and how that distribution changes with cosmic time.

This chapter summarizes the standard ΛCDM picture at the L1/L2 level and sketches the L3 questions HOK must eventually address: the origin of initial conditions, the meaning of the cosmological constant and the interpretation of cosmic horizons.

19.1 11.1 Cosmological Principle and FRW Metric

Observations on very large scales suggest that the universe is approximately homogeneous and isotropic. The *cosmological principle* assumes that, on sufficiently large scales, no region or direction is privileged.

Under this assumption, the most general space–time metric compatible with general relativity is the Friedmann–Robertson–Walker (FRW) metric:

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right], \tag{163}$$

where $a(t)$ is the scale factor and $k = -1, 0, +1$ encodes negative, flat or positive spatial curvature.

Kernel interpretation (L1/L2):

- The FRW metric is the continuum limit of a statistically homogeneous and isotropic kernel graph G_F.
- The scale factor $a(t)$ measures how typical causal distances between nodes change with coarse–grained cosmic time.
- The curvature parameter k summarizes global topological/metric properties of G_F that cannot be removed by local coordinate choices.

At the L2 level, HOK does not try to fix k; it merely demands that coarse–graining of an appropriate G_F reproduces an FRW–type geometry.

19.2 11.2 Friedmann Equations and Cosmic Expansion

Inserting the FRW metric into Einstein's field equations (124) with a perfect fluid stress–energy tensor

$$T^{\mu}{}_{\nu} = \mathrm{diag}(-\rho c^2, p, p, p) \tag{164}$$

leads to the Friedmann equations:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}, \tag{165}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}, \tag{166}$$

supplemented by the continuity equation

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2}\right) = 0, \qquad H = \frac{\dot{a}}{a}. \tag{167}$$

Here ρ and p are the total energy density and pressure; Λ is the cosmological constant.
Kernel interpretation:

- ρ summarizes the average tension density stored in all channels (matter, radiation, vacuum) in a comoving patch of G_F.
- H describes the relative rate at which typical causal distances grow between comoving containers.
- The continuity equation expresses global tension conservation in an expanding background: as the graph "stretches", tension densities redshift according to their equations of state.

At the L2 level, HOK treats (165)–(166) as emergent equations relating coarse–grained tension inventory to the effective geometry of G_F.

19.3 11.3 Cosmic Inventory: Radiation, Matter and Dark Energy

Different components of the cosmic fluid redshift differently with expansion. For a component with equation of state $p = w\rho c^2$, the continuity equation implies

$$\rho(a) \propto a^{-3(1+w)}. \tag{168}$$

Three key cases:

- **Radiation** ($w = 1/3$): $\rho_r \propto a^{-4}$, one factor from volume dilution and one from photon redshift.
- **Nonrelativistic matter** (dust, $w \approx 0$): $\rho_m \propto a^{-3}$, pure volume dilution.
- **Dark energy / cosmological constant** ($w \approx -1$): $\rho_\Lambda = \text{const}$, energy density does not dilute.

It is convenient to define density parameters

$$\Omega_i := \frac{\rho_i}{\rho_{\text{crit}}}, \qquad \rho_{\text{crit}} := \frac{3H_0^2}{8\pi G}, \tag{169}$$

where H_0 is the present–day Hubble parameter. The Friedmann equation at $t = t_0$ becomes

$$1 = \Omega_r + \Omega_m + \Omega_\Lambda + \Omega_k, \qquad \Omega_k := -\frac{kc^2}{(a_0 H_0)^2}. \tag{170}$$

Kernel interpretation (L1/L2):

- Each Ω_i is the fraction of the present–day tension budget stored in a particular channel.
- The near–flatness $\Omega_k \approx 0$ is an observational constraint on the global geometry of G_F.
- The small but nonzero Ω_Λ encodes a "background" tension channel that does not dilute with expansion; its L3 origin is one of the main open questions.

19.4 11.4 Cosmic Horizons and Causal Structure

In an expanding universe, the finite age and finite propagation speed of signals imply that not all regions have been in causal contact. Two important horizon concepts are:

- **Particle horizon:** the maximal comoving distance from which light could have traveled to an observer since the beginning of expansion.
- **Event horizon:** the maximal comoving distance from which light sent *now* will ever be able to reach the observer in the future (which exists in universes with accelerated expansion).

For spatially flat FRW, the comoving particle horizon at scale factor a is

$$\chi_{\text{p}}(a) = c \int_0^{t(a)} \frac{dt'}{a(t')} = c \int_0^a \frac{da'}{a'^2 H(a')}. \tag{171}$$

Similar expressions define the event horizon.

From the HOK perspective:

- Horizons delimit which subsets of G_F can have exchanged tension via lightlike or timelike paths within a given cosmic time budget.
- The apparent homogeneity of the cosmic microwave background (CMB) across regions that were naively outside each other's particle horizons motivates inflationary or other mechanisms that modify the early–time structure of G_F.
- An event horizon in a Λ–dominated universe resembles a cosmological version of a black hole horizon: regions of G_F that become permanently causally disconnected from an observer.

Horizons are therefore global features of the kernel's causal structure, not local forces.

19.5 11.5 Inflation, Initial Conditions and Kernel Initialization (L1/L3)

Inflationary cosmology posits a period of accelerated, quasi–exponential expansion in the early universe, driven by the potential energy of a scalar field (the inflaton). At the L1 level, this helps explain:

- **Horizon problem:** why widely separated regions of the CMB share nearly identical temperatures despite naive causal disconnection.
- **Flatness problem:** why Ω_k is so close to zero today.
- **Seed perturbations:** how quantum fluctuations of the inflaton can generate an approximately scale–invariant spectrum of primordial density perturbations.

In a simple slow–roll model with potential $V(\phi)$, the energy density is dominated by $V(\phi)$ and the scale factor evolves approximately as

$$a(t) \propto e^{H_{\mathrm{inf}} t}, \qquad H_{\mathrm{inf}}^2 \approx \frac{8\pi G}{3c^2} V(\phi). \tag{172}$$

HOK L1 interpretation:

- Inflation corresponds to a transient epoch in which a particular tension channel associated with a scalar field dominates the global budget, causing rapid stretching of causal distances in G_F.
- Quantum fluctuations during this epoch are small stochastic modulations of tension along different branches of G_F that later grow into classical density contrasts.

At the L3 level, inflation touches the deepest kernel questions:

- What does it mean to "initialize" G_F? Are there natural kernel configurations whose coarse–grained description looks like an inflating FRW universe?

- Can the statistics of primordial perturbations (e.g. spectral index, non–Gaussianities) be related to combinatorial properties of kernel update rules rather than free continuous parameters in a potential $V(\phi)$?
- Is inflation necessary at all from an HOK perspective, or can alternative kernel initialization schemes solve the same large–scale puzzles?

This chapter does not answer these questions; it records them as design constraints on any HOK–compatible cosmological model.

19.6 11.6 Dark Energy and the Cosmological Constant Problem (L1/L3)

Observations of supernovae, the CMB and large–scale structure suggest that the universe's expansion is currently accelerating, consistent with a dominant dark energy component with equation of state close to $w = -1$. In GR this is often modeled by a cosmological constant Λ.

At the L1 level, Λ is simply a parameter in the Friedmann equations. At the L2/L3 level it becomes a puzzle:

- Quantum field theory predicts very large vacuum energy densities from zero–point fluctuations, far exceeding the observed value associated with Λ.
- Yet the effective Λ is small but nonzero in our universe, contributing an energy density of order $\rho_\Lambda \sim (10^{-3}\,\mathrm{eV})^4$ in particle–physics units.

From the HOK perspective, several possibilities exist (none yet proven):

1. **Kernel renormalization:** vacuum tension channels may be automatically renormalized in G_F so that only a small residual effective Λ survives at large scales.
2. **Global constraint:** the kernel may enforce a global tension balance condition on G_F that effectively tunes Λ to a value compatible with long–lived, structure–forming universes.
3. **Discreteness bound:** the minimal tension quantum ε^* and node density may combine to bound the allowed range of ρ_Λ.

A successful L3 account of Λ would express it schematically as

$$\rho_\Lambda \sim f_\Lambda\big(l^*, t^*, \varepsilon^*, \rho_{\mathrm{node}}, \mathrm{topology}(G_F)\big), \tag{173}$$

with f_Λ a dimensionless function determined by the kernel architecture. At present, HOK only asserts that such a relation *should* exist if the framework is to qualify as a serious candidate for "explaining constants" (Chapter 20).

19.7 11.7 Summary and Outlook

Cosmology stretches HOK to its largest scales:

- The FRW metric and Friedmann equations provide the L1 continuum description of a statistically homogeneous, isotropic universe.
- The cosmic inventory $(\Omega_r, \Omega_m, \Omega_\Lambda, \Omega_k)$ summarizes how the global tension budget is split among channels.
- Horizons and inflation highlight that the kernel's causal structure and initialization are not free: they must reproduce the observed large–scale coherence and near–flatness of the universe.
- Dark energy and the cosmological constant problem are sharp L3 tests: if HOK cannot say why Λ has its observed order of magnitude, its claim to "derive constants" remains incomplete.

In the rest of the Appendix, we turn from the large–scale behavior of G_F to the microphysical side: Chapter 20 discusses the general strategy for relating physical constants to kernel parameters, and Chapter 21 explores chaos and entanglement entropy, which connect local quantum behavior to global geometric constraints.

20 Physical Constants and Kernel Mapping

Most physical theories contain constants: numerical parameters that must be fixed by experiment. From the HOK point of view this is precisely where the framework is tested: if the kernel graph G_F is more than an elegant vocabulary, it should constrain — and in some cases determine — the values or allowed ranges of key constants.

This chapter has three levels:

- **L1**: Classify constants and restate their roles in standard physics.
- **L2**: Express those roles using the kernel ontology (fields, containers, boundaries, tension).
- **L3**: Outline how kernel scales $(l^*, t^*, \varepsilon^*, I_{\text{bit}}, \dots)$ might *map* to constants like c, $\hbar$, k_B and G, without claiming a completed numerical derivation.

We focus on four pillars:

c (causal speed), $\hbar$ (action quantum), k_B (entropy/temperature link), G (geometry–tension coupling).

20.1 12.1 Dimensional vs Dimensionless Constants (L1)

It is useful to distinguish:

- **Dimensional constants**, which carry units (e.g. c, $\hbar$, k_B, G). Their numerical values depend on our choice of units.
- **Dimensionless constants**, such as the fine-structure constant $\alpha \approx 1/137$ or mass ratios like m_p/m_e. Their numerical values are invariant under unit changes.

Dimensional constants can often be "set to 1" by adopting natural units (e.g. $c = \hbar = k_B = 1$), but this does not remove the underlying physical scales; it merely hides them. Dimensionless constants, by contrast, encode pure numbers that any fundamental theory should ideally explain.

In the HOK program:

- Dimensional constants should map to combinations of kernel scales $(l^*, t^*, \varepsilon^*, I_{\text{bit}}, \dots)$.
- Dimensionless constants should map to topological or combinatorial invariants of G_F (e.g. connectivity patterns, knot invariants of tension channels).

This chapter focuses on the first category, while leaving detailed dimensionless calculations to future work (see Chapter 9 for QFT-related couplings).

20.2 12.2 Natural Units and Kernel Scales (L1/L2)

In many theoretical contexts one sets

$$c = \hbar = k_B = 1, \tag{174}$$

so that velocities, energies and temperatures become dimensionless. This is convenient but can obscure which physical scales are being implicitly fixed.

The HOK kernel introduces its own natural scales:

- A minimal length scale l^* (typical node spacing).
- A minimal time scale t^* (typical update time between generations of G_F).
- A minimal tension/energy quantum ε^* associated with elementary changes in node/edge states.
- A minimal information unit I_{bit} for state distinctions on containers and boundaries (see Chapter 5).

At the L2 level, natural HOK units are those in which

$$l^* = t^* = \varepsilon^* = I_{\text{bit}} = 1, \tag{175}$$

and all emergent constants become dimensionless combinations of these kernels.

The central L3 question is then:

> Given $(l^*, t^*, \varepsilon^*, I_{\text{bit}}, \dots)$ and the update rules of G_F, what combinations manifest as *effective* c, $\hbar$, k_B and G in the continuum limit?

20.3 12.3 Causal Speed c as a Kernel Bound (L1/L2)

Chapter 7 introduced c as the invariant speed appearing in Lorentz transformations, the slope of light cones in Minkowski space and the maximal propagation speed for any field disturbance.

In HOK terms:

- The kernel graph G_F admits a maximal rate at which tension can propagate along edges: each update step advances at most one edge in "distance".
- Coarse–graining over many updates yields an emergent causal speed

$$c_{\text{eff}} \approx \frac{l^*}{t^*}, \tag{176}$$

  up to order–one factors depending on the detailed connectivity and update scheme.
- Special relativity (Chapter 7) is the continuum expression of this discrete causal bound.

At the L2 level we require that:

1. The emergent c_{eff} be the same for all tension channels (electromagnetic, gravitational, etc.).
2. The metric structure induced by c_{eff} be Lorentzian to high precision on accessible scales.

A full L3 derivation would show precisely how l^*/t^* must be tuned (or how it emerges) to match the measured value of c in SI units; in practice this just fixes the conversion between kernel units and human units.

20.4 12.4 Action Quantum $\hbar$ and Kernel Phase (L1/L2)

Quantum mechanics (Chapter 10) and QFT (Chapter 9) weigh histories by phases

$$\exp\left(\frac{i}{\hbar} S\right), \tag{177}$$

with S an action functional. $\hbar$ sets the scale at which phase oscillations become rapid: when $S \gg \hbar$, interference suppresses all but stationary-action histories, recovering classical behavior.

In HOK language:

- Each kernel update that changes the tension configuration by ε^* during a time step t^* contributes an elementary action
$$\Delta S^* \sim \varepsilon^* t^*. \tag{178}$$
- If the kernel also tracks information bits, Landauer-like reasoning (Chapter 5) suggests an association
$$\Delta S^* \propto \varepsilon^* t^* I_{\text{bit}}. \tag{179}$$
- In the continuum limit, many such contributions sum to a macroscopic action $S[\Gamma]$ for a history Γ on G_F.

At the L2 level we simply demand that there exists a constant $\hbar_{\text{eff}}$ such that the coarse–grained path weights are well approximated by $\exp\{iS[\Gamma]/\hbar_{\text{eff}}\}$. Then

$$\hbar \quad \longleftrightarrow \quad \alpha_\hbar \, \varepsilon^* t^* I_{\text{bit}}, \tag{180}$$

for some dimensionless $\alpha_\hbar$ capturing combinatorial factors.

A full L3 derivation would identify $\alpha_\hbar$ from the precise update rules of G_F, matching $\hbar$ numerically in SI units.

20.5 12.5 Landauer Principle and k_B (L1/L3)

In classical thermodynamics and statistical mechanics, Boltzmann's constant k_B links entropy and temperature. For a system with microstate count Ω,

$$S = k_B \ln \Omega, \tag{181}$$

and for small changes at fixed volume,

$$dS = \frac{\delta Q_{\text{rev}}}{T}. \tag{182}$$

The Landauer principle (see Chapter 5) states that erasing one bit of information in a system at temperature T costs at least

$$\Delta Q_{\min} = k_B T \ln 2. \tag{183}$$

Kernel interpretation:

- An information bit I_{bit} corresponds to a minimal distinguishable state change of a container or boundary.

- Erasing a bit means forcing the kernel to collapse multiple micro-configurations onto a single macrostate, which requires dissipating tension/energy into the environment.
- The Landauer bound relates this energy cost to temperature, i.e. to average tension per degree of freedom.

At the L3 level, this suggests a structural mapping

$$k_B \quad \longleftrightarrow \quad \alpha_k \frac{\varepsilon^*}{T^* I_{\text{bit}}}, \qquad T^* := \text{kernel temperature scale}, \tag{184}$$

with α_k a dimensionless factor and T^* an emergent measure of average tension per degree of freedom on G_F.

More concretely, a HOK derivation of k_B should:

1. Define a microscopic entropy S_{ker} for regions of G_F in terms of accessible configurations of σ_i.
2. Show that erasing one kernel bit necessarily releases at least ε^* of energy into tension channels.
3. Relate the macroscopic temperature T of a container to the average kernel tension per degree of freedom, so that
$$\Delta Q_{\min} = \varepsilon^* \iff k_B T \ln 2 \tag{185}$$
for an appropriate mapping.

Section 20.5 is therefore the L1/L3 bridge: k_B is not just a unit conversion factor; it is the proportionality constant between *kernel information* and *thermodynamic tension*.

20.6 12.6 Gravitational Coupling G (L1/L3)

Einstein's equations (Chapter 8) contain Newton's constant G as the coupling between curvature and stress–energy:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \tag{186}$$

Section 8.5 sketched an entropic/emergent view of gravity in which G might arise from statistical properties of boundary degrees of freedom.

From the HOK perspective:

- Boundaries $\mathcal{B}$ carry entropy and tension; their area A measures how many hidden kernel configurations can influence the interior.
- Entanglement entropy tends to scale with area rather than volume (Chapter 21), suggesting that area is the natural measure of boundary information.
- Gravitational attraction may be an effective entropic force arising from the tendency of tension distributions to maximize boundary entropy subject to kernel constraints.

An L3 mapping goal is to express G as

$$G \sim \alpha_G \frac{l^{*\,2} c^3}{\varepsilon^*} \left(\frac{I_{\text{bit}}}{S_{\text{area}}} \right), \tag{187}$$

where S_{area} is an entropy per unit area associated with horizon–like boundaries in G_F, and α_G is a dimensionless factor determined by the detailed combinatorics of tension channels.

The precise functional form is not fixed here; what matters is the *structure*:

1. G must relate geometry (curvature radius $\sim l^*$), causal speed (c) and tension units (ε^*).
2. Boundary area and entropy — not bulk volume — should appear in the fundamental relation (area laws).
3. The same mapping must reproduce both Newtonian gravity at low energies and relativistic gravity near strong–field regimes.

Section 20.6 is thus the counterpart to Section 8.5: the former frames the problem from the constant side; the latter frames it from the gravity side.

20.7 12.7 Dimensionless Constants and Kernel Topology (L2/L3)

Beyond c, $\hbar$, k_B and G, physics contains many dimensionless constants:

- Gauge couplings (e.g. $\alpha = e^2/4\pi\varepsilon_0\hbar c$).
- Yukawa couplings and mass ratios (e.g. m_p/m_e).
- Mixing angles and CP–violating phases in the Standard Model.

Renormalization group flow (Chapter 9) shows that these couplings run with scale, but their values at some reference scale must still be fixed by experiment.

In the HOK vision, such constants should be linked to global features of G_F:

- Topological invariants of tension channels (knots, linking numbers, winding numbers).
- Global connectivity statistics (degree distributions, community structure).
- Consistency conditions for anomaly cancellation and long–range coherence.

For example, one speculative idea (not developed here) is that the reciprocal of the fine-structure constant $1/\alpha$ could be proportional to a "tension Hopf invariant" counting how electromagnetic tension lines wrap around each other in G_F.

At present this remains an L3 aspiration: Chapter 9 provides the effective-field-theory scaffold; future work must supply concrete kernel models and numerical mappings.

20.8 12.8 Scorecard: What HOK Claims and Does Not Yet Claim

To avoid overstatement, we summarize the status of constants in the current HOK physics appendix:

- c **[L2]:** Clearly mapped to a maximal causal speed l^*/t^* on G_F. Matching its numerical value is essentially a unit choice.
- $\hbar$ **[L2]:** Structurally mapped to an action quantum built from $\varepsilon^* t^* I_{\mathrm{bit}}$. A full derivation of the precise coefficient is open.
- k_B **[L1/L3]:** Landauer principle and kernel information provide a clean route: k_B is the proportionality between kernel bits and thermodynamic tension. Completing the derivation requires a detailed kernel model of entropy and temperature.
- G **[L1/L3]:** Entropic/emergent gravity plus area laws strongly constrain the allowed mapping from kernel scales to G, but no quantitative derivation is yet presented.
- **Other dimensionless couplings [L3]:** Hypothesized to arise from topological and combinatorial invariants of tension channels; concrete models remain future work.

The practical takeaway is:

> HOK does not magically "predict all constants" today. What it offers in this appendix is a coherent architecture in which such predictions, if they are possible at all, must live.

Subsequent versions of the HOK Physics Appendix should be evaluated not only by how elegantly they restate known physics, but by whether they can move items in this scorecard from L1/L2 structure to genuine L3 numerical constraints.

21 Chaos, Entanglement and Area Laws

Thermodynamics (Chapter 5) describes how macroscopic entropy increases when we coarse–grain over microscopic degrees of freedom. Quantum mechanics (Chapter 10) adds a different notion of entropy: entanglement between subsystems. General relativity and gravity (Chapter 8) bring in horizons and black holes, where entropy appears to scale with area, not volume.

From the HOK viewpoint, these three threads — classical chaos, quantum entanglement and area laws — are all probes of how tension and information are distributed on the kernel graph G_F:

- **Chaos** measures how sensitively tension flows depend on initial conditions.
- **Entanglement** measures how amplitude/tension is shared between different containers.
- **Area laws** reveal that boundaries, not volumes, are the natural locus of certain information measures.

This chapter works mostly at L1/L2 and frames a few L3 constraints that any serious HOK implementation must satisfy.

21.1 13.1 Classical Chaos and Lyapunov Exponents (L1)

In classical mechanics, a dynamical system with phase–space coordinates $\boldsymbol{x}(t)$ evolves according to

$$\dot{\boldsymbol{x}} = \boldsymbol{F}(\boldsymbol{x}). \tag{188}$$

The system is called *chaotic* if nearby trajectories separate exponentially in time:

$$\delta\boldsymbol{x}(t) \approx \delta\boldsymbol{x}(0)\, e^{\lambda t}, \tag{189}$$

where $\lambda > 0$ is a (maximal) Lyapunov exponent.

Key signatures of classical chaos:

- Sensitivity to initial conditions (positive Lyapunov exponents).
- Mixing in phase space: volumes stretch and fold, leading to effective irreversibility under coarse–graining.
- Complex, fractal–like invariant sets and strange attractors.

From a thermodynamic viewpoint, chaos is one of the mechanisms by which microscopic reversibility leads to macroscopic entropy increase: small uncertainties in initial tension configurations are rapidly amplified, making detailed reconstruction practically impossible.

HOK interpretation (L1/L2):

- Classical state variables $(\boldsymbol{x}, \boldsymbol{p})$ of a subsystem correspond to coarse–grained summaries of many kernel variables $\{\sigma_i\}$.
- Classical chaos reflects exponential sensitivity of coarse summaries to microscopic rearrangements of tension on G_F.
- Lyapunov exponents quantify how quickly two initially similar tension configurations diverge under the kernel update rules when projected onto a low–dimensional description.

At this stage we do not attempt to compute Lyapunov spectra from G_F; we simply require that HOK can reproduce known chaotic systems (e.g. restricted three–body problem, driven pendula) at the coarse–grained level.

21.2 13.2 Quantum Chaos, Scrambling and OTOCs (L1/L2)

Quantum systems do not have trajectories in phase space, so chaos must be characterized differently. Two complementary diagnostics are widely used:

- **Spectral statistics**: energy level spacings of classically chaotic systems tend to follow random–matrix distributions (Wigner–Dyson), while integrable systems show Poisson statistics.
- **Scrambling and out–of–time–ordered correlators (OTOCs)**: operators initially localized in space grow in support under Heisenberg evolution, spreading information across the system.

A typical OTOC is

$$C(t) = -\big\langle [\hat{W}(t), \hat{V}(0)]^2 \big\rangle, \qquad \hat{W}(t) = e^{i\hat{H}t/\hbar}\hat{W}(0)e^{-i\hat{H}t/\hbar}. \tag{190}$$

In many chaotic systems, $C(t)$ grows roughly as

$$C(t) \sim \exp(\lambda_L t) \tag{191}$$

over some time window, defining a quantum Lyapunov exponent λ_L. In thermal systems, there is an upper bound

$$\lambda_L \leq \frac{2\pi k_B T}{\hbar} \tag{192}$$

(the Maldacena–Shenker–Stanford bound), with certain black-hole systems believed to saturate it.

HOK interpretation (L2):

- Scrambling measures how quickly a localized perturbation in tension/amplitude on G_F spreads across many nodes and channels.
- Systems that saturate the quantum chaos bound are the "fastest scramblers": they spread kernel information across containers in the minimal allowed time.
- In this sense, black holes are not just massive containers but extreme scramblers in the kernel architecture.

This connection links chaos directly to the information–theoretic content of gravity and to entanglement generation.

21.3 13.3 Reduced Density Matrices and Entanglement Entropy (L1)

Consider a quantum system partitioned into two subsystems A and B, with Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ and density matrix ρ. The reduced density matrix of A is

$$\rho_A = \mathrm{Tr}_B\, \rho. \tag{193}$$

The von Neumann entropy of A is

$$S_A = -\mathrm{Tr}\,(\rho_A \ln \rho_A). \tag{194}$$

Properties:

- For a pure global state $\rho = |\psi\rangle\langle\psi|$, $S_A = S_B$ measures *entanglement* between A and B.
- For a product state $|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$, $S_A = 0$.
- For a maximally entangled state of two d–dimensional systems, $S_A = \ln d$.

Unlike thermodynamic entropy, entanglement entropy can be nonzero even at zero temperature; it is a purely quantum correlation measure.

HOK interpretation:

- The partition $A|B$ corresponds to two containers C_A and C_B plus their shared boundary $\mathcal{B}_{AB}$ on G_F.
- S_A quantifies how much amplitude/tension information about C_A is encoded nonlocally across C_B and the boundary.
- Entanglement entropy therefore probes the "information capacity" of boundaries, which will be crucial for area laws and gravity.

21.4 13.4 Area Laws vs Volume Laws for Entropy (L1/L2)

In many gapped quantum systems with local interactions, the ground state entanglement entropy of a region A scales approximately as the area of its boundary, not its volume:

$$S_A \sim \alpha\,|\partial A| + \cdots, \tag{195}$$

where $|\partial A|$ is the boundary area (or length in 1D) and α is a constant. This is the *entanglement area law*.

By contrast:

- Highly excited or thermal states often exhibit *volume–law* entanglement: $S_A \sim \beta\,|A|$, proportional to the volume of A.
- Certain critical systems (e.g. 1D conformal field theories) show logarithmic corrections, such as $S_A \sim \frac{c}{3}\ln(L/\epsilon)$ for interval length L.

The most striking area law appears in black-hole physics: the Bekenstein–Hawking entropy

$$S_{\mathrm{BH}} = \frac{k_B c^3}{4G\hbar} A_{\mathrm{horizon}} \tag{196}$$

is proportional to the horizon area, not the volume inside.

HOK interpretation (L2):

- Local interactions on G_F imply that entanglement between a region and its complement is mediated primarily by edges crossing the boundary $\mathcal{B}_A$.
- Each cut edge can carry only a finite amount of entanglement (limited by the local Hilbert space dimension), so S_A naturally scales with the number of boundary edges, i.e. with area.
- Volume–law entanglement corresponds to states where tension and amplitude correlations effectively "percolate" throughout the region, saturating many internal degrees of freedom.

Area laws thus emerge quite naturally from the kernel's locality and finite local Hilbert space, independent of gravity. The black-hole area law is then an extreme, gravitationally dressed example of a more general phenomenon.

21.5 13.5 Chaos, Scrambling and Entanglement Growth (L1/L2)

Entanglement is not static; dynamical systems generate and spread it. In many local quantum many–body systems:

- An initially product state evolves so that entanglement entropy of subregions grows roughly linearly in time before saturating.
- The rate of growth and the pattern of saturation depend on whether the system is integrable, chaotic or many–body localized.

In strongly chaotic systems (fast scramblers):

- Local operators spread rapidly, as reflected in OTOCs.
- Entanglement between any small subset and the rest of the system approaches its maximal value in a time of order $t_{\mathrm{scr}} \sim \frac{1}{\lambda_L} \log N$, where N is the effective number of degrees of freedom.

HOK interpretation:

- Scrambling time measures how quickly a local perturbation to tension in one part of G_F becomes delocalized across many nodes and containers.
- Rapid entanglement growth indicates that the kernel's update rules are efficiently mixing amplitude information across boundaries.
- Systems saturating the chaos bound are those where G_F and the update rules allow the steepest possible gradient in this mixing, subject to the causal constraint set by c.

This links back to thermodynamic irreversibility: once information about a perturbation is scrambled across many degrees of freedom, it becomes practically irrecoverable without global control, even though microscopic evolution is unitary.

21.6 13.6 Entanglement, Horizons and HOK Boundaries (L2/L3)

Horizons — black-hole event horizons, cosmological horizons, Rindler horizons for accelerated observers — are boundaries in space–time that separate regions which cannot exchange signals.

Quantum field theory on such backgrounds suggests that these horizons are associated with entanglement:

- The vacuum state of a quantum field is entangled across any spatial partition; tracing out the unobservable region leads to a thermal density matrix for the accessible region (Unruh effect, Hawking radiation).
- The entropy associated with this reduced density matrix scales with the area of the horizon, echoing the Bekenstein–Hawking formula.

In HOK language:

- A horizon is a boundary $\mathcal{B}_{\text{hor}}$ in G_F beyond which tension signals cannot return to a given container or observer.
- The entanglement between degrees of freedom on either side of $\mathcal{B}_{\text{hor}}$ contributes an entropy proportional to the number of edges crossing it.
- The Bekenstein–Hawking formula can thus be read as a statement about the "entanglement capacity" of the horizon boundary, linking k_B, $\hbar$, G and area.

At the L3 level, HOK aspires to:

1. Derive an effective horizon entanglement entropy S_{hor} directly from the statistics of kernel degrees of freedom and their connectivity across $\mathcal{B}_{\text{hor}}$.
2. Show that this entropy obeys an area law with the precise coefficient $k_B c^3/4G\hbar$, thereby fixing G in terms of kernel scales (see Section 20.6).
3. Explain the Page curve (information loss and recovery) as a competition between scrambling inside the horizon and entanglement transfer to outgoing radiation, all implemented on G_F.

We do not attempt these calculations here; their feasibility is a key test for any future HOK model that claims to "derive" G and black-hole thermodynamics.

21.7 13.7 Summary: Diagnostics for the Kernel

Chaos, entanglement and area laws provide complementary diagnostics of the kernel:

- **Classical chaos** tests how sensitive coarse–grained tension flows are to initial conditions.
- **Quantum chaos and scrambling** test how efficiently the kernel mixes information across containers, bounded by causal constraints and the quantum chaos bound.
- **Entanglement entropy** measures how amplitude/tension is shared across boundaries; its scaling (area vs volume) reveals the locality and connectivity of G_F.
- **Horizon area laws** tie these ideas to gravity, hinting that G, $\hbar$ and k_B are not independent but linked by boundary information content.

For HOK, the message is:

> A viable kernel model must not only reproduce standard equations; it must also reproduce the correct chaos indicators, entanglement patterns and area laws. These are the "stress tests" of the framework, probing how tension and information actually live on G_F.

The final chapter of this Appendix (Chapter 22) collects the mathematical tools — from graph theory to information geometry — that are most useful for making these notions precise on the kernel itself.

22 Mathematical Tools for the Kernel

Throughout this appendix we have moved freely between discrete kernel structures and continuum physics: graphs and fields, containers and manifolds, probabilities and entropies. This final chapter gathers the main mathematical tools into one place.

The goal is not to be exhaustive, but to make explicit which structures HOK actually relies on, and how they fit together. Roughly:

- Graph theory and discrete geometry for the kernel G_F.
- Differential geometry for continuum space–time and fields.
- Linear algebra, Hilbert spaces and operators for quantum theory.
- Probability, information and divergences for entropy and inference.
- Dimensional analysis and scaling for constants and units.
- Numerical and algorithmic considerations for simulating G_F.

22.1 14.1 Graphs, Adjacency and Laplacians

The kernel is built on a graph

$$G_F = (V_N, E_L), \tag{197}$$

where V_N is a set of nodes (events, micro-containers) and E_L is a set of directed or undirected edges (causal links, tension channels).

Adjacency and degree. For a finite graph with $|V_N| = N$, the adjacency matrix A is

$$A_{ij} = \begin{cases} 1, & \text{if there is an edge } i \to j, \\ 0, & \text{otherwise.} \end{cases} \tag{198}$$

The degree matrix D is diagonal, $D_{ii} = \sum_j A_{ij}$.

Graph Laplacian. The (combinatorial) Laplacian is

$$L = D - A. \tag{199}$$

In continuum limits, L approximates $-\nabla^2$. On G_F, L is the basic tool for describing diffusion, wave propagation and coarse–grained curvature.

For weighted graphs, edges carry weights $w_{ij} \geq 0$, and

$$A_{ij} = w_{ij}, \qquad D_{ii} = \sum_j w_{ij}. \tag{200}$$

HOK usage.

- Tension flows often obey discrete diffusion or wave equations expressed using L.
- Effective geometrical quantities (e.g. discrete Ricci curvature or spectral dimension) can be extracted from the spectrum of L.
- Coarse–graining G_F corresponds, at the operator level, to projecting or renormalizing L.

22.2 14.2 Discrete and Continuum Geometry

Gravity (Chapter 8) and cosmology (Chapter 19) use continuum differential geometry. HOK treats this as an effective description of large–scale features of G_F.

Manifolds and metrics. A smooth manifold $\mathcal{M}$ carries a metric tensor $g_{\mu\nu}$, with line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu. \tag{201}$$

Christoffel symbols $\Gamma^{\rho}_{\mu\nu}$, curvature tensors $R^{\rho}{}_{\sigma\mu\nu}$, the Ricci tensor $R_{\mu\nu}$ and scalar R are built from $g_{\mu\nu}$ and its derivatives.

Discrete geometry on G_F. On a graph, we replace smooth structures by:

- Node sets approximating points; edge lengths approximating geodesic distances.
- Discrete notions of curvature (e.g. Ollivier or Forman Ricci curvature) defined from transport or combinatorics of edges.
- Discrete versions of divergence, gradient and Laplacian using incidence and adjacency matrices.

The guiding requirement is that, after coarse–graining over many nodes, these discrete quantities approach their continuum counterparts on an effective manifold.

22.3 14.3 Linear Algebra, Hilbert Spaces and Operators

Quantum mechanics and QFT (Chapters 10 and 9) are formulated in terms of linear operators on Hilbert spaces.

Hilbert spaces. A Hilbert space $\mathcal{H}$ is a complete vector space over $\mathbb{C}$ with inner product $\langle\cdot|\cdot\rangle$. States are rays in $\mathcal{H}$, typically represented by normalized vectors $|\psi\rangle$.

Operators and spectra. An operator $\hat{A}$ on $\mathcal{H}$ is *self–adjoint* if $\hat{A} = \hat{A}^{\dagger}$. Its eigenvalues are real, and in many physical cases $\hat{A}$ can be expanded as

$$\hat{A} = \sum_n a_n |a_n\rangle\langle a_n|, \tag{202}$$

or via an integral over a continuous spectrum.

Unitary evolution. Time evolution is generated by a Hamiltonian $\hat{H}$ via

$$|\psi(t)\rangle = e^{-i\hat{H}(t-t_0)/\hbar}|\psi(t_0)\rangle. \tag{203}$$

Unitarity implements conservation of total probability.

HOK usage.

- Local kernel configurations form a natural finite–dimensional Hilbert space; global states of G_F live in a large tensor product.
- Local update rules can be expressed as unitary maps or quantum channels on these spaces.

- Spectral properties of graph–based operators (e.g. Laplacians, transfer matrices) control propagation, decoherence and entanglement.

22.4 14.4 Probability, Entropy and Information Geometry

Thermodynamics, statistical mechanics and information theory (Chapter 5) all rely on probability distributions over microstates.

Probability distributions. For a discrete set of outcomes, $p = (p_1, \ldots, p_n)$ with $p_i \geq 0$, $\sum_i p_i = 1$. For continuous variables, probability densities $p(x)$ satisfy $\int p(x)\, dx = 1$.

Divergences and distances. A key object is the Kullback–Leibler divergence

$$D_{\mathrm{KL}}(p\|q) = \sum_i p_i \ln \frac{p_i}{q_i}, \tag{204}$$

which is nonnegative and vanishes if and only if $p = q$. Although not symmetric, it measures how costly it is (in log–likelihood terms) to mistake q for p.

Entropy and mutual information. Shannon entropy $H(p) = -\sum_i p_i \log_2 p_i$, thermodynamic entropy $S = k_B \ln \Omega$ and von Neumann entropy $S(\rho) = -\mathrm{Tr}(\rho \ln \rho)$ are different faces of the same idea: uncertainty or information content.

Mutual information between random variables X and Y is

$$I(X : Y) = H(X) + H(Y) - H(X, Y), \tag{205}$$

measuring how much knowing one reduces uncertainty about the other.

Information geometry. The space of probability distributions carries a natural Riemannian metric: the Fisher information metric

$$g_{ij}(\theta) = \mathbb{E}\left[\partial_i \ln p(X|\theta)\, \partial_j \ln p(X|\theta)\right], \tag{206}$$

where θ are parameters. Distances measured by g_{ij} quantify statistical distinguishability between nearby models.

HOK usage.

- Entropy and mutual information measure how tension and amplitude are distributed across containers and boundaries.

- KL divergence and Fisher information measure how sensitive macroscopic behavior is to changes in kernel parameters (useful for L2/L3 identifiability).
- Information geometry provides a natural language for discussing "flows" of models under coarse–graining (renormalization as a trajectory in model space).

22.5 14.5 Dimensional Analysis, Scaling and Natural Units

Chapter 20 emphasized that HOK must eventually *map* kernel scales to physical constants. Dimensional analysis is the first line of defense against nonsense.

Dimensional analysis. Each physical quantity carries dimensions, e.g.

$$[L] = \text{length}, \quad [T] = \text{time}, \quad [M] = \text{mass}, \quad [E] = \text{energy}. \tag{207}$$

For example,

$$[G] = \frac{L^3}{MT^2}, \qquad [\hbar] = \frac{ET}{}, \qquad [k_B] = \frac{E}{\Theta}, \tag{208}$$

where Θ denotes temperature.

Dimensional analysis and the Buckingham Π theorem identify dimensionless combinations (e.g. Reynolds number, fine–structure constant) that control the behavior of systems.

Scaling and renormalization. Renormalization group flow (Chapter 9) describes how effective couplings change with scale. Mathematically this is a flow on parameter space generated by scaling transformations of length, time and energy.

HOK usage.

- Kernel scales l^*, t^*, ε^* define a natural unit system.
- Dimensional analysis constrains how constants like c, $\hbar$, k_B, G can depend on these scales.
- Scaling transformations on G_F correspond to coarse–graining schemes; their fixed points are candidate continuum limits.

22.6 14.6 Numerical and Algorithmic Considerations

Any concrete HOK implementation will require simulation of large discrete systems. A few practical tools and caveats:

Time evolution on graphs. Update rules on G_F can be implemented as:

- Local synchronous or asynchronous update schemes.
- Discrete–time unitary circuits (quantum cellular automata).
- Stochastic kernels describing tension diffusion or jump processes.

Spectral methods. Eigenvalues and eigenvectors of graph Laplacians and other operators are central for:

- Approximating wave and diffusion dynamics.
- Estimating effective dimensions and curvature.
- Identifying metastable structures (communities, coarse containers).

Monte Carlo and sampling. When exact enumeration of configurations is impossible, one typically uses:

- Markov chain Monte Carlo (Metropolis, Gibbs samplers).
- Importance sampling and reweighting.
- Variational approximations (mean field, tensor networks).

Complexity constraints. The state space of G_F grows exponentially with the number of nodes. Any realistic HOK program must therefore:

- Exploit locality and sparsity.
- Use coarse–graining and renormalization to avoid tracking unnecessary microdetails.
- Focus on observables and summary statistics rather than full microstate trajectories.

22.7 14.7 Summary: A Compact Toolbox for HOK

We close the appendix with a compact dictionary of tools:

Tool	**Role in HOK**
Graphs, Laplacians	Kernel structure, tension flow, discrete geometry
Differential geometry	Emergent space–time, curvature, GR limit
Hilbert spaces, operators	Quantum states, dynamics, measurements
Entropy, divergences	Thermodynamics, information, inference
Information geometry	Model space, distinguishability, RG flows
Dimensional analysis	Constraints on constants and scaling laws
Numerical methods	Simulation and coarse–grained exploration of G_F

The philosophy is simple:

> HOK should not require exotic mathematics; it should recombine familiar tools — graphs, fields, entropy, operators — in a way that makes the kernel architecture explicit and testable.

Future versions of the HOK Physics Appendix can deepen any of these sections as needed, but the structures summarized here are the backbone on which all L1 (translation), L2 (structural) and L3 (derivation) claims must ultimately rest.