1)

a)

	X					Total	
Y	0,01	0,02	0,03	0,1	0,1	0,26	x_{51}
	0,05	0,1	0,05	0,07	0,2	0,47	x_{52}
	0,1	0,05	0,03	0,05	0,04	0,27	x_{53}
Total	0,16	0,17	0,11	0,22	0,34	1	
	$P(x_{13})$	x_{23}	x_{33}	x_{43}	x_{53}		

The Marginal Distribution: ↑

$\cancel{p(y) = 0,26 + 0,17 + 0,11 + 0,22 + 0,34 = 1}$

$\cancel{p(x) = 0,26 + 0,47 + 0,27 = 1.}$

b)

The Condition Distribution:

$$p(x \mid y = y_1) = p(x_1 \mid y = y_1) = \frac{0,01}{0,26} \approx 0,038$$

$$= p(x_2 \mid y = y_1) = \frac{0,02}{0,26} \approx 0,770$$

$$= p(x_3 \mid y = y_1) = \frac{0,03}{0,26} = 0,115$$

$$= p(x_4 \mid y = y_1) = \frac{0,1}{0,26} = 0,385$$

$$= p(x_5 \mid y = y_1) = \frac{0,1}{0,26} = 0,385$$

$$p(x \mid y = y_3) = p(x_1 \mid y = y_3) = \frac{0,1}{0,27} = 0,37$$

$$= p(x_2 \mid y = y_3) = \frac{0,05}{0,27} = 0,185$$

$$= p(x_3 \mid y = y_3) = \frac{0,03}{0,27} = 0,11$$

$$= p(x_4 \mid y = y_3) = \frac{0,05}{0,27} = 0,185$$

$$= p(x_5 \mid y = y_3) = \frac{0,04}{0,27} = 0,148.$$

2)
We have :

$$\sum E_x$$

$$E_Y(E_X[X \mid Y]) = \sum E_X[X \mid Y] \cdot p(y) = \sum p(y) \left(\sum x . p(x \mid y)\right)$$

$$= \sum \sum p(y) \cdot p(x \mid y) \cdot x = \sum \sum x . p(x, y) = \sum x \sum p(x, y)$$

$$= \sum x p(x) = E_X[X]$$

3)

$P(X) = 20,7\% = 0,207.$

$P(Y) = 50\% = 0,5$

$P(X/Y) = 36,5\% = 0,365.$

a)

$P(X \text{ and } Y) = P(X).P(X/Y) = 0,207 \times 0,365 = 0,0756$

b)

$$P(Y/\bar{X}) = \frac{P(Y.\bar{X})}{P(\bar{X})} = \frac{P(Y).P(\bar{X}/Y)}{P(\bar{X})}$$

$$= \frac{P(Y).(1 - P(X/Y))}{P(\bar{X})} = \frac{0,5.(1-0,365)}{1-0,207}$$

$\Rightarrow P(Y/\bar{X}) = 0,40038.$

4)

Variance is the expected value of the squared variation of a random variable from its mean value.

$V(X) = E[(X-\mu)]^2$

$= E[(X - E(X))]^2 = E[X^2 - 2XE(X) + (E(X))^2]$

$= E[X^2] - E[2X.E(X)] + E[E(X)]^2$

constant constant

$= E[X]^2 - 2.E[X.E(X)] + E[E(X)]^2$

$= E[X]^2 - 2.E[X].E[X] + E[E(X)]^2$

$= E[X]^2 - 2.(E[X])^2 + (E[X])^2$

$= E[X]^2 - (E[X])^2$ (proved)

$\Rightarrow V(X) = E[X]^2 - (E[X])^2$

5)

Giả sử: Chọn ô 1.

+ Nếu xe nằm ở ô 1 (A) và Monty mở ô số 2 (B)

$\Rightarrow P(A) = \frac{1}{3}$.

$P(B|A) = \frac{1}{2}$ (vì chỉ còn ô 2 or 3)

$P(B) = \frac{1}{2}$

Xác suất Monty mở ô 2

$\Rightarrow$ Xác suất xe nằm ở ô 1 và Monty đã mở ô 2:

$P(A|B) = \frac{1}{3}$.

+ Nếu xe nằm ở cửa số (3) (C) $\Rightarrow P(C) = 1 - P(A) = \frac{2}{3}$

$\Rightarrow$ Đổi cửa ↑ xác suất trúng xe.