1)

Suppose that:

$$X = \{x_1, x_2, \dots, x_n\} \quad (x_i \in R^D)$$

$$X = \begin{bmatrix} \text{---} & x_1^T & \text{---} \\ \text{---} & x_2^T & \text{---} \\ \dots & & \dots \\ \text{---} & x_n^T & \text{---} \end{bmatrix} \in R^{N \times D}$$

weight $\left\langle B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} \in R^{D \times M} \right.$

$$Z^{N \times M} = X . B$$

$$\Leftrightarrow \begin{bmatrix} z_1^T \\ z_2^T \\ \dots \\ z_N^T \end{bmatrix} = \begin{bmatrix} x_1^T \\ x_2^T \\ \dots \\ x_n^T \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_N \end{bmatrix}$$

$$= \begin{bmatrix} x_1^T b_1 & x_2^T b_2 & \dots & x_1^T b_M \\ x_2^T b_1 & x_2^T b_2 & \dots & x_2^T b_M \\ \sim & \sim & \sim & \sim \\ x_N^T b_1 & x_N^T b_2 & \sim & x_N^T b_M \end{bmatrix}.$$

Thứ ... ngày ...

Maximize variance:

$$V_1 = \frac{1}{N} \sum_{n=1}^{N} z_{1n}^2 \qquad (z_{1n}^2 = x_i^T b_1)$$

$$V_1 = \frac{1}{N} \sum_{n=1}^{N} (x_i^T b_1)^2 = \frac{1}{N} \sum_{n=1}^{N} b_1^T x_i x_i^T b_1.$$

$$= b_1^T \left(\frac{1}{N} \sum_{n=1}^{N} x_i x_i^T \right) b_1 = b_1^T S b_1.$$

$\to$ maximize $b_1^T S b_1$ subject to $\|b_1\|^2 = 1$

We have:

$$L = b_1^T S b_1 + \alpha (1 - b_1^T b_1)$$

$$\frac{\partial L}{\partial b_1} = 0 \Leftrightarrow 2Sb_1 - 2\alpha b_1 = 0 \Leftrightarrow Sb_1 = \alpha b_1$$

So we have, α and b_1 : eigenvector

S : eigenvalue.

$$\frac{\partial L}{\partial \alpha} = 0 \Leftrightarrow b_1^T b_1 = 1.$$

$\Rightarrow Var = b_1^T S b_1 = b_1^T \alpha b_1 = \alpha b_1^T b_1 = \alpha$