FACULTY OF COMPUTING
DISCRETE STRUCTURE
(SECI1013)
ASSIGNMENT 1

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SECTION : **SECTION 3**

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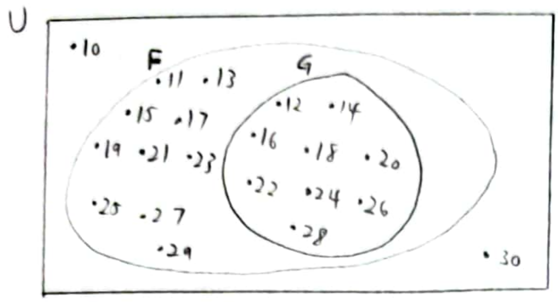
1) a) $F = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$

$|F| = 19$

b) $G = \{12, 14, 16, 18, 20, 22, 24, 26, 28\}$

$|G| = 9$

c)



d) $|G \oplus F| = |G \cup F| - |G \cap F|$

$= 19 - 9 = 10$

2) a) $|A| = 3$, $|P(A)| = 2^3 = 8$

b) $A \cap B \cup C = \{e, n, s, t\}$

c) $A - B = \{b, u\}$

d) $B \times C = \{(s,n), (s,e), (s,t), (e,n), (e,e), (e,t), (t,n), (t,e), (t,t)\}$

3) a) Is a proposition. True.

b) Is a proposition. True.

c) Is a proposition. True.

d) Is a proposition. False.

e) Is a proposition. True.

4) a)

p	q	$p \to q$	$\neg p \leftrightarrow \neg q$	$(p \to q) \land (\neg p \leftrightarrow \neg q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	F
F	F	T	T	T

4 b)

p	q	$p \leftrightarrow q$	$\neg p \to \neg q$	$(p \leftrightarrow q) \vee (\neg p \to \neg q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	F
F	F	T	T	T

5)

					A			B
p	q	r	$\neg p$	$\neg q \vee \neg r$	$\neg p \wedge (\neg q \vee \neg r)$	p	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	F	F	F	T	T	T
T	T	F	F	T	F	T	F	T
T	F	T	F	T	F	T	F	T
T	F	F	F	T	F	T	F	T
F	T	T	T	F	F	F	T	T
F	T	F	T	T	T	F	F	F
F	F	T	T	T	T	F	F	F
F	F	F	T	T	T	F	F	F

A and B are not logically equivalent. ($A \not\equiv B$)

6)

				A	B
p	q	$p \vee q$	$p \wedge q$	$p \wedge (p \vee q)$	$p \vee (p \wedge q)$
T	T	T	T	T	T
T	F	T	F	T	T
F	T	T	F	F	F
F	F	F	F	F	F

A and B are logically equivalent. ($A \equiv B$)

7) a) $\exists x \left(P(x) \wedge R(x) \right)$

b) $\forall x \left(Q(x) \to \neg R(x) \right)$

8. $P(x)$: x is a negative number. ; $Q(x)$: x^2 is a positive number.

Let $x = b-a$, which $a \geqslant b$, a and b are both positive number,

$x^2 = (b-a)^2$

$= b^2 - 2ab + a^2$

$= a^2 - ab + b^2 - ab$

$= a(a-b) + b(b-a)$

$= a(a-b) - b(a-b)$

$= (a-b)(a-b)$

$= (a-b)^2$

Given that $a \geq b$; therefore $a-b \geqslant 0 \Rightarrow$ This shows that $a-b$ is positive number.

As all square of positive number is a positive number, and $x^2 = (a-b)^2$;

therefore we show that a square of any negative number is a positive number.

$\forall x (P(x) \to Q(x))$ shown.

9. $P(x)$: C and D are sets. ; $Q(x)$: $C \cap (D \cap C') = \{\}$

Contradiction : Suppose C and D are sets and $C \cap (D \cap C') \neq \{\}$

$\neg Q(x)$: There exist element in $C \cap (D \cap C') \Rightarrow x \in C \cap (D \cap C')$

As x is in the intersection of set C and set $D \cap C'$,

therefore $x \in C$ and $x \in D \cap C'$.

As x is in the intersection of set D and set C',

therefore $x \in D$ and $x \in C'$.

As $x \in C$ and $x \in C'$, this must be wrong,

as no element can be in a set and its compliment set at the same time,

because $C \cap C' = \emptyset$.

Thus, $x \in C \cap (D \cap C')$ is a contradiction, original supposition could not be true.

Hence, $C \cap (D \cap C') = \{\}$ is proven.

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10. R is irreflexive as there is no pair of (a,b) where $a=b$. This is shown by if $a=b$, $|a-b| = |a-a| = 0 \neq 2$.

R is not reflexive as it is irreflexive.

R is symmetric as all pairs of $(a,b) \in R$ and all pairs of $(b,a) \in R$, when $a \neq b$.

This is shown by $|a-b| = |a-b|$ or $|b-a|$, therefore if $(a,b) \in R$, $(b,a) \in R$.

R is not antisymmetric and not asymmetric because it is symmetric.

R is not transitive because it is irreflexive.

11.

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \Rightarrow$$

R is reflexive as all $M_i = M_j$ (diagonal $= 1$)

R is not symmetric as $(a,b) \in R$ but $(b,a) \in R$, while $a \neq b$

R is not transitive as $(a,b) \in R$, $(b,c) \in R$ but $(a,c) \notin R$.

Therefore R is not an equivalence relation.

12. a) $f(x_1, y_1) = f(x_2, y_2)$

$(2x_1 - y_1, x_1 - 2y_1) = (2x_2 - y_2, x_2 - 2y_2)$

$2x_1 - y_1 = 2x_2 - y_2$

$2x_1 - 2x_2 = y_1 - y_2$ —(1)

$x_1 - 2y_1 = x_2 - 2y_2$

$x_1 - x_2 = 2y_1 - 2y_2$ —(2)

Sub (1) into (2):

$x_1 - x_2 = 2(2x_1 - 2x_2)$

$x_1 - x_2 = 4(x_1 - x_2)$

$3(x_1 - x_2) = 0$

$x_1 - x_2 = 0$

$x_1 = x_2$

Sub (2) into (1):

$2(2y_1 - 2y_2) = y_1 - y_2$

$4(y_1 - y_2) = y_1 - y_2$

$3(y_1 - y_2) = 0$

$y_1 - y_2 = 0$

$y_1 = y_2$

As $x_1 = x_2$ and $y_1 = y_2$, f is an one-to-one function.

12 b) $f(x,y) = (2x-y,\ x-2y)$

$f^{-1}(x,y) = (u,v)$

Let $u = 2x-y$; $v = x-2y$

let $x = u$ and $y = v$.

$\Rightarrow x = 2u - v$; $y = u - 2v$

$x = 2u - v$

$v = 2u - x \quad -(1)$

Sub (1) into y :

$y = u - 2(2u - x)$

$y = -3u + 2x$

$u = \dfrac{2x - y}{3} \quad -(2)$

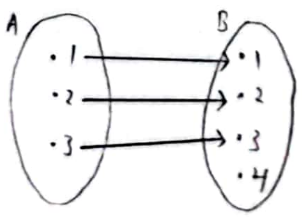
Sub (2) into (1):

$v = 2\left(\dfrac{2x-y}{3}\right) - x$

$v = \dfrac{x - 2y}{3}$

$f^{-1}(x,y) = \left(\dfrac{2x-y}{3},\ \dfrac{x-2y}{3}\right)$

13) a)



b) $(1,1)$, $(3,1)$, $(2,2)$

14) i) $g(f(x)) = f(x) - 1$
$= x^3 - 1$

$f(g(x)) = (g(x))^3$
$= (x-1)^3$
$= x^3 - 3x^2 + 3x - 1$

ii) if $gf(x) = fg(x)$

$gf(x) - fg(x) = 0$

$\Rightarrow x^3 - 1 - (x^3 - 3x^2 + 3x - 1) = 0$

$3x^2 - 3x = 0$

$3x(x-1) = 0$

$fg(x) = gf(x)$ only if $x = 0$, $x = 1$

Therefore $fg(x) \neq gf(x)$, as they are not equivalent for all real number of x.

15. Let a_n = number of strings that do not contain 01.

when $n=1$, $a_n = 2$ $\Rightarrow$ (0 , 1)

when $n=2$, $a_n = 3$ $\Rightarrow$ (00 , 10, 11)

when $n=3$, $a_n = 4$ $\Rightarrow$ (000, 111, 110, 100)

In order to get strings that do not contain 01, 2 cases is discussed;

Case 1 : End with zero

$\rightarrow$ either string consists of all zero $\rightarrow$ 1 way

$\rightarrow$ or string consists of all one before all 0 $\rightarrow$ (n-1) ways

eg (1110, 1100, 1000)

Case 2 : End with one

$\rightarrow$ only when string consists of all one $\rightarrow$ 1 way

$a_n = n+1$

$n = a_n - 1$ — (1)

$a_{n-1} = n-1+1$

$a_{n-1} = n$ — (2)

$a_n - 1 = a_{n-1}$

$a_n = a_{n-1} + 1$; $n \geq 2$, $a_1 = 2$

16. Input : n

Output = C_n

```
Cn {
  if (n=1)
     return 0
  else if (n=2 or n=3)
     return 1
  return Cn-2 + Cn-3
}
```