

## # Ray-cylinder intersection

- Infinite cylinder along y axis of radius r axis has equation

$$x^2 + z^2 - r^2 = 0$$

- The equation for a more general cylinder of radius r oriented along a line  $\mathbf{c} + \mathbf{n} * t$  where  $\mathbf{c}$  is the center of the one surface sphere and  $\mathbf{n}$  is the surface normal has following:

$(\mathbf{d} - \mathbf{n} * \text{dot}(\mathbf{n}, \mathbf{d}))^2 - r^2 = 0$ , where  $\mathbf{d} = \mathbf{x} - \mathbf{c}$ ,  $\mathbf{x}$  is a point on the cylinder

$$\begin{aligned} \text{simplified LHS} &= \text{dot}(\mathbf{d}, \mathbf{d}) - 2 \text{dot}(\mathbf{n}, \mathbf{d})^2 + \text{dot}(\mathbf{n}, \mathbf{d})^2 \\ &- r^2 \\ &= \text{dot}(\mathbf{d}, \mathbf{d}) - \text{dot}(\mathbf{n}, \mathbf{d})^2 - r^2 \end{aligned}$$

To find the intersection points with a ray  $\mathbf{o} + t * \mathbf{dir}$ , substitute in:

$$\mathbf{d} = \mathbf{x} - \mathbf{c} = t * \mathbf{dir} + \mathbf{o} - \mathbf{c} ;$$

reduces to  $A * t^2 + B * t + C = 0$  with following:

$$\begin{aligned} 0 &= \text{normsq}(t * \mathbf{dir} + \mathbf{o} - \mathbf{c}) - \text{dot}(\mathbf{n}, t * \mathbf{dir} \\ &+ \mathbf{o} - \mathbf{c})^2 - r^2 \\ &= \frac{t^2 \text{normsq}(\mathbf{dir}) + 2t \text{dot}(\mathbf{dir}, \mathbf{o} - \mathbf{c}) + \text{normsq}(\mathbf{o} - \mathbf{c}) - t^2 \text{dot}(\mathbf{n}, \mathbf{dir})^2 - 2t \text{dot}(\mathbf{n}, \mathbf{dir}) \text{dot}(\mathbf{n}, \mathbf{o} - \mathbf{c}) - \text{dot}(\mathbf{n}, \mathbf{o} - \mathbf{c})^2 - r^2}{=} \\ &= t^2 [\text{normsq}(\mathbf{dir}) - \text{dot}(\mathbf{n}, \mathbf{dir})^2] + t [2 \text{dot}(\mathbf{dir}, \mathbf{o} - \mathbf{c})] + \text{normsq}(\mathbf{o} - \mathbf{c}) - \text{dot}(\mathbf{n}, \mathbf{o} - \mathbf{c})^2 - r^2 \end{aligned}$$

$$\rightarrow A * t^2 + B * t + C = 0$$

$$\text{Thus, } A = \text{dot}(\mathbf{dir}, \mathbf{dir}) - \text{dot}(\mathbf{n}, \mathbf{dir})^2 = (\mathbf{dir} - \mathbf{n} * \text{dot}(\mathbf{n}, \mathbf{dir}))^2$$

$$B = 2 * \text{dot}(\mathbf{dir}, \mathbf{o} - \mathbf{c}) - 2 * \text{dot}(\mathbf{n}, \mathbf{dir}) \text{dot}(\mathbf{n}, \mathbf{o} - \mathbf{c})$$

$$C = \text{dot}(\mathbf{o} - \mathbf{c}, \mathbf{o} - \mathbf{c}) - \text{dot}(\mathbf{n}, \mathbf{o} - \mathbf{c})^2 - r^2 = (\mathbf{o} - \mathbf{c} - \mathbf{n} * \text{dot}(\mathbf{n}, \mathbf{o} - \mathbf{c}))^2 - r^2$$

calculate  $t$  with  $A, B, C$  we get, then the intersection point.

# Normal deriviations

Let  $\mathbf{I}$  be the intersection point, and  $\mathbf{T}$  be the point on the axis through center of cylinder

There is  $\|\mathbf{I} - \mathbf{T}\|^2 - r^2 = 0$  where  $\mathbf{I} = t*\mathbf{dir} + \mathbf{o}$  and  $\mathbf{T} = a*\mathbf{n} + \mathbf{c}$

To simplify, substitute in  $\mathbf{I}$  and  $\mathbf{T}$ , reduces to  $A*t^2 + B*t + C = 0$  with following:

$$\begin{aligned} & \|t*\mathbf{dir} + \mathbf{o} - (a*\mathbf{n} + \mathbf{c})\|^2 - r^2 = 0 \\ & (t*\mathbf{dir} + \mathbf{oc} - a*\mathbf{n})^2 - r^2 = 0 \\ & a^2 - 2n(t*\mathbf{dir} + \mathbf{oc})*a + (t*\mathbf{dir} + \mathbf{oc})^2 - r^2 \\ & = 0 \end{aligned}$$

$$\rightarrow A*t^2 + B*t + C = 0$$

$$\begin{aligned} \text{Thus,} \quad A &= 1 \\ B &= -2n(t*\mathbf{dir} + \mathbf{oc}) \\ C &= (t*\mathbf{dir} + \mathbf{oc})^2 - r^2 \end{aligned}$$

calculate  $a$  with  $A, B, C$  we get, then the point  $\mathbf{T}$  on the axis  $\mathbf{n}$  through  $\mathbf{c}$ .

With equation for points  $\mathbf{I}$  and  $\mathbf{T}$  we calculated above, the normal at intersection point  $\mathbf{I}$  is:

$$\text{Normal} = (\mathbf{I} - \mathbf{T})/r$$