MH1200 Problem Set 9

October 26, 2017

Elementary consequences

Problem 1. Let V be a vector space with zero element $\mathbf{0}$. Using the defining properties of a vector space, show the following:

1. $v + (-1) \cdot v = \mathbf{0}$ for any $v \in V$.
2. if $a \cdot v = \mathbf{0}$ then either $a = 0$ or $v = \mathbf{0}$.
3. If $a \cdot v = b \cdot v$ and $v \neq \mathbf{0}$ then $a = b$.

Subspaces in $\mathbb{R}^n$

Problem 2. Determine if the following subsets of $\mathbb{R}^3$ are subspaces

(a) $\{(x_1, x_2, x_3) : x_1 \geq 0, x_2 \geq 0, x_3 \geq 0\}$
(b) $\{(1, x_2, x_3) : x_2, x_3 \in \mathbb{R}\}$
(c) $\{(x_1, x_2, x_3) : x_1 \leq x_2 \leq x_3\}$
(d) $\{(x_1, x_2, x_3) : x_1 + x_2 = 0, x_3 + x_2 = 0\}$
(e) $\{(x_1, x_2, x_3) : x_1 \cdot x_3 = 0\}$
(f) $\{(0, 0, c) : c$ is an integer$\}$

Problem 3. Let $S \subseteq \mathbb{R}^3$ be a subspace. Define $T \subseteq \mathbb{R}^2$ as

$$T = \{(x, y) : \text{there is a } z \text{ such that } (x, y, z) \in S\}.$$

Show that T is a subspace of $\mathbb{R}^2$.

Subspaces in other vector spaces

Problem 4. Let V be the vector space of 3-by-3 matrices.

1. Find a subspace of V that contains no nonzero diagonal matrices.
2. Let $S \subseteq V$ be the set of *symmetric* 3-by-3 matrices. Show that S is a subspace. Find a set of matrices that span S. How small can you make your spanning set?

3. Let $S \subseteq V$ be the set of all matrices of the form

$$\begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$$

for some $a, b, c \in \mathbb{R}$. Matrices of this form are called *circulant*. Show that S is a subspace. Find a set of matrices that span S in this case.

Problem 5. Let V be the vector space of all functions $f : \mathbb{R} \to \mathbb{R}$. Show that the subset $S \subseteq V$ of *even* functions, those satisfying $f(x) = f(-x)$ for all $x \in \mathbb{R}$, is a subspace.

Problem 6. Let V be a vector space and $U, W \subseteq V$ be subspaces of V. Show that $U \cap W$ is also a subspace of V.

Span

Problem 7. Consider the three vectors

$$\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \vec{u} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \vec{v} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad .$$

Is span$(\{\vec{w}, \vec{u}\})$ equal to span$(\{\vec{w}, \vec{u}, \vec{v}\})$?