MH1200 Problem Set 10

Nov 2, 2017

Problem 1. Determine the nullspace of the matrix

$$A = \begin{bmatrix} 1 & 3 & 6 & 9 \\ 2 & 4 & 12 & 15 \\ 1 & 5 & 6 & 12 \end{bmatrix} \quad .$$

What is a set of vectors that span the nullspace?

Problem 2. Is it possible to construct a matrix whose column space contains $(1, 1, 0)$ and $(0, 1, 1)$ and whose nullspace contains $(1, 0, 1)$ and $(0, 0, 1)$?

Problem 3. Construct a 2-by-2 matrix whose column space is equal to its nullspace.

Problem 4. Show that the polynomials $1, x, 2x^2 - 1, 4x^3 - 3x$ are linearly independent.

Problem 5. Show that the functions $\cos(t), \sin(t), \cos(2t), \sin(2t)$ are linearly independent.

Problem 6. Let the sequence of vectors $\vec{u}_1, \ldots, \vec{u}_k \in \mathbb{R}^n$ be linearly independent. Let $\vec{v} \in \mathbb{R}^n$ be a vector not in $\text{span}(\{\vec{u}_1, \ldots, \vec{u}_k\})$. Show that the sequence of vectors $\vec{u}_1, \ldots, \vec{u}_k, \vec{v}$ is linearly independent.

Problem 7. Let $\vec{v}_1, \ldots, \vec{v}_k \in \mathbb{R}^n$ be a sequence of linearly independent vectors and A be an n-by-n *invertible* matrix. Show that the sequence of vectors $A\vec{v}_1, \ldots, A\vec{v}_k$ is also linearly independent.

Problem 8. Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$ be three vectors, none of them equal to the all-zero vector $\vec{0}_n$. We say that $\vec{u}, \vec{v}, \vec{w}$ are *mutually perpendicular* if all of their pairwise inner products are zero. That is,

$$\langle \vec{u}, \vec{v} \rangle = \langle \vec{u}, \vec{w} \rangle = \langle \vec{v}, \vec{w} \rangle = 0 \quad .$$

Show that in this case $\vec{u}, \vec{v}, \vec{w}$ are linearly independent.