

## Exercises for Chapter 9

**Exercise 83.** Consider the set  $A = \{a, b, c\}$  with power set  $P(A)$  and  $\cap: P(A) \times P(A) \rightarrow P(A)$ . What is its domain? its co-domain? its range? What is the cardinality of the pre-image of  $\{a\}$ ?

**Exercise 84.** Show that  $\sin: \mathbb{R} \rightarrow \mathbb{R}$  is not one-to-one.

**Exercise 85.** Show that  $\sin: \mathbb{R} \rightarrow \mathbb{R}$  is not onto, but  $\sin: \mathbb{R} \rightarrow [-1, 1]$  is.

**Exercise 86.** Is  $h: \mathbb{Z} \rightarrow \mathbb{Z}$ ,  $h(n) = 4n - 1$ , onto (surjective)?

**Exercise 87.** Is  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x^3$ , a bijection (one-to-one correspondence)?

**Exercise 88.** Consider  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x^2$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$ ,  $g(x) = x + 5$ . What is  $g \circ f$ ? What is  $f \circ g$ ?

**Exercise 89.** Consider  $f: \mathbb{Z} \rightarrow \mathbb{Z}$ ,  $f(n) = n + 1$  and  $g: \mathbb{Z} \rightarrow \mathbb{Z}$ ,  $g(n) = n^2$ . What is  $g \circ f$ ? What is  $f \circ g$ ?

**Exercise 90.** Given two functions  $f: X \rightarrow Y$ ,  $g: Y \rightarrow Z$ . If  $g \circ f: X \rightarrow Z$  is one-to-one, must both  $f$  and  $g$  be one-to-one? Prove or give a counter-example.

**Exercise 91.** Show that if  $f: X \rightarrow Y$  is invertible with inverse function  $f^{-1}: Y \rightarrow X$ , then  $f^{-1} \circ f = i_X$  and  $f \circ f^{-1} = i_Y$ .

**Exercise 92.** Prove or disprove  $\lceil x+y \rceil = \lceil x \rceil + \lceil y \rceil$ , for  $x, y$  two real numbers.

**Exercise 93.** If you pick five cards from a deck of 52 cards, prove that at least two will be of the same suit.

**Exercise 94.** If you have 10 black socks and 10 white socks, and you are picking socks randomly, you will only need to pick three to find a matching pair.

**Exercise 95.** Prove that the set of all integers is countable.