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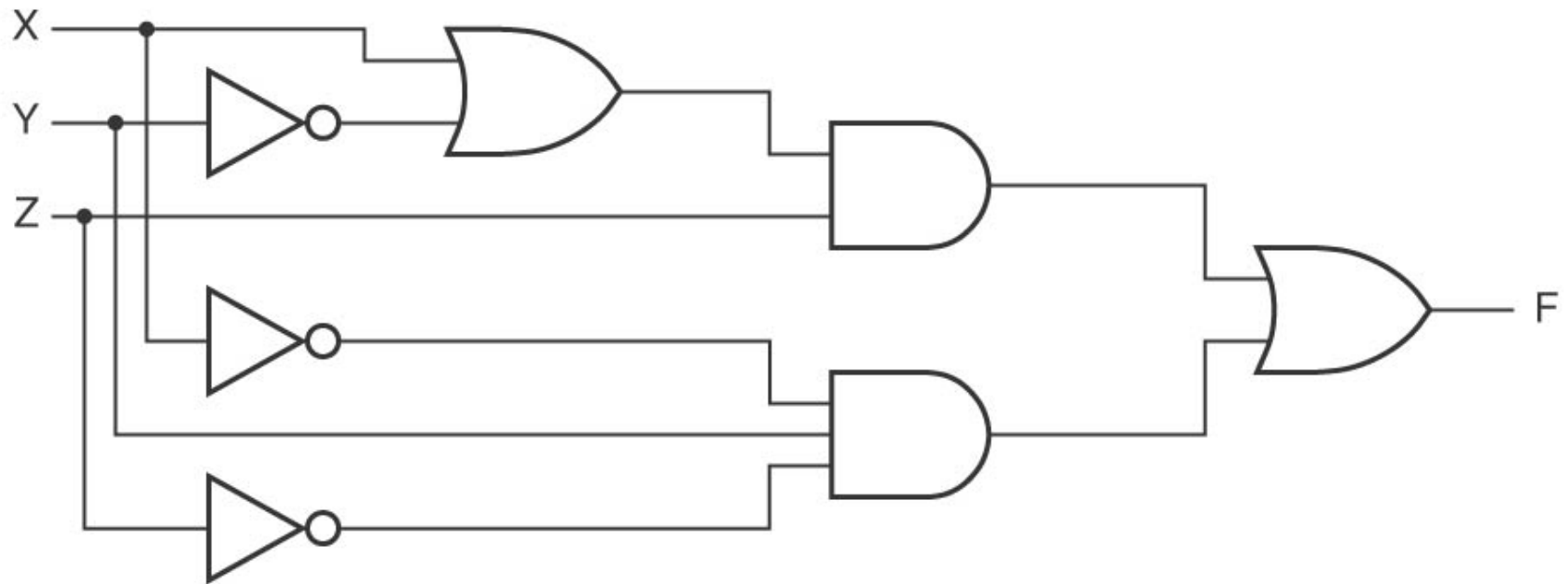
L1: Key concepts

- **Boolean Constants: 0 and 1**
- **Boolean variables, logic signals**
- **Logic levels represented by voltage**
- **Basic logic gates: AND, OR, NOT**
- **Logic symbols, logic expression, truth table**
- **Logic circuits has inputs and outputs, need power supply to operate**

L1: Key concepts (cont)

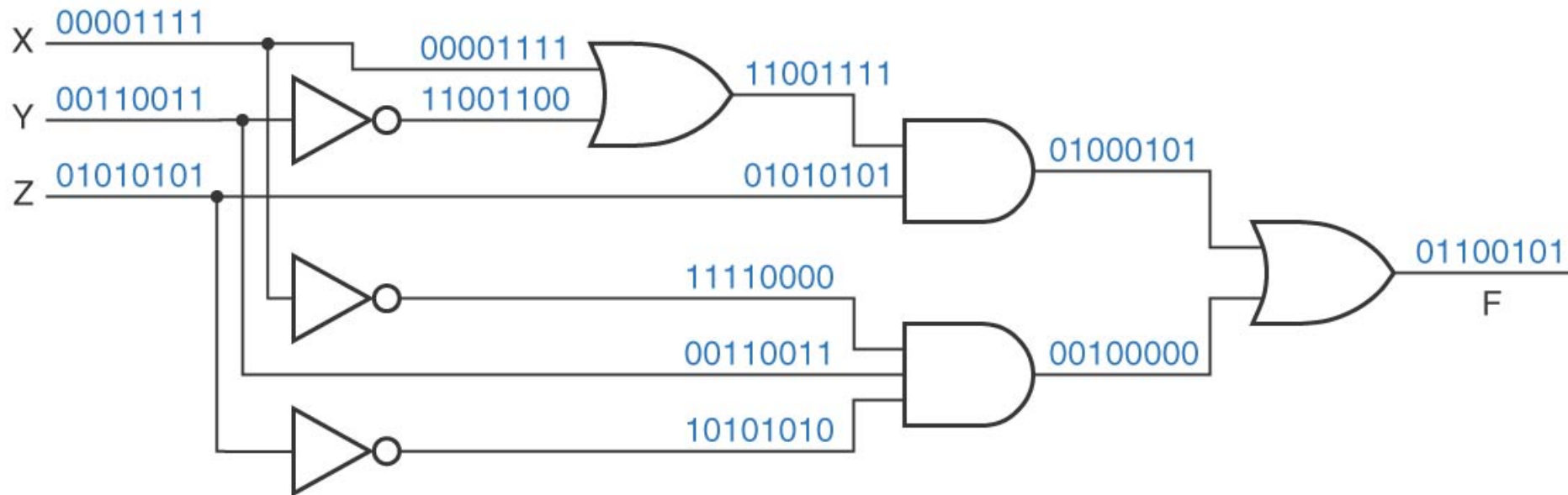
- **An output can connect to one or more inputs**
- **Outputs should not be connected together unless one is very sure**
- **Timing diagram/waveform**
- **A logic circuit can be described by truth table, logic expression, or circuit diagram**
- **Evaluation of logic expression: order of precedence**

Example: describe by circuit diagram



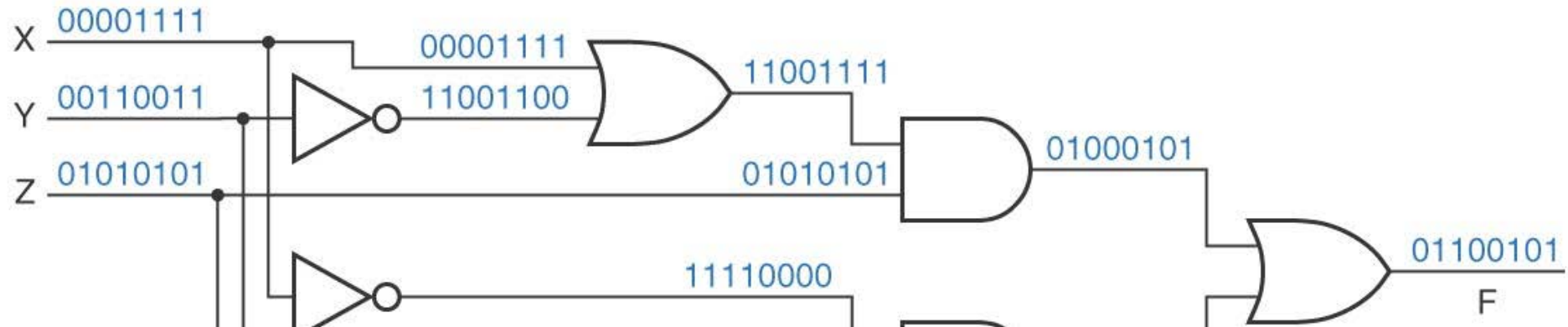
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Example: evaluate output from inputs



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Sketch timing waveforms

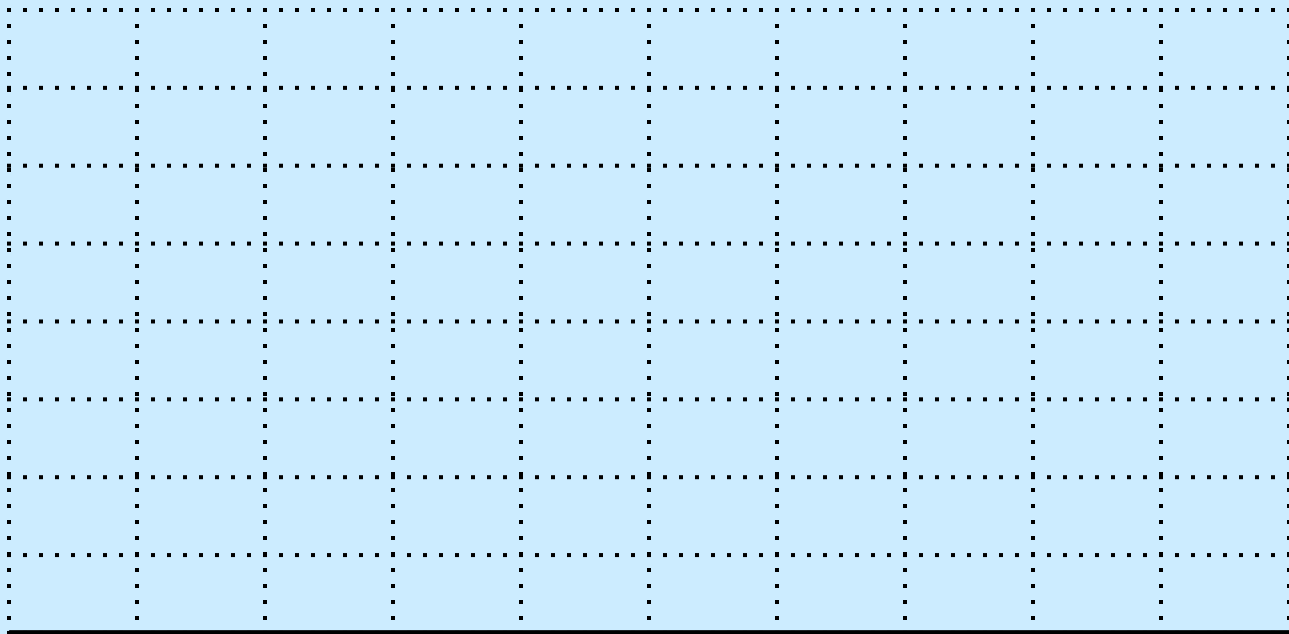


X

Y

Z

F



time

Example: describe by truth table

<i>Row</i>	X	Y	Z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

How many different 2-input truth tables can be constructed? Assume each has 1 output.

A. 4

B. 8

C. 12

✓ D. 16



Inputs		Possible outputs					
a	b	x					
0	0	0	0	0	...	1	1
0	1	0	0	0		1	1
1	0	0	0	1		1	1
1	1	0	1	0		0	1

2^n

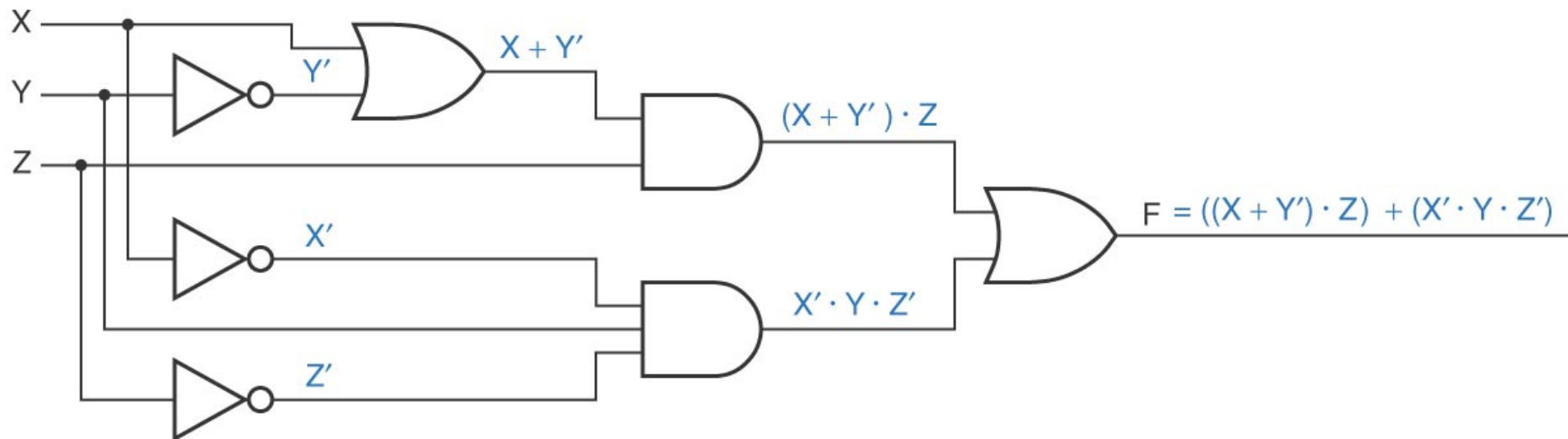
$2^{(2^n)}$

Commonly-used 2-input logic gates:

- AND, NAND
- OR, NOR
- XOR, XNOR

Example: describe by logic expression

$$F = (X + Y') Z + X' Y Z'$$



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How about describe by timing diagram?

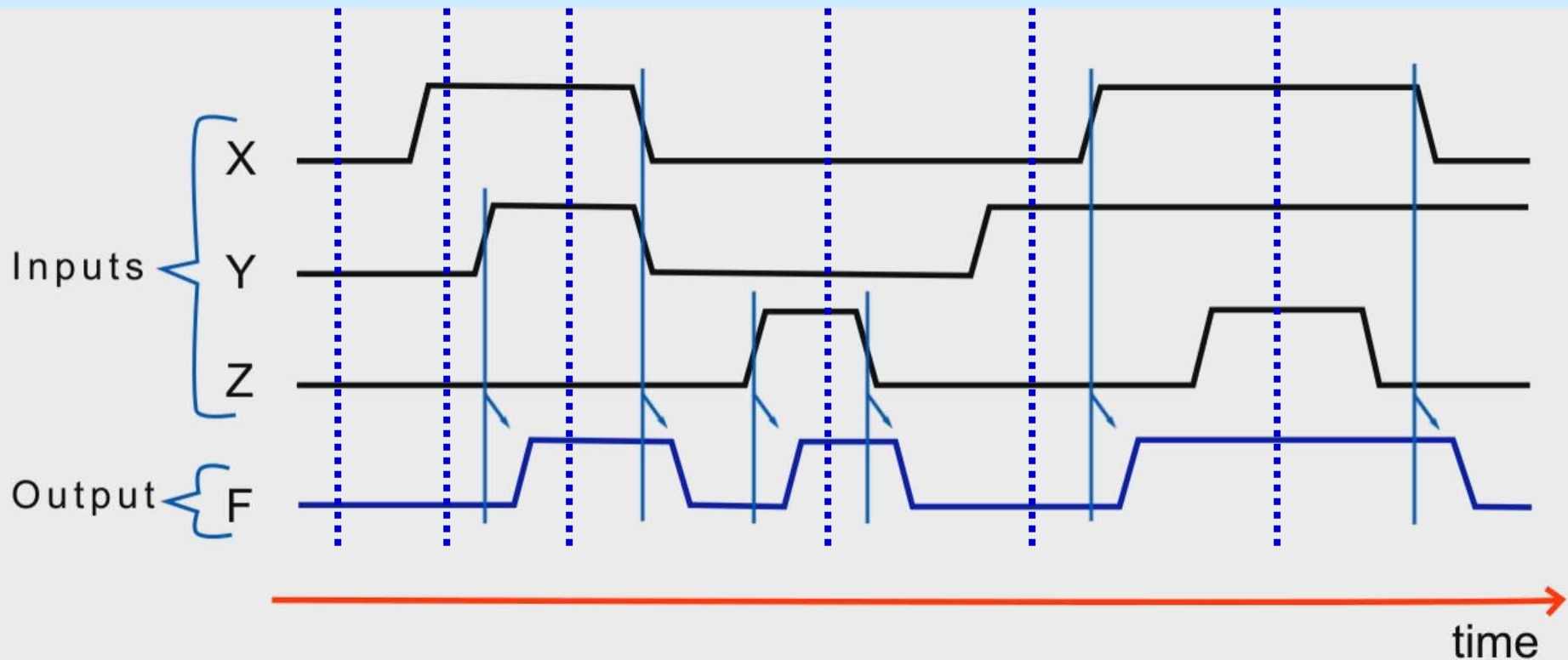


Fig. 3.17 (taken from Wakerly)

Truth table

inputs			output
X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	?
1	0	0	0
1	0	1	?
1	1	0	1
1	1	1	1

Lecture 2: Key concepts

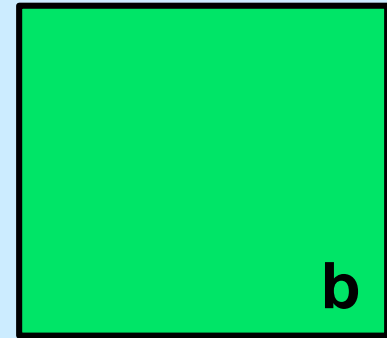
- **Boolean theorems: single and multivariable, DeMorgan's theorems**
- **Theorems are required for algebraic manipulation and simplification**
- **NOR and NAND gates: universal gates can replace basic logic gates AND, OR, NOT**

Boolean theorems

- **Essential to know and apply the theorems**
- **For those interested in more proofs on Boolean theorems (optional):**
 - <http://www.electrical4u.com/boolean-algebra-theorems-and-laws-of-boolean-algebra/>
 - http://mines.humanoriented.com/410/books/boolean_algebra.pdf

Absorption laws

Absorption laws:

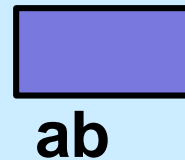
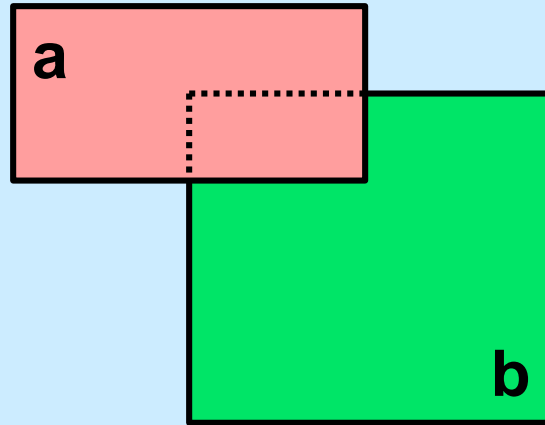


$$a + ab = a$$

Absorption laws (cont)

Absorption laws:

$$a + ab = a$$



ab is absorbed in a

Simple example for illustration

Let

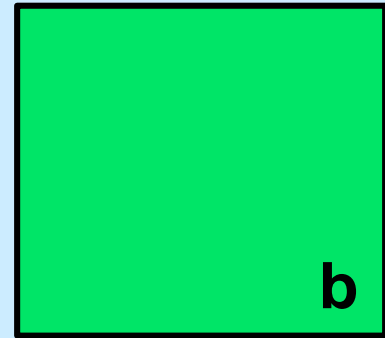
- $A=1$ means a person likes apples ($A=0$ means a person dislikes apples)
- $B=1$ means a person likes oranges ($B=0$ means a person dislikes oranges)

$$A + AB = A(1+B) = A(1) = A$$

A person who likes apples ($A=1$) may also like oranges ($B=1$)

Absorption laws (cont)

Absorption laws:

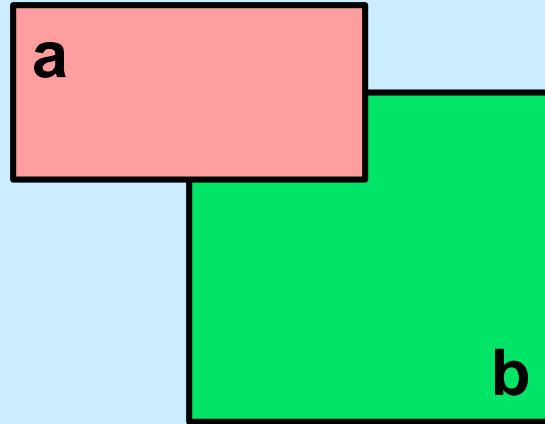


$$a + a'b = a + b$$

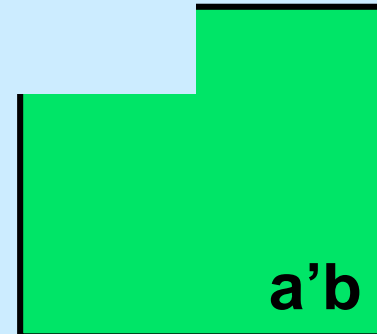
Absorption laws (cont)

Absorption laws:

$$a + a'b = a + b$$



a'b is absorbed in b



Simple example for illustration

Let

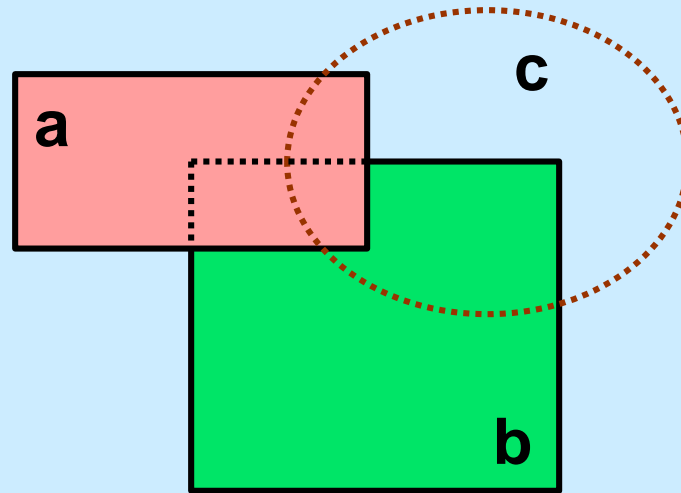
- $A=1$ means a person likes apples ($A=0$ means a person dislikes apples)
- $B=1$ means a person likes oranges ($B=0$ means a person dislikes oranges)

$$X = A + A'B = A + B$$

A person who likes apples or oranges may like either one or both

Consensus law

Consensus law:



$$\begin{aligned} bc &= (ab + a'b) c \\ &= abc + a'bc \end{aligned}$$



$$ab + a'c + bc = ab + a'c$$

A part of bc is absorbed in ab ,
the other part is absorbed in $a'c$

Simple example for illustration

Let

- $A=1$ means a person likes apples ($A=0$ means a person dislikes apples)
- $B=1$ means a person likes oranges ($B=0$ means a person dislikes oranges)
- $C=1$ means a person likes pears ($C=0$ means a person dislikes pears)

Example (continue)

$AB + A'C$ means a person who likes both apples and oranges, or likes pears but dislikes apples.

BC means a person who likes both oranges and pears.

- **$AB + A'C + BC = AB + A'C$**

a person who likes both oranges and pears may like or dislike apples

- **Important: it does NOT imply $BC = 0$**

DeMorgans theorems

$$V = (a + b + \dots + g)' = a' b' \dots g'$$

Let $V=1$ means need a visa

- Don't need a visa ($V=0$) if one is from country a or b or c ... or g
- Need a visa ($V=1$) if one is not from country a, and not from b, and not from c ... and not from g

DeMorgans theorems (cont)

$$J = (a \ b \ c \ \dots \ g)' = a' + b' + c' + \dots + g'$$

Let $J=1$ means need an immunization jab

- Don't need a jab ($J=0$) if one is already immunized for viruses a and b and c ... and g
- Need a jab ($J=1$) if one is not immunized from virus a, or not from b, or not from c ... or not from g

$$A(B+C)' = AB' + AC'$$

True or false?

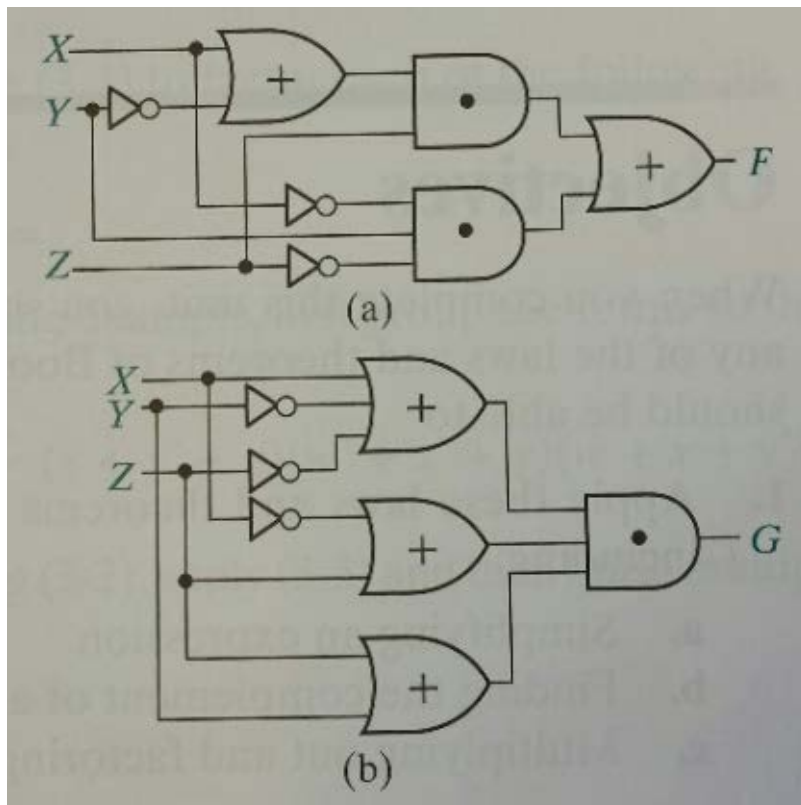
A. True

✓ **B. False**

$$\begin{aligned} A(B+C)' &= A(B'C') \\ &= AB'C' \end{aligned}$$



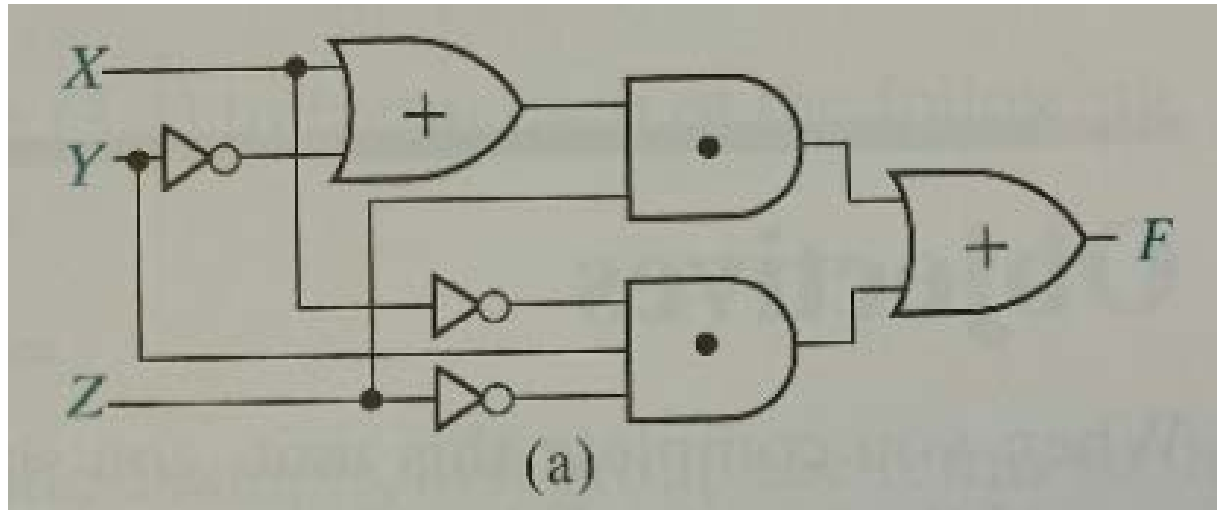
Are these two circuits algebraically equivalent?



✓ **A. Yes**
B. No

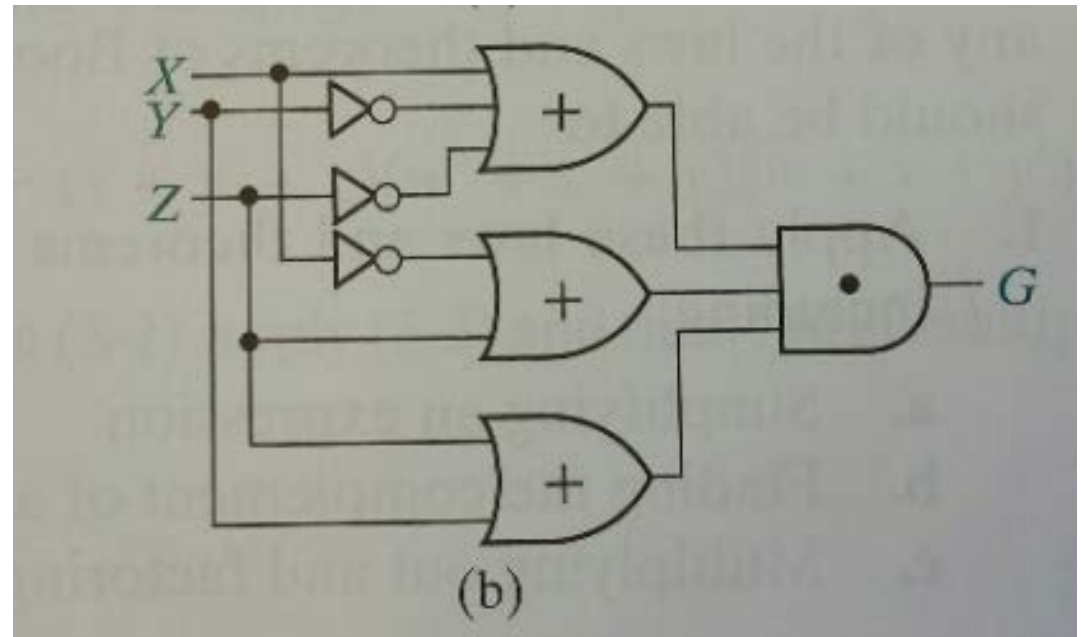


The answer is Yes.



$$F = (X + Y')Z + X'YZ'$$

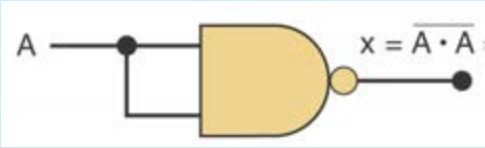
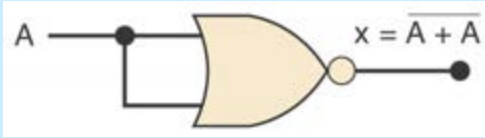
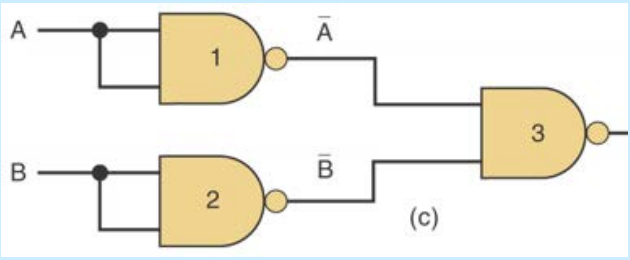
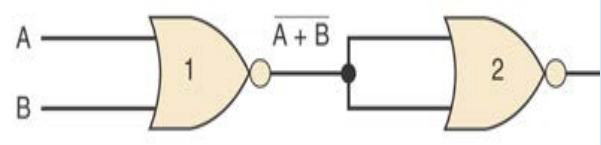
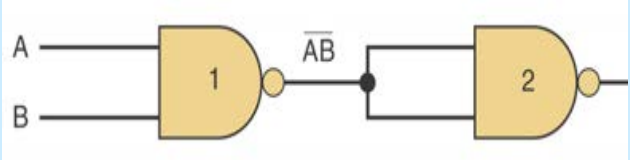
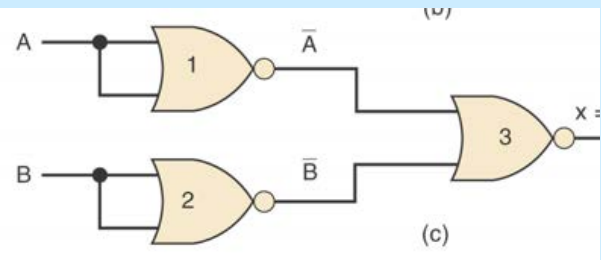
$$\begin{aligned}
G &= (X+Y'+Z')(X'+Z)(Y+Z) \\
&= (X+Y'+Z')(X'Y + X'Z + YZ + ZZ) \\
&= (X+Y'+Z')[X'Y + Z(X'+Y+1)] \\
&= (X+Y'+Z')[X'Y + Z] \\
&= XZ + Y'Z + X'YZ' \\
&= (X+Y')Z + X'YZ' \\
&= F
\end{aligned}$$



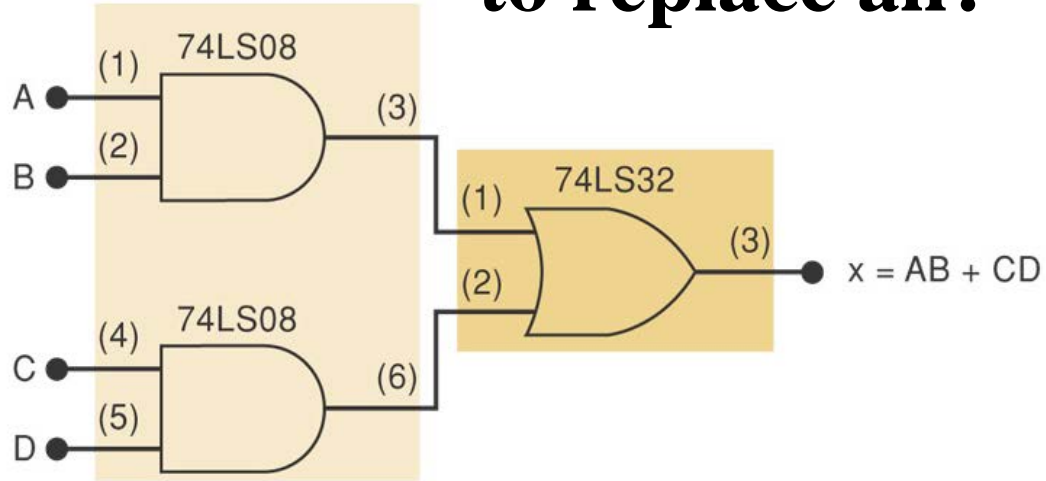
Replacing AND, OR, NOT with purely NAND, or purely NOR

Basic gate	NAND only	NOR only
NOT	$(XX)' = X'$ 1	$(X+X)' = X'$ 1
OR	$X+Y = [(X') (Y')]'$ 3	$X+Y = [(X+Y)']'$ 2
AND	$XY = [(XY)']'$ 2	$XY = [(X') + (Y')]'$ 3

Logic symbol approach

Basic gate	NAND only	NOR only
NOT		
OR		
AND		

How many NAND gates are needed in total to replace all?

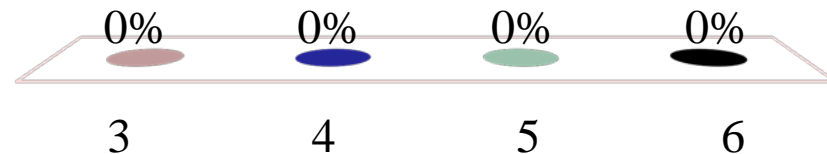


✓ **A. 3**

B. 4

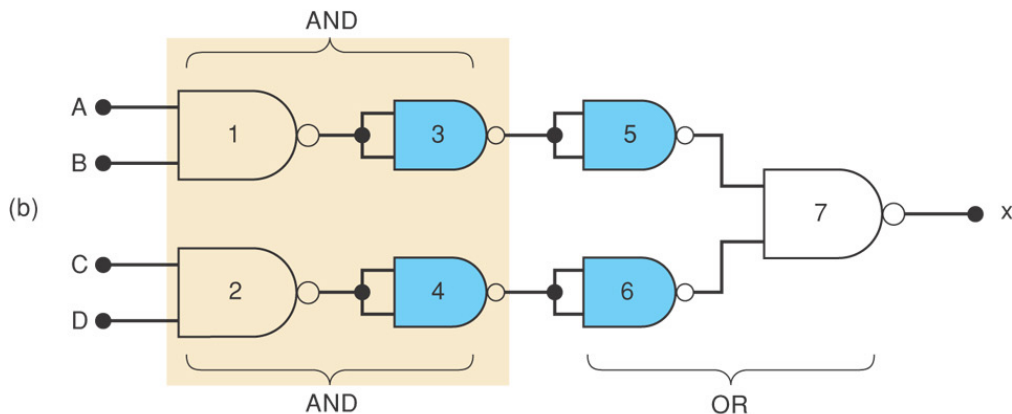
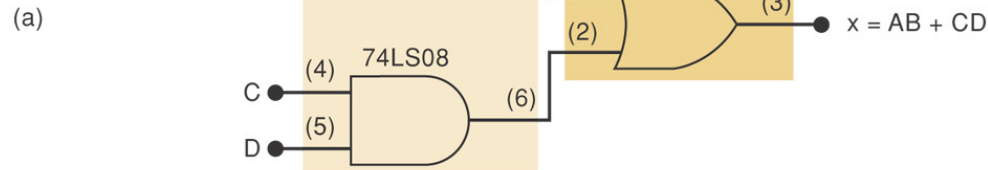
C. 5

D. 6



Example: NAND gates replace AND, OR

By diagram



After eliminating
double inversions

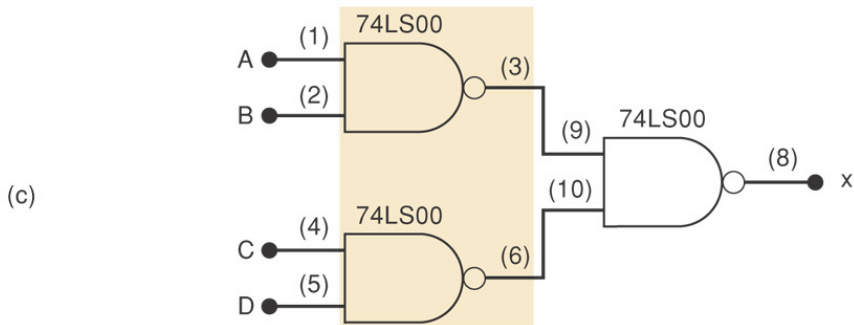
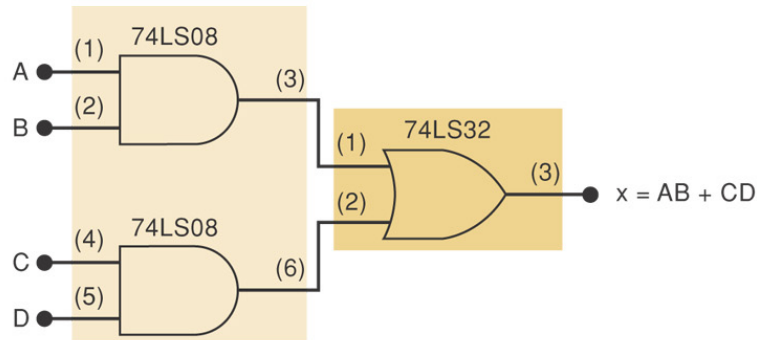


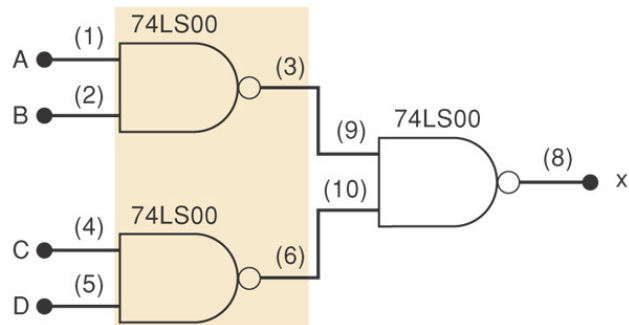
Figure 3-32

(a)



$$\begin{aligned} AB + CD &= [(AB + CD)']' \\ &= [(AB)' (CD)']' \end{aligned}$$

(c)



Example:
NAND gates
replace AND,
OR

By Boolean
expression

Figure 3-32

**Universal gates always reduce
the number of gates used.
True or false?**

A. True

✓ B. False



End of L1, L2 summary