MH1200 Problem Set 7

October 12, 2017

Problem 1. Compute the determinant of each of these matrices.

(a) $\begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$, (b) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, (c) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}$.

Problem 2. Let A be a 4-by-4 matrix such that $\det(A) = 9$. Find

(a) $\det(3A)$, (b) $\det(A^{-1})$, (c) $\det(3A^{-1})$, (d) $\det((3A)^{-1})$, (e) $\det(-A)$.

Problem 3. Let A and B be 3×3 matrices such that

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} B.$$

Given $\det(A) = 4$, determine $\det(B)$.

Problem 4. Show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b) \ .$$

Problem 5. Let A be a n-by-n matrix. Let B be the matrix where $B(i,j) = \frac{i}{j}A(i,j)$. For example, in the 3-by-3 case, if

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad B = \begin{bmatrix} a_{11} & \frac{1}{2}a_{12} & \frac{1}{3}a_{13} \\ 2a_{21} & a_{22} & \frac{2}{3}a_{23} \\ 3a_{31} & \frac{3}{2}a_{32} & a_{33} \end{bmatrix} \ .$$

How does the determinant of B relate to that of A?

Problem 6 (3-by-3 determinant)**.** Consider a 3-by-3 matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Show that the function

$$aei + bfg + cdh - (afh + bdi + ceg)$$

satisfies the three defining properties of the determinant.

Problem 7. Prove or give a counter example:

$$\det(A + B) = \det(A) + \det(B) \ .$$