MH1200:
Linear Algebra I

Course Info

Lectures: Tue 14:30-15:30, Wed 8:30-10:30, LT27

Tutorials: Thu beginning Aug 24.

Instructor: Troy Lee

Office hours: Tue, Thu 16:00-17:00, SPMS-MAS-05-02

Contact me: troyjlee@gmail.com

Grading

§ Quizzes: 20% of your grade

- Short understanding checks given in tutorial

Sept. 7	Quiz 1
Sept. 21	Quiz 2
Oct. 19	Quiz 3
Nov. 2	Quiz 4
Nov. 16	Quiz 5

Grading

§ Midterm: 30% of your grade

- Tuesday 10 October. 14:30-15:30 in Exam Hall C

§ Final: 50% of your grade

- 2 hour closed book exam.

§ Problem sets: **not graded.**

- But for learning they are the most important part of the class!

Resources

§ NTUlearn

- All the previous info can be found there.
- Course content, recorded lectures
- Subscribe to the discussion forum!

§ YouTube Channel

- Search for MH1200 on YouTube

§ Textbook: **Introduction to Linear Algebra, Gilbert Strang**

- Book website: http://math.mit.edu/~gs/linearalgebra/
- You can use 4th or 5th edition.

Additional Resources

§ http://ocw.mit.edu/18-06S05

- MIT linear algebra course
- Lectures by Gilbert Strang himself

§ Other References

- Anton and Rorres, Elementary Linear Algebra
- Jim Hefferon, Linear Algebra

∞ Freely available:
http://joshua.smcvt.edu/linearalgebra/

Additional Resources

§ Khan Academy

- Generally at an easier level than our course

§ Matlab

- Software that is very convenient for doing linear algebra. Should be on school computers.

§ Python

- Has a library for doing linear algebra called numpy.
- I will demonstrate some basic functions in the course.

Motivation

What is a matrix?

The fundamental object of study in linear algebra is a matrix.

A matrix is simply a 2D array of numbers.

$$\begin{bmatrix} 5 & 0 & -2 \\ 1.67 & \pi & 0 \\ -1 & 4 & 5.89 \end{bmatrix}$$

Lots of data is naturally represented as a 2D array.

Linear algebra can help analyze it!

Images!

An image is naturally a 2D object.

If we look at the intensity of each pixel, we get a matrix!

182	182	183	183	184	184	184	184	184	184	184	184	183	183	183	183	181	182	182	183	185
183	185	183	183	183	182	183	184	183	182	182	183	183	183	182	184	183	182	183	181	182
184	184	182	183	184	183	184	184	184	184	186	187	183	182	184	184	183	183	183	182	182
182	181	181	184	185	183	182	180	184	183	178	179	181	183	184	180	181	181	180	181	182
150	150	148	150	149	151	152	152	146	152	148	149	153	149	148	150	153	153	151	151	152
144	151	153	155	153	154	149	147	155	157	155	156	155	150	151	152	144	147	150	150	151
147	152	150	149	150	154	151	154	159	152	153	156	152	154	159	156	153	151	154	148	145
158	161	156	157	160	162	154	158	162	160	165	162	157	159	160	159	153	151	163	162	160
158	159	153	159	167	169	158	164	166	168	165	157	162	165	158	160	157	149	163	163	162
159	156	148	156	168	171	169	161	159	157	161	167	163	156	164	163	168	167	162	158	153
160	166	157	158	154	158	159	164	173	159	179	139	68	62	68	69	82	122	149	160	174
160	166	170	168	166	168	168	164	150	164	165	56	31	35	31	35	33	32	53	99	140
160	162	168	170	165	174	168	153	159	179	143	41	37	42	33	34	36	34	42	47	67
169	166	166	178	165	165	156	160	170	173	121	44	35	32	39	33	32	27	36	34	33
160	170	169	156	157	156	160	163	172	175	106	31	38	34	39	33	31	29	26	100	124
167	164	165	176	183	168	166	174	172	171	84	44	34	39	30	33	26	27	29	37	50
167	168	148	177	161	185	172	173	171	160	68	40	35	35	34	30	30	25	24	24	21
173	177	155	178	159	168	175	172	168	148	56	36	33	32	31	30	26	25	24	24	22
177	178	170	177	174	178	166	171	177	135	45	34	34	30	31	29	25	24	22	23	22
177	180	182	185	174	169	174	170	178	114	39	36	34	29	31	28	25	23	22	22	21
178	180	187	179	173	172	166	171	175	89	36	34	32	32	31	26	24	24	23	23	19

upper 20-by-20 pixel section of image

50-by-50 pixel section of image

Image compression

The intensity of each pixel is a number between 0-255.

Storage: 8 bits = 1 byte for each pixel.

369 x 590 ~ 218 KB

Represent the image in a different basis. Throw away low order terms.

(Done using SVD)

~ 50 KB

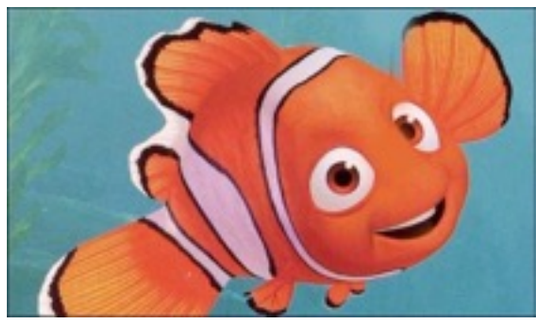
The Movie Matrix

Not that movie matrix!

The Movie Matrix

Movie Matrix

Finding Nemo



Pulp Fiction

Wolverine

Alice

	8	4	?

Bob

	3	9	5

Collaborative filtering

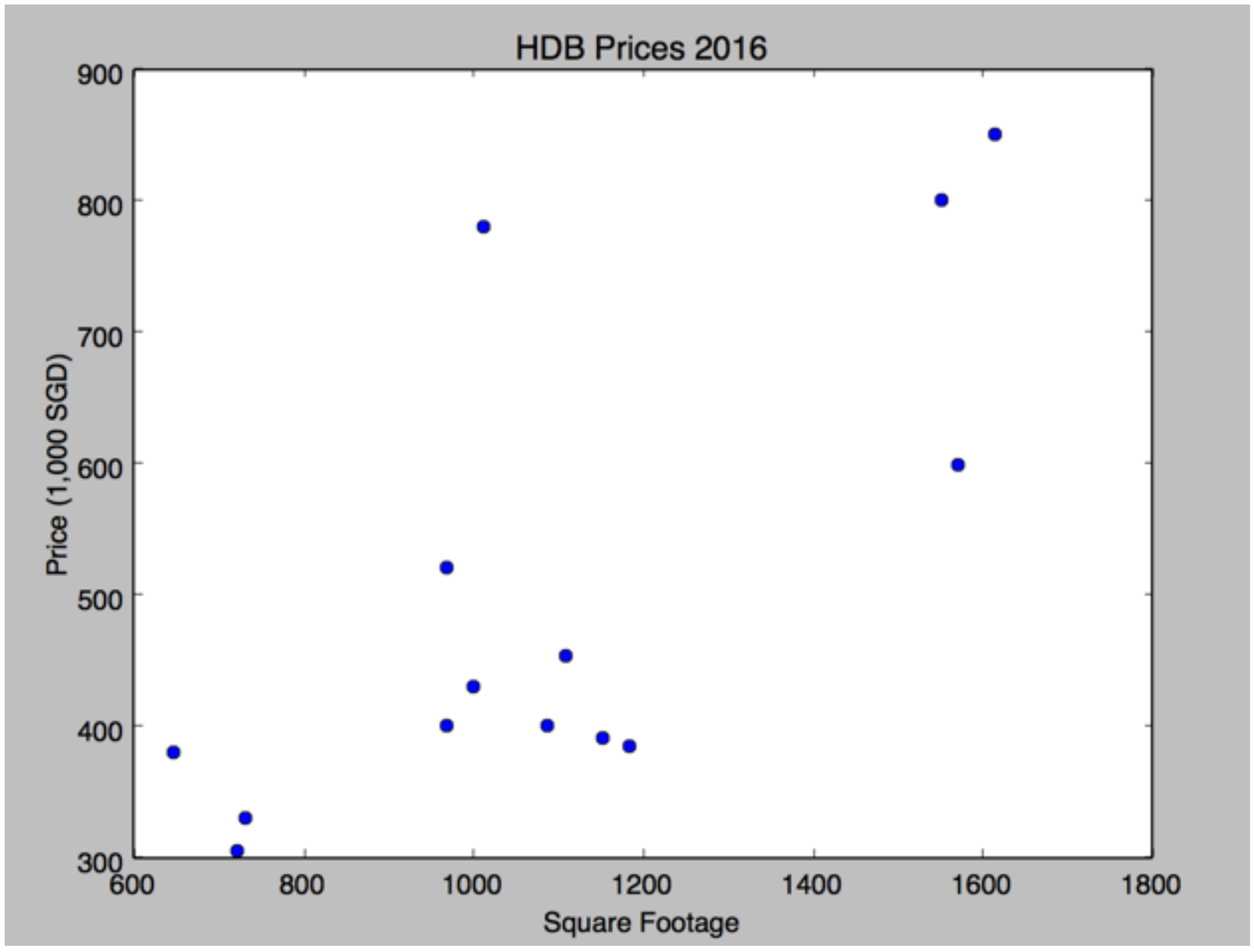
If you like Finding Nemo, you might also like...

Such recommendations can be found by looking at factorizations of the movie matrix.

We will soon see the use of factorizations with the LU decomposition, given by Gaussian elimination.

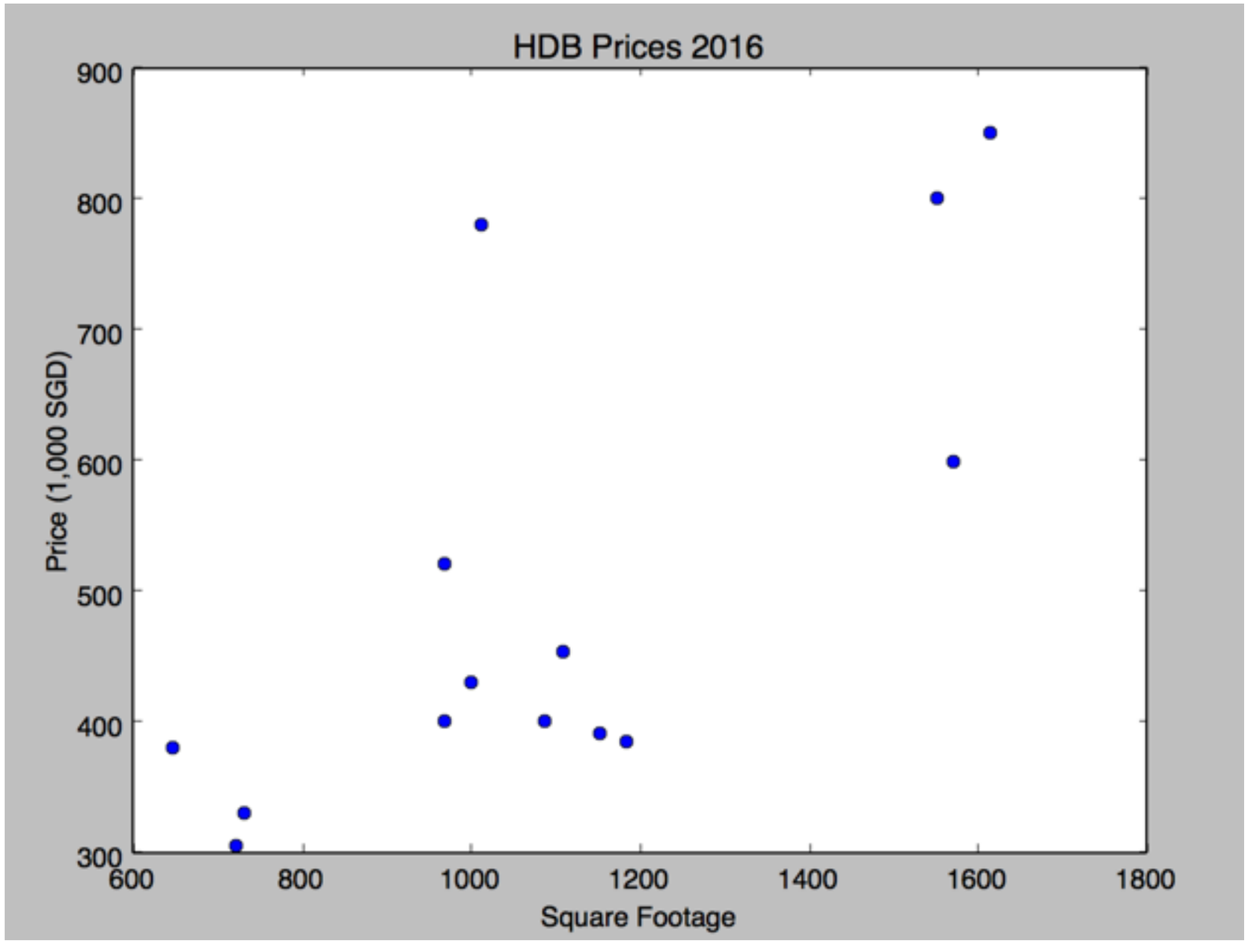
Housing Prices

Here is a plot of the square footage versus price of 14 HDB flats.



Housing Prices

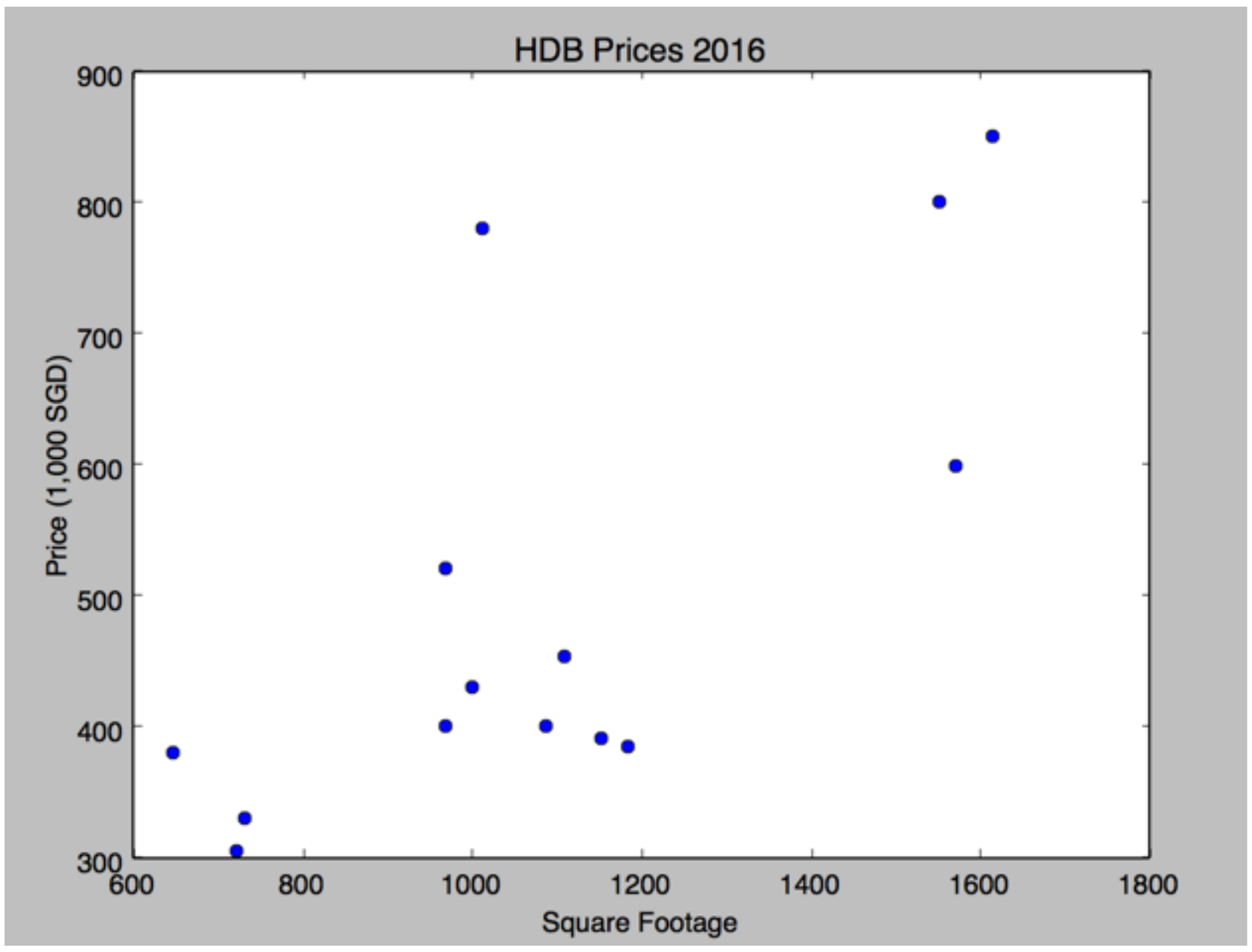
Do you think there is a line going through all these points?



Housing Prices

We would like to find the "best" formula of the form

$$\text{price} = a \cdot \text{square footage} + b$$



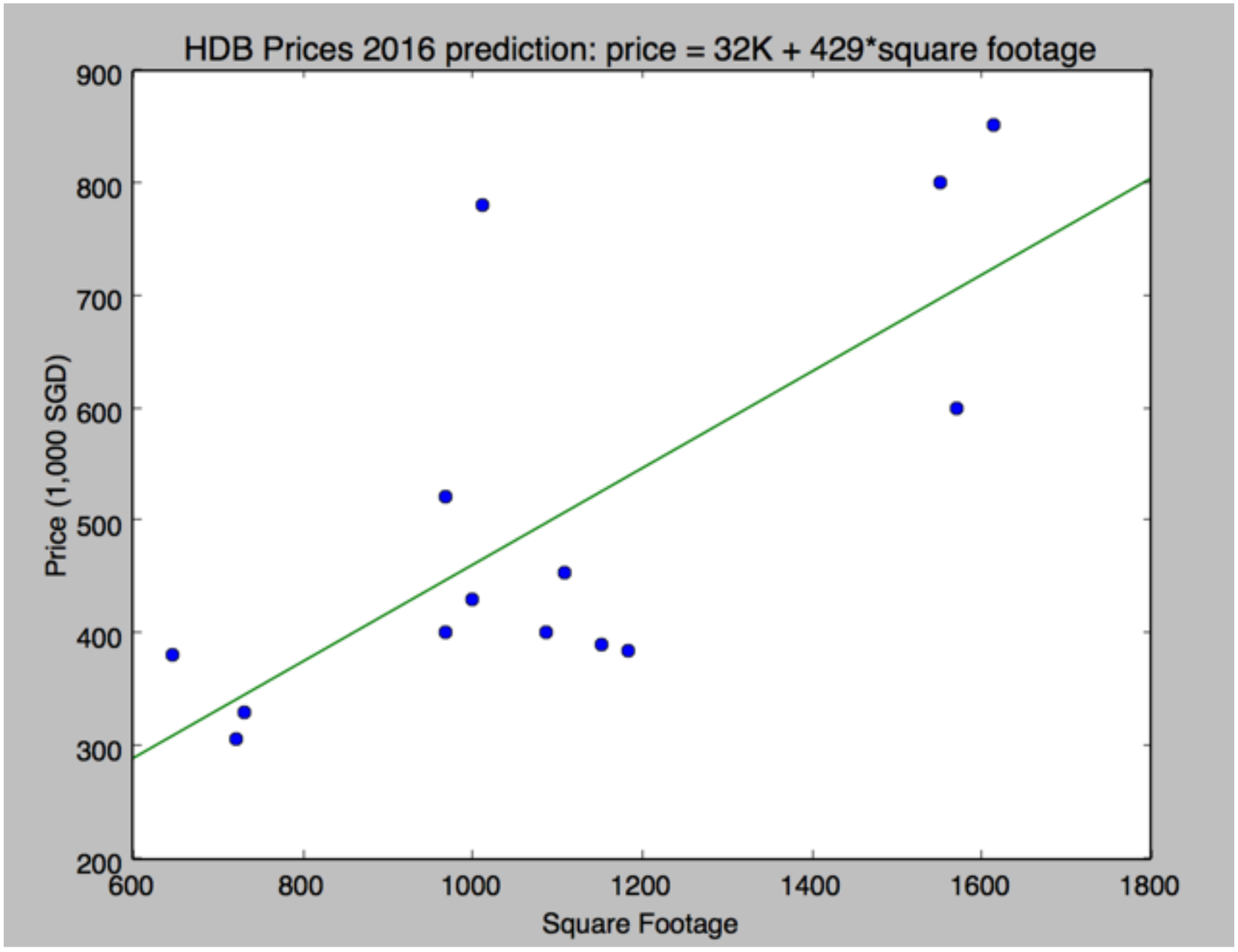
How to solve for a and b?

This is a classic linear algebra problem!

Housing Prices

The "best" line fitting these points is

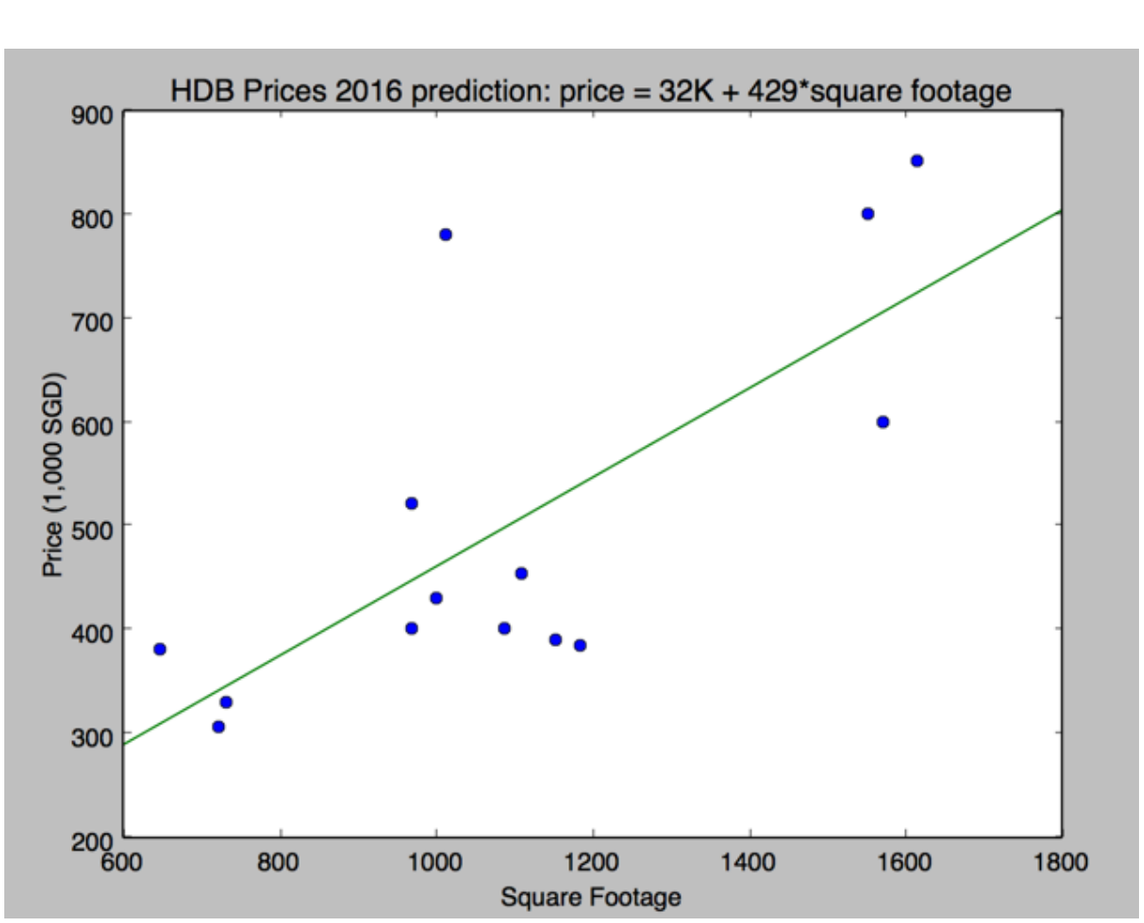
$$\text{price} = 429 \cdot \text{square footage} + 32,000$$



Housing Prices

This line is quite off for several flats.

There are factors other than square footage that affect the price of a flat!



What other factors might influence the price of a flat?

The linear algebra approach easily extends to the case of multiple factors!

Deep Learning

One of the most exciting technological developments of our time!

Used by AlphaGo to beat the best human Go players.

Used in self driving cars.

Linear algebra is one of the key prerequisites for this field!

Neural Networks

Deep learning is about neural networks with many layers.

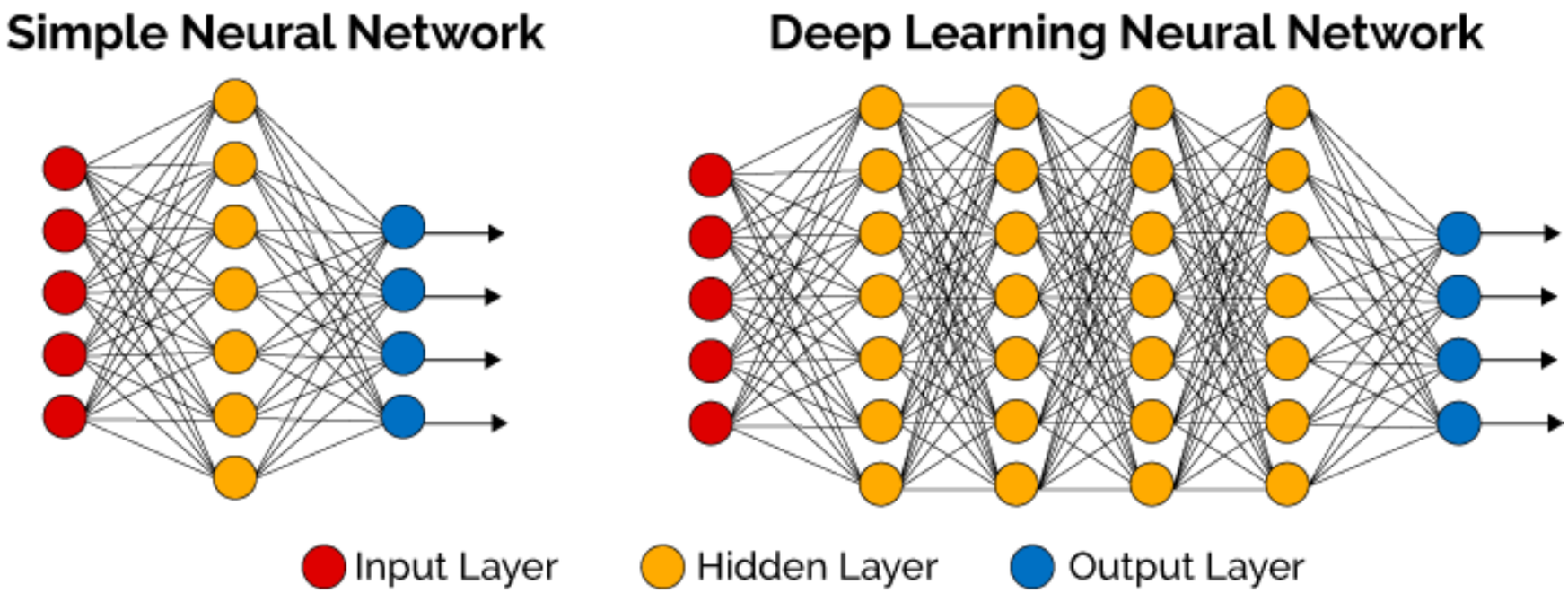


image credit: hackernoon.com

Each layer of the network computes a matrix multiplication.

Being able to manipulate matrices is extremely beneficial to understanding these networks.

Image Classification

If we have time at the end of the module, we will apply the theoretical knowledge we learn to a fun project.

We will see how we can use linear algebra to read handwritten digits!

This will involve solving a system of linear equations with 50,000 equations.

After this course you will know how this can be done!

Perspective

At times the course will seem very theoretical, full of proofs and definitions.

I hope you keep these applications in mind as we go along.

Linear algebra is a beautiful subject full of connections to other areas. I hope that you will enjoy learning about it.

Tomorrow we start with vectors!