Lis User Guide Version 1.7.28



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Contents

.4.19 etest6 .4.20 etest7 .4.21 spmvtest1 .4.22 spmvtest2 .4.23 spmvtest2b

4	Q ua 4.1	druple Precision Operations Using Quadruple Precision Operations	 	. 42 . 42
5	Mat	rix Storage Formats		44
•	5.1	Compressed Sparse Row (CSR)		
	0.1	5.1.1 Creating Matrices (for Serial and Multithreaded Environments)		
		5.1.2 Creating Matrices (for Multiprocessing Environment)		
		5.1.3 Associating Arrays		
	5.2	Compressed Sparse Column (CSC)		
	5.2	5.2.1 Creating Matrices (for Serial and Multithreaded Environments)		
		0 (1		
	- 0	5.2.3 Associating Arrays		
	5.3	Modified Compressed Sparse Row (MSR)		
		5.3.1 Creating Matrices (for Serial and Multithreaded Environments)		
		5.3.2 Creating Matrices (for Multiprocessing Environment)		
		5.3.3 Associating Arrays		
	5.4	Diagonal (DIA)		
		5.4.1 Creating Matrices (for Serial Environment)		
		5.4.2 Creating Matrices (for Multithreaded Environment)		
		5.4.3 Creating Matrices (for Multiprocessing Environment)	 	. 52
		5.4.4 Associating Arrays	 	. 52
	5.5	Ellpack-Itpack Generalized Diagonal (ELL)	 	. 53
		5.5.1 Creating Matrices (for Serial and Multithreaded Environments)	 	. 53
		5.5.2 Creating Matrices (for Multiprocessing Environment)	 	. 54
		5.5.3 Associating Arrays		
	5.6	Jagged Diagonal (JAD)		
		5.6.1 Creating Matrices (for Serial Environment)		
		5.6.2 Creating Matrices (for Multithreaded Environment)		
		5.6.3 Creating Matrices (for Multiprocessing Environment)		
		5.6.4 Associating Arrays		
	5.7	Block Sparse Row (BSR)		
	0.1	5.7.1 Creating Matrices (for Serial and Multithreaded Environments)		
		5.7.2 Creating Matrices (for Multiprocessing Environment)		
		5.7.3 Associating Arrays		
	5.8	Block Sparse Column (BSC)		
	5.6			
		·		
		5.8.2 Creating Matrices (for Multiprocessing Environment)		
	- 0	5.8.3 Associating Arrays	 	. 62
	5.9	Variable Block Row (VBR)		
		5.9.1 Creating Matrices (for Serial and Multithreaded Environments)		
		5.9.2 Creating Matrices (for Multiprocessing Environment)		
		5.9.3 Associating Arrays		
	5.10	Coordinate (COO)		
		5.10.1~ Creating Matrices (for Serial and Multithreaded Environments)		
		5.10.2 Creating Matrices (for Multiprocessing Environment)		
		5.10.3 Associating Arrays		
	5.11	Dense (DNS)		
		5.11.1 Creating Matrices (for Serial and Multithreaded Environments)	 	. 69
		5.11.2 Creating Matrices (for Multiprocessing Environment)	 	. 70
		5.11.3 Associating Arrays	 	. 70

6	Fun	ctions		7 1
	6.1	Operatin	Vector Elements	72
		6.1.1 li	_vector_create	72
		6.1.2 li	_vector_destroy	72
		6.1.3 li	_vector_duplicate	73
			_vector_set_size	73
			_vector_get_size	74
			_vector_get_range	74
			_vector_set_value	75
			_vector_get_value	75
			_vector_set_values	76
			_vector_get_values	77
			_vector_scatter	77
			_vector_gather	78
	6.2		Matrix Elements	79
	0.2	-	_matrix_create	79
			_matrix_destroy	79
		_	_matrix_duplicate	80
			_matrix_malloc	80
			_matrix_manoc	81
				81
			_matrix_assemble	
			_matrix_set_size	82
			_matrix_get_size	82
			_matrix_get_range	83
			_matrix_get_nnz	83
			matrix_set_type	84
			_matrix_get_type	84
			_matrix_set_csr	85
			_matrix_set_csc	85
			_matrix_set_msr	86
			_matrix_set_dia	86
			_matrix_set_ell	87
			_matrix_set_jad	87
			_matrix_set_bsr	88
			_matrix_set_bsc	88
			_matrix_set_vbr	89
			_matrix_set_coo	90
			_matrix_set_dns	91
			_matrix_unset	91
	6.3	-	g with Vectors and Matrices	92
			_vector_swap	92
			_vector_copy	92
			_vector_axpy	93
		6.3.4 li	_vector_xpay	93
		6.3.5 li	_vector_axpyz	94
		6.3.6 li	_vector_scale	94
		6.3.7 li	_vector_pmul	95
		6.3.8 li	_vector_pdiv	95
		6.3.9 li	_vector_set_all	96
		6.3.10 li	_vector_abs	96
		6.3.11 li	_vector_reciprocal	97
			_vector_conjugate	97
			vector shift	98

	6.3.14	lis_vector_dot	99
	6.3.15	lis_vector_nhdot	99
	6.3.16	lis_vector_nrm1	00
	6.3.17	lis_vector_nrm2	00
		lis_vector_nrmi	
		lis_vector_sum	
		lis_matrix_set_blocksize	
		lis_matrix_convert	
		lis_matrix_copy	
		lis_matrix_axpy	
		lis_matrix_xpay	
		lis_matrix_axpyz	
		lis_matrix_scale	
		lis_matrix_get_diagonal	
		lis_matrix_shift_diagonal	
		lis_matvec	
		lis_matvect	
6.4		g Linear Equations	
		lis_solver_create	
		lis_solver_destroy	
	6.4.3	lis_precon_create	
	6.4.4	lis_precon_destroy	
	6.4.5	lis_solver_set_option	1
	6.4.6	lis_solver_set_optionC	4
	6.4.7	lis_solve	4
	6.4.8	lis_solve_kernel	15
	6.4.9	lis_solver_get_status	6
		lis_solver_get_iter	
		lis_solver_get_iterex	
		lis_solver_get_time	
		lis_solver_get_timeex	
		lis_solver_get_residualnorm	
		lis_solver_get_rhistory	
		lis_solver_get_solver	
		lis_solver_get_precon	
		lis_solver_get_solvername	
c =		lis_solver_get_preconname	
6.0		g Eigenvalue Problems	
	6.5.1	lis_esolver_create	
	6.5.2	lis_esolver_destroy	
	6.5.3	lis_esolver_set_option	
	6.5.4	lis_esolver_set_optionC	
	6.5.5	lis_esolve	
	6.5.6	lis_gesolve	
	6.5.7	lis_esolver_get_status	28
	6.5.8	lis_esolver_get_iter	28
	6.5.9	lis_esolver_get_iterex	29
	6.5.10	lis_esolver_get_time	29
	6.5.11	lis_esolver_get_timeex	30
		lis_esolver_get_residualnorm	30
		lis_esolver_get_rhistory	31
		lis_esolver_get_evalues	
		lis_esolver_get_evectors	
		₩	

	6.5.16	lis_esolver_get_residualnorms	132
	6.5.17	lis_esolver_get_iters	133
	6.5.18	lis_esolver_get_esolver	133
	6.5.19	lis_esolver_get_esolvername	133
6.6		uting with Arrays	
		lis_array_swap	
		lis_array_copy	
		lis_array_axpy	
	6.6.4	lis_array_xpay	
	6.6.5	lis_array_axpyz	
	6.6.6	lis_array_scale	
	6.6.7	lis_array_pmul	
	6.6.8	lis_array_pdiv	
		lis_array_set_all	
		lis_array_abs	
		lis_array_reciprocal	
		lis_array_conjugate	
		lis_array_shift	
		lis_array_dot	
		lis_array_nhdot	
		lis_array_nrm1	
		lis_array_nrm2	
		lis_array_nrmi	
		lis_array_sum	
		lis_array_matvec	
		lis_array_matvect	
		lis_array_matvec_ns	
		lis_array_matmat	
		lis_array_matmat_ns	
		lis_array_ge	
		lis_array_solve	
		lis_array_cgs	
		lis_array_mgs	
		lis_array_qr	
6.7		ting External Files	
٠.,		lis_input	
		lis_input_vector	
		lis_input_matrix	
	6.7.4	lis_output	
	6.7.5	lis_output_vector	
	6.7.6		
6.8		Functions	154
0.0	6.8.1	lis_initialize	154
	6.8.2	lis_finalize	154
	6.8.3	lis_wtime	155
	6.8.4	CHKERR	155
	6.8.5	lis_printf	
	0.0.0	по-риши	190

157

References

Α	File	Formats	162
	A.1	Extended Matrix Market Format	162
	A.2	Harwell-Boeing Format	163
	A.3	Extended Matrix Market Format for Vectors	164
	A.4	PLAIN Format for Vectors	164

Changes from Version 1.0

- 1. Addition: Support for double-double (quadruple) precision operations.
- 2. Addition: Support for Fortran compilers.
- 3. Addition: Support for Autotools.
- 4. Changes:
 - (a) Structure of solvers.
 - (b) Arguments of the functions lis_matrix_create() and lis_vector_create().
 - (c) Notation of command line options.

Changes from Version 1.1

- 1. Addition: Support for standard eigenvalue problems.
- 2. Addition: Support for 64bit integers.
- 3. Changes:
 - (a) Names of the functions lis_output_residual_history() and lis_get_residual_history() to lis_solver_output_rhistory() and lis_solver_get_rhistory(), respectively.
 - (b) Origin of the Fortran interfaces lis_vector_set_value() and lis_vector_get_value() to 1.
 - (c) Origin of the Fortran interface lis_vector_set_size() to 1.
 - (d) Name of the precision flag -precision to -f.
- 4. Change: Specification of the function lis_solve_kernel() to return the residual computed by the function lis_solve_execute().
- 5. Changes: Specifications of integer types:
 - (a) Replacement: The type of integer in C programs with LIS_INT, which is equivalent to int by default. If the preprossor macro _LONGLONG is defined, it is replaced with long long int.
 - (b) Replacement: The type of integer in Fortran programs with LIS_INTEGER, which is equivalent to integer by default. If the preprossor macro LONGLONG is defined, it is replaced with integer*8.
- 6. Change: Names of the matrix storage formats CRS (Compressed Row Storage) and CCS (Compressed Column Storage) to CSR (Compressed Sparse Row) and CSC (Compressed Sparse Column), respectively.
- 7. Change: Names of the functions lis_get_solvername(), lis_get_preconname(), and lis_get_esolvername() to lis_solver_get_solvername(), lis_solver_get_preconname(), and lis_esolver_get_esolvername(), respectively.

Changes from Version 1.2

- 1. Addition: Support for nmake.
- 2. Change: Name of the file lis_config_win32.h to lis_config_win.h.
- 3. Change: Name of the matrix storage format JDS (Jagged Diagonal Storage) to JAD (Jagged Diagonal).
- 4. Change: Names of the functions lis_fscan_double() and lis_bswap_double() to lis_fscan_scalar() and lis_bswap_scalar(), respectively.

Changes from Version 1.3

- 1. Addition: Support for long double (quadruple) precision operations.
- 2. Addition: Support for pointer operations in Fortran.
- 3. Change: Name of the members residual of the structs LIS_SOLVER and LIS_ESOLVER to rhistory.
- 4. Change: Names of the members iters and iters2 of the structs LIS_SOLVER and LIS_ESOLVER to iter and iter2, respectively.
- 5. Change: Names of the functions lis_solver_get_iters(), lis_solver_get_itersex(), lis_esolver_get_iters(), and lis_esolver_get_itersex() to lis_solver_get_iter(), lis_solver_get_iterex(), lis_esolver_get_iter(), and lis_esolver_get_iterex(), respectively.
- 6. Change: Names of the members *times of the structs LIS_SOLVER and LIS_ESOLVER to *time, respectively.
- 7. Addition: Member intvalue to the struct LIS_VECTOR.
- 8. Change: Specifications of the functions lis_output_vector*() and lis_output_mm_vec() to allow integer data.
- 9. Change: Names of the functions lis_matrix_scaling*() to lis_matrix_scale*(), respectively.
- 10. Change: Names of the functions lis_array_dot2() and lis_array_invGauss() to lis_array_dot() and lis_array_ge(), respectively.

Changes from Version 1.4

- 1. Addition: Support for array operations.
- 2. Addition: Support for alternative workflow PSD (Preconditioner and Solver Decoupled).
- 3. Change: Specification of the function <code>lis_array_qr()</code> to return the number of iterations and error of the QR algorithm.
- 4. Change: Names of the functions lis_array_matvec2() and lis_array_matmat2() to lis_array_matvec_ns() and lis_array_matmat_ns(), respectively.
- 5. Change: Names of the preprossor macros _LONGLONG and LONGLONG to _LONG__LONG and LONG__LONG, respectively.

Changes from Version 1.5

1. Addition: Support for complex arithmetic.

Changes from Version 1.6

- 1. Addition: Support for generalized eigenvalue problems.
- 2. Addition: Support for GCC libquadmath.
- 3. Change: Sign of shift for eigensolvers in accordance with custom.
- 4. Change: Sign of shift in lis_matrix_shift_diagonal(), lis_vector_shift(), and lis_array_shift(), respectively.

1 Introduction

Lis (Library of Iterative Solvers for linear systems, pronounced [lis]) is a parallel software library for solving the linear equations

$$Ax = b$$

and the eigenvalue problems

$$Ax = \lambda Bx$$

with sparse matrices, which arise in the numerical solution of partial differential equations, using iterative methods[1]. The solvers available in Lis are listed in Table 1 and 2, and the preconditioners are listed in Table 3. The supported matrix storage formats are listed in Table 4.

Table 1: Linear Solvers

Table 1: Line	ear sorvers
CG[2, 3]	CR[2]
BiCG[4]	BiCR[5]
CGS[6]	CRS[7]
BiCGSTAB[8]	BiCRSTAB[7]
GPBiCG[9]	GPBiCR[7]
BiCGSafe[10]	BiCRSafe[11]
BiCGSTAB(1)[12]	TFQMR[13]
Jacobi[14]	Orthomin(m)[15]
Gauss-Seidel[16, 17]	GMRES(m)[18]
SOR[19, 20]	FGMRES(m)[21]
IDR(s)[22]	MINRES[23]
COCG[24]	COCR[25]

Table 2: Eigensolvers

Power[26]
Inverse[27]
Rayleigh Quotient[28]
CG[29]
CR[30]
Jacobi-Davidson[31]
Subspace[32]
Lanczos[33]
Arnoldi[34]

Table 3: Preconditioners

Table 4: Matrix Storage Formats

Table 9: 1 reconditioners	Table 1. Matrix Storage Formats			
Jacobi[35]	Compressed Sparse Row	(CSR)		
SSOR[35]	Compressed Sparse Column	(CSC)		
ILU(k)[36, 37]	Modified Compressed Sparse Row	(MSR)		
ILUT[38, 39]	Diagonal	(DIA)		
Crout ILU[39, 40]	Ellpack-Itpack Generalized Diagonal	(ELL)		
I+S[41]	Jagged Diagonal	(JAD)		
SA-AMG[42]	Block Sparse Row	(BSR)		
Hybrid[43]	Block Sparse Column	(BSC)		
SAINV[44]	Variable Block Row	(VBR)		
Additive Schwarz[45, 46]	Coordinate	(COO)		
User defined	Dense	(DNS)		

2 Installation

This section describes the instructions for installing and testing Lis.

2.1 System Requirements

The installation of Lis requires a C compiler. The Fortran interface requires a Fortran compiler, and the algebraic multigrid preconditioner requires a Fortran 90 compiler. For parallel computing environments, an OpenMP[88] or MPI-1[82] library is used[47, 48]. Both the Harwell-Boeing[74] and Matrix Market[78] formats are supported to import and export user data. Lis has been tested in the environments listed in Table 5 (see also Table 7).

C Compilers OS Intel C/C++ Compiler 7.0, 8.0, 9.1, 10.1, 11.1, 12.1, 14.0, 16.0 Linux Windows IBM XL C/C++ V7.0, 9.0 AIX Linux Sun WorkShop 6, Sun ONE Studio 7, Solaris Sun Studio 11, 12 PGI C++ 6.0, 7.1, 10.5, 16.10 Linux gcc 3.3, 4.4, 5.4 Linux Mac OS X Windows Microsoft Visual C++ 2008, 2010, 2012, 2013, 2015, 2017 Windows OS Fortran Compilers (Optional) Intel Fortran Compiler 8.1, 9.1, 10.1, 11.1, 12.1, 14.0, 16.0 Linux Windows IBM XL Fortran V9.1, 11.1 AIX Linux Sun WorkShop 6, Sun ONE Studio 7, Solaris Sun Studio 11, 12 PGI Fortran 6.0, 7.1, 10.5, 16.10 Linux g77 3.3 Linux gfortran 4.4, 5.4 Mac OS X Windows

Table 5: Major Tested Platforms

2.2 Installing on UNIX and Compatible Systems

2.2.1 Extracting Archive

Enter the following command to extract the archive, where (\$VERSION) represents the version: > gunzip -c lis-(\$VERSION).tar.gz | tar xvf -

This creates a directory lis-(\$VERSION) together with its subfolders as shown in Figure 1.

2.2.2 Configuring Source Tree

In the directory lis-(\$VERSION), run the following command to configure the source tree:

- default: > ./configure
- specify the installation destination: > ./configure --prefix=<install-dir>

Figure 1: Files contained in lis-(\$VERSION).tar.gz

Table 6 shows the major options that can be specified for the configuration, and Table 7 shows the major computing environments that can be specified by TARGET.

2.2.3 Compiling

In the directory lis-(\$VERSION), run the following command to generate the executable files:

> make

To ensure that the library has been built successfully, enter

> make check

in lis-(\$VERSION). This runs a test script using the executable files created in lis-(\$VERSION)/test, which reads the data of the coefficient matrix and the right-hand side vector from the file test/testmat.mtx and solve the linear equation Ax = b by the BiCG method. The result on the SGI Altix 3700 is shown below. Options --enable-omp and --enable-mpi can be combined.

```
– Default –
matrix size = 100 x 100 (460 nonzero entries)
initial vector x
precision
                     : double
linear solver
                     : BiCG
preconditioner
                    : none
convergence condition : ||b-Ax||_2 \le 1.0e-12 * ||b-Ax_0||_2
matrix storage format : CSR
linear solver status : normal end
BiCG: number of iterations = 15 (double = 15, quad = 0)
BiCG: elapsed time = 5.178690e-03 sec.
BiCG:
        preconditioner
                        = 1.277685e-03 sec.
BiCG:
        matrix creation = 1.254797e-03 sec.
BiCG: linear solver = 3.901005e-03 sec.
BiCG: relative residual = 6.327297e-15
```

Table 6: Major Configuration Options (see ./configure --help for the complete list)

enable-omp	Build with OpenMP library
enable-mpi	Build with MPI library
enable-fortran	Enable FORTRAN 77 compatible interface
enable-f90	Enable Fortran 90 compatible interface
enable-saamg	Enable SA-AMG preconditioner
enable-quad	Enable double-double (quadruple) precision support
enable-longdouble	Enable long double (quadruple) precision support
enable-longlong	Enable 64bit integer support
enable-complex	Enable complex scalar support
enable-debug	Enable debugging
enable-shared	Enable dynamic linking
enable-gprof	Enable profiling
prefix= <install-dir></install-dir>	Specify installation destination
TARGET= <target></target>	Specify computing environment
CC= <c_compiler></c_compiler>	Specify C compiler
CFLAGS= <c_flags></c_flags>	Specify options for C compiler
F77= <f77_compiler></f77_compiler>	Specify FORTRAN 77 compiler
F77FLAGS= <f77_flags></f77_flags>	Specify options for FORTRAN 77 compiler
FC= <f90_compiler></f90_compiler>	Specify Fortran 90 compiler
FCFLAGS= <f90_flags></f90_flags>	Specify options for Fortran 90 compiler
LDFLAGS= <ld_flags></ld_flags>	Specify link options

Table 7: Examples of Targets (see lis-(\$VERSION)/configure.ac for details)

<target></target>	Equivalent options
cray_xt3_cross	./configure CC=cc FC=ftn CFLAGS="-03 -B -fastsse -tp k8-64"
	FCFLAGS="-03 -fastsse -tp k8-64 -Mpreprocess" FCLDFLAGS="-Mnomain"
	ac_cv_sizeof_void_p=8 cross_compiling=yes
	ax_f77_mangling="lower case, no underscore, extra underscore"
fujitsu_fx10_cross	./configure CC=fccpx FC=frtpx CFLAGS="-Kfast,ocl,preex"
	FCFLAGS="-Kfast,ocl,preex -Cpp -fs" FCLDFLAGS="-mlcmain=main"
	ac_cv_sizeof_void_p=8 cross_compiling=yes
	ax_f77_mangling="lower case, underscore, no extra underscore"
hitachi_sr16k	./configure CC=cc FC=f90 CFLAGS="-Os -noparallel"
	FCFLAGS="-Oss -noparallel" FCLDFLAGS="-1f90s"
	ac_cv_sizeof_void_p=8
	ax_f77_mangling="lower case, underscore, no extra underscore"
ibm_bgl_cross	./configure CC=blrts_xlc FC=blrts_xlf90
	CFLAGS="-03 -qarch=440d -qtune=440 -qstrict"
	FCFLAGS="-03 -qarch=440d -qtune=440 -qsuffix=cpp=F90"
	ac_cv_sizeof_void_p=4 cross_compiling=yes
	ax_f77_mangling="lower case, no underscore, no extra underscore"
nec_sx9_cross	./configure CC=sxmpic++ FC=sxmpif90 AR=sxar RANLIB=true
	ac_cv_sizeof_void_p=8 ax_vector_machine=yes cross_compiling=yes
	ax_f77_mangling="lower case, no underscore, extra underscore"

```
--enable-omp
max number of threads = 32
number of threads = 2
matrix size = 100 x 100 (460 nonzero entries)
 initial vector x
                       : 0
precision
                          : double
linear solver : BiCG preconditioner : none
convergence condition : ||b-Ax||_2 \le 1.0e-12 * ||b-Ax_0||_2
\verb|matrix| \verb| storage| \verb| format| : CSR
 linear solver status : normal end
BiCG: number of iterations = 15 (double = 15, quad = 0)
BiCG: elapsed time = 8.960009e-03 sec.
BiCG: preconditioner = 2.297878e-03 sec.
         matrix creation = 2.072096e-03 sec.
 BiCG:
BiCG: linear solver = 6.662130e-03 sec.
BiCG: relative residual = 6.221213e-15
```

```
— --enable-mpi —
number of processes = 2
matrix size = 100 x 100 (460 nonzero entries)
                     : 0
initial vector x
precision
                     : double
linear solver
                     : BiCG
preconditioner : none
convergence condition : ||b-Ax||_2 \le 1.0e-12 * ||b-Ax_0||_2
matrix storage format : CSR
linear solver status : normal end
BiCG: number of iterations = 15 (double = 15, quad = 0)
BiCG: elapsed time = 2.911400e-03 sec.
BiCG: preconditioner = 1.560780e-04 sec.
BiCG:
        matrix creation = 1.459997e-04 sec.
BiCG: linear solver = 2.755322e-03 sec.
BiCG: relative residual = 6.221213e-15
```

2.2.4 Installing

```
| +liblis.a
+share
```

+doc/lis examples/lis man

lis_config.h is the header file required to build the library, and lis.h and lisf.h are the header files required by the C and Fortran compilers, respectively. liblis.a is the library. To ensure that the library has been installed successfully, enter

> make installcheck

in lis-(\$VERSION). This runs a test script using the executable files installed in examples/lis. test1, etest5, getest5, test3b, and spmvtest3b in examples/lis are copied in (\$INSTALLDIR)/bin as lsolve, esolve, gesolve, hpcg_kernel, and hpcg_spmvtest, respectively. examples/lis/spmvtest* are also copied in

(\$INSTALLDIR)/bin.

To remove the copied files in (\$INSTALLDIR), enter

> make uninstall

To remove the generated library and executable files in lis-(\$VERSION), enter

> make clean

To remove the configuration files in addition to the other generated files, enter

> make distclean

2.3 Installing on Windows Systems

Use an appropriate tool to extract the archive. To use the Microsoft Build Engine, run the following command in the directory lis-(\$VERSION)\win and generate the configuration file Makefile (See configure.bat --help for details):

> configure.bat

The default configuration of Makefile is defined in Makefile.in. To build the library, run the following command in the directory lis-(\$VERSION)\win:

> nmake

To ensure that the library has been built successfully, enter

> nmake check

The following command copies the library to (\$INSTALLDIR)\lin, the executable files to (\$INSTALLDIR)\bin, the header files to (\$INSTALLDIR)\include, and the PDF documents to (\$INSTALLDIR)\doc, respectively:

> nmake install

To remove the copied files in (\$INSTALLDIR), enter

> nmake uninstall

To remove the generated library and executables files in lis-(\$VERSION)\win, enter

> nmake clean

To remove the configuration files in addition to the other generated files, enter

> nmake distclean

To use UNIX compatible environments, follow the instructions in the previous section.

2.4 Testing

Test programs are located in lis-(\$VERSION)/test.

2.4.1 test1

Usage: test1 matrix_filename rhs_setting solution_filename rhistory_filename [options]

This program inputs the data of the coefficient matrix from matrix_filename and solves the linear equation Ax = b with the solver specified by options. It outputs the solution to solution_filename in the extended Matrix Market format and the residual history to rhistory_filename in the PLAIN

format (see Appendix). Both the extended Matrix Market format and the Harwell-Boeing format are supported for matrix_filename. One of the following values can be specified by rhs_setting:

0 Use the right-hand side vector b included in the data file

1 Use $b = (1, ..., 1)^T$

2 Use $b = A \times (1, \dots, 1)^T$

rhs_filename The filename for the right-hand side vector

The PLAIN and Matrix Market formats are supported for rhs_filename. test1f.F is the Fortran version of test1.c.

2.4.2 test2

Usage: test2 m n matrix_type solution_filename rhistory_filename [options]

This program solves the linear equation Ax = b, where the coefficient matrix A of size mn is a discretized two dimensional Laplacian using the five point central difference scheme, with the coefficient matrix in the storage format specified by matrix_type and the solver specified by options. It outputs the solution to solution_filename in the extended Matrix Market format and the residual history to rhistory_filename in the PLAIN format. The right-hand side vector b is set such that the values of the elements of the solution x are 1. The values m and n represent the numbers of grid points in each dimension. test2f.F90 is the Fortran 90 version of test2.c.

2.4.3 test2b

Usage: test2b m n matrix_type solution_filename rhistory_filename [options]

This program solves the linear equation Ax = b, where the coefficient matrix A of size mn is a discretized two dimensional Laplacian using the nine point central difference scheme, with the coefficient matrix in the storage format specified by matrix_type and the solver specified by options. It outputs the solution to solution_filename in the extended Matrix Market format and the residual history to rhistory_filename in the PLAIN format. The right-hand side vector b is set such that the values of the elements of the solution x are 1. The values m and n represent the numbers of grid points in each dimension.

2.4.4 test3

Usage: test3 1 m n matrix_type solution_filename rhistory_filename [options]

This program solves the linear equation Ax = b, where the coefficient matrix A of size lmn is a discretized three dimensional Laplacian using the seven point central difference scheme, with the coefficient matrix in the storage format specified by matrix_type and the solver specified by options. It outputs the solution to solution_filename in the extended Matrix Market format and the residual history to rhistory_filename in the PLAIN format. The right-hand side vector b is set such that the values of the elements of the solution x are 1. The values 1, m and n represent the numbers of grid points in each dimension.

2.4.5 test3b

Usage: test3b 1 m n matrix_type solution_filename rhistory_filename [options]

This program solves the linear equation Ax = b, where the coefficient matrix A of size lmn is a discretized three dimensional Laplacian using the twenty-seven point central difference scheme, with the

coefficient matrix in the storage format specified by $\mathtt{matrix_type}$ and the solver specified by $\mathtt{options}$. It outputs the solution to $\mathtt{solution_filename}$ in the extended Matrix Market format and the residual history to $\mathtt{rhistory_filename}$ in the PLAIN format. The right-hand side vector b is set such that the values of the elements of the solution x are 1. The values 1, m and m represent the numbers of grid points in each dimension.

2.4.6 test3c

Usage: test3c 1 m n step [options]

This program solves the linear equation Ax = b for step steps, where the coefficient matrix A of size lmn is a discretized three dimensional Laplacian using the seven point central difference scheme, with the coefficient matrix in the CSR format and the solver specified by options. The right-hand side vector b is set such that the values of the elements of the solution x are 1. The values 1, m and m represent the numbers of grid points in each dimension. This program sets matrix values using the function lis_matrix_set_csr, and measures the running time of the functions. See also test4b.

2.4.7 test4

This program solves the linear equation Ax = b with a specified solver and a preconditioner, where A is a tridiagonal matrix

$$\begin{pmatrix}
2 & -1 & & & & \\
-1 & 2 & -1 & & & & \\
& \ddots & \ddots & \ddots & & \\
& & -1 & 2 & -1 & \\
& & & -1 & 2
\end{pmatrix}$$

of size 12. The right-hand side vector b is set such that the values of the elements of the solution x are 1. test4f.F is the Fortran version of test4.c.

2.4.8 test4b

Usage: test4b 1 m n step [options]

This program solves the linear equation Ax = b for step steps, where the coefficient matrix A of size lmn is a discretized three dimensional Laplacian using the seven point central difference scheme, with the coefficient matrix in the CSR format and the solver specified by options. The right-hand side vector b is set such that the values of the elements of the solution x are 1. The values 1, m and m represent the numbers of grid points in each dimension. This program sets matrix values using the function lis_matrix_set_value, and measures the running time of the functions. See also test3c.

2.4.9 test5

Usage: test5 n gamma [options]

This program solves a linear equation Ax = b, where A is a Toeplitz matrix

$$\begin{pmatrix} 2 & 1 & & & & \\ 0 & 2 & 1 & & & \\ \gamma & 0 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \ddots & \\ & & \gamma & 0 & 2 & 1 \\ & & & \gamma & 0 & 2 \end{pmatrix}$$

of size n, with the solver specified by options. Note that the right-hand vector is set such that the values of the elements of the solution x are 1.

2.4.10 test6

Usage: test6 m n

test6.c is the array version of test2.c. This program solves the linear equation Ax = b using the direct method, where the coefficient matrix A of size mn is a discretized two dimensional Laplacian using the five point central difference scheme. The right-hand side vector b is set such that the values of the elements of the solution x are 1. The values m and n represent the numbers of grid points in each dimension. test6f.F90 is the Fortran 90 version of test6.c.

2.4.11 test7

Usage: test7

This program show examples of complex arithmetic. test7f.F is the Fortran version of test7.c.

2.4.12 etest1

Usage: etest1 matrix_filename evector_filename rhistory_filename [options]

This program inputs the matrix data from matrix_filename and solves the standard eigenvalue problem $Ax = \lambda x$ with the solver specified by options. It outputs the specified eigenvalue to the standard output, the associated eigenvector to evector_filename in the extended Matrix Market format, and the residual history to rhistory_filename in the PLAIN format. Both the Matrix Market format and the Harwell-Boeing format are supported for the matrix filename. etest1f.F is the Fortran version of etest1.c. See etest5 for obtaining multiple eigenpairs.

2.4.13 getest1

Usage: getest1 matrix_a_filename matrix_b_filename evector_filename rhistory_filename
[options]

This program inputs the matrix data from matrix_a_filename and matrix_b_filename, and solves the generalized eigenvalue problem $Ax = \lambda Bx$ with the solver specified by options. It outputs the specified eigenvalue to the standard output, the associated eigenvector to evector_filename in the extended Matrix Market format, and the residual history to rhistory_filename in the PLAIN format. Both the Matrix Market format and the Harwell-Boeing format are supported for the matrix filenames. See getest5 for obtaining multiple eigenpairs.

2.4.14 etest2

Usage: etest2 m n matrix_type evector_filename rhistory_filename [options]

This program solves the eigenvalue problem $Ax = \lambda x$, where the coefficient matrix A of size mn is a discretized two dimensional Laplacian using the five point central difference scheme, with the coefficient matrix in the storage format specified by matrix_type and the solver specified by options. It outputs the specified eigenvalue to the standard output, the associated eigenvector to evector_filename in the extended Matrix Market format, and the residual history to rhistory_filename in the PLAIN format. The values m and n represent the numbers of grid points in each dimension.

2.4.15 etest3

Usage: etest3 1 m n matrix_type evector_filename rhistory_filename [options]

This program solves the eigenvalue problem $Ax = \lambda x$, where the coefficient matrix A of size lmn is a discretized three dimensional Laplacian using the seven point central difference scheme, with the coefficient matrix in the storage format specified by matrix_type and the solver specified by options. It outputs the specified eigenvalue to the standard output, the associated eigenvector to evector_filename in the extended Matrix Market format, and the residual history to rhistory_filename in the PLAIN format. The values 1, m and n represent the numbers of grid points in each dimension. See etest6 for obtaining multiple eigenpairs.

2.4.16 etest4

Usage: etest4 n [options]

This program solves the eigenvalue problem $Ax = \lambda x$ with a specified solver, where A is a tridiagonal matrix

$$A = \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{pmatrix}$$

of size $n \times n$. etest4f.F is the Fortran version of etest4.c.

2.4.17 etest5

Usage: etest5 matrix_filename evalues_filename evectors_filename residuals_filename iters_filename [options]

This program inputs the matrix data from matrix_filename and solves the standard eigenvalue problem $Ax = \lambda x$ with the solver specified by options. It outputs the eigenvalues specified by options to evalues_filename and the associated eigenvectors, residual norms, and numbers of iterations to evectors_filename, residuals_filename, and iters_filename respectively in the extended Matrix Market format. Both the Matrix Market format and the Harwell-Boeing format are supported for the matrix filename.

2.4.18 getest5

Usage: getest5 matrix_a_filename matrix_b_filename evalues_filename evectors_filename residuals_filename iters_filename [options]

This program inputs the matrix data from matrix_a_filename and matrix_b_filename, and solves the generalized eigenvalue problem $Ax = \lambda Bx$ with the solver specified by options. It outputs the eigenvalues specified by options to evalues_filename and the associated eigenvectors, residual norms, and numbers of iterations to

evectors_filename, residuals_filename, and iters_filename respectively in the extended Matrix Market format. Both the Matrix Market format and the Harwell-Boeing format are supported for the matrix filenames.

2.4.19 etest6

Usage: etest6 l m n matrix_type evalues_filename evectors_filename residuals_filename
 iters_filename [options]

This program solves the eigenvalue problem $Ax = \lambda x$, where the coefficient matrix A of size lmn is a discretized three dimensional Laplacian using the seven point central difference scheme, with the coefficient matrix in the storage format specified by matrix_type and the solver specified by options. It outputs the eigenvalues specified by options to evalues_filename and the associated eigenvectors and residual norms to evectors_filename, residuals_filename, and iters_filename respectively in the extended Matrix Market format. The values 1, m and n represent the numbers of grid points in each dimension.

2.4.20 etest7

Usage: etest7 m n

etest7.c is the array version of etest2.c. This program solves the eigenvalue problem $Ax = \lambda x$ using the QR algorithm, where the coefficient matrix A of size mn is a discretized two dimensional Laplacian using the five point central difference scheme. The values m and n represent the numbers of grid points in each dimension.

2.4.21 spmvtest1

Usage: spmvtest1 n iter [matrix_type]

This program computes the matrix-vector multiply of a discretized one dimensional Laplacian

$$\begin{pmatrix}
2 & -1 & & & & \\
-1 & 2 & -1 & & & & \\
& \ddots & \ddots & \ddots & & \\
& & -1 & 2 & -1 & \\
& & & -1 & 2
\end{pmatrix}$$

of size n using the three point central difference scheme and a vector $(1, ..., 1)^T$. The FLOPS performance is measured as the average of iter iterations. If necessary, one of the following values can be specified by matrix_type:

0 Measure the performance for the available matrix storage formats

1-11 The number of the matrix storage format

2.4.22 spmvtest2

Usage: spmvtest2 m n iter [matrix_type]

This program computes the matrix-vector multiply of a discretized two dimensional Laplacian of size mn using the five point central difference scheme and a vector $(1, ..., 1)^T$. The FLOPS performance is measured as the average of iter iterations. If necessary, one of the following values can be specified by matrix_type:

0 Measure the performance for the available matrix storage formats

1-11 The number of the matrix storage format

The values m and n represent the numbers of grid points in each dimension.

2.4.23 spmvtest2b

Usage: spmvtest2b m n iter [matrix_type]

This program computes the matrix-vector multiply of a discretized two dimensional Laplacian of size mn using the nine point central difference scheme and a vector $(1, ..., 1)^T$. The FLOPS performance is measured as the average of iter iterations. If necessary, one of the following values can be specified by matrix_type:

0 Measure the performance for the available matrix storage formats

1-11 The number of the matrix storage format

The values m and n represent the numbers of grid points in each dimension.

2.4.24 spmvtest3

Usage: spmvtest3 1 m n iter [matrix_type]

This program computes the matrix-vector multiply of a discretized three dimensional Laplacian of size lmn using the seven point central difference scheme and a vector $(1, ..., 1)^T$. The values 1, m and n represent the numbers of grid points in each dimension. The FLOPS performance is measured as the average of iter iterations. If necessary, one of the following values can be specified by matrix_type:

0 Measure the performance for the available matrix storage formats

1-11 The number of the matrix storage format

2.4.25 spmvtest3b

Usage: spmvtest3b 1 m n iter [matrix_type]

This program computes the matrix-vector multiply of a discretized three dimensional Laplacian of size lmn using the twenty-seven point central difference scheme and a vector $(1, ..., 1)^T$. The values 1, m and n represent the numbers of grid points in each dimension. The FLOPS performance is measured as the average of iter iterations. If necessary, one of the following values can be specified by matrix_type:

0 Measure the performance for the available matrix storage formats

1-11 The number of the matrix storage format

2.4.26 spmvtest4

Usage: spmvtest4 matrix_filename_list iter [block]

This program inputs the matrix data from the files listed in $\mathtt{matrix_filename_list}$, and computes the multiplies of matrices in available matrix storage formats and a vector $(1,\ldots,1)^T$. The FLOPS performance is measured as the average of iter iterations. Both the Matrix Market format and the Harwell-Boeing format are supported for the matrix filenames. If necessary, the block size of the BSR and BSC formats can be specified by block.

2.4.27 spmvtest5

Usage: spmvtest5 matrix_filename matrix_type iter [block]

This program inputs the matrix data from $\mathtt{matrix_filename}$ and compute the multiply of the matrix with $\mathtt{matrix_type}$ and a vector $(1,\ldots,1)^T$. The FLOPS performance is measured as the average of iter iterations. Both the Matrix Market format and the Harwell-Boeing format are supported for $\mathtt{matrix_filename}$. If necessary, the block size of the BSR and BSC formats can be specified by block.

2.5 Limitations

The current version has the following limitations:

- Matrix storage formats
 - The VBR format does not support the multiprocessing environment.
 - The SA-AMG preconditioner supports only the CSR format.
 - In the multiprocessing environment, the CSR is the only accepted format for user defined arrays.
- Double-double (quadruple) precision operations (see Section 4)
 - The Jacobi, Gauss-Seidel, SOR, and IDR(s) methods do not support the double-double precision operations.
 - The CG, CR, and Jacobi-Davidson methods for the eigenvalue problems do not support the double-double precision operations.
 - The Jacobi, Gauss-Seidel and SOR methods in the hybrid preconditioner do not support the double-double precision operations.
 - The I+S and SA-AMG preconditioners do not support the double-double precision operations.

• Preconditioners

- If a preconditioner other than the Jacobi or SSOR is selected and matrix A is not in the CSR format, a new matrix is created in the CSR format for preconditioning.
- The SA-AMG preconditioner does not support the BiCG method for unsymmetric matrices.
- The SA-AMG preconditioner does not support multithreading.
- The assembly of the matrices in the SAINV preconditioner is not parallelized.
- The user defined preconditioner cannot be used.

3 Basic Operations

This section describes how to use the library. A program requires the following statements:

- Initialization
- Matrix creation
- Vector creation
- Solver creation
- Value assignment for matrices and vectors
- Solver assignment
- Solver execution
- Finalization

In addition, it must include one of the following compiler directives:

```
• C #include "lis.h"
```

• Fortran #include "lisf.h"

When Lis is installed in (\$INSTALLDIR), lis.h and lisf.h are located in (\$INSTALLDIR)/include.

3.1 Initializing and Finalizing

The functions for initializing and finalizing the execution environment must be called at the top and bottom of the program, respectively, as follows:

```
1: #include "lis.h"
2: LIS_INT main(LIS_INT argc, char* argv[])
3: {
4: lis_initialize(&argc, &argv);
5: ...
6: lis_finalize();
7: }
```

```
Fortran

1: #include "lisf.h"

2: call lis_initialize(ierr)

3: ...

4: call lis_finalize(ierr)
```

Initializing

For initializing, the following functions are used:

- C LIS_INT lis_initialize(LIS_INT* argc, char** argv[])
- Fortran subroutine lis_initialize(LIS_INTEGER ierr)

This function initializes the MPI execution environment, and specifies the options on the command line. The default type of the integer in the C programs is LIS_INT, which is equivalent to int. If the preprossor macro _LONG__LONG is defined, it is replaced with long long int. The default type of the integer in the Fortran programs is LIS_INTEGER, which is equivalent to integer. If the preprocessor

macro LONG_LONG is defined, it is replaced with integer*8.

Finalizing

For finalizing, the following functions are used:

- C LIS_INT lis_finalize()
- Fortran subroutine lis_finalize(LIS_INTEGER ierr)

3.2 Operating Vectors

Assume that the size of vector v is $global_n$, and the size of each partial vector stored on nprocs processing elements is $local_n$. If $global_n$ is divisible, then $local_n$ is equal to $global_n / nprocs$. For example, when vector v is stored on two processing elements, as shown in Equation (3.1), $global_n$ and $local_n$ are 4 and 2, respectively.

$$v = \begin{pmatrix} 0 \\ \frac{1}{2} \\ 3 \end{pmatrix} \text{ PE0}$$
PE1 (3.1)

In the case of creating vector v in Equation (3.1), vector v itself is created for the serial and multithreaded environments, while the partial vectors are created and stored on a given number of processing elements for the multiprocessing environment.

Programs to create vector v are as follows, where the number of processing elements for the multiprocessing environment is assumed to be two:

```
C (for serial and multithreaded environments)

1: LIS_INT i,n;
2: LIS_VECTOR v;
3: n = 4;
4: lis_vector_create(0,&v);
5: lis_vector_set_size(v,0,n); /* or lis_vector_set_size(v,n,0); */
6:
7: for(i=0;i<n;i++)
8: {
9: lis_vector_set_value(LIS_INS_VALUE,i,(double)i,v);
10: }
```

```
Fortran (for serial and multithreaded environments)

1: LIS_INTEGER i,n

2: LIS_VECTOR v

3: n = 4

4: call lis_vector_create(0,v,ierr)

5: call lis_vector_set_size(v,0,n,ierr)

6:

7: do i=1,n

9: call lis_vector_set_value(LIS_INS_VALUE,i,DBLE(i),v,ierr)

10: enddo
```

```
Fortran (for multiprocessing environment)

1: LIS_INTEGER i,n,is,ie

2: LIS_VECTOR v

3: n = 4

4: call lis_vector_create(MPI_COMM_WORLD,v,ierr)

5: call lis_vector_set_size(v,0,n,ierr)

6: call lis_vector_get_range(v,is,ie,ierr)

7: do i=is,ie-1

8: call lis_vector_set_value(LIS_INS_VALUE,i,DBLE(i),v,ierr);

9: enddo
```

Creating Vectors

To create vector v, the following functions are used:

- C LIS_INT lis_vector_create(LIS_Comm comm, LIS_VECTOR *v)
- Fortran subroutine lis_vector_create(LIS_Comm comm, LIS_VECTOR v, LIS_INTEGER ierr)

For the example program above, comm must be replaced with the MPI communicator. For the serial and multithreaded environments, the value of comm is ignored.

Assigning Sizes

To assign a size to vector v, the following functions are used:

- C LIS_INT lis_vector_set_size(LIS_VECTOR v, LIS_INT local_n, LIS_INT global_n)
- Fortran subroutine lis_vector_set_size(LIS_VECTOR v, LIS_INTEGER local_n, LIS_INTEGER global_n, LIS_INTEGER ierr)

Either $local_n$ or $global_n$ must be provided.

For the serial and multithreaded environments, $local_n$ is equal to $global_n$. Therefore, both $lis_vector_set_size(v,n,0)$ and $lis_vector_set_size(v,0,n)$ create a vector of size n.

For the multiprocessing environment, $lis_{vector_set_size(v,n,0)}$ creates a partial vector of size n on each processing element. On the other hand, $lis_{vector_set_size(v,0,n)}$ creates a partial vector of size m_p on processing element p. The values of m_p are determined by the library.

Assigning Values

To assign a value to the i-th element of vector v, the following functions are used:

- C LIS_INT lis_vector_set_value(LIS_INT flag, LIS_INT i, LIS_SCALAR value, LIS_VECTOR v)
- Fortran subroutine lis_vector_set_value(LIS_INTEGER flag, LIS_INTEGER i, LIS_SCALAR value, LIS_VECTOR v, LIS_INTEGER ierr)

For the multiprocessing environment, the i-th row of the global vector must be specified. Either

 $LIS_{INS_{VALUE}} : v[i] = value, or$

 $\texttt{LIS_ADD_VALUE} \, : \, v[i] = v[i] + value$

must be provided for flag.

Duplicating Vectors

To create a vector that has the same information as the existing vector, the following functions are used:

- C LIS_INT lis_vector_duplicate(LIS_VECTOR vin, LIS_VECTOR *vout)
- Fortran subroutine lis_vector_duplicate(LIS_VECTOR vin, LIS_VECTOR vout, LIS_INTEGER ierr)

This function does not copy the values of the vector. To copy the values as well, the following functions must be called after the above functions:

- C LIS_INT lis_vector_copy(LIS_VECTOR vsrc, LIS_VECTOR vdst)
- Fortran subroutine lis_vector_copy(LIS_VECTOR vsrc, LIS_VECTOR vdst, LIS_INTEGER ierr)

Destroying Vectors

To destroy the vector, the following functions are used:

- C LIS_INT lis_vector_destroy(LIS_VECTOR v)
- Fortran subroutine lis_vector_destroy(LIS_VECTOR v, LIS_INTEGER ierr)

3.3 Operating Matrices

Assume that the size of matrix A is $global_n \times global_n$, and that the size of each row block of matrix A stored on nprocs processing elements is $local_n \times global_n$. If $global_n$ is divisible, then $local_n$ is equal to $global_n / nprocs$. For example, when the row block of matrix A is stored on two processing elements, as shown in Equation (3.2), $global_n$ and $local_n$ are 4 and 2, respectively.

$$A = \begin{pmatrix} 2 & 1 & & \\ 1 & 2 & 1 & & \\ \hline & 1 & 2 & 1 & \\ & & 1 & 2 & \end{pmatrix} \text{ PE0}$$
(3.2)

A matrix in a specific storage format can be created in one of the following three ways:

Method 1: Define Arrays in a Specific Storage Format with Library Functions

For creating matrix A in Equation (3.2) in the CSR format, matrix A itself is created for the serial and multithreaded environments, while partial matrices are created and stored on the given number of processing elements for the multiprocessing environment.

Programs to create matrix A in the CSR format are as follows, where the number of processing elements for the multiprocessing environment is assumed to be two:

```
— C (for serial and multithreaded environments) —
 1: LIS_INT i,n;
2: LIS_MATRIX A;
3: n = 4;
 4: lis_matrix_create(0,&A);
                                            /* or lis_matrix_set_size(A,n,0); */
5: lis_matrix_set_size(A,0,n);
 6: for(i=0;i<n;i++) {
        if( i>0 ) lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0,A);
7:
        if( i<n-1 ) lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0,A);</pre>
8:
9:
        lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0,A);
10: }
11: lis_matrix_set_type(A,LIS_MATRIX_CSR);
12: lis_matrix_assemble(A);
```

```
- C (for multiprocessing environment) -
1: LIS_INT i,n,gn,is,ie;
2: LIS_MATRIX A;
3: gn = 4;
                                              /* or n=2 */
4: lis_matrix_create(MPI_COMM_WORLD,&A);
5: lis_matrix_set_size(A,0,gn);
                                              /* lis_matrix_set_size(A,n,0); */
6: lis_matrix_get_size(A,&n,&gn);
7: lis_matrix_get_range(A,&is,&ie);
8: for(i=is;i<ie;i++) {
9:
        if( i>0 ) lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0,A);
        if( i<gn-1 ) lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0,A);</pre>
10:
        lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0,A);
11:
12: }
13: lis_matrix_set_type(A,LIS_MATRIX_CSR);
14: lis_matrix_assemble(A);
```

```
    Fortran (for multiprocessing environment) -

 1: LIS_INTEGER i,n,gn,is,ie
2: LIS_MATRIX A
3: gn = 4
4: call lis_matrix_create(MPI_COMM_WORLD,A,ierr)
5: call lis_matrix_set_size(A,0,gn,ierr)
6: call lis_matrix_get_size(A,n,gn,ierr)
7: call lis_matrix_get_range(A,is,ie,ierr)
8: do i=is,ie-1
        if( i>1 ) call lis_matrix_set_value(LIS_INS_VALUE,i,i-1,1.0d0,A,ierr)
10:
        if( i<gn ) call lis_matrix_set_value(LIS_INS_VALUE,i,i+1,1.0d0,A,ierr)</pre>
        call lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0d0,A,ierr)
11:
12: enddo
13: call lis_matrix_set_type(A,LIS_MATRIX_CSR,ierr)
14: call lis_matrix_assemble(A,ierr)
```

Creating Matrices

To create matrix A, the following functions are used:

- C LIS_INT lis_matrix_create(LIS_Comm comm, LIS_MATRIX *A)
- Fortran subroutine lis_matrix_create(LIS_Comm comm, LIS_MATRIX A, LIS_INTEGER ierr)

comm must be replaced with the MPI communicator. For the serial and multithreaded environments, the value of comm is ignored.

Assigning Sizes

To assign a size to matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_size(LIS_MATRIX A, LIS_INT local_n, LIS_INT global_n)
- Fortran subroutine lis_matrix_set_size(LIS_MATRIX A, LIS_INTEGER local_n, LIS_INTEGER global_n, LIS_INTEGER ierr)

Either $local_n$ or $global_n$ must be provided.

For the serial and multithreaded environments, $local_n$ is equal to $global_n$. Therefore, both $lis_matrix_set_size(A,n,0)$ and $lis_matrix_set_size(A,0,n)$ create a matrix of size $n \times n$.

For the multiprocessing environment, $lis_matrix_set_size(A,n,0)$ creates a partial matrix of size $n \times N$ on each processing element, where N is the total sum of n. On the other hand, $lis_matrix_set_size(A,0,n)$ creates a partial matrix of size $m_p \times n$ on processing element p. The values of m_p are determined by the library.

Assigning Values

To assign a value to the element at the i-th row and the j-th column of matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_value(LIS_INT flag, LIS_INT i, LIS_INT j, LIS_SCALAR value, LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_value(LIS_INTEGER flag, LIS_INTEGER i, LIS_INTEGER j, LIS_SCALAR value, LIS_MATRIX A, LIS_INTEGER ierr)

For the multiprocessing environment, the i-th row and the j-th column of the global matrix must be specified. Either

```
\begin{split} & \texttt{LIS\_INS\_VALUE} \,:\, A(i,j) = value, \, \text{or} \\ & \texttt{LIS\_ADD\_VALUE} \,:\, A(i,j) = A(i,j) + value \end{split}
```

must be provided for the parameter flag.

Assigning Storage Formats

To assign a storage format to matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_type(LIS_MATRIX A, LIS_INT matrix_type)
- Fortran subroutine lis_matrix_set_type(LIS_MATRIX A, LIS_INTEGER matrix_type, LIS_INTEGER ierr)

where matrix_type is LIS_MATRIX_CSR when the matrix is created. The following storage formats are supported:

Storage format		matrix_type
Compressed Sparse Row	(CSR)	{LIS_MATRIX_CSR 1}
Compressed Sparse Column	(CSC)	{LIS_MATRIX_CSC 2}
Modified Compressed Sparse Row	(MSR)	{LIS_MATRIX_MSR 3}
Diagonal	(DIA)	{LIS_MATRIX_DIA 4}
Ellpack-Itpack Generalized Diagonal	(ELL)	{LIS_MATRIX_ELL 5}
Jagged Diagonal	(JAD)	{LIS_MATRIX_JAD 6}
Block Sparse Row	(BSR)	{LIS_MATRIX_BSR 7}
Block Sparse Column	(BSC)	{LIS_MATRIX_BSC 8}
Variable Block Row	(VBR)	{LIS_MATRIX_VBR 9}
Coordinate	(COO)	{LIS_MATRIX_COO 10}
Dense	(DNS)	{LIS_MATRIX_DNS 11}

Assembling Matrices

After assigning values and storage formats, the following functions must be called:

- C LIS_INT lis_matrix_assemble(LIS_MATRIX A)
- Fortran subroutine lis_matrix_assemble(LIS_MATRIX A, LIS_INTEGER ierr)

 $\verb|lis_matrix_assemble| assembles A into the storage format specified by \verb|lis_matrix_set_type|.$

Destroying Matrices

To destroy the matrix, the following functions are used:

- C LIS_INT lis_matrix_destroy(LIS_MATRIX A)
- Fortran subroutine lis_matrix_destroy(LIS_MATRIX A, LIS_INTEGER ierr)

Method 2: Define Arrays in a Specific Storage Format Directly

For creating matrix A in Equation (3.2) in the CSR format, matrix A itself is created for the serial and multithreaded environments, while the partial matrices are created and stored on the given number of processing elements for the multiprocessing environment.

Programs to create matrix A in the CSR format are as follows, where the number of processing elements for the multiprocessing environment is assumed to be two:

```
— C (for serial and multithreaded environments) –
 1: LIS_INT i,k,n,nnz;
2: LIS_INT *ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnz = 10; k = 0;
6: lis_matrix_malloc_csr(n,nnz,&ptr,&index,&value);
7: lis_matrix_create(0,&A);
                                           /* or lis_matrix_set_size(A,n,0); */
8: lis_matrix_set_size(A,0,n);
9:
10: for(i=0;i<n;i++)
11: {
12:
        if( i>0 ) {index[k] = i-1; value[k] = 1; k++;}
13:
        index[k] = i; value[k] = 2; k++;
14:
        if( i < n-1 ) {index[k] = i+1; value[k] = 1; k++;}
15:
        ptr[i+1] = k;
16: }
17: ptr[0] = 0;
18: lis_matrix_set_csr(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);
```

```
— C (for multiprocessing environment) —
1: LIS_INT i,k,n,nnz,is,ie;
2: LIS_INT *ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 2; nnz = 5; k = 0;
6: lis_matrix_malloc_csr(n,nnz,&ptr,&index,&value);
7: lis_matrix_create(MPI_COMM_WORLD,&A);
8: lis_matrix_set_size(A,n,0);
9: lis_matrix_get_range(A,&is,&ie);
10: for(i=is;i<ie;i++)</pre>
11: {
12:
        if( i>0 ) {index[k] = i-1; value[k] = 1; k++;}
13:
        index[k] = i; value[k] = 2; k++;
14:
        if( i < n-1 ) {index[k] = i+1; value[k] = 1; k++;}
15:
        ptr[i-is+1] = k;
16: }
17: ptr[0] = 0;
18: lis_matrix_set_csr(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);
```

Associating Arrays

To associate the arrays in the CSR format with matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_csr(LIS_INT nnz, LIS_INT ptr[], LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_csr(LIS_INTEGER nnz, LIS_INTEGER ptr(), LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

Method 3: Read Matrix and Vector Data from External Files

Programs to read matrix A in Equation (3.2) in the CSR format and vector b in Equation (3.1) from an external file are as follows:

```
C (for serial, multithreaded and multiprocessing environments)

1: LIS_MATRIX A;
2: LIS_VECTOR b,x;
3: lis_matrix_create(LIS_COMM_WORLD,&A);
4: lis_vector_create(LIS_COMM_WORLD,&b);
5: lis_vector_create(LIS_COMM_WORLD,&x);
6: lis_matrix_set_type(A,LIS_MATRIX_CSR);
7: lis_input(A,b,x,"matvec.mtx");
```

```
Fortran (for serial, multithreaded and multiprocessing environments)

1: LIS_MATRIX A

2: LIS_VECTOR b,x

3: call lis_matrix_create(LIS_COMM_WORLD,A,ierr)

4: call lis_vector_create(LIS_COMM_WORLD,b,ierr)

5: call lis_vector_create(LIS_COMM_WORLD,x,ierr)

6: call lis_matrix_set_type(A,LIS_MATRIX_CSR,ierr)

7: call lis_input(A,b,x,'matvec.mtx',ierr)
```

The content of the destination file matvec.mtx is:

```
%%MatrixMarket matrix coordinate real general
4 4 10 1 0
1 2 1.0e+00
1 1 2.0e+00
2 3 1.0e+00
2 1 1.0e+00
2 2 2.0e+00
3 4 1.0e+00
3 2 1.0e+00
3 3 2.0e+00
4 4 2.0e+00
4 3 1.0e+00
1 0.0e+00
2 1.0e+00
3 2.0e+00
4 3.0e+00
```

Reading from External Files

To input the matrix data for A from an external file, the following functions are used:

```
• C LIS_INT lis_input_matrix(LIS_MATRIX A, char *filename)
```

```
• Fortran subroutine lis_input(LIS_MATRIX A, character filename, LIS_INTEGER ierr)
```

filename must be replaced with the file path. The following file formats are supported:

- The Matrix Market format
- The Harwell-Boeing format

To read the data for matrix A and vectors b and x from external files, the following functions are used:

- C LIS_INT lis_input(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, char *filename)
- Fortran subroutine lis_input(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, character filename, LIS_INTEGER ierr)

filename must be replaced with the file path. The following file formats are supported:

- The Extended Matrix Market format (extended to allow vector data)
- The Harwell-Boeing format

3.4 Solving Linear Equations

A program to solve the linear equation Ax = b with a specified solver is as follows:

```
C (for serial, multithreaded and multiprocessing environments)

1: LIS_MATRIX A;
2: LIS_VECTOR b,x;
3: LIS_SOLVER solver;
4:
5: /* Create matrix and vector */
6:
7: lis_solver_create(&solver);
8: lis_solver_set_option("-i bicg -p none",solver);
9: lis_solver_set_option("-tol 1.0e-12",solver);
10: lis_solve(A,b,x,solver);
```

```
Fortran (for serial, multithreaded and multiprocessing environments)

1: LIS_MATRIX A

2: LIS_VECTOR b,x

3: LIS_SOLVER solver

4:

5: /* Create matrix and vector */

6:

7: call lis_solver_create(solver,ierr)

8: call lis_solver_set_option('-i bicg -p none',solver,ierr)

9: call lis_solver_set_option('-tol 1.0e-12',solver,ierr)

10: call lis_solve(A,b,x,solver,ierr)
```

Creating Solvers

To create a solver, the following functions are used:

- C LIS_INT lis_solver_create(LIS_SOLVER *solver)
- Fortran subroutine lis_solver_create(LIS_SOLVER solver, LIS_INTEGER ierr)

Specifying Options

To specify options, the following functions are used:

- C LIS_INT lis_solver_set_option(char *text, LIS_SOLVER solver)
- Fortran subroutine lis_solver_set_option(character text, LIS_SOLVER solver, LIS_INTEGER ierr)

or

• C LIS_INT lis_solver_set_optionC(LIS_SOLVER solver)

• Fortran subroutine lis_solver_set_optionC(LIS_SOLVER solver, LIS_INTEGER ierr)

<code>lis_solver_set_optionC</code> is a function that sets the options specified on the command line, and passes them to <code>solver</code> when the program is run.

The table below shows the available command line options, where $-i \{cg|1\}$ means -i cg or -i 1 and -maxiter [1000] indicates that -maxiter defaults to 1,000.

Options for Linear Solvers (Default: -i bicg)

Options for Linear Solvers (Default: -i bicg)					
Solver	Option	Auxiliary Options			
CG	-i {cg 1}				
BiCG	-i {bicg 2}				
CGS	-i {cgs 3}				
BiCGSTAB	-i {bicgstab 4}				
BiCGSTAB(l)	-i {bicgstabl 5}	-ell [2]	The degree l		
GPBiCG	-i {gpbicg 6}				
TFQMR	-i {tfqmr 7}				
Orthomin(m)	-i {orthomin 8}	-restart [40]	The restart value m		
GMRES(m)	-i {gmres 9}	-restart [40]	The restart value m		
Jacobi	-i {jacobi 10}				
Gauss-Seidel	-i {gs 11}				
SOR	-i {sor 12}	-omega [1.9]	The relaxation coefficient		
		$\omega \ (0 < \omega < 2)$			
BiCGSafe	-i {bicgsafe 13}				
CR	-i {cr 14}				
BiCR	-i {bicr 15}				
CRS	-i {crs 16}				
BiCRSTAB	-i {bicrstab 17}				
GPBiCR	-i {gpbicr 18}				
BiCRSafe	-i {bicrsafe 19}				
FGMRES(m)	-i {fgmres 20}	-restart [40]	The restart value m		
IDR(s)	-i {idrs 21}	-irestart [2]	The restart value s		
IDR(1)	-i {idr1 22}				
MINRES	-i {minres 23}				
COCG	-i {cocg 24}				
COCR	-i {cocr 25}				

Options for Preconditioners (Default: -p none)

Preconditioner	Option	Auxiliary Options	
None	-p {none 0}		
Jacobi	-p {jacobi 1}		
ILU(k)	-p {ilu 2}	-ilu_fill [0]	The fill level k
SSOR	-p {ssor 3}	-ssor_w [1.0]	The relaxation coefficient
			$\omega \ (0 < \omega < 2)$
Hybrid	-p {hybrid 4}	-hybrid_i [sor]	The linear solver
		-hybrid_maxiter [25]	The maximum number of iterations
		-hybrid_tol [1.0e-3]	The convergence tolerance
		-hybrid_w [1.5]	The relaxation coefficient ω
			of the SOR $(0 < \omega < 2)$
		-hybrid_ell [2]	The degree l of the BiCGSTAB(l)
		-hybrid_restart [40]	The restart values
			of the GMRES and Orthomin
I+S	-p {is 5}	-is_alpha [1.0]	The parameter α of $I + \alpha S^{(m)}$
		-is_m [3]	The parameter m of $I + \alpha S^{(m)}$
SAINV	-p {sainv 6}	-sainv_drop [0.05]	The drop criterion
SA-AMG	-p {saamg 7}	-saamg_unsym [false]	Select the unsymmetric version
			(The matrix structure must be symmetric)
		-saamg_theta [0.05 0.12]	The drop criterion $a_{ij}^2 \le \theta^2 a_{ii} a_{jj} $ (symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05]	The drop criterion
		-iluc_rate [5.0]	The ratio of the maximum fill-in
ILUT	-p {ilut 9}		
Additive Schwarz	-adds true	-adds_iter [1]	The number of iterations

Other Options

		-			
Option					
-maxiter [1000]	The maximum number of iterations				
-tol [1.0e-12]	The convergence tolerance tol				
-tol_w [1.0]	The convergence tolerance tol_w				
-print [0]	The output of the residual history				
	-print {none 0}	None			
	-print {mem 1}	Save the residual history			
	-print {out 2}	Output it to the standard output			
	-print {all 3}	Save the residual history and output it			
		to the standard output			
-scale [0]	The scaling				
	(The result will overwrite	te the original matrix and vectors)			
	-scale {none 0}	No scaling			
	-scale {jacobi 1}	The Jacobi scaling $D^{-1}Ax = D^{-1}b$			
		(D represents the diagonal of $A = (a_{ij})$)			
	-scale {symm_diag 2}	The diagonal scaling $D^{-1/2}AD^{-1/2}x = D^{-1/2}b$			
		$(D^{-1/2}$ represents the diagonal matrix			
		with $1/\sqrt{a_{ii}}$ as the diagonal)			
-initx_zeros [1]	The behavior of the init				
	-initx_zeros {false	9 0			
		of the function lis_solve()			
	-initx_zeros {true 1				
-conv_cond [0]	The convergence condition				
	-conv_cond {nrm2_r 0	11 11 11 11 11			
	-conv_cond {nrm2_b 1	11 11-			
	-conv_cond {nrm1_b 2	$ b - Ax _1 \le tol_w * b _1 + tol$			
-omp_num_threads [t]	The number of threads				
543	(t represents the maximum number of threads)				
-storage [0]	The matrix storage format				
-storage_block [2]		The block size of the BSR and BSC formats			
-f [0]	The precision of the linear solver				
	-f {double 0}	Double precision			
	-f {quad 1}	Quadruple precision			

Solving Linear Equations

To solve the linear equation Ax = b, the following functions are used:

- C LIS_INT lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, LIS_SOLVER solver)
- Fortran subroutine lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x, LIS_SOLVER solver, LIS_INTEGER ierr)

3.5 Solving Eigenvalue Problems

A program to solve the standard eigenvalue problem $Ax = \lambda x$ with a specified solver is as follows:

```
C (for serial, multithreaded and multiprocessing environments)

1: LIS_MATRIX A;
2: LIS_VECTOR x;
3: LIS_REAL evalue;
4: LIS_ESOLVER esolver;
5:
6: /* Create matrix and vector */
7:
8: lis_esolver_create(&esolver);
9: lis_esolver_set_option("-e ii -i bicg -p none",esolver);
10: lis_esolver_set_option("-etol 1.0e-12 -tol 1.0e-12",esolver);
11: lis_esolve(A,x,evalue,esolver);
```

```
Fortran (for serial, multithreaded and multiprocessing environments)

1: LIS_MATRIX A

2: LIS_VECTOR x

3: LIS_REAL evalue

4: LIS_ESOLVER esolver

5:

6: /* Create matrix and vector */

7:

8: call lis_esolver_create(esolver,ierr)

9: call lis_esolver_set_option('-e ii -i bicg -p none',esolver,ierr)

10: call lis_esolver_set_option('-etol 1.0e-12 -tol 1.0e-12',esolver,ierr)

11: call lis_esolve(A,x,evalue,esolver,ierr)
```

Creating Eigensolvers

To create an eigensolver, the following functions are used:

- C LIS_INT lis_esolver_create(LIS_ESOLVER *esolver)
- Fortran subroutine lis_esolver_create(LIS_ESOLVER esolver, LIS_INTEGER ierr)

Specifying Options

To specify options, the following functions are used:

- C LIS_INT lis_esolver_set_option(char *text, LIS_ESOLVER esolver)
- Fortran subroutine lis_esolver_set_option(character text, LIS_ESOLVER esolver, LIS_INTEGER ierr)

or

- C LIS_INT lis_esolver_set_optionC(LIS_ESOLVER esolver)
- Fortran subroutine lis_esolver_set_optionC(LIS_ESOLVER esolver, LIS_INTEGER ierr)

lis_esolver_set_optionC is a function that sets the options specified in the command line, and passes them to esolver when the program is run.

The table below shows the available command line options, where -e {pi|1} means -e pi or -e 1 and -emaxiter [1000] indicates that -emaxiter defaults to 1,000.

Options for Eigensolvers (Default: -e pi)

Eigensolver	Option	Auxiliary Options	
Power	-e {pi 1}		
Inverse	-e {ii 2}	-i [bicg]	The linear solver
Rayleigh Quotient	-e {rqi 3}	-i [bicg]	The linear solver
CG	-e {cg 4}	-i [cg]	The linear solver
CR	-e {cr 5}	-i [bicg]	The linear solver
Jacobi-Davidson	-e {jd 6}	-i [cg]	The linear solver
Subspace	-e {si 7}	-ss [1]	The size of the subspace
Lanczos	-e {li 8}	-ss [1]	The size of the subspace
Arnoldi	-e {ai 9}	-ss [1]	The size of the subspace
Generalized Power	-e {gpi 10}	-i [bicg]	The linear solver
Generalized Inverse	-e {gii 11}	-i [bicg]	The linear solver
Generalized Rayleigh Quotient	-e {grqi 12}	-i [bicg]	The linear solver
Generalized CG	-e {gcg 13}	-i [cg]	The linear solver
Generalized CR	-e {gcr 14}	-i [bicg]	The linear solver
Generalized Subspace	-e {gsi 15}	-ss [1]	The size of the subspace
Generalized Lanczos	-e {gli 16}	-ss [1]	The size of the subspace
Generalized Arnoldi	-e {gai 17}	-ss [1]	The size of the subspace

Options for Preconditioners (Default: -p none)

Preconditioner	Option	Auxiliary Options	
None	-p {none 0}		
Jacobi	-p {jacobi 1}		
ILU(k)	-p {ilu 2}	-ilu_fill [0]	The fill level k
SSOR	-p {ssor 3}	-ssor_w [1.0]	The relaxation coefficient
			$\omega \ (0 < \omega < 2)$
Hybrid	-p {hybrid 4}	-hybrid_i [sor]	The linear solver
		-hybrid_maxiter [25]	The maximum number of iterations
		-hybrid_tol [1.0e-3]	The convergence tolerance
		-hybrid_w [1.5]	The relaxation coefficient ω
			of the SOR $(0 < \omega < 2)$
		-hybrid_ell [2]	The degree l of the BiCGSTAB(l)
		-hybrid_restart [40]	The restart values of the GMRES
			and Orthomin
I+S	-p {is 5}	-is_alpha [1.0]	The parameter α of $I + \alpha S^{(m)}$
		-is_m [3]	The parameter m of $I + \alpha S^{(m)}$
SAINV	-p {sainv 6}	-sainv_drop [0.05]	The drop criterion
SA-AMG	-p {saamg 7}	-saamg_unsym [false]	Select the unsymmetric version
			(The matrix structure must be
			symmetric)
		-saamg_theta [0.05 0.12]	The drop criterion $a_{ij}^2 \le \theta^2 a_{ii} a_{jj} $
			(symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05]	The drop criterion
		-iluc_rate [5.0]	The ratio of the maximum fill-in
ILUT	-p {ilut 9}		
Additive	-adds true	-adds_iter [1]	The number of iterations
Schwarz			

Other Options

Option				
-emaxiter [1000]	The maximum number of iterations			
-etol [1.0e-12]	The convergence tolerance			
-eprint [0]	The output of the residual history			
	-eprint {none 0}	None		
	-eprint {mem 1}	Save the residual history		
	-eprint {out 2}	Output it to the standard output		
	-print {all 3}	Save the residual history and output it		
		to the standard output		
-ie [ii]	The inner eigensolver us	sed in Subspace, Lanczos, and Arnoldi		
-ige [gii]	The inner eigensolver us	sed in Generalized Subspace, Generalized Lanczos,		
	and Generalized Arnold	i		
-shift [0.0]	The amount of the shift	σ		
-initx_ones [1]	The behavior of the init	ial vector x_0		
	-initx_ones {false 0	Values are given by the argument x		
		of the function lis_esolve()		
	-initx_ones {true 1}	All values are set to 1		
-omp_num_threads [t]	The number of threads			
	(t represents the maxim	num number of threads)		
-estorage [0]	The matrix storage format			
-estorage_block [2]	The block size of the BSR and BSC formats			
-ef [0]	The precision of the eige	ensolver		
	-ef {double 0}	Double precision		
	-ef {quad 1}	Quadruple precision		

Solving Eigenvalue Problems

To solve the standard eigenvalue problem $Ax = \lambda x$, the following functions are used:

- C LIS_INT lis_esolve(LIS_MATRIX A, LIS_VECTOR x, LIS_REAL evalue, LIS_ESOLVER esolver)
- Fortran subroutine lis_esolve(LIS_MATRIX A, LIS_VECTOR x, LIS_REAL evalue, LIS_ESOLVER esolver, LIS_INTEGER ierr)

To solve the generalized eigenvalue problem $Ax = \lambda Bx$, the following functions are used instead:

- C LIS_INT lis_gesolve(LIS_MATRIX A, LIS_MATRIX B, LIS_VECTOR x, LIS_REAL evalue, LIS_ESOLVER esolver)
- Fortran subroutine lis_gesolve(LIS_MATRIX A, LIS_MATRIX B, LIS_VECTOR x, LIS_REAL evalue, LIS_ESOLVER esolver, LIS_INTEGER ierr)

3.6 Writing Programs

The following are the programs for solving the linear equation Ax = b, where matrix A is a tridiagonal matrix

$$\begin{pmatrix}
2 & -1 & & & \\
-1 & 2 & -1 & & & \\
& \ddots & \ddots & \ddots & \\
& & -1 & 2 & -1 \\
& & & -1 & 2
\end{pmatrix}$$

of size 12. The the right-hand side vector b is set such that the values of the elements of the solution x are 1. The program is located in the directory lis-(\$VERSION)/test.

```
Test program: test4.c -
 1: #include <stdio.h>
 2: #include "lis.h"
3: main(LIS_INT argc, char *argv[])
4: {
5:
        LIS_INT i,n,gn,is,ie,iter;
 6:
        LIS_MATRIX A;
7:
        LIS_VECTOR b,x,u;
8:
        LIS_SOLVER solver;
9:
        n = 12;
10:
        lis_initialize(&argc,&argv);
11:
        lis_matrix_create(LIS_COMM_WORLD,&A);
12:
        lis_matrix_set_size(A,0,n);
13:
        lis_matrix_get_size(A,&n,&gn)
14:
        lis_matrix_get_range(A,&is,&ie)
15:
        for(i=is;i<ie;i++)</pre>
16:
17:
            if( i>0 ) lis_matrix_set_value(LIS_INS_VALUE,i,i-1,-1.0,A);
18:
            if( i<gn-1 ) lis_matrix_set_value(LIS_INS_VALUE,i,i+1,-1.0,A);</pre>
            lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0,A);
19:
20:
21:
        lis_matrix_set_type(A,LIS_MATRIX_CSR);
22:
        lis_matrix_assemble(A);
23:
24:
        lis_vector_duplicate(A,&u);
25:
        lis_vector_duplicate(A,&b);
26:
        lis_vector_duplicate(A,&x);
27:
        lis_vector_set_all(1.0,u);
28:
        lis_matvec(A,u,b);
29:
30:
        lis_solver_create(&solver);
31:
        lis_solver_set_optionC(solver);
32:
        lis_solve(A,b,x,solver);
33:
        lis_solver_get_iter(solver,&iter);
34:
        printf("number of iterations = %d\n",iter);
        lis_vector_print(x);
35:
36:
        lis_matrix_destroy(A);
37:
        lis_vector_destroy(u);
38:
        lis_vector_destroy(b);
39:
        lis_vector_destroy(x);
40:
        lis_solver_destroy(solver);
41:
        lis_finalize();
42:
        return 0;
43: }
}
```

```
– Test program: test4f.F –
         implicit none
 2:
3:#include "lisf.h"
 4:
         LIS_INTEGER i,n,gn,is,ie,iter,ierr
 5:
         LIS_MATRIX A
 6:
7:
         LIS_VECTOR b,x,u
         LIS_SOLVER solver
8:
9:
         n = 12
10:
         call lis_initialize(ierr)
11:
         call lis_matrix_create(LIS_COMM_WORLD,A,ierr)
12:
         call lis_matrix_set_size(A,0,n,ierr)
13:
         call lis_matrix_get_size(A,n,gn,ierr)
14:
         call lis_matrix_get_range(A,is,ie,ierr)
         do i=is,ie-1
15:
16:
           if( i>1 ) call lis_matrix_set_value(LIS_INS_VALUE,i,i-1,-1.0d0,
17:
                                                  A,ierr)
           if( i<gn ) call lis_matrix_set_value(LIS_INS_VALUE,i,i+1,-1.0d0,</pre>
18:
19:
                                                  A,ierr)
20:
           call lis_matrix_set_value(LIS_INS_VALUE,i,i,2.0d0,A,ierr)
21:
         enddo
22:
         call lis_matrix_set_type(A,LIS_MATRIX_CSR,ierr)
23:
         call lis_matrix_assemble(A,ierr)
24:
25:
         call lis_vector_duplicate(A,u,ierr)
26:
         call lis_vector_duplicate(A,b,ierr)
27:
         call lis_vector_duplicate(A,x,ierr)
28:
         call lis_vector_set_all(1.0d0,u,ierr)
29:
         call lis_matvec(A,u,b,ierr)
30:
31:
         call lis_solver_create(solver,ierr)
32:
         call lis_solver_set_optionC(solver,ierr)
33:
         call lis_solve(A,b,x,solver,ierr)
34:
         call lis_solver_get_iter(solver,iter,ierr)
35:
         write(*,*) 'number of iterations = ',iter
36:
         call lis_vector_print(x,ierr)
37:
         call lis_matrix_destroy(A,ierr)
38:
         call lis_vector_destroy(b,ierr)
39:
         call lis_vector_destroy(x,ierr)
40:
         call lis_vector_destroy(u,ierr)
41:
         call lis_solver_destroy(solver,ierr)
42:
         call lis_finalize(ierr)
43:
44:
         stop
45:
         end
```

3.7 An Alternate Workflow: PSD

The previous section represents one of two workflows in Lis. The present workflow represents a decoupling of the preconditioner and the solver (Preconditioner and Solver Decoupled, or PSD), in the sense that the preconditioner can be updated separately from the solver; in the workflow presented in the previous section, the preconditioner is updated with every call to lis_solve. This property (conditional updating of the preconditioner) turns out to be quite useful in solving certain problems, including some nonlinear partial differential equations. For nonlinear PDEs, iterative solution methods are used (e.g., Newton-Raphson) to calculate a solution. In the current context, each Newton-Raphson iteration calculates an incremental improvement to a solution vector via the solution of a linear system of equations. To support this functionality, the following (additional) functions must be used:

- lis_matrix_psd_set_value: Re-define the value of an existing matrix component.
- lis_matrix_psd_reset_scale: Set the matrix "scaled" status to *false*. Note that this call and the following one are only needed for the case non-trivial scaling.
- lis_vector_psd_reset_scale: Set the vector "scaled" status to false.
- lis_solver_set_matrix: Associate the given matrix and solver. Note that this must be done before the call to lis_precon_psd_create.
- lis_precon_psd_create: Create data structures associated with the chosen preconditioner.
- lis_precon_psd_update: Evaluate the preconditioner.

It is noted that this workflow currently has the following restrictions:

- The only matrix format currently supported is CSR.
- The only solver currently supported is GMRES.
- The only preconditioners currently supported are ILU(k) and SA-AMG.

Error checking has been implemented to prevent any unsupported cases from running.

The following pseudo-code listing presents an example of the workflow. An actual implementation can be seen in test problem test8f.F90.

```
- Pseudo-code example of workflow -
        PROGRAM psd_driver
 2:
 3:
        implicit none
 5:#include "lisf.h"
       LIS_INTEGER i,n,gn,is,ie,iter,ierr
 7:
 8:
       LIS_MATRIX A
 9:
       LIS_VECTOR b,x
10:
       LIS_SOLVER solver
11:
        REAL :: u(:),du
12:
13:
        CALL lis_initialize(ierr)
14:
       15:
       ! initialization, only done once
16:
       17:
18:
       CALL lis_matrix_create(LIS_COMM_WORLD,A,ierr)
        CALL lis_matrix_set_size(A,0,n,ierr)
19:
20:
        CALL lis_matrix_get_size(A,n,gn,ierr)
21:
        CALL lis_matrix_get_range(A,is,ie,ierr)
22:
23:
        CALL UpdateLinearSystem(RHS,LHS)
24:
        DO i=is,ie-1
25:
        DO j=1,gn
26:
              IF (LHS(i,j) exists) THEN
27:
                   CALL lis_matrix_set_value(LIS_INS_VALUE,i,j,LHS(i,j),A,ierr)
28:
             END IF
29:
          END DO
30:
       END DO
31:
        CALL lis_matrix_set_type(A,LIS_MATRIX_CSR,ierr)
32:
        CALL lis_matrix_assemble(A,ierr)
33:
34:
        CALL lis_vector_duplicate(A,b,ierr)
35:
        CALL lis_vector_duplicate(A,x,ierr)
36:
        DO i=is,ie-1
37:
            CALL lis_vector_set_value(LIS_INS_VALUE,i,RHS(i),b,ierr)
38:
       END DO
39:
       u = u_{initial}
40:
41:
        CALL lis_solver_create(solver,ierr)
        WRITE(UNIT=options,FMT='(a)') "-p ilu -i gmres -print out -scale none"
42:
43:
       CALL lis_solver_set_option(TRIM(options),solver,ierr)
44:
45:
       !-----
46:
       ! everything up to this point is more or less identical to the standard workflow.
47:
       ! Now comes the preconditioner initialization, and the Newton-Raphson
48:
       ! iteration.
49:
       50:
       CALL lis_solver_set_matrix(A,solver,ierr)
51:
       CALL lis_precon_psd_create(solver,precon,ierr)
52:
       ! evaluate the preconditioner, at least once . . .
53:
       CALL lis_precon_psd_update(solver,precon,ierr)
54:
```

```
- Pseudo-code example of workflow (contd.) -
 55:
 56:
 57:
             IF (UpdateLHS) THEN
 58:
                 DO i=is,ie-1
59:
                     DO j=1,gn
                         IF (component (i,j) exists) THEN
 60:
                             CALL lis_matrix_psd_set_value(LIS_INS_VALUE,i,j,LHS(i,j),A,ierr)
 61:
 62:
                         END IF
 63:
                     END DO
 64:
                  END DO
 65:
                  CALL lis_matrix_psd_reset_scale(A,ierr)
 66:
             END IF
 67:
 68:
             ! update RHS every iteration
 69:
             DO i=is,ie-1
                 CALL lis_vector_set_value(LIS_INS_VALUE,i,RHS(i),b,ierr)
70:
71:
             END DO
72:
             CALL lis_vector_psd_reset_scale(A,ierr)
73:
74:
             IF (UpdateLHS) THEN
75:
                 CALL lis_precon_psd_update(solver,precon,ierr)
76:
77:
             CALL lis_solve_kernel(A,b,x,solver,precon,ierr)
78:
             CALL lis_solver_get_iter(solver,iter,ierr)
79:
             write(*,*) 'number of iterations = ',iter
80:
             CALL lis_vector_print(x,ierr)
81:
             ! update the solution
82:
83:
             DO i=is,ie-1
84:
                 CALL lis_vector_get_value(x,i,du,ierr)
85:
                 u(i)=u(i)-du
             END DO
86:
87:
88:
             CALL UpdateLinearSystem(RHS,LHS)
89:
90:
             IF (termination criteria satisfied) EXIT
91:
92:
         END DO
93:
94:
          CALL lis_matrix_destroy(A,ierr)
95:
96:
          CALL lis_vector_destroy(b,ierr)
97:
          CALL lis_vector_destroy(x,ierr)
98:
          CALL lis_vector_destroy(u,ierr)
99:
          CALL lis_solver_destroy(solver,ierr)
100:
101:
          CALL lis_finalize(ierr)
102:
103:
          END PROGRAM psd_driver
```

3.8 Compiling and Linking

Provided below is an example test4.c located in the directory lis-(\$VERSION)/test, compiled on the SGI Altix 3700 using the Intel C/C++ Compiler (icc). Since the library includes some Fortran 90 codes when the SA-AMG preconditioner is selected, a Fortran 90 compiler must be used for the linking. The preprocessor macro USE_MPI must be defined for the multiprocessing environment. The preprocessor macros LONG_LONG for C and LONG_LONG for Fortran must be defined when using the 64bit integer.

```
For the serial environment

Compiling

> icc -c -I($INSTALLDIR)/include test4.c

Linking

> icc -o test4 test4.o -L($INSTALLDIR)/lib -llis

Linking (with SA-AMG)

> ifort -nofor_main -o test4 test4.o -L($INSTALLDIR)/lib -llis
```

```
For multithreaded environment

Compiling

> icc -c -openmp -I($INSTALLDIR)/include test4.c

Linking

> icc -openmp -o test4 test4.o -L($INSTALLDIR)/lib -llis

Linking (with SA-AMG)

> ifort -nofor_main -openmp -o test4 test4.o -L($INSTALLDIR)/lib -llis
```

```
Compiling

> icc -c -DUSE_MPI -I($INSTALLDIR)/include test4.c

Linking

> icc -o test4 test4.o -L($INSTALLDIR)/lib -llis -lmpi

Linking (with SA-AMG)

> ifort -nofor_main -o test4 test4.o -L($INSTALLDIR)/lib -llis -lmpi
```

```
Compiling

> icc -c -openmp -DUSE_MPI -I($INSTALLDIR)/include test4.c

Linking

> icc -openmp -o test4 test4.o -L($INSTALLDIR)/lib -llis -lmpi

Linking (with SA-AMG)

> ifort -nofor_main -openmp -o test4 test4.o -L($INSTALLDIR)/lib -llis -lmpi
```

Provided below is an example test4f.F located in the directory lis-(\$VERSION)/test, compiled on the SGI Altix 3700 using the Intel Fortran Compiler (ifort). Since compiler directives are used in the program, an appropriate compiler option should be specified to use the preprocessor. -fpp is the option for the Intel compiler.

```
For serial environment

Compiling

> ifort -c -fpp -I($INSTALLDIR)/include test4f.F

Linking

> ifort -o test4f test4f.o -L($INSTALLDIR)/lib -llis
```

3.9 Running

The test programs test4 and test4f in the directory lis-(\$VERSION)/test are run as follows:

For serial environment

> ./test4 -i bicgstab

For multithreaded environment

> env OMP_NUM_THREADS=2 ./test4 -i bicgstab

For multiprocessing environment

> mpirun -np 2 ./test4 -i bicgstab

For multithreaded and multiprocessing environment

> mpirun -np 2 env OMP_NUM_THREADS=2 ./test4 -i bicgstab

The solution will be returned:

initial vector x : 0
precision : double
linear solver : BiCGSTAB
preconditioner : none

convergence condition : $||b-Ax||_2 \le 1.0e-12 * ||b-Ax_0||_2$

matrix storage format : CSR

linear solver status : normal end

0 1.000000e+000 1 1.000000e+000

2 1.000000e+000

3 1.000000e+000 4 1.000000e+000

5 1.000000e+000

6 1.000000e+000

7 1.000000e+000

8 1.000000e+000

9 1.000000e+000

10 1.000000e+000

11 1.000000e+000

3.10 Zero DOFs on a Process

For the case of multiple MPI processes, it is possible to specify $global_n = 0$ and $local_n \ge 0$. This will allow one or more MPI processes to have zero DOFs. However, it is still the case that, for $global_n = 0$, the summation of $local_n$ over all processes should be greater than zero.

4 Quadruple Precision Operations

Double precision operations sometimes require a large number of iterations because of the rounding error. Besides long double precision operations, Lis supports "double-double" precision operations, or quadruple precision operations by combining two double precision floating point numbers[49, 50]. To use the double-double precision with the same interface as the double precision operations, both the matrix and vectors are assumed to be double precision. Lis also supports the performance acceleration of the double-double precision operations with the SIMD instructions, such as Intel's Streaming SIMD Extensions (SSE)[55].

4.1 Using Quadruple Precision Operations

The test program test5.c solves a linear equation Ax = b, where A is a Toeplitz matrix

$$\begin{pmatrix} 2 & 1 & & & & \\ 0 & 2 & 1 & & & \\ \gamma & 0 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \ddots & \\ & & \gamma & 0 & 2 & 1 \\ & & & \gamma & 0 & 2 \end{pmatrix}.$$

The right-hand vector is set such that the values of the elements of the solution x are 1. The value n is the size of matrix A. test5 with option \neg f is run:

Double precision

```
By entering > ./test5 200 2.0 -f double the following results will be returned:
```

```
n = 200, gamma = 2.000000
                     : 0
initial vector x
precision
                     : double
linear solver
                     : BiCG
preconditioner
                     : none
convergence condition : ||b-Ax||_2 \le 1.0e-12 * ||b-Ax_0||_2
matrix storage format : CSR
linear solver status : normal end
BiCG: number of iterations = 1001 (double = 1001, quad = 0)
BiCG: elapsed time = 2.044368e-02 sec.
                        = 4.768372e-06 sec.
BiCG:
       preconditioner
BiCG:
         matrix creation = 4.768372e-06 sec.
BiCG:
       linear solver = 2.043891e-02 sec.
BiCG: relative residual
                          = 8.917591e+01
```

Quadruple precision

```
By entering > ./test5 200 2.0 -f quad the following results will be returned:
```

```
n = 200, gamma = 2.000000
```

initial vector x : 0
precision : quad
linear solver : BiCG

preconditioner : none

convergence condition : ||b-Ax||_2 <= 1.0e-12 * ||b-Ax_0||_2

matrix storage format : CSR

linear solver status : normal end

BiCG: number of iterations = 230 (double = 230, quad = 0)

BiCG: elapsed time = 2.267408e-02 sec. BiCG: preconditioner = 4.549026e-04 sec. BiCG: matrix creation = 5.006790e-06 sec. BiCG: linear solver = 2.221918e-02 sec.

BiCG: relative residual = 6.499145e-11

5 Matrix Storage Formats

This section describes the matrix storage formats supported by the library. Assume that the matrix row (column) number begins with 0 and that the number of nonzero elements of matrix A of size $n \times n$ is nnz.

5.1 Compressed Sparse Row (CSR)

The CSR format uses three arrays ptr, index and value to store data.

- ullet value is a double precision array of length nnz, which stores the nonzero elements of matrix A along the row.
- index is an integer array of length nnz, which stores the column numbers of the nonzero elements stored in the array value.
- ptr is an integer array of length n+1, which stores the starting points of the rows of the arrays value and index.

5.1.1 Creating Matrices (for Serial and Multithreaded Environments)

The diagram on the right in Figure 2 shows how matrix A in Figure 2 is stored in the CSR format. A program to create the matrix in the CSR format is as follows:

Figure 2: Data structure of CSR format (for serial and multithreaded environments).

```
- For serial and multithreaded environments
 1: LIS_INT n,nnz;
 2: LIS_INT *ptr,*index;
3: LIS_SCALAR *value;
 4: LIS_MATRIX A;
 5: n = 4; nnz = 8;
 6: ptr = (LIS_INT *)malloc( (n+1)*sizeof(LIS_INT) );
7: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
8: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
12: ptr[0] = 0; ptr[1] = 1; ptr[2] = 3; ptr[3] = 5; ptr[4] = 8;
13: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 1;
14: index[4] = 2; index[5] = 0; index[6] = 2; index[7] = 3;
15: value[0] = 11; value[1] = 21; value[2] = 22; value[3] = 32;
16: value[4] = 33; value[5] = 41; value[6] = 43; value[7] = 44;
18: lis_matrix_set_csr(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);
```

5.1.2 Creating Matrices (for Multiprocessing Environment)

Figure 3 shows how matrix A in Figure 2 is stored in the CSR format on two processing elements. A program to create the matrix in the CSR format on two processing elements is as follows:



Figure 3: Data structure of CSR format (for multiprocessing environment).

```
    For multiprocessing environment -

 1: LIS_INT i,k,n,nnz,my_rank;
2: LIS_INT *ptr,*index;
3: LIS_SCALAR *value;
 4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3;}
7: else
                     {n = 2; nnz = 5;}
8: ptr = (LIS_INT *)malloc( (n+1)*sizeof(LIS_INT) );
9: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
10: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
14:
        ptr[0] = 0; ptr[1] = 1; ptr[2] = 3;
15:
        index[0] = 0; index[1] = 0; index[2] = 1;
        value[0] = 11; value[1] = 21; value[2] = 22;}
16:
17: else {
        ptr[0] = 0; ptr[1] = 2; ptr[2] = 5;
18:
19:
        index[0] = 1; index[1] = 2; index[2] = 0; index[3] = 2; index[4] = 3;
        value[0] = 32; value[1] = 33; value[2] = 41; value[3] = 43; value[4] = 44;}
21: lis_matrix_set_csr(nnz,ptr,index,value,A);
22: lis_matrix_assemble(A);
```

5.1.3 Associating Arrays

To associate the arrays in the CSR format with matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_csr(LIS_INT nnz, LIS_INT ptr[], LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_csr(LIS_INTEGER nnz, LIS_INTEGER ptr(), LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.2 Compressed Sparse Column (CSC)

The CSS format uses three arrays ptr, index and value to store data.

- value is a double precision array of length nnz, which stores the nonzero elements of matrix A along the column.
- index is an integer array of length nnz, which stores the row numbers of the nonzero elements stored in the array value.
- ptr is an integer array of length n+1, which stores the starting points of the rows of the arrays value and index.

5.2.1 Creating Matrices (for Serial and Multithreaded Environments)

The diagram on the right in Figure 4 shows how matrix A in Figure 4 is stored in the CSC format. A program to create the matrix in the CSC format is as follows:

Figure 4: Data structure of CSC format (for serial and multithreaded environments).

```
- For serial and multithreaded environments -
 1: LIS_INT n,nnz;
 2: LIS_INT *ptr,*index;
3: LIS_SCALAR *value;
 4: LIS_MATRIX A;
5: n = 4; nnz = 8;
 6: ptr = (LIS_INT *)malloc( (n+1)*sizeof(LIS_INT) );
7: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
8: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: ptr[0] = 0; ptr[1] = 3; ptr[2] = 5; ptr[3] = 7; ptr[4] = 8;
13: index[0] = 0; index[1] = 1; index[2] = 3; index[3] = 1;
14: index[4] = 2; index[5] = 2; index[6] = 3; index[7] = 3;
15: value[0] = 11; value[1] = 21; value[2] = 41; value[3] = 22;
16: value[4] = 32; value[5] = 33; value[6] = 43; value[7] = 44;
18: lis_matrix_set_csc(nnz,ptr,index,value,A);
19: lis_matrix_assemble(A);
```

5.2.2 Creating Matrices (for Multiprocessing Environment)

Figure 5 shows how matrix A in Figure 4 is stored on two processing elements. A program to create the matrix in the CSC format on two processing elements is as follows:



Figure 5: Data structure of CSC format (for multiprocessing environment).

```
- For multiprocessing environment -
 1: LIS_INT i,k,n,nnz,my_rank;
2: LIS_INT *ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3;}
7: else
                     {n = 2; nnz = 5;}
8: ptr = (LIS_INT *)malloc( (n+1)*sizeof(LIS_INT) );
9: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
10: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
14:
        ptr[0] = 0; ptr[1] = 3; ptr[2] = 5;
        index[0] = 0; index[1] = 1; index[2] = 3; index[3] = 1; index[4] = 2;
15:
        value[0] = 11; value[1] = 21; value[2] = 41; value[3] = 22; value[4] = 32}
16:
17: else {
        ptr[0] = 0; ptr[1] = 2; ptr[2] = 3;
18:
19:
        index[0] = 2; index[1] = 3; index[2] = 3;
        value[0] = 33; value[1] = 43; value[2] = 44;}
21: lis_matrix_set_csc(nnz,ptr,index,value,A);
22: lis_matrix_assemble(A);
```

5.2.3 Associating Arrays

To associate the arrays in the CSC format with matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_csc(LIS_INT nnz, LIS_INT row[], LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_csc(LIS_INTEGER nnz, LIS_INTEGER row(), LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.3 Modified Compressed Sparse Row (MSR)

The MSR format uses two arrays index and value to store data. Assume that ndz represents the number of zero elements of the diagonal.

- value is a double precision array of length nnz + ndz + 1, which stores the diagonal of matrix A down to the n-th element. The n + 1-th element is not used. For the n + 2-th and after, the values of the nonzero elements except the diagonal of matrix A are stored along the row.
- index is an integer array of length nnz + ndz + 1, which stores the starting points of the rows of the off-diagonal elements of matrix A down to the n + 1-th element. For the n + 2-th and after, it stores the row numbers of the off-diagonal elements of matrix A stored in the array value.

5.3.1 Creating Matrices (for Serial and Multithreaded Environments)

The diagram on the right in Figure 6 shows how matrix A is stored in the MSR format. A program to create the matrix in the MSR format is as follows:

Figure 6: Data structure of MSR format (for serial and multithreaded environments).

```
    For serial and multithreaded environments -

 1: LIS_INT n,nnz,ndz;
 2: LIS_INT *index;
3: LIS_SCALAR *value;
 4: LIS_MATRIX A;
5: n = 4; nnz = 8; ndz = 0;
 6: index = (LIS_INT *)malloc( (nnz+ndz+1)*sizeof(LIS_INT) );
7: value = (LIS_SCALAR *)malloc( (nnz+ndz+1)*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
11: index[0] = 5; index[1] = 5; index[2] = 6; index[3] = 7;
12: index[4] = 9; index[5] = 0; index[6] = 1; index[7] = 0; index[8] = 2;
13: value[0] = 11; value[1] = 22; value[2] = 33; value[3] = 44;
14: value[4] = 0; value[5] = 21; value[6] = 32; value[7] = 41; value[8] = 43;
16: lis_matrix_set_msr(nnz,ndz,index,value,A);
17: lis_matrix_assemble(A);
```

5.3.2 Creating Matrices (for Multiprocessing Environment)

Figure 7 shows how matrix A in Figure 6 is stored in the MSR format on two processing elements. A program to create the matrix in the MSR format on two processing element is as follows:



Figure 7: Data structure of MSR format (for multiprocessing environment).

```
- For multiprocessing environment -
 1: LIS_INT i,k,n,nnz,ndz,my_rank;
2: LIS_INT *index;
3: LIS_SCALAR *value;
 4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; nnz = 3; ndz = 0;}
                     {n = 2; nnz = 5; ndz = 0;}
7: else
8: index = (LIS_INT *)malloc( (nnz+ndz+1)*sizeof(LIS_INT) );
9: value = (LIS_SCALAR *)malloc( (nnz+ndz+1)*sizeof(LIS_SCALAR) );
10: lis_matrix_create(MPI_COMM_WORLD,&A);
11: lis_matrix_set_size(A,n,0);
12: if( my_rank==0 ) {
        index[0] = 3; index[1] = 3; index[2] = 4; index[3] = 0;
13:
        value[0] = 11; value[1] = 22; value[2] = 0; value[3] = 21;}
14:
15: else {
        index[0] = 3; index[1] = 4; index[2] = 6; index[3] = 1;
16:
        index[4] = 0; index[5] = 2;
17:
        value[0] = 33; value[1] = 44; value[2] = 0; value[3] = 32;
18:
19:
        value[4] = 41; value[5] = 43;}
20: lis_matrix_set_msr(nnz,ndz,index,value,A);
21: lis_matrix_assemble(A);
```

5.3.3 Associating Arrays

To associate the arrays in the MSR format with matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_msr(LIS_INT nnz, LIS_INT ndz, LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_msr(LIS_INTEGER nnz, LIS_INTEGER ndz, LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

Diagonal (DIA)

The DIA format uses two arrays index and value to store data. Assume that nnd represents the number of nonzero diagonal elements of matrix A.

- value is a double precision array of length $nnd \times n$, which stores the values of the nonzero diagonal elements of matrix A.
- index is an integer array of length nnd, which stores the offsets from the main diagonal.

For the multithreaded environment, the following modifications have been made: the format uses two arrays index and value to store data. Assume that nprocs represents the number of threads. nnd_n is the number of nonzero diagonal elements of the partial matrix into which the row block of matrix A is divided. maxnnd is the maximum value nnd_p .

- value is a double precision array of length $maxnnd \times n$, which stores the values of the nonzero diagonal elements of matrix A.
- index is an integer array of length nprocs × maxnnd, which stores the offsets from the main diagonal.

5.4.1 Creating Matrices (for Serial Environment)

The diagram on the right in Figure 8 shows how matrix A in Figure 8 is stored in the DIA format. A program to create the matrix in the DIA format is as follows:

$$A = \begin{pmatrix} 11 & & & \\ 21 & 22 & & & \\ & 32 & 33 & \\ 41 & & 43 & 44 \end{pmatrix} \begin{bmatrix} -3 & -1 & 0 & & \\ \hline 0 & 0 & 041 & 021 & 32 & 43 & 11 & 22 & 33 & 44 \end{bmatrix}$$
 A.index A.value

Figure 8: Data structure of DIA format (for serial environment).

```
- For serial environment -
 1: LIS_INT n,nnd;
 2: LIS_INT *index;
 3: LIS_SCALAR *value;
 4: LIS_MATRIX A;
5: n = 4; nnd = 3;
 6: index = (LIS_INT *)malloc( nnd*sizeof(LIS_INT) );
7: value = (LIS_SCALAR *)malloc( n*nnd*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
10:
11: index[0] = -3; index[1] = -1; index[2] = 0;
12: value[0] = 0; value[1] = 0; value[2] = 0; value[3] = 41;
13: value[4] = 0; value[5] = 21; value[6] = 32; value[7] = 43;
14: value[8] = 11; value[9] = 22; value[10] = 33; value[11] = 44;
15:
16: lis_matrix_set_dia(nnd,index,value,A);
17: lis_matrix_assemble(A);
```

5.4.2 Creating Matrices (for Multithreaded Environment)

Figure 9 shows how matrix A in Figure 8 is stored in the DIA format on two threads. A program to create the matrix in the DIA format on two threads is as follows:

-1	0		-3	-1	0							A.index
0	21	11	22			0	41	32	43	33	44	A.value

Figure 9: Data structure of DIA format (for multithreaded environment).

```
- For multithreaded environment -
 1: LIS_INT n,maxnnd,nprocs;
2: LIS_INT *index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; maxnnd = 3; nprocs = 2;
6: index = (LIS_INT *)malloc( maxnnd*sizeof(LIS_INT) );
7: value = (LIS_SCALAR *)malloc( n*maxnnd*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
10:
11: index[0] = -1; index[1] = 0; index[2] = 0; index[3] = -3; index[4] = -1; index[5] = 0;
12: value[0] = 0; value[1] = 21; value[2] = 11; value[3] = 22; value[4] = 0; value[5] = 0;
13: value[6] = 0; value[7] = 41; value[8] = 32; value[9] = 43; value[10] = 33; value[11] = 44;
15: lis_matrix_set_dia(maxnnd,index,value,A);
16: lis_matrix_assemble(A);
```

5.4.3 Creating Matrices (for Multiprocessing Environment)

Figure 10 shows how matrix A in Figure 8 is stored in the DIA format on two processing elements. A program to create the matrix in the DIA format on two processing elements is as follows:



Figure 10: Data structure of DIA format (for multiprocessing environment).

```
    For multiprocessing environment ——

 1: LIS_INT i,n,nnd,my_rank;
2: LIS_INT *index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; nnd = 2;}
7: else
                     {n = 2; nnd = 3;}
8: index = (LIS_INT *)malloc( nnd*sizeof(LIS_INT) );
9: value = (LIS_SCALAR *)malloc( n*nnd*sizeof(LIS_SCALAR) );
10: lis_matrix_create(MPI_COMM_WORLD,&A);
11: lis_matrix_set_size(A,n,0);
12: if( my_rank==0 ) {
13:
        index[0] = -1; index[1] = 0;
        value[0] = 0; value[1] = 21; value[2] = 11; value[3] = 22;}
14:
15: else {
16:
        index[0] = -3; index[1] = -1; index[2] = 0;
        value[0] = 0; value[1] = 41; value[2] = 32; value[3] = 43; value[4] = 33;
17:
        value[5] = 44;
18:
19: lis_matrix_set_dia(nnd,index,value,A);
20: lis_matrix_assemble(A);
```

5.4.4 Associating Arrays

To associate the arrays in the DIA format with matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_dia(LIS_INT nnd, LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_dia(LIS_INTEGER nnd, LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.5 Ellpack-Itpack Generalized Diagonal (ELL)

The ELL format uses two arrays index and value to store data. Assume that maxnzr is the maximum value of the number of nonzero elements in the rows of matrix A.

- value is a double precision array of length $maxnzr \times n$, which stores the values of the nonzero elements of the rows of matrix A along the column. The first column consists of the first nonzero elements of each row. If there is no nonzero elements to be stored, then 0 is stored.
- index is an integer array of length $maxnzr \times n$, which stores the column numbers of the nonzero elements stored in the array value. If the number of nonzero elements in the *i*-th row is nnz, then index $[nnz \times n + i]$ stores row number *i*.

5.5.1 Creating Matrices (for Serial and Multithreaded Environments)

The diagram on the right in Figure 11 shows how matrix A in Figure 11 is stored in the ELL format. A program to create the matrix in the ELL format is as follows:

Figure 11: Data structure of ELL format (for serial and multithreaded environments).

```
- For serial and multithreaded environments -
 1: LIS_INT n,maxnzr;
 2: LIS_INT *index;
3: LIS_SCALAR *value;
 4: LIS_MATRIX A;
 5: n = 4; maxnzr = 3;
 6: index = (LIS_INT *)malloc( n*maxnzr*sizeof(LIS_INT) );
7: value = (LIS_SCALAR *)malloc( n*maxnzr*sizeof(LIS_SCALAR) );
8: lis_matrix_create(0,&A);
9: lis_matrix_set_size(A,0,n);
11: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 0; index[4] = 0; index[5] = 1;
12: index[6] = 2; index[7] = 2; index[8] = 0; index[9] = 1; index[10] = 2; index[11] = 3;
13: value[0] = 11; value[1] = 21; value[2] = 32; value[3] = 41; value[4] = 0; value[5] = 22;
14: value[6] = 33; value[7] = 43; value[8] = 0; value[9] = 0; value[10] = 0; value[11] = 44;
16: lis_matrix_set_ell(maxnzr,index,value,A);
17: lis_matrix_assemble(A);
```

5.5.2 Creating Matrices (for Multiprocessing Environment)

Figure 12 shows how matrix A in Figure 11 is stored in the ELL format. A program to create the matrix in the ELL format on two processing elements is as follows:

0 0 0 1	1 0 2 2 3	A.inde
11 21 0 22	32 41 33 43 0 44	A.valu
PE0	PE1	

Figure 12: Data structure of ELL format (for multiprocessing environment).

```
    For multiprocessing environment —

 1: LIS_INT i,n,maxnzr,my_rank;
2: LIS_INT *index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; maxnzr = 2;}
7: else
                     {n = 2; maxnzr = 3;}
8: index = (LIS_INT *)malloc( n*maxnzr*sizeof(LIS_INT) );
9: value = (LIS_SCALAR *)malloc( n*maxnzr*sizeof(LIS_SCALAR) );
10: lis_matrix_create(MPI_COMM_WORLD,&A);
11: lis_matrix_set_size(A,n,0);
12: if( my_rank==0 ) {
13:
        index[0] = 0; index[1] = 0; index[2] = 0; index[3] = 1;
        value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;}
14:
15: else {
16:
        index[0] = 1; index[1] = 0; index[2] = 2; index[3] = 2; index[4] = 2;
        index[5] = 3;
17:
        value[0] = 32; value[1] = 41; value[2] = 33; value[3] = 43; value[4] = 0;
18:
        value[5] = 44;
19:
20: lis_matrix_set_ell(maxnzr,index,value,A);
21: lis_matrix_assemble(A);
```

5.5.3 Associating Arrays

To associate an array required by the ELL format with matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_ell(LIS_INT maxnzr, LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_ell(LIS_INTEGER maxnzr, LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.6 Jagged Diagonal (JAD)

The JAD format first sorts the nonzero elements of the rows in decreasing order of size, and then stores them along the column. The JAD format uses four arrays, perm, ptr, index, and value, to store data. Assume that maxnzr represents the maximum value of the number of nonzero elements of matrix A.

- \bullet perm is an integer array of length n, which stores the sorted row numbers.
- value is a double precision array of length nnz, which stores the values of the jagged diagonal elements of the sorted matrix A. The first jagged diagonal consists of the values of the first nonzero elements of each row. The next jagged diagonal consists of the values of the second nonzero elements, and so on.
- index is an integer array of length nnz, which stores the row numbers of the nonzero elements stored in the array value.
- ptr is an integer array of length maxnzr+1, which stores the starting points of the jagged diagonal elements.

For the multithreaded environment, the following modifications have been made: the format uses four arrays, perm, ptr, index, and value, to store data. Assume that nprocs is the number of threads. $maxnzr_p$ is the number of nonzero diagonal elements of the partial matrix into which the row block of matrix A is divided. maxmaxnzr is the maximum value of $maxnzr_p$.

- perm is an integer array of length n, which stores the sorted row numbers.
- value is a double precision array of length nnz, which stores the values of the jagged diagonal elements of the sorted matrix A. The first jagged diagonal consists of the values of the first nonzero elements of each row. The next jagged diagonal consist of the values of the second nonzero elements of each row, and so on.
- index is an integer array of length nnz, which stores the row numbers of the nonzero elements stored in the array value.
- ptr is an integer array of length $nprocs \times (maxmaxnzr + 1)$, which stores the starting points of the jagged diagonal elements.

5.6.1 Creating Matrices (for Serial Environment)

The diagram on the right in Figure 13 shows how matrix A in Figure 13 is stored in the JAD format. A program to create the matrix in the JAD format is as follows:



Figure 13: Data structure of JAD format (for serial environment).

```
- For serial environment -
 1: LIS_INT n,nnz,maxnzr;
 2: LIS_INT *perm,*ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnz = 8; maxnzr = 3;
 6: perm = (LIS_INT *)malloc( n*sizeof(LIS_INT) );
7: ptr = (LIS_INT *)malloc( (maxnzr+1)*sizeof(LIS_INT) );
8: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
9: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
10: lis_matrix_create(0,&A);
11: lis_matrix_set_size(A,0,n);
12:
13: perm[0] = 3; perm[1] = 1; perm[2] = 2; perm[3] = 0;
14: ptr[0] = 0; ptr[1] = 4; ptr[2] = 7; ptr[3] = 8;
15: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 0;
16: index[4] = 2; index[5] = 1; index[6] = 2; index[7] = 3;
17: value[0] = 41; value[1] = 21; value[2] = 32; value[3] = 11;
18: value[4] = 43; value[5] = 22; value[6] = 33; value[7] = 44;
20: lis_matrix_set_jad(nnz,maxnzr,perm,ptr,index,value,A);
21: lis_matrix_assemble(A);
```

5.6.2 Creating Matrices (for Multithreaded Environment)

Figure 14 shows how matrix A in Figure 13 is stored in the JAD format on two threads. A program to create the matrix in the JAD format on two threads is as follows:



Figure 14: Data structure of JAD format (for multithreaded environment).

```
- For multithreaded environment -
 1: LIS_INT n,nnz,maxmaxnzr,nprocs;
 2: LIS_INT *perm,*ptr,*index;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; nnz = 8; maxmaxnzr = 3; nprocs = 2;
 6: perm = (LIS_INT *)malloc( n*sizeof(LIS_INT) );
7: ptr = (LIS_INT *)malloc( nprocs*(maxmaxnzr+1)*sizeof(LIS_INT) );
8: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
9: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
10: lis_matrix_create(0,&A);
11: lis_matrix_set_size(A,0,n);
13: perm[0] = 1; perm[1] = 0; perm[2] = 3; perm[3] = 2;
14: ptr[0] = 0; ptr[1] = 2; ptr[2] = 3; ptr[3] = 0;
15: ptr[4] = 3; ptr[5] = 5; ptr[6] = 7; ptr[7] = 8;
16: index[0] = 0; index[1] = 0; index[2] = 1; index[3] = 0;
17: index[4] = 1; index[5] = 2; index[6] = 2; index[7] = 3;
18: value[0] = 21; value[1] = 11; value[2] = 22; value[3] = 41;
19: value[4] = 32; value[5] = 43; value[6] = 33; value[7] = 44;
20:
21: lis_matrix_set_jad(nnz,maxmaxnzr,perm,ptr,index,value,A);
22: lis_matrix_assemble(A);
```

5.6.3 Creating Matrices (for Multiprocessing Environment)

Figure 15 shows how matrix A in Figure 13 is stored in the JAD format on two processing elements. A program to create the matrix in the JAD format on two processing elements is as follows:



Figure 15: Data structure of JAD format (for multiprocessing environment).

```
- For multiprocessing environment -
 1: LIS_INT i,n,nnz,maxnzr,my_rank;
 2: LIS_INT *perm,*ptr,*index;
3: LIS_SCALAR *value;
 4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; nnz = 3; maxnzr = 2;}
                     {n = 2; nnz = 5; maxnzr = 3;}
8: perm = (LIS_INT *)malloc( n*sizeof(LIS_INT) );
9: ptr = (LIS_INT *)malloc( (maxnzr+1)*sizeof(LIS_INT) );
10: index = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
11: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
12: lis_matrix_create(MPI_COMM_WORLD,&A);
13: lis_matrix_set_size(A,n,0);
14: if( my_rank==0 ) {
15:
        perm[0] = 1; perm[1] = 0;
16:
        ptr[0] = 0; ptr[1] = 2; ptr[2] = 3;
        index[0] = 0; index[1] = 0; index[2] = 1;
17:
18:
        value[0] = 21; value[1] = 11; value[2] = 22;}
19: else {
20:
        perm[0] = 3; perm[1] = 2;
        ptr[0] = 0; ptr[1] = 2; ptr[2] = 4; ptr[3] = 5;
21:
        index[0] = 0; index[1] = 1; index[2] = 2; index[3] = 2; index[4] = 3;
22:
        value[0] = 41; value[1] = 32; value[2] = 43; value[3] = 33; value[4] = 44;}
24: lis_matrix_set_jad(nnz,maxnzr,perm,ptr,index,value,A);
25: lis_matrix_assemble(A);
```

5.6.4 Associating Arrays

To associate an array required by the JAD format with matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_jad(LIS_INT nnz, LIS_INT maxnzr, LIS_INT perm[], LIS_INT ptr[], LIS_INT index[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_jad(LIS_INTEGER nnz, LIS_INTEGER maxnzr, LIS_INTEGER perm(), LIS_INTEGER ptr(), LIS_INTEGER index(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.7 Block Sparse Row (BSR)

The BSR format breaks down matrix A into partial matrices called blocks of size $r \times c$. The BSR format stores the nonzero blocks, in which at least one nonzero element exists, in a format similar to that of CSR. Assume that nr = n/r and bnnz are the numbers of nonzero blocks of A. The BSR format uses three arrays bptr, bindex and value to store data.

- value is a double precision array of length $bnnz \times r \times c$, which stores the values of the elements of the nonzero blocks.
- \bullet bindex is an integer array of length bnnz, which stores the block column numbers of the nonzero blocks.
- bptr is an integer array of length nr + 1, which stores the starting points of the block rows in the array bindex.

5.7.1 Creating Matrices (for Serial and Multithreaded Environments)

The diagram on the right in Figure 16 shows how matrix A in Figure 16 is stored in the BSR format. A program to create the matrix in the BSR format is as follows:

$$A = \begin{pmatrix} 11 & & & & & \\ 21 & 22 & & & & \\ \hline & 32 & 33 & & \\ 41 & & 43 & 44 \end{pmatrix} \begin{pmatrix} 0 & 1 & 3 & & & \\ \hline & 0 & 0 & 1 & & & \\ \hline & 0 & 0 & 1 & & & \\ \hline & 11 & 21 & 0 & 22 & 0 & 41 & 32 & 0 & 33 & 43 & 0 & 44 & \\ \hline & A. \text{ bindex} \\ & A. \text{value} \end{pmatrix}$$

Figure 16: Data structure of BSR format (for serial and multithreaded environments).

```
- For serial and multithreaded environments
 1: LIS_INT n,bnr,bnc,nr,nc,bnnz;
 2: LIS_INT *bptr,*bindex;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: n = 4; bnr = 2; bnc = 2; bnnz = 3; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;
          = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
 7: bindex = (LIS_INT *)malloc( bnnz*sizeof(LIS_INT) );
8: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: bptr[0] = 0; bptr[1] = 1; bptr[2] = 3;
13: bindex[0] = 0; bindex[1] = 0; bindex[2] = 1;
14: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
15: value[4] = 0; value[5] = 41; value[6] = 32; value[7] = 0;
16: value[8] = 33; value[9] = 43; value[10] = 0; value[11] = 44;
18: lis_matrix_set_bsr(bnr,bnc,bnnz,bptr,bindex,value,A);
19: lis_matrix_assemble(A);
```

5.7.2 Creating Matrices (for Multiprocessing Environment)

Figure 17 shows how matrix A in Figure 16 is stored in the BSR format on two processing elements. A program to create the matrix in the BSR format on two processing elements is as follows:



Figure 17: Data structure of BSR format (for multiprocessing environment).

```
    For multiprocessing environment -

 1: LIS_INT n,bnr,bnc,nr,nc,bnnz,my_rank;
2: LIS_INT *bptr,*bindex;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; bnr = 2; bnc = 2; bnnz = 1; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
                     {n = 2; bnr = 2; bnc = 2; bnnz = 2; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
           = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
9: bindex = (LIS_INT *)malloc( bnnz*sizeof(LIS_INT) );
10: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
14:
        bptr[0] = 0; bptr[1] = 1;
15:
        bindex[0] = 0;
        value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;}
16:
17: else {
18:
        bptr[0] = 0; bptr[1] = 2;
19:
        bindex[0] = 0; bindex[1] = 1;
        value[0] = 0; value[1] = 41; value[2] = 32; value[3] = 0;
20:
        value[4] = 33; value[5] = 43; value[6] = 0; value[7] = 44;}
21:
22: lis_matrix_set_bsr(bnr,bnc,bnnz,bptr,bindex,value,A);
23: lis_matrix_assemble(A);
```

5.7.3 Associating Arrays

To associate the arrays in the BSR format with matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_bsr(LIS_INT bnr, LIS_INT bnc, LIS_INT bnnz, LIS_INT bptr[], LIS_INT bindex[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_bsr(LIS_INTEGER bnr, LIS_INTEGER bnc, LIS_INTEGER bnnz, LIS_INTEGER bptr(), LIS_INTEGER bindex(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.8 Block Sparse Column (BSC)

The BSC format breaks down matrix A into partial matrices called blocks of size $r \times c$. The BSC format stores the nonzero blocks, in which at least one nonzero element exists, in a format similar to that of CSC. Assume that nc = n/c and bnnz are the numbers of the nonzero blocks of A. The BSC format uses three arrays bptr, bindex and value to store data.

- value is a double precision array of length $bnnz \times r \times c$, which stores the values of the elements of the nonzero blocks.
- bindex is an integer array of length bnnz, which stores the block row numbers of the nonzero blocks.
- bptr is an integer array of length nc + 1, which stores the starting points of the block columns in the array bindex.

5.8.1 Creating Matrices (for Serial and Multithreaded Environments)

The diagram on the right in Figure 18 shows how matrix A in Figure 18 is stored in the BSC format. A program to create the matrix in the BSC format is as follows:

$$A = \begin{pmatrix} 11 & & & & \\ 21 & 22 & & & \\ \hline & 32 & 33 & \\ 41 & & 43 & 44 \end{pmatrix} \begin{pmatrix} \textbf{0} & \textbf{1} & \textbf{3} & & \\ \textbf{0} & \textbf{1} & \textbf{1} & & \\ \hline & \textbf{0} & \textbf{1} & \textbf{21} & \textbf{0} & \textbf{22} & \textbf{0} & \textbf{41} & \textbf{32} & \textbf{0} & \textbf{33} & \textbf{43} & \textbf{0} & \textbf{44} \\ \hline & \textbf{11} & \textbf{21} & \textbf{0} & \textbf{22} & \textbf{0} & \textbf{41} & \textbf{32} & \textbf{0} & \textbf{33} & \textbf{43} & \textbf{0} & \textbf{44} \\ \hline \end{pmatrix} & \textbf{A.bindex}$$

Figure 18: Data structure of BSC format (for serial and multithreaded environments).

```
- For serial and multithreaded environments -
 1: LIS_INT n,bnr,bnc,nr,nc,bnnz;
 2: LIS_INT *bptr,*bindex;
 3: LIS_SCALAR *value;
 4: LIS_MATRIX A;
5: n = 4; bnr = 2; bnc = 2; bnnz = 3; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;
         = (LIS_INT *)malloc( (nc+1)*sizeof(LIS_INT) );
7: bindex = (LIS_INT *)malloc( bnnz*sizeof(LIS_INT) );
8: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: bptr[0] = 0; bptr[1] = 1; bptr[2] = 3;
13: bindex[0] = 0; bindex[1] = 1; bindex[2] = 1;
14: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
15: value[4] = 0; value[5] = 41; value[6] = 32; value[7] = 0;
16: value[8] = 33; value[9] = 43; value[10] = 0; value[11] = 44;
17:
18: lis_matrix_set_bsc(bnr,bnc,bnnz,bptr,bindex,value,A);
19: lis_matrix_assemble(A);
```

5.8.2 Creating Matrices (for Multiprocessing Environment)

Figure 19 shows how matrix A in Figure 18 is stored in the BSC format on two processing elements. A program to create the matrix in the BSC format on two processing elements is as follows:



Figure 19: Data structure of BSC format (for multiprocessing environment).

```
    For multiprocessing environment -

 1: LIS_INT n,bnr,bnc,nr,nc,bnnz,my_rank;
2: LIS_INT *bptr,*bindex;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_n = 0 ) {n = 2; bnr = 2; bnc = 2; bnnz = 2; nr = (n-1)/bnr + 1; nc = (n-1)/bnc + 1;}
                     {n = 2; bnr = 2; bnc = 2; bnnz = 1; nr = (n-1)/bnr+1; nc = (n-1)/bnc+1;}
           = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
9: bindex = (LIS_INT *)malloc( bnnz*sizeof(LIS_INT) );
10: value = (LIS_SCALAR *)malloc( bnr*bnc*bnnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
        bptr[0] = 0; bptr[1] = 2;
14:
15:
        bindex[0] = 0; bindex[1] = 1;
        value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
16:
        value[4] = 0; value[5] = 41; value[6] = 32; value[7] = 0;}
17:
18: else {
19:
        bptr[0] = 0; bptr[1] = 1;
20:
        bindex[0] = 1;
        value[0] = 33; value[1] = 43; value[2] = 0; value[3] = 44;}
21:
22: lis_matrix_set_bsc(bnr,bnc,bnnz,bptr,bindex,value,A);
23: lis_matrix_assemble(A);
```

5.8.3 Associating Arrays

To associate the arrays in the BSC format with matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_bsc(LIS_INT bnr, LIS_INT bnc, LIS_INT bnnz, LIS_INT bptr[], LIS_INT bindex[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_bsc(LIS_INTEGER bnr, LIS_INTEGER bnc, LIS_INTEGER bnnz, LIS_INTEGER bptr(), LIS_INTEGER bindex(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.9 Variable Block Row (VBR)

The VBR format is the generalized version of the BSR format. The division points of the rows and columns are given by the arrays row and col. The VBR format stores the nonzero blocks (the blocks in which at least one nonzero element exists) in a format similar to that of CSR. Assume that nr and nc are the numbers of row and column divisions, respectively, and that bnnz denotes the number of nonzero blocks of A, and nnz denotes the total number of elements of the nonzero blocks. The VBR format uses six arrays, bptr, bindex, row, col, ptr, and value, to store data.

- row is an integer array of length nr + 1, which stores the starting row number of the block rows.
- col is an integer array of length nc + 1, which stores the starting column number of the block columns.
- bindex is an integer array of length *bnnz*, which stores the block column numbers of the nonzero blocks.
- bptr is an integer array of length nr + 1, which stores the starting points of the block rows in the array bindex.
- \bullet value is a double precision array of length nnz, which stores the values of the elements of the nonzero blocks.
- ptr is an integer array of length bnnz + 1, which stores the starting points of the nonzero blocks in the array value.

5.9.1 Creating Matrices (for Serial and Multithreaded Environments)

The diagram on the right in Figure 20 shows how matrix A in Figure 20 is stored in the VBR format. A program to create the matrix in the VBR format is as follows:



Figure 20: Data structure of VBR format (for serial and multithreaded environments).

```
- For serial and multithreaded environments -
 1: LIS_INT n,nnz,nr,nc,bnnz;
2: LIS_INT *row,*col,*ptr,*bptr,*bindex;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
 5: n = 4; nnz = 11; bnnz = 6; nr = 3; nc = 3;
 6: bptr = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
           = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
8: col
           = (LIS_INT *)malloc( (nc+1)*sizeof(LIS_INT) );
           = (LIS_INT *)malloc( (bnnz+1)*sizeof(LIS_INT) );
9: ptr
10: bindex = (LIS_INT *)malloc( bnnz*sizeof(LIS_INT) );
11: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
12: lis_matrix_create(0,&A);
13: lis_matrix_set_size(A,0,n);
15: bptr[0] = 0; bptr[1] = 1; bptr[2] = 3; bptr[3] = 6;
16: row[0] = 0; row[1] = 1; row[2] = 3; row[3] = 4;
17: col[0] = 0; col[1] = 1; col[2] = 3; col[3] = 4;
18: bindex[0] = 0; bindex[1] = 0; bindex[2] = 1; bindex[3] = 0;
19: bindex[4] = 1; bindex[5] = 2;
20: ptr[0]
              = 0; ptr[1]
                               = 1; ptr[2]
                                               = 3; ptr[3]
                                                                   7;
                               = 10; ptr[6]
                                                = 11;
21: ptr[4]
              = 8; ptr[5]
22: value[0] = 11; value[1] = 21; value[2] = 0; value[3]
23: value[4] = 32; value[5] = 0; value[6] = 33; value[7]
24: value[8] = 0; value[9] = 43; value[10] = 44;
26: lis_matrix_set_vbr(nnz,nr,nc,bnnz,row,col,ptr,bptr,bindex,value,A);
27: lis_matrix_assemble(A);
```

5.9.2 Creating Matrices (for Multiprocessing Environment)

Figure 21 shows how matrix A in Figure 20 is stored in the VBR format on two processing elements. A program to create the matrix in the VBR format on two processing elements is as follows:



Figure 21: Data structure of VBR format (for multiprocessing environment).

```
- For multiprocessing environment
 1: LIS_INT n,nnz,nr,nc,bnnz,my_rank;
 2: LIS_INT *row,*col,*ptr,*bptr,*bindex;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
 6: if( my_rank==0 ) {n = 2; nnz = 7; bnnz = 3; nr = 2; nc = 3;}
                     {n = 2; nnz = 4; bnnz = 3; nr = 1; nc = 3;}
7: else
           = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
8: bptr
          = (LIS_INT *)malloc( (nr+1)*sizeof(LIS_INT) );
9: row
          = (LIS_INT *)malloc( (nc+1)*sizeof(LIS_INT) );
10: col
          = (LIS_INT *)malloc( (bnnz+1)*sizeof(LIS_INT) );
12: bindex = (LIS_INT *)malloc( bnnz*sizeof(LIS_INT) );
13: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
14: lis_matrix_create(MPI_COMM_WORLD,&A);
15: lis_matrix_set_size(A,n,0);
16: if( my_rank==0 ) {
        bptr[0] = 0; bptr[1] = 1; bptr[2] = 3;
17:
        row[0] = 0; row[1] = 1; row[2] = 3;
18:
        col[0] = 0; col[1] = 1; col[2] = 3; col[3] = 4;
19:
        bindex[0] = 0; bindex[1] = 0; bindex[2] = 1;
20:
                 = 0; ptr[1]
                               = 1; ptr[2]
                                                 = 3; ptr[3]
21:
        value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
22:
23:
        value[4] = 32; value[5] = 0; value[6] = 33;}
24: else {
        bptr[0] = 0; bptr[1] = 3;
25:
        row[0] = 3; row[1] = 4;
26:
27:
        col[0] = 0; col[1] = 1; col[2] = 3; col[3] = 4;
28:
        bindex[0] = 0; bindex[1] = 1; bindex[2] = 2;
29:
        ptr[0]
                 = 0; ptr[1]
                                 = 1; ptr[2]
                                                 = 3; ptr[3]
        value[0] = 41; value[1] = 0; value[2] = 43; value[3] = 44;}
31: lis_matrix_set_vbr(nnz,nr,nc,bnnz,row,col,ptr,bptr,bindex,value,A);
32: lis_matrix_assemble(A);
```

5.9.3 Associating Arrays

To associate the arrays in the VBR format with matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_vbr(LIS_INT nnz, LIS_INT nr, LIS_INT nc, LIS_INT bnnz, LIS_INT row[], LIS_INT col[], LIS_INT ptr[], LIS_INT bptr[], LIS_INT bindex[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_vbr(LIS_INTEGER nnz, LIS_INTEGER nr, LIS_INTEGER nc, LIS_INTEGER bnnz, LIS_INTEGER row(), LIS_INTEGER col(), LIS_INTEGER ptr(), LIS_INTEGER bptr(), LIS_INTEGER bindex(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.10 Coordinate (COO)

The COO format uses three arrays row, col and value to store data.

- value is a double precision array of length nnz, which stores the values of the nonzero elements.
- row is an integer array of length nnz, which stores the row numbers of the nonzero elements.
- col is an integer array of length nnz, which stores the column numbers of the nonzero elements.

5.10.1 Creating Matrices (for Serial and Multithreaded Environments)

The diagram on the right in Figure 22 shows how matrix A in Figure 22 is stored in the COO format. A program to create the matrix in the COO format is as follows:

Figure 22: Data structure of COO format (for serial and multithreaded environments).

```
    For serial and multithreaded environments -

 1: LIS_INT n,nnz;
 2: LIS_INT *row,*col;
 3: LIS_SCALAR *value;
 4: LIS_MATRIX A;
5: n = 4; nnz = 8;
 6: row = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
         = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
8: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
9: lis_matrix_create(0,&A);
10: lis_matrix_set_size(A,0,n);
11:
12: row[0] = 0; row[1] = 1; row[2] = 3; row[3] = 1;
13: row[4] = 2; row[5] = 2; row[6] = 3; row[7] = 3;
14: col[0] = 0; col[1] = 0; col[2] = 0; col[3] = 1;
15: col[4] = 1; col[5] = 2; col[6] = 2; col[7] = 3;
16: value[0] = 11; value[1] = 21; value[2] = 41; value[3] = 22;
17: value[4] = 32; value[5] = 33; value[6] = 43; value[7] = 44;
19: lis_matrix_set_coo(nnz,row,col,value,A);
20: lis_matrix_assemble(A);
```

5.10.2 Creating Matrices (for Multiprocessing Environment)

Figure 23 shows how matrix A in Figure 22 is stored in the COO format on two processing elements. A program to create the matrix in the COO format on two processing elements is as follows:

0	1	1	3	2	2	3	3	A.row
0	0	1	0	1	2	2	3	A.col
11	21	22	41	32	33	43	44	A.value
PE	Ξ0		PE	Ξ1				

Figure 23: Data structure of COO format (for multiprocessing environment).

```
- For multiprocessing environment -
 1: LIS_INT n,nnz,my_rank;
2: LIS_INT *row,*col;
3: LIS_SCALAR *value;
4: LIS_MATRIX A;
5: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
6: if( my_rank==0 ) {n = 2; nnz = 3;}
                     {n = 2; nnz = 5;}
7: else
         = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
8: row
9: col = (LIS_INT *)malloc( nnz*sizeof(LIS_INT) );
10: value = (LIS_SCALAR *)malloc( nnz*sizeof(LIS_SCALAR) );
11: lis_matrix_create(MPI_COMM_WORLD,&A);
12: lis_matrix_set_size(A,n,0);
13: if( my_rank==0 ) {
14:
        row[0] = 0; row[1] = 1; row[2] = 1;
15:
        col[0] = 0; col[1] = 0; col[2] = 1;
        value[0] = 11; value[1] = 21; value[2] = 22;}
16:
17: else {
18:
        row[0] = 3; row[1] = 2; row[2] = 2; row[3] = 3; row[4] = 3;
19:
        col[0] = 0; col[1] = 1; col[2] = 2; col[3] = 2; col[4] = 3;
        value[0] = 41; value[1] = 32; value[2] = 33; value[3] = 43; value[4] = 44;}
21: lis_matrix_set_coo(nnz,row,col,value,A);
22: lis_matrix_assemble(A);
```

5.10.3 Associating Arrays

To associate the arrays in the COO format with matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_coo(LIS_INT nnz, LIS_INT row[], LIS_INT col[], LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_coo(LIS_INTEGER nnz, LIS_INTEGER row(), LIS_INTEGER col(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

5.11 Dense (DNS)

The DNS format uses one array value to store data.

• value is a double precision array of length $n \times n$, which stores the values of the elements with priority given to the columns.

5.11.1 Creating Matrices (for Serial and Multithreaded Environments)

The right diagram in Figure 24 shows how matrix A in Figure 24 is stored in the DNS format. A program to create the matrix in the DNS format is as follows:

Figure 24: Data structure of DNS format (for serial and multithreaded environments).

```
To serial and multithreaded environments

1: LIS_INT n;

2: LIS_SCALAR *value;

3: LIS_MATRIX A;

4: n = 4;

5: value = (LIS_SCALAR *)malloc( n*n*sizeof(LIS_SCALAR) );

6: lis_matrix_create(0,&A);

7: lis_matrix_set_size(A,0,n);

8:

9: value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 41;

10: value[4] = 0; value[5] = 22; value[6] = 32; value[7] = 0;

11: value[8] = 0; value[9] = 0; value[10]= 33; value[11]= 43;

12: value[12]= 0; value[13]= 0; value[14]= 0; value[15]= 44;

13:

14: lis_matrix_set_dns(value,A);

15: lis_matrix_assemble(A);
```

5.11.2 Creating Matrices (for Multiprocessing Environment)

Figure 25 shows how matrix A in Figure 24 is stored in the DNS format on two processing elements. A program to create the matrix in the DNS format on two processing elements is as follows:



Figure 25: Data structure of DNS format (for multiprocessing environment).

```
- For multiprocessing environment -
 1: LIS_INT n,my_rank;
 2: LIS_SCALAR *value;
3: LIS_MATRIX A;
4: MPI_Comm_rank(MPI_COMM_WORLD,&my_rank);
5: if( my_n = 0 ) n = 2;
                     {n = 2;}
6: else
7: value = (LIS_SCALAR *)malloc( n*n*sizeof(LIS_SCALAR) );
8: lis_matrix_create(MPI_COMM_WORLD,&A);
9: lis_matrix_set_size(A,n,0);
10: if( my_rank==0 ) {
11:
        value[0] = 11; value[1] = 21; value[2] = 0; value[3] = 22;
        value[4] = 0; value[5] = 0; value[6] = 0; value[7] = 0;}
12:
13: else {
        value[0] = 0; value[1] = 41; value[2] = 32; value[3] = 0;
14:
        value[4] = 33; value[5] = 43; value[6] = 0; value[7] = 44;}
15:
16: lis_matrix_set_dns(value,A);
17: lis_matrix_assemble(A);
```

5.11.3 Associating Arrays

To associate the arrays in the DNS format with matrix A, the following functions are used:

- C LIS_INT lis_matrix_set_dns(LIS_SCALAR value[], LIS_MATRIX A)
- Fortran subroutine lis_matrix_set_dns(LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)

6 Functions

This section describes the functions which can be employed by the user. The statuses of the solvers are defined as follows:

LIS_SUCCESS(0) Normal termination

LIS_ILL_OPTION(1) Illegal option

LIS_BREAKDOWN(2) Breakdown (division by zero)

LIS_OUT_OF_MEMORY(3) Out of working memory

LIS_MAXITER(4) Maximum number of iterations

LIS_NOT_IMPLEMENTED(5) Not implemented

LIS_ERR_FILE_IO(6) File I/O error

6.1 Operating Vector Elements

Assume that the size of vector v is $global_n$ and that the size of the partial vectors stored on nprocs processing elements is $local_n$. $global_n$ and $local_n$ are called the global size and the local size, respectively.

6.1.1 lis_vector_create

```
C LIS_INT lis_vector_create(LIS_Comm comm, LIS_VECTOR *v)
Fortran subroutine lis_vector_create(LIS_Comm comm, LIS_VECTOR v, LIS_INTEGER ierr)
```

Description

Create vector v.

Input

comm The MPI communicator

Output

v The vector

ierr The return code

Note

For the serial and multithreaded environments, the value of comm is ignored.

6.1.2 lis_vector_destroy

```
C LIS_INT lis_vector_destroy(LIS_VECTOR v)
Fortran subroutine lis_vector_destroy(LIS_VECTOR v, LIS_INTEGER ierr)
```

Description

Destroy vector v.

Input

v The vector to be destroyed

Output

6.1.3 lis_vector_duplicate

```
C LIS_INT lis_vector_duplicate(void *vin, LIS_VECTOR *vout)
Fortran subroutine lis_vector_duplicate(LIS_VECTOR vin, LIS_VECTOR vout,
LIS_INTEGER ierr)
```

Description

Create vector v_{out} , which has the same information as v_{in} .

Input

vin The source vector or matrix

Output

vout The destination vector

ierr The return code

Note

The function lis_vector_duplicate does not copy the values, but allocates only the memory. To copy the values as well, the function lis_vector_copy must be called after this function.

6.1.4 lis_vector_set_size

Description

Assign the size of vector v.

Input

v The vector

local_n The size of the partial vector

global_n The size of the global vector

Output

ierr The return code

Note

Either $local_n$ or $global_n$ must be provided.

For the serial and multithreaded environments, $local_n$ is equal to $global_n$. Therefore, both $lis_vector_set_size(v,n,0)$ and $lis_vector_set_size(v,0,n)$ create a vector of size n.

For the multiprocessing environment, $lis_vector_set_size(v,n,0)$ creates a partial vector of size n on each processing element. On the other hand, $lis_vector_set_size(v,0,n)$ creates a partial vector of size m_p on processing element p. The values of m_p are determined by the library.

6.1.5 lis_vector_get_size

Description

Get the size of vector v.

Input

v The vector

Output

local_n The size of the partial vector global_n The size of the global vector

ierr The return code

Note

For the serial and multithreaded environments, local_n is equal to global_n.

6.1.6 lis_vector_get_range

```
C LIS_INT lis_vector_get_range(LIS_VECTOR v, LIS_INT *is, LIS_INT *ie)
Fortran subroutine lis_vector_get_range(LIS_VECTOR v, LIS_INTEGER is,
LIS_INTEGER ie, LIS_INTEGER ierr)
```

Description

Get the location of the partial vector v in the global vector.

Input

v The partial vector

Output

is The location where the partial vector v starts in the global vector ie The location where the partial vector v ends in the global vector

ierr The return code

Note

For the serial and multithreaded environments, a vector of size n results in is = 0 and ie = n in the C version, and is = 1 and ie = n + 1 in the Fortran version.

6.1.7 lis_vector_set_value

```
C LIS_INT lis_vector_set_value(LIS_INT flag, LIS_INT i, LIS_SCALAR value, LIS_VECTOR v)

Fortran subroutine lis_vector_set_value(LIS_INTEGER flag, LIS_INTEGER i, LIS_SCALAR value, LIS_VECTOR v, LIS_INTEGER ierr)
```

Description

Assign the scalar value to the i-th row of vector v.

Input

 ${\tt flag} \qquad \qquad {\tt LIS_INS_VALUE} \, : \, v[i] = value$

 $\texttt{LIS_ADD_VALUE} : v[i] = v[i] + value$

i The location where the value is assigned

value The scalar value to be assigned

v The vector

Output

 \mathbf{v} The vector with the scalar value assigned to the i-th row

ierr The return code

Note

For the multiprocessing environment, the i-th row of the global vector must be specified instead of the i-th row of the partial vector.

6.1.8 lis_vector_get_value

Description

Get the scalar value of the i-th row of vector v.

Input

i The location where the value is assigned

v The source vector

Output

value The value of the *i*-th row

ierr The return code

Note

For the multiprocessing environment, the i-th row of the global vector must be specified.

6.1.9 lis_vector_set_values

Description

Assign scalar value[i] to the index[i]-th row of vector v, where i = 0, 1, ..., count - 1.

Input

 $\texttt{flag} \qquad \qquad \texttt{LIS_INS_VALUE} \, : \, v[index[i]] = value[i]$

 $\texttt{LIS_ADD_VALUE} \ : \ v[index[i]] = v[index[i]] + value[i]$

The number of elements in the array that stores the scalar values

to be assigned

index The array that stores the location where the scalar values are as-

signed

value The array that stores the scalar values to be assigned

v The vector

Output

v The vector with scalar value[i] assigned to the index[i]-th row

ierr The return code

Note

For the multiprocessing environment, the index[i]-th row of the global vector must be specified instead of the index[i]-th row of the partial vector.

6.1.10 lis_vector_get_values

```
C LIS_INT lis_vector_get_values(LIS_VECTOR v, LIS_INT start, LIS_INT count, LIS_SCALAR value[])
Fortran subroutine lis_vector_get_values(LIS_VECTOR v, LIS_INTEGER start, LIS_INTEGER count, LIS_SCALAR value(), LIS_INTEGER ierr)
```

Description

Get scalar value[i] of the start + i-th row of vector v, where i = 0, 1, ..., count - 1.

Input

start The starting location

count The number of values to get

v The source vector

Output

value The array to store the scalar values

ierr The return code

Note

For the multiprocessing environment, the start + i-th row of the global vector must be specified.

6.1.11 lis_vector_scatter

Description

Assign scalar value[i] to the *i*-th row of vector v, where $i = 0, 1, ..., global_n - 1$.

Input

value The array that stores the scalar values to be assigned

Output

v The vector

ierr The return code

Note

6.1.12 lis_vector_gather

Description

Get scalar value[i] of the i-th row of vector v, where $i = 0, 1, ..., global_n - 1$.

Input

v The source vector

Output

value The array that stores the scalar values

ierr The return code

Note

6.2 Operating Matrix Elements

Assume that the size of matrix A is $global_n \times global_n$ and that the size of each partial matrix stored on nprocs processing elements is $local_n \times global_n$. Here, $global_n$ and $local_n$ are called the number of rows of the global matrix and the number of rows of the partial matrix, respectively.

6.2.1 lis_matrix_create

C LIS_INT lis_matrix_create(LIS_Comm comm, LIS_MATRIX *A)
Fortran subroutine lis_matrix_create(LIS_Comm comm, LIS_MATRIX A, LIS_INTEGER ierr)

Description

Create matrix A.

Input

comm The MPI communicator

Output

A The matrix

ierr The return code

Note

For the serial and multithreaded environments, the value of comm is ignored.

6.2.2 lis_matrix_destroy

C LIS_INT lis_matrix_destroy(LIS_MATRIX A)
Fortran subroutine lis_matrix_destroy(LIS_MATRIX A, LIS_INTEGER ierr)

Description

Destroy matrix A.

Input

A The matrix to be destroyed

Output

ierr The return code

Note

The function $lis_{matrix_destroy}$ frees the memory for the set of arrays associated with matrix A.

6.2.3 lis_matrix_duplicate

Description

Create matrix A_{out} which has the same information as A_{in} .

Input

Ain The source matrix

Output

Aout The destination matrix

ierr The return code

Note

The function <code>lis_matrix_duplicate</code> does not copy the values of the elements of the matrix, but allocates only the memory. To copy the values of the elements as well, the function <code>lis_matrix_copy</code> must be called after this function.

6.2.4 lis_matrix_malloc

Description

Allocate memory for matrix A.

Input

A The matrix

nnz_row The average number of nonzero elements in each row

nnz The array of numbers of nonzero elements in each row

Output

ierr The return code

Note

Either nnz_row or nnz must be provided.

This function allocates memory for the function lis_matrix_set_value.

6.2.5 lis_matrix_set_value

Description

Assign the scalar value to the (i, j)-th element of matrix A.

Input

Output

A The matrix

ierr The return code

Note

For the multiprocessing environment, the i-th row and the j-th column of the global matrix must be specified.

The function lis_matrix_set_value stores the assigned value in a temporary internal format. Therefore, after lis_matrix_set_value is called, the function lis_matrix_assemble must be called.

To improve performance of this function, the introduction of the function <code>lis_matrix_malloc</code> should be considered.

For large matrices, the introduction of the function <code>lis_matrix_set_type</code> should be considered. See <code>lis-(\$VERSION)/test/test2.c</code> and <code>lis-(\$VERSION)/test/test2f.F90</code> for details.

6.2.6 lis_matrix_assemble

```
C LIS_INT lis_matrix_assemble(LIS_MATRIX A)
Fortran subroutine lis_matrix_assemble(LIS_MATRIX A, LIS_INTEGER ierr)
```

Description

Assemble matrix A into the specified storage format.

Input

A The matrix

Output

A The matrix assembled into the specified storage format

6.2.7 lis_matrix_set_size

```
C LIS_INT lis_matrix_set_size(LIS_MATRIX A, LIS_INT local_n,
LIS_INT global_n)

Fortran subroutine lis_matrix_set_size(LIS_MATRIX A, LIS_INTEGER local_n,
LIS_INTEGER global_n, LIS_INTEGER ierr)
```

Description

Assign the size of matrix A.

Input

A The matrix

local_n The number of rows of the partial matrix global_n The number of rows of the global matrix

Output

ierr The return code

Note

Either $local_n$ or $global_n$ must be provided.

For the serial and multithreaded environments, $local_n$ is equal to $global_n$. Therefore, both $lis_matrix_set_size(A,n,0)$ and $lis_matrix_set_size(A,0,n)$ create a matrix of size $n \times n$.

For the multiprocessing environment, $lis_matrix_set_size(A,n,0)$ creates a partial matrix of size $n \times N$ on each processing element, where N is the total sum of n. On the other hand, $lis_matrix_set_size(A,0,n)$ creates a partial matrix of size $m_p \times n$ on processing element p. The

lis_matrix_set_size(A,0,n) creates a partial matrix of size $m_p \times n$ on processing element p. T values of m_p are determined by the library.

6.2.8 lis_matrix_get_size

```
C LIS_INT lis_matrix_get_size(LIS_MATRIX A, LIS_INT *local_n,
LIS_INT *global_n)
Fortran subroutine lis_matrix_get_size(LIS_MATRIX A, LIS_INTEGER local_n,
LIS_INTEGER global_n, LIS_INTEGER ierr)
```

Description

Get the size of matrix A.

Input

A The matrix

Output

local_n The number of rows of the partial matrix
global_n The number of rows of the global matrix

ierr The return code

Note

For the serial and multithreaded environments, *local_n* is equal to *global_n*.

6.2.9 lis_matrix_get_range

```
C LIS_INT lis_matrix_get_range(LIS_MATRIX A, LIS_INT *is, LIS_INT *ie)
Fortran subroutine lis_matrix_get_range(LIS_MATRIX A, LIS_INTEGER is,
LIS_INTEGER ie, LIS_INTEGER ierr)
```

Description

Get the location of partial matrix A in the global matrix.

Input

A The partial matrix

Output

is The location where partial matrix A starts in the global matrix

ie The location where partial matrix A ends in the global matrix

ierr The return code

Note

For the serial and multithreaded environments, a matrix of $n \times n$ results in is = 0 and ie = n in the C version, and is = 1 and ie = n + 1 in the Fortran version.

6.2.10 lis_matrix_get_nnz

Description

Get the number of nonzero elements of matrix A.

Input

A The matrix

Output

nnz The number of nonzero elements

ierr The return code

Note

For the multiprocessing environment, this function gets the number of nonzero elements of partial matrix A.

6.2.11 lis_matrix_set_type

Description

Assign the storage format.

Input

A The matrix

matrix_type The storage format

Output

ierr The return code

Note

 $\mathtt{matrix_type}$ of A is $\mathtt{LIS_MATRIX_CSR}$ when the matrix is created. The table below shows the available storage formats for $\mathtt{matrix_type}$.

Storage format	matrix_type		
Compressed Sparse Row	(CSR)	{LIS_MATRIX_CSR 1}	
Compressed Sparse Column	(CSC)	{LIS_MATRIX_CSC 2}	
Modified Compressed Sparse Row	(MSR)	{LIS_MATRIX_MSR 3}	
Diagonal	(DIA)	{LIS_MATRIX_DIA 4}	
Ellpack-Itpack Generalized Diagonal	(ELL)	{LIS_MATRIX_ELL 5}	
Jagged Diagonal	(JAD)	{LIS_MATRIX_JAD 6}	
Block Sparse Row	(BSR)	{LIS_MATRIX_BSR 7}	
Block Sparse Column	(BSC)	{LIS_MATRIX_BSC 8}	
Variable Block Row	(VBR)	{LIS_MATRIX_VBR 9}	
Coordinate	(COO)	{LIS_MATRIX_COO 10}	
Dense	(DNS)	{LIS_MATRIX_DNS 11}	

6.2.12 lis_matrix_get_type

Description

Get the storage format.

Input

A The matrix

Output

matrix_type The storage format

6.2.13 lis_matrix_set_csr

Description

Associate the arrays in the CSR format with matrix A.

Input

nnz The number of nonzero elements

ptr, index, value The arrays in the CSR format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_csr is called, the function lis_matrix_assemble must be called. Array indexing must be zero-origin in the Fortran version.

6.2.14 lis_matrix_set_csc

Description

Associate the arrays in the CSC format with matrix A.

Input

nnz The number of nonzero elements

ptr, index, value The arrays in the CSC format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_csc is called, the function lis_matrix_assemble must be called. Array indexing must be zero-origin in the Fortran version.

6.2.15 lis_matrix_set_msr

Description

Associate the arrays in the MSR format with matrix A.

Input

nnz The number of nonzero elements

ndz The number of nonzero elements in the diagonal

index, value The arrays in the MSR format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_msr is called, the function lis_matrix_assemble must be called. Array indexing must be zero-origin in the Fortran version.

6.2.16 lis_matrix_set_dia

Description

Associate the arrays in the DIA format with matrix A.

Input

nnd The number of nonzero diagonal elements

index, value The arrays in the DIA format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_dia is called, the function lis_matrix_assemble must be called. Array indexing must be zero-origin in the Fortran version.

6.2.17 lis_matrix_set_ell

Description

Associate the arrays in the ELL format with matrix A.

Input

maxnzr The maximum number of nonzero elements in each row

index, value The arrays in the ELL format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_ell is called, the function lis_matrix_assemble must be called. Array indexing must be zero-origin in the Fortran version.

6.2.18 lis_matrix_set_jad

Description

Associate the arrays in the JAD format with matrix A.

Input

nnz The number of nonzero elements

maxnzr The maximum number of nonzero elements in each row

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_jad is called, the function lis_matrix_assemble must be called. Array indexing must be zero-origin in the Fortran version.

6.2.19 lis_matrix_set_bsr

```
C LIS_INT lis_matrix_set_bsr(LIS_INT bnr, LIS_INT bnc, LIS_INT bnnz, LIS_INT bptr[], LIS_INT bindex[], LIS_SCALAR value[], LIS_MATRIX A)

Fortran subroutine lis_matrix_set_bsr(LIS_INTEGER bnr, LIS_INTEGER bnc, LIS_INTEGER bnnz, LIS_INTEGER bptr(), LIS_INTEGER bindex(), LIS_SCALAR value(), LIS_MATRIX A, LIS_INTEGER ierr)
```

Description

Associate the arrays in the BSR format with matrix A.

Input

bnr The row block size
bnc The column block size

bnnz The number of nonzero blocks
bptr, bindex, value The arrays in the BSR format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_bsr is called, the function lis_matrix_assemble must be called. Array indexing must be zero-origin in the Fortran version.

6.2.20 lis_matrix_set_bsc

Description

Associate the arrays in the BSC format with matrix A.

Input

bnr The row block size
bnc The column block size

bnnz The number of nonzero blocks
bptr, bindex, value The arrays in the BSC format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_bsc is called, the function lis_matrix_assemble must be called. Array indexing must be zero-origin in the Fortran version.

6.2.21 lis_matrix_set_vbr

```
C LIS_INT lis_matrix_set_vbr(LIS_INT nnz, LIS_INT nr, LIS_INT nc,
    LIS_INT bnnz, LIS_INT row[], LIS_INT col[], LIS_INT ptr[],
    LIS_INT bptr[], LIS_INT bindex[], LIS_SCALAR value[],
    LIS_MATRIX A)

Fortran subroutine lis_matrix_set_vbr(LIS_INTEGER nnz, LIS_INTEGER nr,
    LIS_INTEGER nc, LIS_INTEGER bnnz, LIS_INTEGER row(),
    LIS_INTEGER col(), LIS_INTEGER ptr(), LIS_INTEGER bptr(),
    LIS_INTEGER bindex(), LIS_SCALAR value(), LIS_MATRIX A,
    LIS_INTEGER ierr)
```

Description

Associate the arrays in the VBR format with matrix A.

Input

nnz
The number of nonzero elements

The number of row blocks

The number of column blocks

The number of nonzero blocks

Tow, col, ptr, bptr, bindex, value The arrays in the VBR format

The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_vbr is called, the function lis_matrix_assemble must be called. Array indexing must be zero-origin in the Fortran version.

6.2.22 lis_matrix_set_coo

Description

Associate the arrays in the COO format with matrix A.

Input

nnz The number of nonzero elements

row, col, value The arrays in the COO format

A The matrix

Output

A The matrix associated with the arrays

Note

After lis_matrix_set_coo is called, the function lis_matrix_assemble must be called. Array indexing must be zero-origin in the Fortran version.

6.2.23 lis_matrix_set_dns

C LIS_INT lis_matrix_set_dns(LIS_SCALAR value[], LIS_MATRIX A)
Fortran subroutine lis_matrix_set_dns(LIS_SCALAR value(), LIS_MATRIX A,
LIS_INTEGER ierr)

Description

Associate the array in the DNS format with matrix A.

Input

value The array in the DNS format

A The matrix

Output

A The matrix associated with the array

Note

After lis_matrix_set_dns is called, the function lis_matrix_assemble must be called. Array indexing must be zero-origin in the Fortran version.

6.2.24 lis_matrix_unset

```
C LIS_INT lis_matrix_unset(LIS_MATRIX A)
Fortran subroutine lis_matrix_unset(LIS_MATRIX A, LIS_INTEGER ierr)
```

Description

Unassociate the arrays from matrix A without deallocating memory.

Input

A The matrix associated with the arrays

Output

A The unassociated matrix

Note

After lis_matrix_unset is called, the function lis_matrix_destroy must be called.

6.3 Computing with Vectors and Matrices

6.3.1 lis_vector_swap

```
C LIS_INT lis_vector_swap(LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_vector_swap(LIS_VECTOR x, LIS_VECTOR y, LIS_INTEGER ierr)
```

Description

Swap the values of the vector elements.

Input

x, y The source vectors

Output

x, y

The destination vectors

ierr The return code

6.3.2 lis_vector_copy

```
C LIS_INT lis_vector_copy(LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_vector_copy(LIS_VECTOR x, LIS_VECTOR y, LIS_INTEGER ierr)
```

Description

Copy the values of the vector elements.

Input

x The source vector

Output

y The destination vector

6.3.3 lis_vector_axpy

C LIS_INT lis_vector_axpy(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_vector_axpy(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y,
LIS_INTEGER ierr)

Description

Calculate the sum of the vectors $y = \alpha x + y$.

Input

alpha The scalar value

x, y The vectors

Output

y $\alpha x + y$ (vector y is overwritten)

ierr The return code

6.3.4 lis_vector_xpay

C LIS_INT lis_vector_xpay(LIS_VECTOR x, LIS_SCALAR alpha, LIS_VECTOR y)
Fortran subroutine lis_vector_xpay(LIS_VECTOR x, LIS_SCALAR alpha, LIS_VECTOR y,
LIS_INTEGER ierr)

Description

Calculate the sum of the vectors $y = x + \alpha y$.

Input

alpha The scalar value

x, y The vectors

Output

y $x + \alpha y$ (vector y is overwritten)

6.3.5 lis_vector_axpyz

C LIS_INT lis_vector_axpyz(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y, LIS_VECTOR z)

Fortran subroutine lis_vector_axpyz(LIS_SCALAR alpha, LIS_VECTOR x, LIS_VECTOR y, LIS_VECTOR z, LIS_INTEGER ierr)

Description

Calculate the sum of the vectors $z = \alpha x + y$.

Input

alpha The scalar value

x, y The vectors

Output

 \mathbf{z} $\alpha x + y$

ierr The return code

6.3.6 lis_vector_scale

Description

Multiply vector x by scalar α .

${\bf Input}$

alpha The scalar value

x The vector

Output

 αx (vector x is overwritten)

6.3.7 lis_vector_pmul

C LIS_INT lis_vector_pmul(LIS_VECTOR x, LIS_VECTOR y, LIS_VECTOR z)
Fortran subroutine lis_vector_pmul(LIS_VECTOR x, LIS_VECTOR y, LIS_VECTOR z,
LIS_INTEGER ierr)

Description

Multiply each element of vector x by the corresponding element of y.

Input

x, y The vectors

Output

 ${f z}$ The vector that stores the multiplied elements of x

ierr The return code

6.3.8 lis_vector_pdiv

C LIS_INT lis_vector_pdiv(LIS_VECTOR x, LIS_VECTOR y, LIS_VECTOR z)
Fortran subroutine lis_vector_pdiv(LIS_VECTOR x, LIS_VECTOR y, LIS_VECTOR z,
LIS_INTEGER ierr)

Description

Divide each element of vector x by the corresponding element of y.

Input

x, y The vectors

Output

z The vector that stores the divided elements of x

6.3.9 lis_vector_set_all

Description

Assign the scalar value to the elements of vector x.

Input

value The scalar value to be assigned

x The vector

Output

x The vector with the value assigned to the elements

ierr The return code

6.3.10 lis_vector_abs

```
C LIS_INT lis_vector_abs(LIS_VECTOR x)
Fortran subroutine lis_vector_abs(LIS_VECTOR x, LIS_INTEGER ierr)
```

Description

Get the absolute values of the elements of vector x.

Input

x The vector

Output

x The vector that stores the absolute values

6.3.11 lis_vector_reciprocal

```
C LIS_INT lis_vector_reciprocal(LIS_VECTOR x)
Fortran subroutine lis_vector_reciprocal(LIS_VECTOR x, LIS_INTEGER ierr)
```

Description

Get the reciprocal values of the elements of vector x.

Input

x The vector

Output

x The vector that stores the reciprocal values

ierr The return code

6.3.12 lis_vector_conjugate

```
C LIS_INT lis_vector_conjugate(LIS_VECTOR x)
Fortran subroutine lis_vector_conjugate(LIS_VECTOR x, LIS_INTEGER ierr)
```

Description

Get the conjugate complex values of the elements of vector x.

Input

x The vector

Output

x The vector that stores the conjugate complex values

6.3.13 lis_vector_shift

Description

Get the shifted values of the elements of vector x.

Input

alpha The amount of the shift

x The vector

Output

x The vector that stores the shifted values

6.3.14 lis_vector_dot

Description

Calculate the Hermitian inner product $y^H x$.

Input

x, y The vectors

Output

value The inner product

ierr The return code

$6.3.15 \quad lis_vector_nhdot$

Description

Calculate the non-Hermitian inner product y^Tx .

Input

x, y The vectors

Output

value The inner product

6.3.16 lis_vector_nrm1

C LIS_INT lis_vector_nrm1(LIS_VECTOR x, LIS_SCALAR *value)
Fortran subroutine lis_vector_nrm1(LIS_VECTOR x, LIS_SCALAR value, LIS_INTEGER ierr)

Description

Calculate the 1-norm of vector x.

Input

x The vector

Output

value The 1-norm of the vector

ierr The return code

6.3.17 lis_vector_nrm2

C LIS_INT lis_vector_nrm2(LIS_VECTOR x, LIS_REAL *value)
Fortran subroutine lis_vector_nrm2(LIS_VECTOR x, LIS_REAL value, LIS_INTEGER ierr)

Description

Calculate the 2-norm of vector x.

Input

x The vector

Output

value The 2-norm of the vector

6.3.18 lis_vector_nrmi

C LIS_INT lis_vector_nrmi(LIS_VECTOR x, LIS_SCALAR *value)
Fortran subroutine lis_vector_nrmi(LIS_VECTOR x, LIS_SCALAR value, LIS_INTEGER ierr)

Description

Calculate the infinity norm of vector x.

Input

x The vector

Output

value The infinity norm of the vector

ierr The return code

6.3.19 lis_vector_sum

C LIS_INT lis_vector_sum(LIS_VECTOR x, LIS_SCALAR *value)
Fortran subroutine lis_vector_sum(LIS_VECTOR x, LIS_SCALAR value, LIS_INTEGER ierr)

Description

Calculate the sum of the elements of vector x.

Input

x The vector

Output

value The sum of the vector elements

6.3.20 lis_matrix_set_blocksize

C LIS_INT lis_matrix_set_blocksize(LIS_MATRIX A, LIS_INT bnr, LIS_INT bnc, LIS_INT row[], LIS_INT col[])
Fortran subroutine lis_matrix_set_blocksize(LIS_MATRIX A, LIS_INTEGER bnr, LIS_INTEGER bnc, LIS_INTEGER row[], LIS_INTEGER col[], LIS_INTEGER ierr)

Description

Assign the block size of the BSR, BSC, and VBR formats.

Input

The row block size of the BSR (BSC) format or the number of row blocks of the VBR format The olumn block size of the BSR (BSC) format or the number of column blocks of the VBR format The array of the row division information about the VBR format The array of the column division information about the VBR format	A	The matrix
column blocks of the VBR format row The array of the row division information about the VBR format col The array of the column division information about the VBR for-	bnr	
The array of the column division information about the VBR for-	bnc	,
·	row	The array of the row division information about the VBR format
	col	·

Output

6.3.21 lis_matrix_convert

Description

Convert matrix A_{in} into A_{out} of the format specified by lis_matrix_set_type.

Input

Ain The source matrix

Output

Aout The destination matrix

ierr The return code

Note

The storage format of A_{out} is set by lis_matrix_set_type. The block size of the BSR, BSC, and VBR formats is set by lis_matrix_set_blocksize. See lis-(\$VERSION)/test/test2.c and lis-(\$VERSION)/test/test2f.F90.

The conversions indicated by 1 in the table below are performed directly, and the others are performed via the indicated formats. The conversions with no indication are performed via the CSR format.

Src \Dst	CSR	CSC	MSR	DIA	ELL	JAD	BSR	BSC	VBR	COO	DNS
CSR		1	1	1	1	1	1	CSC	1	1	1
COO	1	1	1	CSR	CSR	CSR	CSR	CSC	CSR		CSR

6.3.22 lis_matrix_copy

Description

Copy the values of the matrix elements.

Input

Ain The source matrix

Output

Aout The destination matrix

ierr The return code

6.3.23 lis_matrix_axpy

C LIS_INT lis_matrix_axpy(LIS_SCALAR alpha, LIS_MATRIX A, LIS_MATRIX B)
Fortran subroutine lis_matrix_axpy(LIS_SCALAR alpha, LIS_MATRIX A, LIS_MATRIX B,
LIS_INTEGER ierr)

Description

Calculate the sum of the matrices $B = \alpha A + B$.

Input

alpha The scalar value

A, B The matrices

Output

B $\alpha A + B$ (matrix B is overwritten)

ierr The return code

Matrices $A,\,B$ must be in the DNS format.

6.3.24 lis_matrix_xpay

C LIS_INT lis_matrix_xpay(LIS_SCALAR alpha, LIS_MATRIX A, LIS_MATRIX B)
Fortran subroutine lis_matrix_xpay(LIS_SCALAR alpha, LIS_MATRIX A, LIS_MATRIX B,
LIS_INTEGER ierr)

Description

Calculate the sum of the matrices $B = A + \alpha B$.

Input

alpha The scalar value

A, B The matrices

Output

B $A + \alpha B$ (matrix B is overwritten)

ierr The return code

Note

Matrices A, B must be in the DNS format.

6.3.25 lis_matrix_axpyz

C LIS_INT lis_matrix_axpyz(LIS_SCALAR alpha, LIS_MATRIX A, LIS_MATRIX B, LIS_MATRIX C)

Fortran subroutine lis_matrix_axpyz(LIS_SCALAR alpha, LIS_MATRIX A, LIS_MATRIX B, LIS_MATRIX C, LIS_INTEGER ierr)

Description

Calculate the sum of the matrices $C = \alpha A + B$.

Input

alpha The scalar value

A, B The matrices

Output

 \mathbf{C} $\alpha A + B$

ierr The return code

Note

Matrices A, B, and C must be in the DNS format.

6.3.26 lis_matrix_scale

C LIS_INT lis_matrix_scale(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR d, LIS_INT action)

Fortran subroutine lis_matrix_scale(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR d, LIS_INTEGER action, LIS_INTEGER ierr)

Description

Scale matrix A and vector b.

Input

A The matrix
b The vector

action LIS_SCALE_JACOBI : Jacobi scaling $D^{-1}Ax = D^{-1}b$, where D rep-

resents the diagonal of $A = (a_{ij})$

LIS_SCALE_SYMM_DIAG : Diagonal scaling $D^{-1/2}AD^{-1/2}x=D^{-1/2}b$, where $D^{-1/2}$ represents a diagonal matrix with $1/\sqrt{a_{ii}}$ as the

diagona

Output

A The scaled matrix

b The scaled vector

d The vector that stores the diagonal elements of D^{-1} or $D^{-1/2}$

6.3.27 lis_matrix_get_diagonal

C LIS_INT lis_matrix_get_diagonal(LIS_MATRIX A, LIS_VECTOR d)
Fortran subroutine lis_matrix_get_diagonal(LIS_MATRIX A, LIS_VECTOR d,
LIS_INTEGER ierr)

Description

Store the diagonal elements of matrix A to vector d.

Input

A The matrix

Output

d The vector that stores the diagonal elements of the matrix

ierr The return code

6.3.28 lis_matrix_shift_diagonal

Description

Shift the diagonal of matrix A.

Input

alpha The amount of the shift

A The matrix

Output

A The shifted matrix

6.3.29 lis_matvec

C LIS_INT lis_matvec(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_matvec(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y,
LIS_INTEGER ierr)

Description

Calculate the matrix-vector product y = Ax.

Input

A The matrix

x The vector

Output

 \mathbf{y} Ax

ierr The return code

6.3.30 lis_matvect

C LIS_INT lis_matvect(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y)
Fortran subroutine lis_matvect(LIS_MATRIX A, LIS_VECTOR x, LIS_VECTOR y,
LIS_INTEGER ierr)

Description

Calculate the matrix-vector product $y = A^T x$.

Input

A The matrix

x The vector

Output

 \mathbf{y} $A^T x$

6.4 Solving Linear Equations

6.4.1 lis_solver_create

C LIS_INT lis_solver_create(LIS_SOLVER *solver)
Fortran subroutine lis_solver_create(LIS_SOLVER solver, LIS_INTEGER ierr)

Description

Create the solver.

Input

None

Output

solver The solver

ierr The return code

Note

solver has the information on the solver, the preconditioner, etc.

6.4.2 lis_solver_destroy

C LIS_INT lis_solver_destroy(LIS_SOLVER solver)
Fortran subroutine lis_solver_destroy(LIS_SOLVER solver, LIS_INTEGER ierr)

Description

Destroy the solver.

Input

solver The solver to be destroyed

Output

6.4.3 lis_precon_create

C LIS_INT lis_precon_create(LIS_SOLVER solver, LIS_PRECON *precon)
Fortran subroutine lis_precon_create(LIS_SOLVER solver, LIS_PRECON precon,
LIS_INTEGER ierr)

Description

Create the preconditioner.

Input

None

Output

solver The solver

precon The preconditioner

ierr The return code

${\bf 6.4.4 \quad lis_precon_destroy}$

C LIS_INT lis_precon_destroy(LIS_PRECON precon)
Fortran subroutine lis_precon_destroy(LIS_PRECON precon, LIS_INTEGER ierr)

Description

Destroy the preconditioner.

Input

precon The preconditioner to be destroyed

Output

6.4.5 lis_solver_set_option

Description

Set the options for the solver.

Input

text The command line options

Output

solver The solver

ierr The return code

Note

The table below shows the available command line options, where $-i \{cg|1\}$ means -i cg or -i 1 and -maxiter [1000] indicates that -maxiter defaults to 1,000.

Options for Linear Solvers (Default: -i bicg)

Solver	Option	Auxiliary Options	<u> </u>
CG	-i {cg 1}	J 21	
BiCG	-i {bicg 2}		
CGS	-i {cgs 3}		
BiCGSTAB	-i {bicgstab 4}		
BiCGSTAB(l)	-i {bicgstabl 5}	-ell [2]	The degree l
GPBiCG	-i {gpbicg 6}		
TFQMR	-i {tfqmr 7}		
Orthomin(m)	-i {orthomin 8}	-restart [40]	The restart value m
GMRES(m)	-i {gmres 9}	-restart [40]	The restart value m
Jacobi	-i {jacobi 10}		
Gauss-Seidel	-i {gs 11}		
SOR	-i {sor 12}	-omega [1.9]	The relaxation coefficient
			$\omega \ (0 < \omega < 2)$
BiCGSafe	-i {bicgsafe 13}		
CR	-i {cr 14}		
BiCR	-i {bicr 15}		
CRS	-i {crs 16}		
BiCRSTAB	-i {bicrstab 17}		
GPBiCR	-i {gpbicr 18}		
BiCRSafe	-i {bicrsafe 19}		
FGMRES(m)	-i {fgmres 20}	-restart [40]	The restart value m
IDR(s)	-i {idrs 21}	-irestart [2]	The restart value s
IDR(1)	-i {idr1 22}		
MINRES	-i {minres 23}		
COCG	-i {cocg 24}		
COCR	-i {cocr 25}		

Options for Preconditioners (Default: -p none)

Preconditioner	Option	Auxiliary Options	
None	-p {none 0}		
Jacobi	-p {jacobi 1}		
ILU(k)	-p {ilu 2}	-ilu_fill [0]	The fill level k
SSOR	-p {ssor 3}	-ssor_w [1.0]	The relaxation coefficient
			$\omega \ (0 < \omega < 2)$
Hybrid	-p {hybrid 4}	-hybrid_i [sor]	The linear solver
		-hybrid_maxiter [25]	The maximum number of iterations
		-hybrid_tol [1.0e-3]	The convergence tolerance
		-hybrid_w [1.5]	The relaxation coefficient ω
			of the SOR $(0 < \omega < 2)$
		-hybrid_ell [2]	The degree l of the BiCGSTAB(l)
		-hybrid_restart [40]	The restart values
			of the GMRES and Orthomin
I+S	-p {is 5}	-is_alpha [1.0]	The parameter α of $I + \alpha S^{(m)}$
		-is_m [3]	The parameter m of $I + \alpha S^{(m)}$
SAINV	-p {sainv 6}	-sainv_drop [0.05]	The drop criterion
SA-AMG	-p {saamg 7}	-saamg_unsym [false]	Select the unsymmetric version
			(The matrix structure must be
			symmetric)
		-saamg_theta [0.05 0.12]	The drop criterion $a_{ij}^2 \le \theta^2 a_{ii} a_{jj} $
			(symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05]	The drop criterion
		-iluc_rate [5.0]	The ratio of the maximum fill-in
ILUT	-p {ilut 9}		
Additive	-adds true	-adds_iter [1]	The number of iterations
Schwarz			

Other Options

Option					
-maxiter [1000]	The maximum number of iterations				
-tol [1.0e-12]	The convergence tolerance tol				
-tol_w [1.0]	The convergence tolers	ance tol_w			
-print [0]	The output of the resi	dual history			
	-print {none 0}	None			
	-print {mem 1}	Save the residual history			
	-print {out 2}	Output it to the standard output			
	-print {all 3}	Save the residual history and output it			
	_	to the standard output			
-scale [0]	The scaling				
	(The result will overw	rite the original matrix and vectors)			
	-scale {none 0}	No scaling			
	-scale {jacobi 1}	The Jacobi scaling $D^{-1}Ax = D^{-1}b$			
		(D represents the diagonal of $A = (a_{ij})$)			
	-scale {symm_diag 2} The diagonal scaling $D^{-1/2}AD^{-1/2}x = D^{-1/2}b$				
	$(D^{-1/2}$ represents the diagonal matrix				
	with $1/\sqrt{a_{ii}}$ as the diagonal)				
-initx_zeros [1]	The behavior of the initial vector x_0				
	-initx_zeros {false	· · · · · · · · · · · · · · · · · · ·			
		of the function lis_solve()			
	-initx_zeros {true				
-conv_cond [0]	The convergence cond				
	-conv_cond {nrm2_r				
	-conv_cond {nrm2_b	11 - 11			
	-conv_cond {nrm1_b	11 112 11 112			
-omp_num_threads [t]	The number of thread				
1 = -	(t represents the maximum number of threads)				
-storage [0]	The matrix storage format				
-storage_block [2]	The block size of the I				
-f [0]	The precision of the linear solver				
- -	-f {double 0}	Double precision			
	-f {quad 1}	Quadruple precision			

6.4.6 lis_solver_set_optionC

```
C LIS_INT lis_solver_set_optionC(LIS_SOLVER solver)
Fortran subroutine lis_solver_set_optionC(LIS_SOLVER solver, LIS_INTEGER ierr)
```

Description

Set the options for the solver on the command line.

Input

None

Output

solver The solver

ierr The return code

6.4.7 lis_solve

```
C LIS_INT lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
LIS_SOLVER solver)

Fortran subroutine lis_solve(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
LIS_SOLVER solver, LIS_INTEGER ierr)
```

Description

Solve the linear equation Ax = b with the specified solver.

Input

A The coefficient matrix

b The right-hand side vector

x The initial vector

solver The solver

Output

x The solution

The number of iterations, the execution time, etc.

ierr The return code

Note

If the option $-initx_zeros$ {false|0} is specified, the initial vector is given by the argument x. Otherwise, all values of the initial vector are set to 0.

This function returns 0 if the status of the solver (solver->retcode) is LIS_BREAKDOWN or LIS_MAXITER. See also the function lis_solver_get_status().

6.4.8 lis_solve_kernel

```
C LIS_INT lis_solve_kernel(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
LIS_SOLVER solver, LIS_PRECON, precon)

Fortran subroutine lis_solve_kernel(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
LIS_SOLVER solver, LIS_PRECON precon, LIS_INTEGER ierr)
```

Description

Solve the linear equation Ax = b with the specified solver and the predefined preconditioner.

Input

A The coefficient matrix

b The right-hand side vector

x The initial vector

solver The solver

precon The preconditioner

Output

x The solution

The number of iterations, the execution time, etc.

ierr The return code

Note

See lis-(\$VERSION)/src/esolver/lis_esolver_ii.c, which computes the smallest eigenvalue by calling lis_solve_kernel multiple times, for example.

This function returns 0 if the status of the solver (solver->retcode) is LIS_BREAKDOWN or LIS_MAXITER. See also the function lis_solver_get_status().

6.4.9 lis_solver_get_status

C LIS_INT lis_solver_get_status(LIS_SOLVER solver, LIS_INT *status)
Fortran subroutine lis_solver_get_status(LIS_SOLVER solver, LIS_INTEGER status,
LIS_INTEGER ierr)

Description

Get the status from the solver.

Input

solver The solver

Output

status The status

ierr The return code

Note

This function returns the status of the solver (solver->retcode).

6.4.10 lis_solver_get_iter

Description

Get the number of iterations from the solver.

Input

solver The solver

Output

iter The number of iterations

6.4.11 lis_solver_get_iterex

Description

Get the detailed information on the number of iterations from the solver.

Input

solver The solver

Output

iter The number of iterations

iter_double The number of double precision iterations

iter_quad The number of quadruple precision iterations

ierr The return code

6.4.12 lis_solver_get_time

Description

Get the execution time from the solver.

Input

solver The solver

Output

time The time in seconds of the execution

6.4.13 lis_solver_get_timeex

```
C LIS_INT lis_solver_get_timeex(LIS_SOLVER solver, double *time, double *time, double *ptime, double *pc_time, double *p_i_time)

Fortran subroutine lis_solver_get_timeex(LIS_SOLVER solver, real*8 time, real*8 itime, real*8 ptime, real*8 p_c_time, real*8 p_i_time, LIS_INTEGER ierr)
```

Description

Get the detailed information on the execution time from the solver.

Input

solver The solver

Output

time The total time in seconds

itime The time in seconds of the iterations

ptime The time in seconds of the preconditioning

p_c_time The time in seconds of the creation of the preconditioner

p_i_time The time in seconds of the iterations in the preconditioner

ierr The return code

6.4.14 lis_solver_get_residualnorm

Description

Get the relative residual norm $||b - Ax||_2/||b||_2$ from the solver.

Input

solver The solver

Output

residual The relative residual norm $||b - Ax||_2/||b||_2$

6.4.15 lis_solver_get_rhistory

```
C LIS_INT lis_solver_get_rhistory(LIS_SOLVER solver, LIS_VECTOR v)
Fortran subroutine lis_solver_get_rhistory(LIS_SOLVER solver, LIS_VECTOR v,
LIS_INTEGER ierr)
```

Description

Get the residual history from the solver.

Input

solver The solver

Output

v The vector

ierr The return code

Note

Vector v must be created in advance with the function lis_vector_create . When vector v is shorter than the residual history, it stores the residual history in order to vector v.

6.4.16 lis_solver_get_solver

C LIS_INT lis_solver_get_solver(LIS_SOLVER solver, LIS_INT *nsol)
Fortran subroutine lis_solver_get_solver(LIS_SOLVER solver, LIS_INTEGER nsol,
LIS_INTEGER ierr)

Description

Get the solver number from the solver.

Input

solver The solver

Output

nsol The solver number ierr The return code

6.4.17 lis_solver_get_precon

Description

Get the preconditioner number from the solver.

Input

solver The solver

Output

precon_type The preconditioner number

6.4.18 lis_solver_get_solvername

Description

Get the solver name from the solver number.

Input

nsol The solver number

Output

 $\begin{array}{ccc} {\tt name} & & {\tt The \ solver \ name} \\ {\tt ierr} & & {\tt The \ return \ code} \end{array}$

6.4.19 lis_solver_get_preconname

Description

Get the preconditioner name from the preconditioner number.

Input

precon_type The preconditioner number

Output

name The preconditioner name

6.5 Solving Eigenvalue Problems

$\bf 6.5.1 \quad lis_esolver_create$

C LIS_INT lis_esolver_create(LIS_ESOLVER *esolver)
Fortran subroutine lis_esolver_create(LIS_ESOLVER esolver, LIS_INTEGER ierr)

Description

Create the eigensolver.

Input

None

Output

esolver The eigensolver ierr The return code

Note

esolver has the information on the eigensolver, the preconditioner, etc.

6.5.2 lis_esolver_destroy

C LIS_INT lis_esolver_destroy(LIS_ESOLVER esolver)
Fortran subroutine lis_esolver_destroy(LIS_ESOLVER esolver, LIS_INTEGER ierr)

Description

Destroy the eigensolver.

Input

esolver The eigensolver to be destoyed

Output

6.5.3 lis_esolver_set_option

Description

Set the options for the eigensolver.

Input

text The command line options

Output

esolver The eigensolver ierr The return code

Note

The table below shows the available command line options, where -e {pi|1} means -e pi or -e 1 and -emaxiter [1000] indicates that -emaxiter defaults to 1,000.

Options for Eigensolvers (Default: -e pi)

Eigensolver	Option	Auxiliary Options	
Power	-e {pi 1}		
Inverse	-e {ii 2}	-i [bicg]	The linear solver
Rayleigh Quotient	-e {rqi 3}	-i [bicg]	The linear solver
\overline{CG}	-e {cg 4}	-i [cg]	The linear solver
CR	-e {cr 5}	-i [bicg]	The linear solver
Jacobi-Davidson	-e {jd 6}	-i [bicg]	The linear solver
Subspace	-e {si 7}	-ss [1]	The size of the subspace
Lanczos	-e {li 8}	-ss [1]	The size of the subspace
Arnoldi	-e {ai 9}	-ss [1]	The size of the subspace
Generalized Power	-e {gpi 10}	-i [bicg]	The linear solver
Generalized Inverse	-e {gii 11}	-i [bicg]	The linear solver
Generalized Rayleigh Quotient	-e {grqi 12}	-i [bicg]	The linear solver
Generalized CG	-e {gcg 13}	-i [cg]	The linear solver
Generalized CR	-e {gcr 14}	-i [bicg]	The linear solver
Generalized Subspace	-e {gsi 15}	-ss [1]	The size of the subspace
Generalized Lanczos	-e {gli 16}	-ss [1]	The size of the subspace
Generalized Arnoldi	-e {gai 17}	-ss [1]	The size of the subspace

Options for Preconditioners (Default: -p none)

Preconditioner	Option	Auxiliary Options	
None	-p {none 0}		
Jacobi	-p {jacobi 1}		
ILU(k)	-p {ilu 2}	-ilu_fill [0]	The fill level k
SSOR	-p {ssor 3}	-ssor_w [1.0]	The relaxation coefficient
			$\omega \ (0 < \omega < 2)$
Hybrid	-p {hybrid 4}	-hybrid_i [sor]	The linear solver
		-hybrid_maxiter [25]	The maximum number of iterations
		-hybrid_tol [1.0e-3]	The convergence tolerance
		-hybrid_w [1.5]	The relaxation coefficient ω
			of the SOR $(0 < \omega < 2)$
		-hybrid_ell [2]	The degree l of the BiCGSTAB(l)
		-hybrid_restart [40]	The restart values
			of the GMRES and Orthomin
I+S	-p {is 5}	-is_alpha [1.0]	The parameter α of $I + \alpha S^{(m)}$
		-is_m [3]	The parameter m of $I + \alpha S^{(m)}$
SAINV	-p {sainv 6}	-sainv_drop [0.05]	The drop criterion
SA-AMG	-p {saamg 7}	-saamg_unsym [false]	Select the unsymmetric version
			(The matrix structure must be
			symmetric)
		-saamg_theta [0.05 0.12]	The drop criterion $a_{ij}^2 \le \theta^2 a_{ii} a_{jj} $
			(symmetric or unsymmetric)
Crout ILU	-p {iluc 8}	-iluc_drop [0.05]	The drop criterion
		-iluc_rate [5.0]	The ratio of the maximum fill-in
ILUT	-p {ilut 9}		
Additive	-adds true	-adds_iter [1]	The number of iterations
Schwarz			

Other Options

Option				
-emaxiter [1000]	The maximum number of iterations			
-etol [1.0e-12]	[1.0e-12] The convergence tolerance			
-eprint [0]	The output of the residual history			
	-eprint {none 0}	None		
	-eprint {mem 1}	Save the residual history		
	-eprint {out 2}	Output it to the standard output		
	-eprint {all 3}	Save the residual history and output it		
		to the standard output		
-ie [ii]	The inner eigensolver used in Subspace, Lanczos, and Arnoldi			
-ige [gii]	The inner eigensolver us	sed in Generalized Subspace, Generalized Lanczos,		
	and Generalized Arnold	i		
-shift [0.0]	The amount of the shift σ			
-initx_ones [1]	The behavior of the initial vector x_0			
	-initx_ones {false 0	Values are given by the argument x		
		of the function lis_esolve()		
	-initx_ones {true 1}	All values are set to 1		
-omp_num_threads [t]	The number of threads			
	(t represents the maximum number of threads)			
-estorage [0]	The matrix storage format			
-estorage_block [2]	The block size of the BSR and BSC formats			
-ef [0]	The precision of the eig			
	-ef {double 0}	Double precision		
	-ef {quad 1}	Quadruple precision		

6.5.4 lis_esolver_set_optionC

```
C LIS_INT lis_esolver_set_optionC(LIS_ESOLVER esolver)
Fortran subroutine lis_esolver_set_optionC(LIS_ESOLVER esolver, LIS_INTEGER ierr)
```

Description

Set the options for the eigensolver on the command line.

Input

None

Output

esolver The eigensolver ierr The return code

6.5.5 lis_esolve

Description

Solve the standard eigenvalue problem $Ax = \lambda x$ with the specified eigensolver.

Input

A The matrix

x The initial vector

esolver The eigensolver

Output

evalue The eigenvalue of mode 0

x The associated eigenvector

esolver The number of iterations, the execution time, etc.

ierr The return code

Note

If the option $-initx_ones$ {false|0} is specified, the initial vector is given by the argument x. Otherwise, all values of the initial vector are set to 1.

This function returns 0 if the status of the eigensolver (esolver->retcode) is LIS_MAXITER. See also the function lis_esolver_get_status().

6.5.6 lis_gesolve

```
C LIS_INT lis_gesolve(LIS_MATRIX A, LIS_MATRIX B,
LIS_VECTOR x, LIS_REAL evalue, LIS_ESOLVER esolver)

Fortran subroutine lis_gesolve(LIS_MATRIX A, LIS_MATRIX B,
LIS_VECTOR x, LIS_REAL evalue, LIS_ESOLVER esolver, LIS_INTEGER ierr)
```

Description

Solve the generalized eigenvalue problem $Ax = \lambda Bx$ with the specified eigensolver.

Input

A The matrix
B The matrix
The initial vector

Output

esolver

evalue The eigenvalue of mode 0

The associated eigenvector

esolver The number of iterations, the execution time, etc.

ierr The return code

The eigensolver

Note

If the option $-initx_ones$ {false|0} is specified, the initial vector is given by the argument x. Otherwise, all values of the initial vector are set to 1.

This function returns 0 if the status of the eigensolver (esolver->retcode) is LIS_MAXITER. See also the function lis_esolver_get_status().

6.5.7 lis_esolver_get_status

C LIS_INT lis_esolver_get_status(LIS_ESOLVER esolver, LIS_INT *status)
Fortran subroutine lis_esolver_get_status(LIS_ESOLVER esolver, LIS_INTEGER status,
LIS_INTEGER ierr)

Description

Get the status for the specified eigenpair from the eigensolver.

Input

esolver The eigensolver

Output

status The status

ierr The return code

Note

This function returns the status of the eigensolver (esolver->retcode).

${\bf 6.5.8 \quad lis_esolver_get_iter}$

Description

Get the number of iterations for the specified eigenpair from the eigensolver.

Input

esolver The eigensolver

Output

iter The number of iterations

6.5.9 lis_esolver_get_iterex

Description

Get the detailed information on the number of iterations for the specified eigenpair from the eigensolver.

Input

esolver The eigensolver

Output

iter The number of iterations

iter_double The number of double precision iterations

iter_quad The number of quadruple precision iterations

ierr The return code

6.5.10 lis_esolver_get_time

Description

Get the execution time for the specified eigenpair from the eigensolver.

Input

esolver The eigensolver

Output

time The time in seconds of the execution

6.5.11 lis_esolver_get_timeex

```
C LIS_INT lis_esolver_get_timeex(LIS_ESOLVER esolver, double *time, double *time, double *ptime, double *ptime, double *ptime, double *ptime, double *ptime)

Fortran subroutine lis_esolver_get_timeex(LIS_ESOLVER esolver, real*8 time, real*8 itime, real*8 ptime, real*8 p_c_time, real*8 p_i_time, LIS_INTEGER ierr)
```

Description

Get the detailed information on the execution time for the specified eigenpair from the eigensolver.

Input

esolver The eigensolver

Output

time The total time in seconds

itime The time in seconds of the iterations

ptime The time in seconds of the preconditioning

p_c_timeThe time in seconds of the creation of the preconditionerp_i_timeThe time in seconds of the iterations in the preconditioner

ierr The return code

6.5.12 lis_esolver_get_residualnorm

Description

Get the relative residual norm $||\lambda x - (B^{-1})Ax||_2/||\lambda x||_2$ for the specified eigenpair from the eigensolver.

Input

esolver The eigensolver

Output

residual norm $||\lambda x - Ax||_2/||\lambda x||_2$

6.5.13 lis_esolver_get_rhistory

```
C LIS_INT lis_esolver_get_rhistory(LIS_ESOLVER esolver, LIS_VECTOR v)
Fortran subroutine lis_esolver_get_rhistory(LIS_ESOLVER esolver, LIS_VECTOR v,
LIS_INTEGER ierr)
```

Description

Get the residual history for the specified eigenpair from the eigensolver.

Input

esolver The eigensolver

Output

v The vector

ierr The return code

Note

Vector v must be created in advance with the function lis_vector_create . When vector v is shorter than the residual history, it stores the residual history in order to vector v.

6.5.14 lis_esolver_get_evalues

Description

Get all the eigenvalues from the eigensolver.

Input

esolver The eigensolver

Output

v The vector which stores the eigenvalues

ierr The return code

Note

Vector v must be created in advance with the function lis_vector_create.

6.5.15 lis_esolver_get_evectors

Description

Get all the eigenvectors from the eigensolver.

Input

esolver The eigensolver

Output

M The matrix in the COO format which stores the eigenvectors

ierr The return code

Note

Matrix M must be created in advance with the function lis_matrix_create .

$\bf 6.5.16 \quad lis_esolver_get_residual norms$

Description

Get the relative residual norms $||\lambda x - (B^{-1})Ax||_2/||\lambda x||_2$ of all the eigenpairs from the eigensolver.

Input

esolver The eigensolver

Output

v The vector which stores the residual norms

ierr The return code

Note

Vector v must be created in advance with the function lis_vector_create .

6.5.17 lis_esolver_get_iters

Description

Get the numbers of iterations of all the eigenpairs from the eigensolver.

Input

esolver The eigensolver

Output

v The vector which stores the numbers of iterations

ierr The return code

Note

Vector v must be created in advance with the function lis_vector_create .

6.5.18 lis_esolver_get_esolver

Description

Get the eigensolver number from the eigensolver.

Input

esolver The eigensolver

Output

nesol The eigensolver number

ierr The return code

6.5.19 lis_esolver_get_esolvername

Description

Get the eigensolver name from the eigensolver number.

Input

nesol The eigensolver number

Output

ename The eigensolver name
ierr The return code

6.6 Computing with Arrays

The following functions, which are not parallelized, are for local processing. Array data are stored in column-major order. Array indexing is zero-origin. See lis-(\$VERSION)/test/test6.c and lis-(\$VERSION)/test/test6f.F90.

6.6.1 lis_array_swap

Description

Swap the values of the vector elements.

Input

n The size of the vectors

x, y The source arrays that store vectors x, y of size n

Output

x, y The destination arrays

ierr The return code

6.6.2 lis_array_copy

Description

Copy the values of the vector elements.

Input

n The size of the vectors

 ${\tt x}$ The source array that stores vector x

Output

y The destination array

6.6.3 lis_array_axpy

C LIS_INT lis_array_axpy(LIS_INT n, LIS_SCALAR alpha, LIS_SCALAR x[], LIS_SCALAR y[])

Fortran subroutine lis_array_axpy(LIS_INTEGER n, LIS_SCALAR alpha, LIS_SCALAR x(), LIS_SCALAR y(), LIS_INTEGER ierr)

Description

Calculate the sum of the vectors $y = \alpha x + y$.

Input

n The size of the vectors

alpha The scalar value

x, y The arrays that store vectors x, y

Output

y $\alpha x + y$ (vector y is overwritten)

ierr The return code

6.6.4 lis_array_xpay

C LIS_INT lis_array_xpay(LIS_INT n, LIS_SCALAR x[], LIS_SCALAR alpha, LIS_SCALAR y[])

Fortran subroutine lis_array_xpay(LIS_INTEGER n, LIS_SCALAR x(), LIS_SCALAR alpha, LIS_SCALAR y(), LIS_INTEGER ierr)

Description

Calculate the sum of the vectors $y = x + \alpha y$.

Input

n The size of the vectors

alpha The scalar value

x, y The arrays that store vectors x, y

Output

y $x + \alpha y$ (vector y is overwritten)

$6.6.5 \quad lis_array_axpyz$

Description

Calculate the sum of the vectors $z = \alpha x + y$.

Input

n The size of the vectors

alpha The scalar value

x, y The arrays that store vectors x, y

Output

z $\alpha x + y$

ierr The return code

6.6.6 lis_array_scale

Description

Multiply vector x by scalar α .

Input

n The size of the vector

alpha The scalar value

 \mathbf{x} The array that stores vector x

Output

 αx (vector x is overwritten)

6.6.7 lis_array_pmul

```
C LIS_INT lis_array_pmul(LIS_INT n, LIS_SCALAR x[], LIS_SCALAR y[],
LIS_SCALAR z[])

Fortran subroutine lis_array_pmul(LIS_INTEGER n, LIS_SCALAR x(), LIS_SCALAR y(),
LIS_SCALAR z(), LIS_INTEGER ierr)
```

Description

Multiply each element of vector x by the corresponding element of y.

Input

n The size of the vectors

x, y The arrays that store vectors x, y

Output

z The array that stores the multiplied elements of x

ierr The return code

6.6.8 lis_array_pdiv

C LIS_INT lis_array_pdiv(LIS_INT n, LIS_SCALAR x[], LIS_SCALAR y[],
LIS_SCALAR z[])
Fortran subroutine lis_array_pdiv(LIS_INTEGER n, LIS_SCALAR x(), LIS_SCALAR y(),
LIS_SCALAR z(), LIS_INTEGER ierr)

Description

Divide each element of vector x by the corresponding element of y.

Input

n The size of the vectors

x, y The arrays that store vectors x, y

Output

z The array that stores the divided elements of x

6.6.9 lis_array_set_all

Description

Assign the scalar value to the elements of vector x.

Input

n The size of the vector

value The scalar value to be assigned

 \mathbf{x} The array that stores vector \mathbf{x}

Output

The array with the value assigned to the elements

ierr The return code

6.6.10 lis_array_abs

```
C LIS_INT lis_array_abs(LIS_INT n, LIS_SCALAR x[])
Fortran subroutine lis_array_abs(LIS_INTEGER n, LIS_SCALAR x(), LIS_INTEGER ierr)
```

Description

Get the absolute values of the elements of vector x.

Input

n The size of the vector

 \mathbf{x} The array that stores vector \mathbf{x}

Output

x The array that stores the absolute values

6.6.11 lis_array_reciprocal

```
C LIS_INT lis_array_reciprocal(LIS_INT n, LIS_SCALAR x[])
Fortran subroutine lis_array_reciprocal(LIS_INTEGER n, LIS_SCALAR x(),
LIS_INTEGER ierr)
```

Description

Get the reciprocal values of the elements of vector x.

Input

n The size of the vector

 \mathbf{x} The array that stores vector \mathbf{x}

Output

x The array that stores the reciprocal values

ierr The return code

${\bf 6.6.12}\quad {\bf lis_array_conjugate}$

Description

Get the conjugate values of the elements of vector x.

Input

n The size of the vector

 \mathbf{x} The array that stores vector x

Output

x The array that stores the conjugate values

6.6.13 lis_array_shift

Description

Get the shifted values of the elements of vector x.

Input

n The size of the vector

alpha The amount of the shift

 \mathbf{x} The array that stores vector \mathbf{x}

Output

x The array that stores the shifted values

ierr The return code

6.6.14 lis_array_dot

C LIS_INT lis_array_dot(LIS_INT n, LIS_SCALAR x[], LIS_SCALAR y[], LIS_SCALAR *value)

Fortran subroutine lis_array_dot(LIS_INTEGER n, LIS_SCALAR x(), LIS_SCALAR y(), LIS_SCALAR value, LIS_INTEGER ierr)

Description

Calculate the Hermitian inner product $y^H x$ of vectors x, y.

Input

n The size of the vectors

 ${\tt x}$, ${\tt y}$ The arrays that store vectors x,y

Output

value The Hermitian inner product

6.6.15 lis_array_nhdot

C LIS_INT lis_array_nhdot(LIS_INT n, LIS_SCALAR x[], LIS_SCALAR y[], LIS_SCALAR *value)

Fortran subroutine lis_array_nhdot(LIS_INTEGER n, LIS_SCALAR x(), LIS_SCALAR y(), LIS_SCALAR value, LIS_INTEGER ierr)

Description

Calculate the non-Hermitian inner product $y^T x$ of vectors x, y.

Input

n The size of the vectors

x, y The arrays that store vectors x, y

Output

value The non-Hermitian inner product

ierr The return code

6.6.16 lis_array_nrm1

Description

Calculate the 1-norm of vector x.

Input

n The size of the vector

 \mathbf{x} The array that stores vector \mathbf{x}

Output

value The 1-norm of the vector

6.6.17 lis_array_nrm2

Description

Calculate the 2-norm of vector x.

Input

n The size of the vector

 \mathbf{x} The array that stores vector \mathbf{x}

Output

value The 2-norm of the vector

ierr The return code

6.6.18 lis_array_nrmi

Description

Calculate the infinity norm of vector x.

Input

n The size of the vector

 \mathbf{x} The array that stores vector x

Output

value The infinity norm of the vector

6.6.19 lis_array_sum

Description

Calculate the sum of the elements of vector x.

Input

n The size of the vector

 \mathbf{x} The array that stores vector x

Output

value The sum of the vector elements

6.6.20 lis_array_matvec

Description

Calculate the matrix-vector product Ax.

Input

n The size of the matrix and vectors

The array that stores matrix A of size $n \times n$

 \mathbf{x} The array that stores vector x of size n

The array that stores vector y of size n

op LIS_INS_VALUE : y = Ax

 $\mathtt{LIS_SUB_VALUE} \, : \, y = y - Ax$

Output

У

ierr The return code

6.6.21 lis_array_matvect

```
C LIS_INT lis_array_matvect(LIS_INT n, LIS_SCALAR a[], LIS_SCALAR x[], LIS_SCALAR y[], LIS_INT op)

Fortran subroutine lis_array_matvect(LIS_INTEGER n, LIS_SCALAR a(), LIS_SCALAR x(), LIS_SCALAR y(), LIS_INTEGER op, LIS_INTEGER ierr)
```

Description

Calculate the matrix-vector product A^Tx .

Input

n The size of the matrix and vectors

a The array that stores matrix A of size $n \times n$

 \mathbf{x} The array that stores vector x of size n

The array that stores vector y of size n

op LIS_INS_VALUE : $y = A^T x$

 $\mathtt{LIS_SUB_VALUE} \, : \, y = y - A^T x$

Output

y y

6.6.22 lis_array_matvec_ns

```
C LIS_INTEGER lis_array_matvec_ns(LIS_INT m, LIS_INT n, LIS_SCALAR a[],
LIS_INT lda, LIS_SCALAR x[], LIS_SCALAR y[], LIS_INT op)

Fortran subroutine lis_array_matvec_ns(LIS_INTEGER m, LIS_INTEGER n, LIS_SCALAR a()
LIS_INTEGER lda, LIS_SCALAR x(), LIS_SCALAR y(), LIS_INTEGER op,
LIS_INTEGER ierr)
```

Description

Calculate the matrix-vector product Ax, where matrix A is not square.

Input

m, n	The sizes of the matrix and vectors
a	The array that stores matrix A of size $m \times n$
lda	The size of the leading dimension of array ${\cal A}$
x	The array that stores vector x of size n
У	The array that stores vector y of size m
ор	${\tt LIS_INS_VALUE}:y=Ax$
	$\mathtt{LIS_SUB_VALUE} : y = y - Ax$

Output

У	y

6.6.23 lis_array_matmat

```
C LIS_INT lis_array_matmat(LIS_INT n, LIS_SCALAR a[], LIS_SCALAR b[],
    LIS_SCALAR c[], LIS_INT op)

Fortran subroutine lis_array_matmat(LIS_INTEGER n, LIS_SCALAR a(), LIS_SCALAR b(),
    LIS_SCALAR c(), LIS_INTEGER op, LIS_INTEGER ierr)
```

Description

Calculate the matrix-matrix product AB.

Input

n	The size of the matrices
a	The array that stores matrix A of size $n \times n$
b	The array that stores matrix B of size $n \times n$
С	The array that stores matrix C of size $n \times n$
ор	${\tt LIS_INS_VALUE}:C=AB$
	LIS_SUB_VALUE : $C = C - AB$

Output

 c

6.6.24 lis_array_matmat_ns

```
C LIS_INT lis_array_matmat_ns(LIS_INT 1, LIS_INT m, LIS_INT n,
LIS_SCALAR a[], LIS_INT lda, LIS_SCALAR b[], LIS_INT ldb, LIS_SCALAR c[],
LIS_INT ldc, LIS_INT op)

Fortran subroutine lis_array_matmat_ns(LIS_INTEGER 1, LIS_INTEGER m, LIS_INTEGER n,
LIS_SCALAR a(), LIS_INTEGER lda, LIS_SCALAR b(), LIS_INTEGER ldb,
LIS_SCALAR c(), LIS_INTEGER ldc, LIS_INTEGER op, LIS_INTEGER ierr)
```

Description

Calculate the matrix-matrix product AB, where matrices A,B are not square.

Input

m, n	The sizes of the matrices
a	The array that stores matrix A of size $l \times m$
lda	The size of the leading dimension of array ${\cal A}$
b	The array that stores matrix B of size $m \times n$
ldb	The size of the leading dimension of array ${\cal B}$
С	The array that stores matrix C of size $l \times n$
ldc	The size of the leading dimension of array ${\cal C}$
ор	${\tt LIS_INS_VALUE} : C = AB$
	${\tt LIS_SUB_VALUE}:C=C-AB$
.	

Output

 c

6.6.25 lis_array_ge

```
C LIS_INT lis_array_ge(LIS_INT n, LIS_SCALAR a[])
Fortran subroutine lis_solve(LIS_INTEGER n, LIS_SCALAR a(), LIS_INTEGER ierr)
```

Description

Calculate the inverse of matrix A with the Gaussian elimination.

Input

n The size of the matrix

The array that stores matrix A of size $n \times n$

Output

a The inverse A^{-1} ierr The return code

6.6.26 lis_array_solve

```
C LIS_INT lis_array_solve(LIS_INT n, LIS_SCALAR a[], LIS_SCALAR b[],
LIS_SCALAR x[], LIS_SCALAR w[])

Fortran subroutine lis_array_solve(LIS_INTEGER n, LIS_SCALAR a(), LIS_SCALAR b(),
LIS_SCALAR x(), LIS_SCALAR w(), LIS_INTEGER ierr)
```

Description

Solve the linear equation Ax = b with the direct method.

Input

n The size of the matrix

The array that stores coefficient matrix A of size $n \times n$

b The array that stores right-hand side vector b of size n

 \mathbf{w} The work array of size $n \times n$

Output

 \mathbf{x} The array that stores solution x

6.6.27 lis_array_cgs

```
C LIS_INT lis_array_cgs(LIS_INT n, LIS_SCALAR a[], LIS_SCALAR q[],
LIS_SCALAR r[])
Fortran subroutine lis_array_cgs(LIS_INTEGER n, LIS_SCALAR a(), LIS_SCALAR q(),
LIS_SCALAR r(), LIS_INTEGER ierr)
```

Description

Calculate the QR factorization QR = A with the classical Gram-Schmidt process.

Input

n The size of the matrices

a The array that stores matrix A of size $n \times n$

Output

q The array that stores orthogonal matrix Q of size $n \times n$

The array that stores upper-triangular matrix R of size $n \times n$

ierr The return code

6.6.28 lis_array_mgs

```
C LIS_INT lis_array_mgs(LIS_INT n, LIS_SCALAR a[], LIS_SCALAR q[],
LIS_SCALAR r[])

Fortran subroutine lis_array_mgs(LIS_INTEGER n, LIS_SCALAR a(), LIS_SCALAR q(),
LIS_SCALAR r(), LIS_INTEGER ierr)
```

Description

Calculate the QR factorization QR = A with the modified Gram-Schmidt process.

Input

n The size of the matrices

a The array that stores matrix A of size $n \times n$

Output

q The array that stores orthogonal matrix Q of size $n \times n$

The array that stores upper-triangular matrix R of size $n \times n$

6.6.29 lis_array_qr

```
С
       LIS_INT lis_array_qr(LIS_INT n, LIS_SCALAR a[], LIS_SCALAR q[],
         LIS_SCALAR r[], LIS_INT *qriter, LIS_REAL *qrerr)
Fortran subroutine lis_array_qr(LIS_INTEGER n, LIS_SCALAR a(), LIS_SCALAR q(),
         LIS_SCALAR r(), LIS_INTEGER qriter, LIS_REAL qrerr, LIS_INTEGER ierr)
```

Description

Calculate the eigenvalues of matrix A with the QR algorithm.

1

ierr

Input	
n	The size of the matrices
a	The array that stores symmetric matrix A of size $n \times n$
q	The work array of size $n \times n$
r	The work array of size $n \times n$
Output	
a	The array that stores the block upper-triangular matrix with eigenvalues in the block diagonal elements after similarity transformation
qriter	The number of iterations of the QR algorithm
qrerr	The 2-norm of the first subdiagonal element $A(2,1)$ after similarity transformation

The return code

6.7 Operating External Files

6.7.1 lis_input

Description

Read the matrix and vector data from the external file.

Input

filename The source file

Output

A The matrix in the specified storage format

b The right-hand side vector

x The solution

ierr The return code

Note

The following file formats are supported:

- \bullet The extended Matrix Market format (extended to allow vector data)
- $\bullet\,$ The Harwell-Boeing format

6.7.2 lis_input_vector

Description

Read the vector data from the external file.

Input

filename The source file

Output

v The vector

ierr The return code

Note

The following file formats are supported:

- $\bullet\,$ The PLAIN format
- The extended Matrix Market format (extended to allow vector data)

6.7.3 lis_input_matrix

Description

Read the matrix data from the external file.

Input

filename The source file

Output

A The matrix in the specified storage format

The return code

x The solution

Note

ierr

The following file formats are supported:

- The Matrix Market format
- The Harwell-Boeing format

6.7.4 lis_output

```
C LIS_INT lis_output(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
LIS_INT format, char *filename)

Fortran subroutine lis_output(LIS_MATRIX A, LIS_VECTOR b, LIS_VECTOR x,
LIS_INTEGER format, character filename, LIS_INTEGER ierr)
```

Description

Write the matrix and vector data into the external file.

Input

A The matrix

b The right-hand side vector (If no vector is written to the external

file, then NULL must be input.)

The solution (If no vector is written to the external file, then NULL

must be input.)

format The file format

LIS_FMT_MM The Matrix Market format

filename The destination file

Output

6.7.5 lis_output_vector

Description

Write the vector data into the external file.

Input

v The vector

format The file format

LIS_FMT_PLAIN The PLAIN format

LIS_FMT_MM The Matrix Market format

filename The destination file

Output

ierr The return code

6.7.6 lis_output_matrix

Description

Write the matrix data into the external file.

Input

A The matrix

format The file format

LIS_FMT_MM The Matrix Market format

filename The destination file

Output

6.8 Other Functions

6.8.1 lis_initialize

```
C LIS_INT lis_initialize(LIS_INT* argc, char** argv[])
Fortran subroutine lis_initialize(LIS_INTEGER ierr)
```

Description

Initialize the execution environment.

Input

argc The number of command line arguments

argv The command line argument

Output

ierr The return code

6.8.2 lis_finalize

```
C LIS_INT lis_finalize()
Fortran subroutine lis_finalize(LIS_INTEGER ierr)
```

Description

Finalize the execution environment.

Input

None

Output

6.8.3 lis_wtime

```
C double lis_wtime()
Fortran function lis_wtime()
```

Description

Measure the elapsed time.

Input

None

Output

The elapsed time in seconds from the given point is returned as the double precision number.

Note

To measure the processing time, call <code>lis_wtime</code> to get the starting time, call it again to get the ending time, and calculate the difference.

6.8.4 CHKERR

```
C void CHKERR(LIS_INT ierr)
Fortran subroutine CHKERR(LIS_INTEGER ierr)
```

Description

Check the value of the return code.

Input

ierr

The return code

Output

None

Note

If the value of the return code is not 0, it calls lis_finalize and terminates the program.

$\bf 6.8.5 \quad lis_printf$

C LIS_INT lis_printf(LIS_Comm comm, const char *mess, ...)

Description

Print message on processing element 0.

Input

comm The MPI communicator

Output

mess The message

Note

The string '%D' is replaced with '%lld' if LIS_INT is long long int, and is replaced with '%d' if it is int.

For the serial and multithreaded environments, the value of comm is ignored.

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A File Formats

This section describes the file formats available for the library. Note that both the upper and lower triangular entries need to be stored irrespective of whether the matrix is symmetric or not in Harwell-Boeing Format.

A.1 Extended Matrix Market Format

The Matrix Market format does not support the vector data. The extended Matrix Market format is the extension of the Matrix Market format to handle the matrix and vector data. Assume that the number of nonzero elements of matrix $A = (a_{ij})$ of size $M \times N$ is L and that $a_{ij} = A(I, J)$. The format is as follows:

```
%%MatrixMarket matrix coordinate real general
                                                   <-- Header
                                                    <-+
%
                                                      | Comment lines with 0 or more lines
%
{\tt M} {\tt N} {\tt L} {\tt B} {\tt X}
                                                    <-- Numbers of rows, columns, and
I1 J1 A(I1,J1)
                                                         nonzero elements (0 or 1) (0 or 1)
I2 J2 A(I2,J2)
                                                      | Row and column number values
                                                      | The index is one-origin
IL JL A(IL, JL)
                                                    <-+
I1 B(I1)
I2 B(I2)
                                                      | Exists only when B=1
                                                      | Row number value
 . . .
IM B(IM)
I1 X(I1)
I2 X(I2)
                                                      | Exists only when X=1
                                                      | Row number value
IM X(IM)
```

The extended Matrix Market format for matrix A and vector b in Equation (A.1) is as follows:

$$A = \begin{pmatrix} 2 & 1 & & \\ 1 & 2 & 1 & & \\ & 1 & 2 & 1 \\ & & 1 & 2 \end{pmatrix} \qquad b = \begin{pmatrix} 0 & & \\ 1 & & \\ 2 & & \\ 3 & & \end{pmatrix}$$
 (A.1)

%%MatrixMarket matrix coordinate real general

```
4 4 10 1 0
1 2 1.00e+00
1 1 2.00e+00
2 3 1.00e+00
2 1
    1.00e+00
2 2
    2.00e+00
3 4 1.00e+00
3 2
    1.00e+00
3 3 2.00e+00
4 4 2.00e+00
4 3 1.00e+00
1 0.00e+00
2 1.00e+00
3 2.00e+00
```

4 3.00e+00

A.2 Harwell-Boeing Format

The Harwell-Boeing format stores the matrix in the CSC format. Assume that the array value stores the values of the nonzero elements of matrix A, the array index stores the row indices of the nonzero elements and the array ptr stores pointers to the top of each column in the arrays value and index. The format is as follows:

```
Line 1 (A72,A8)
   1 - 72 Title
  73 - 80 Key
Line 2 (5I14)
  1 - 14 Total number of lines excluding header
  15 - 28 Number of lines for ptr
 29 - 42 Number of lines for index
 43 - 56 Number of lines for value
 57 - 70 Number of lines for right-hand side vectors
Line 3 (A3,11X,4I14)
   1 - 3 Matrix type
            Col.1: R Real matrix
                   C Complex matrix
                   P Pattern only (Not supported)
            Col.2: S Symmetric (Not supported)
                   U Unsymmetric
                   H Hermitian (Not supported)
                   Z Skew symmetric (Not supported)
                   R Rectangular (Not supported)
            Col.3: A Assembled
                   E Elemental matrices (Not supported)
  4 - 14 Blank space
  15 - 28 Number of rows
  29 - 42 Number of columns
  43 - 56 Number of nonzero elements
  57 - 70 0
Line 4 (2A16,2A20)
  1 - 16 Format for ptr
  17 - 32 Format for index
  33 - 52 Format for value
 53 - 72 Format for right-hand side vectors
Line 5 (A3,11X,2I14) Only presents if there are right-hand side vectors
          right-hand side vector type
   1
            F for full storage
            M for same format as matrix (Not supported)
          G if a starting vector is supplied
          X if an exact solution is supplied
  4 - 14 Blank space
  15 - 28 Number of right-hand side vectors
  29 - 42 Number of nonzero elements
```

The Harwell-Boeing format for matrix A and vector b in Equation (A.1) is as follows:

```
1 - - - - - 10 - - - - - 20 - - - - - 30 - - - - - 40 - - - - - 50 - - - - - 60 - - - - - 70 - - - - - 80
Harwell-Boeing format sample
                                                                                       Lis
                                                                  4
                                                                                   2
                                                 1
               8
                                1
RUA
                                                                 10
                                                 4
                                                                                   4
                                4
(11i7)
                   (13i6)
                                       (3e26.18)
                                                               (3e26.18)
                                1
               3
                        6
       1
                                   3
                                          2
                                                 3
                                                                       4
```

A.3 Extended Matrix Market Format for Vectors

The extended Matrix Market format for vectors is the extension of the Matrix Market format to handle the vector data. Assume that vector $b = (b_i)$ is a vector of size N and that $b_i = B(I)$. The format is as follows:

```
      %//MatrixMarket vector coordinate real general
      <-- Header</td>

      %
      <-+</td>

      %
      | Comment lines with 0 or more lines

      %
      <-+</td>

      N
      <-- Number of rows</td>

      I1 B(I1)
      <-+</td>

      I2 B(I2)
      | Row number value

      . . .
      | The index is one-origin

      IN B(IN)
      <-+</td>
```

The extended Matrix Market format for vector b in Equation (A.1) is as follows:

```
%%MatrixMarket vector coordinate real general
4
1 0.00e+00
2 1.00e+00
3 2.00e+00
4 3.00e+00
```

A.4 PLAIN Format for Vectors

The PLAIN format for vectors is designed to write vector values in order. Assume that vector $b = (b_i)$ is a vector of size N and that b_i is equal to B(I). The format is as follows:

```
B(1) <-+
B(2) | Vector value
... |
B(N) <-+
```

The PLAIN format for vector b in Equation (A.1) is as follows:

```
0.00e+00
1.00e+00
2.00e+00
3.00e+00
```