

Research Design

09: statistical power, sample size, simulations

Initial questions?

statistical power — probability of identifying statistical significance, given an effect

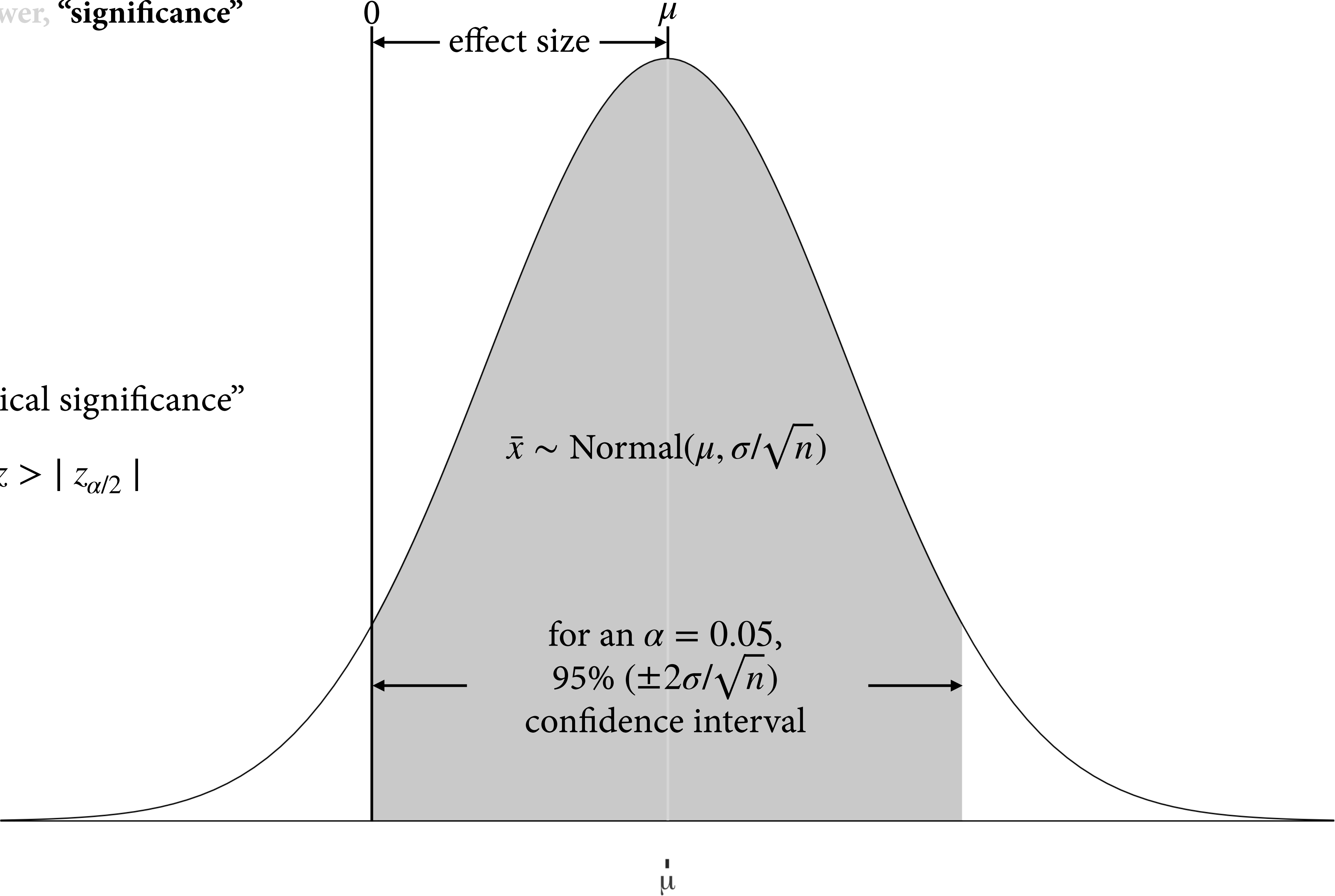
STATISTICAL POWER | the probability, before a study is performed, that a particular comparison will achieve “statistical significance” at some predetermined level (typically a p-value below 0.05), given some assumed true effect size.

NOTE | a typical threshold for statistical power $1 - \beta$ is 0.8 but — as with choosing a level of confidence α — choice β should inform good decisions.

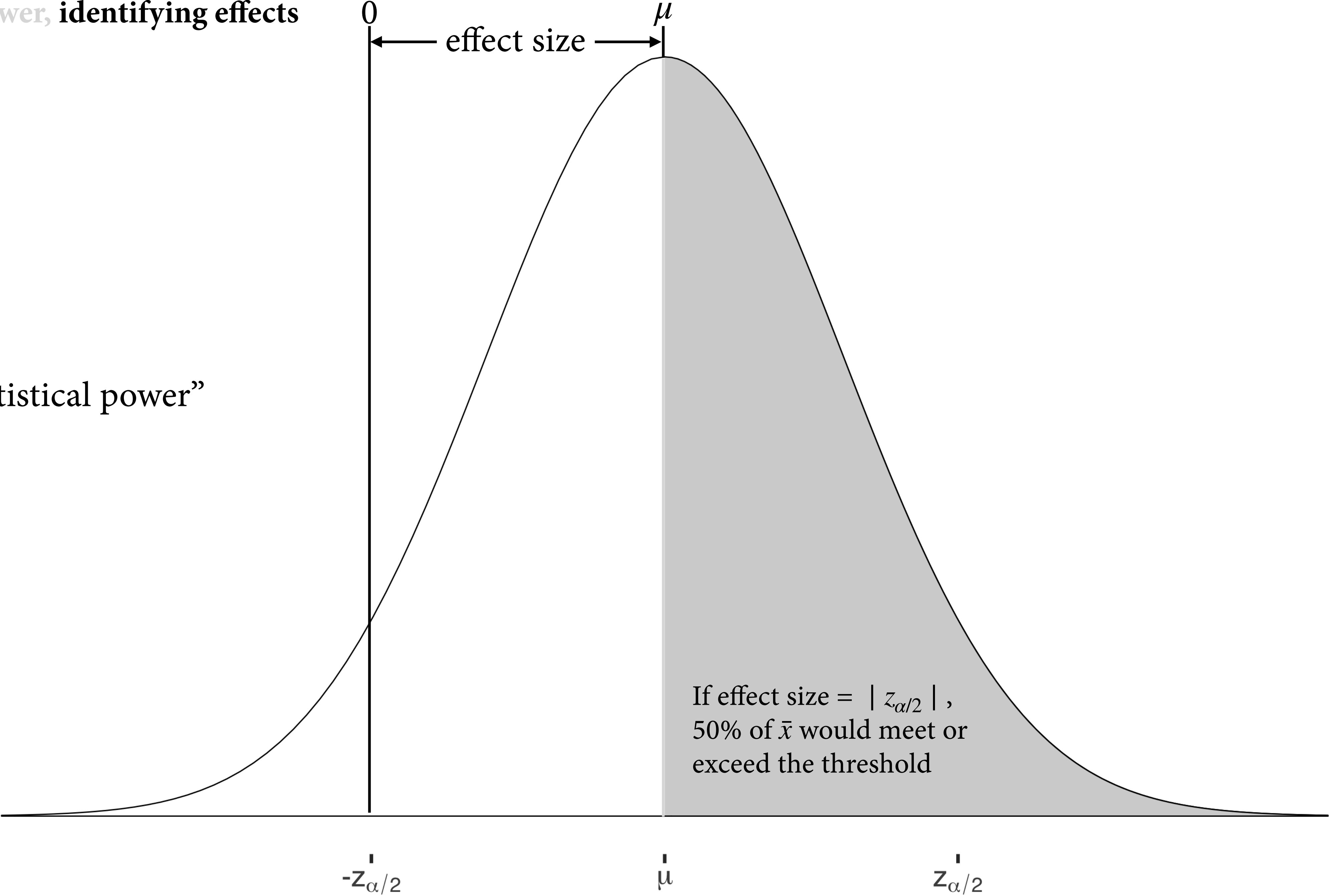
statistical power, “significance”

“statistical significance”

$z > |z_{\alpha/2}|$



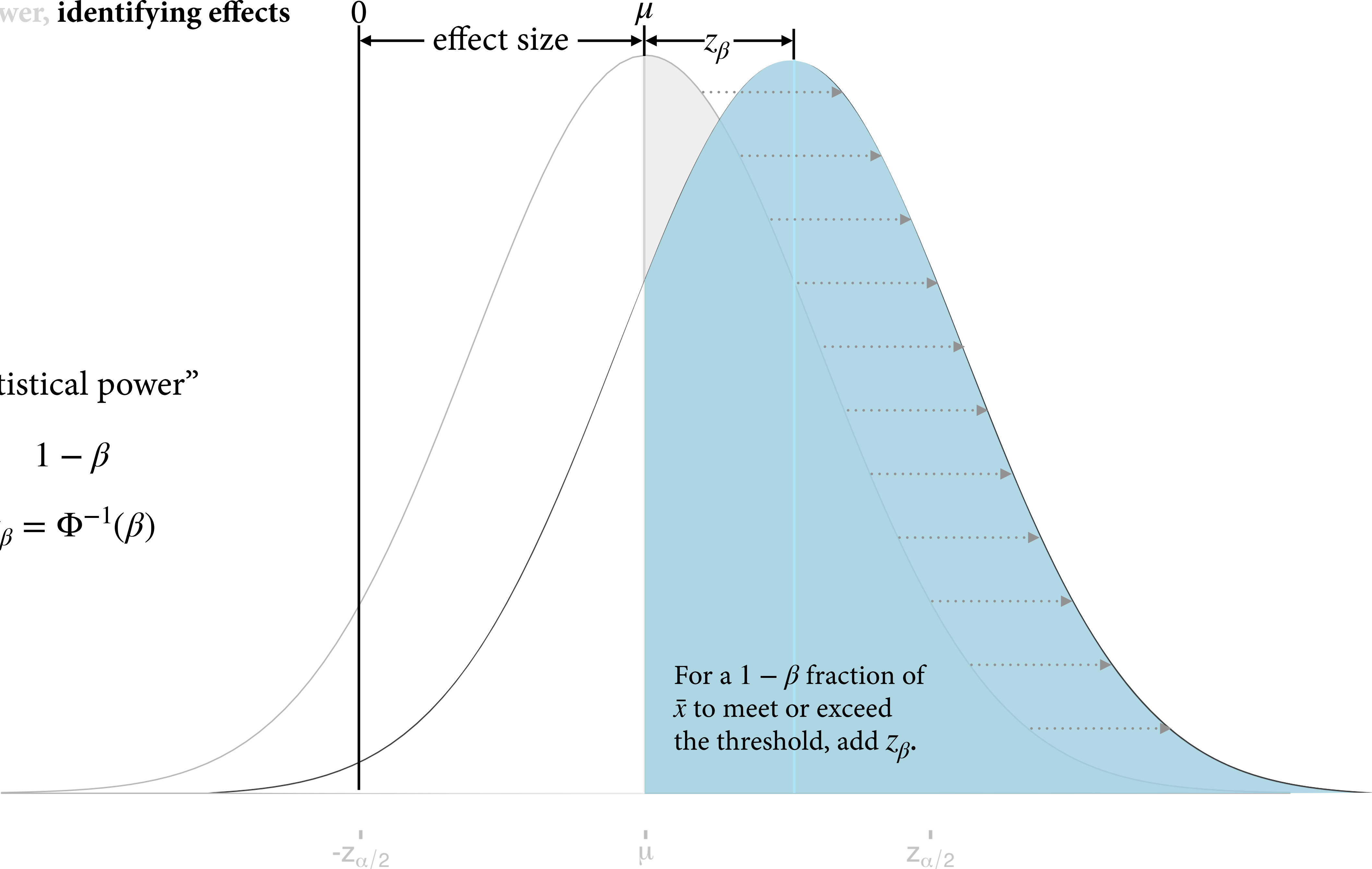
“statistical power”



“statistical power”

$$1 - \beta$$

$$z_\beta = \Phi^{-1}(\beta)$$



statistical power, an example — calculating probability of finding an effect

Consider a hypothesis to test

$$H_0 : \mu = 0 \text{ , } H_a : \mu > 0$$

Choose an appropriate test statistic and reference distribution (probability model)

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} > |z_\alpha| \text{ compared against } F_\Phi$$

Choose a meaningful effect size and variation to test

$$\bar{x} > \frac{\sigma}{\sqrt{n}} |z_\alpha| + \mu$$

Choose a sample size

$$n$$

Calculate the probability of identifying an underlying effect

$$p(\bar{x} > \frac{\sigma}{\sqrt{n}} |z_\alpha| + \mu) = F_\Phi(\frac{\sigma}{\sqrt{n}} |z_\alpha| + \mu)$$

sample size to achieve a specified probability of obtaining statistical significance

statistical power, estimating sample size to have p chance of finding an effect — solve for n

For a given $\bar{x} - \mu$, α , and β : $\mu_0 + |Z_{\alpha/2}| \frac{s}{\sqrt{n}} = \bar{x} - |Z_{\beta}| \frac{s}{\sqrt{n}}$

rearrange:

$$(|z_{\alpha/2}| + |z_{\beta}|) \cdot \frac{s}{\sqrt{n}} = \bar{x} - \mu_0$$

solve for n :

$$n = \left[\frac{(|z_{\alpha/2}| + |z_{\beta}|) \cdot s}{\bar{x} - \mu_0} \right]^2$$

For a given $\bar{x} - \mu$, α , and β : $\mu_0 + |Z_{\alpha/2}| \frac{s}{\sqrt{n}} = \bar{x} - |Z_{\beta}| \frac{s}{\sqrt{n}}$

Let $\bar{x} = 0.6$, $\mu = 0.5$, $\alpha = 0.05$, and $\beta = 0.2$.

rearrange:

$$(|z_{\alpha/2}| + |z_{\beta}|) \cdot \frac{s}{\sqrt{n}} = \bar{x} - \mu_0$$

$$(1.96 + 0.84) \frac{\sqrt{0.6(1-0.6)}}{\sqrt{n}} = 0.6 - 0.5$$

solve for n :

$$n = \left[\frac{(|z_{\alpha/2}| + |z_{\beta}|) \cdot s}{\bar{x} - \mu_0} \right]^2$$

$$n = \left[\frac{(1.96 + 0.84) \cdot 0.49}{0.1} \right]^2 = 196$$

simulations

statistical power, estimating sample size — a toy example, estimate sample size for a proportion, simulating experiments

Let $\bar{x} = 0.6$, $\mu = 0.5$, $\alpha = 0.05$, and try $\beta = \{0.2, 0.5\}$

Using $n_{\beta_{0.5}} = 96$ and $n_{\beta_{0.2}} = 196$, simulate experiments.

```
p0    <- 0.5
p      <- 0.6
alpha <- 0.05 / 2

z_alpha_2 <- qnorm(alpha, 0, 1, lower.tail = F)

# get n for 80 percent power
beta    <- 0.2
z_beta  <- qnorm(1 - beta, 0, 1, lower.tail = T)
n_pwr80 <- ( (z_alpha_2 + z_beta) * sqrt( p0 * (1 - p0) ) / (p - p0) ) ^ 2

# get n for 50 percent power
beta    <- 0.5
z_beta  <- qnorm(1 - beta, 0, 1, lower.tail = T)
n_pwr50 <- ( (z_alpha_2 + z_beta) * sqrt( p0 * (1 - p0) ) / (p - p0) ) ^ 2
```

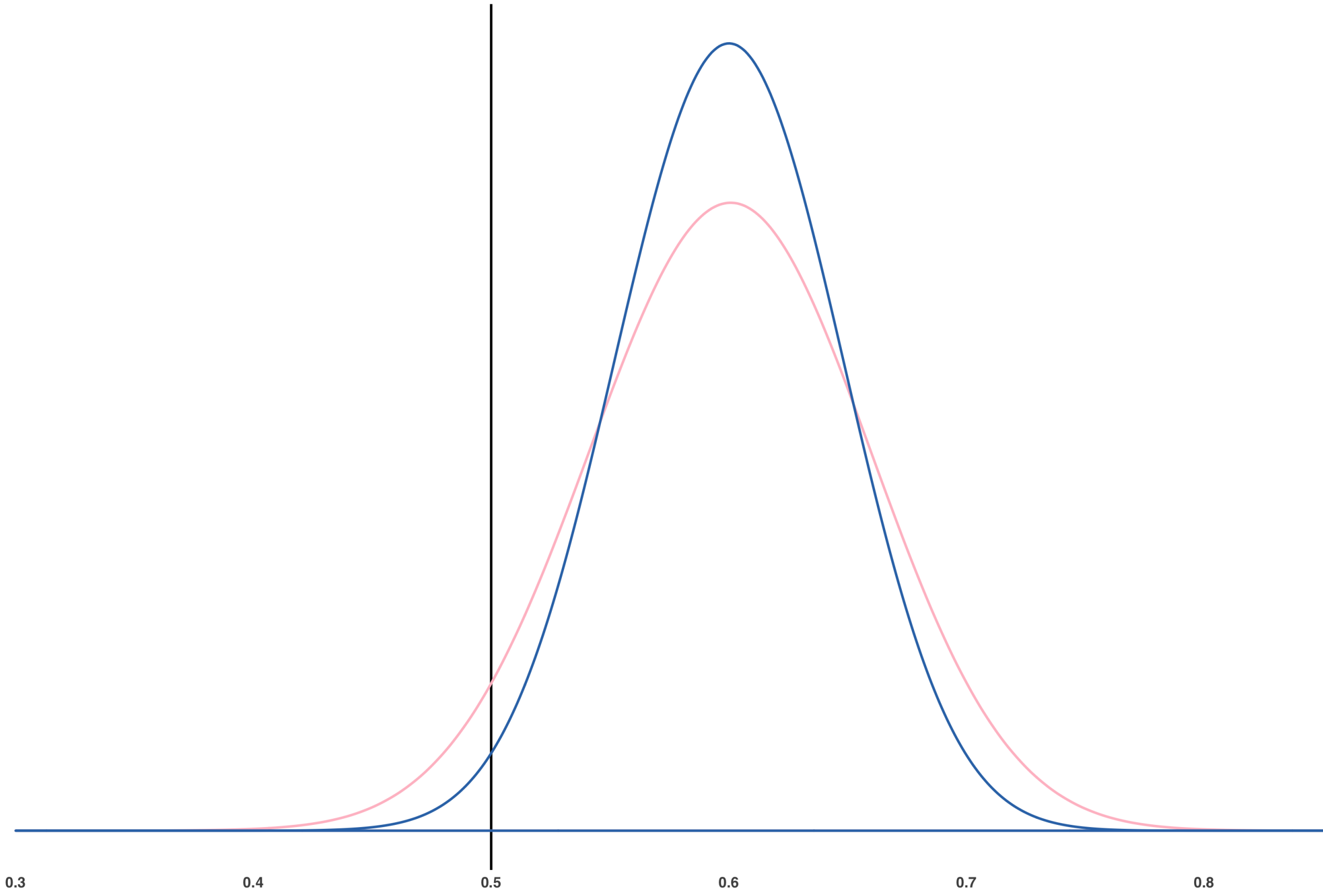
```
# simulate experiments
survey <- function(n, p) {
  x = ( rbinom(n = n, size = 1, prob = p) )
  x_bar = mean(x)
  se = sd(x) / sqrt(n)
  c(x_bar, se)
}

set.seed(1)
p_hat_96 <- replicate( 1e5, survey(n = n_pwr50, p) )

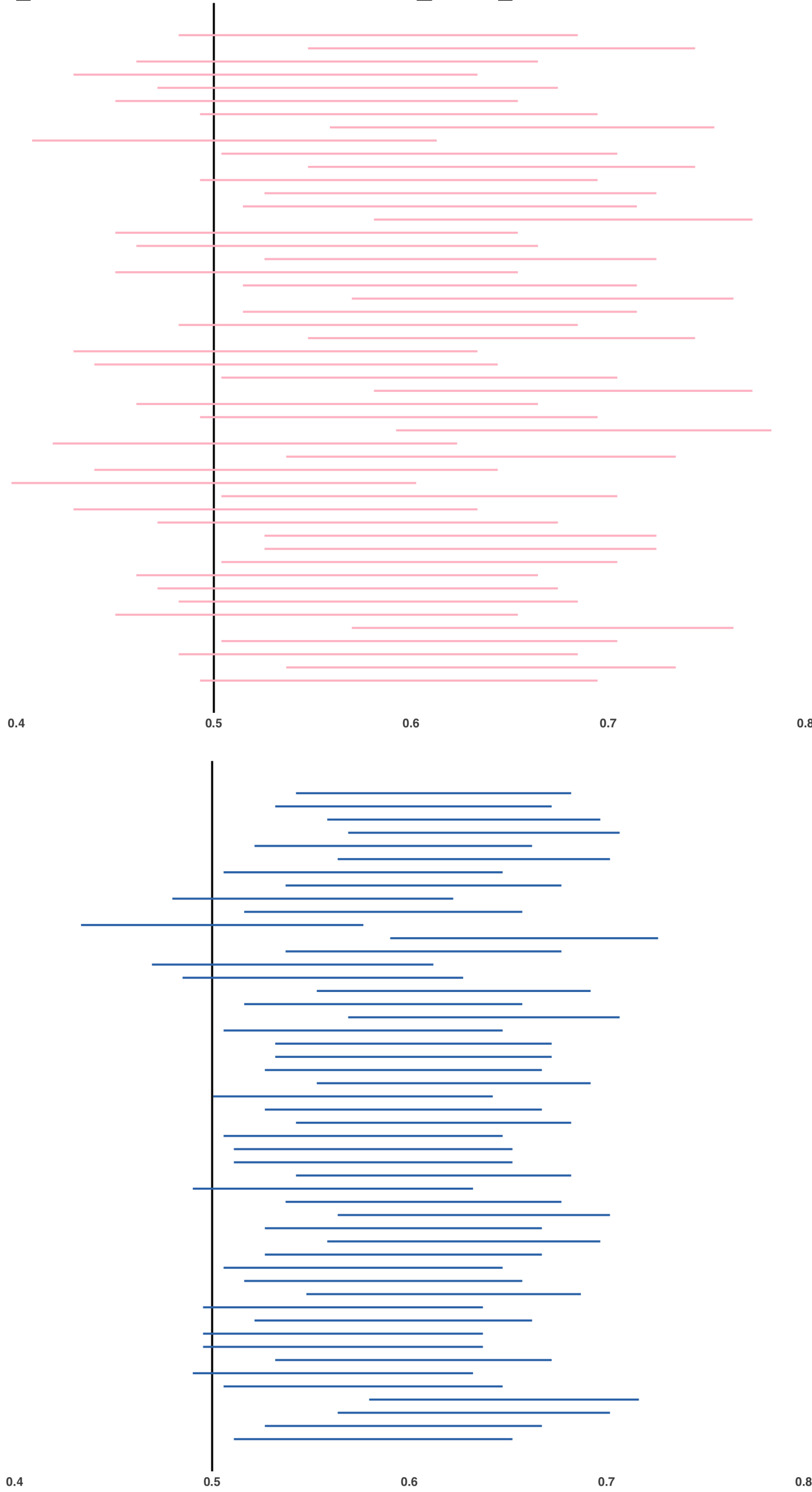
set.seed(1)
p_hat_196 <- replicate( 1e5, survey(n = n_pwr80, p) )
```

\hat{p} , with $n = 96$

\hat{p} , with $n = 196$



$$\hat{p} \pm 2 \cdot s/\sqrt{n}$$



References

Cox, D. R., and N. Reid. “Precision and power, Section 8.1.2.” In *The Theory of the Design of Experiments*. Monographs on Statistics and Applied Probability 86. Boca Raton: Chapman & Hall/CRC, 2000.

Gelman, Andrew, and John Carlin. “*Beyond Power Calculations*.” Perspectives on Psychological Science 9, no. 6 (November 2014): 641–51.

Gelman, Andrew, Jennifer Hill, and Aki Ventari. “Design and sample size decisions, Chp. 16.” In *Regression and Other Stories*. S.l.: Cambridge University Press, 2020.

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