EE123
Digital Signal Processing

Lecture 14B
Compressed Sensing II

M. Lustig, EECS UC Berkeley

From Samples to Measurements

- Shanon-Nyquist sampling
  - –Worst case for ANY bandlimited data

- Compressive sampling (CS)

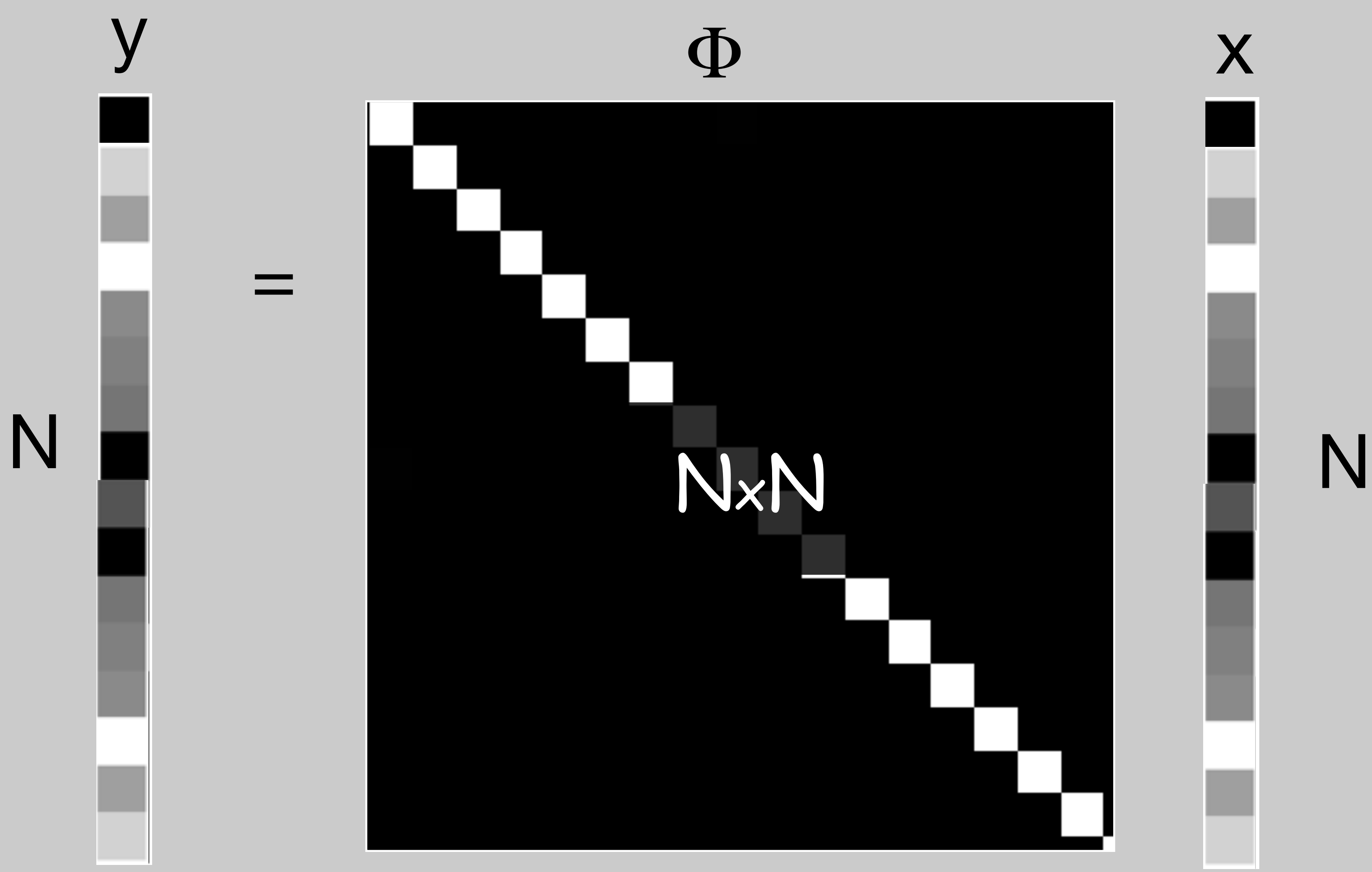
  "Sparse signals statistics can be recovered from a small number of non-adaptive linear measurements"

  - –Integrated sensing, compression and processing.
  - –Based on concepts of incoherency between signal and measurements

Traditional Sensing

Desktop scanner/ digital camera sensing

- $x \in \Re^N$ is a signal
  - Make N linear measurements

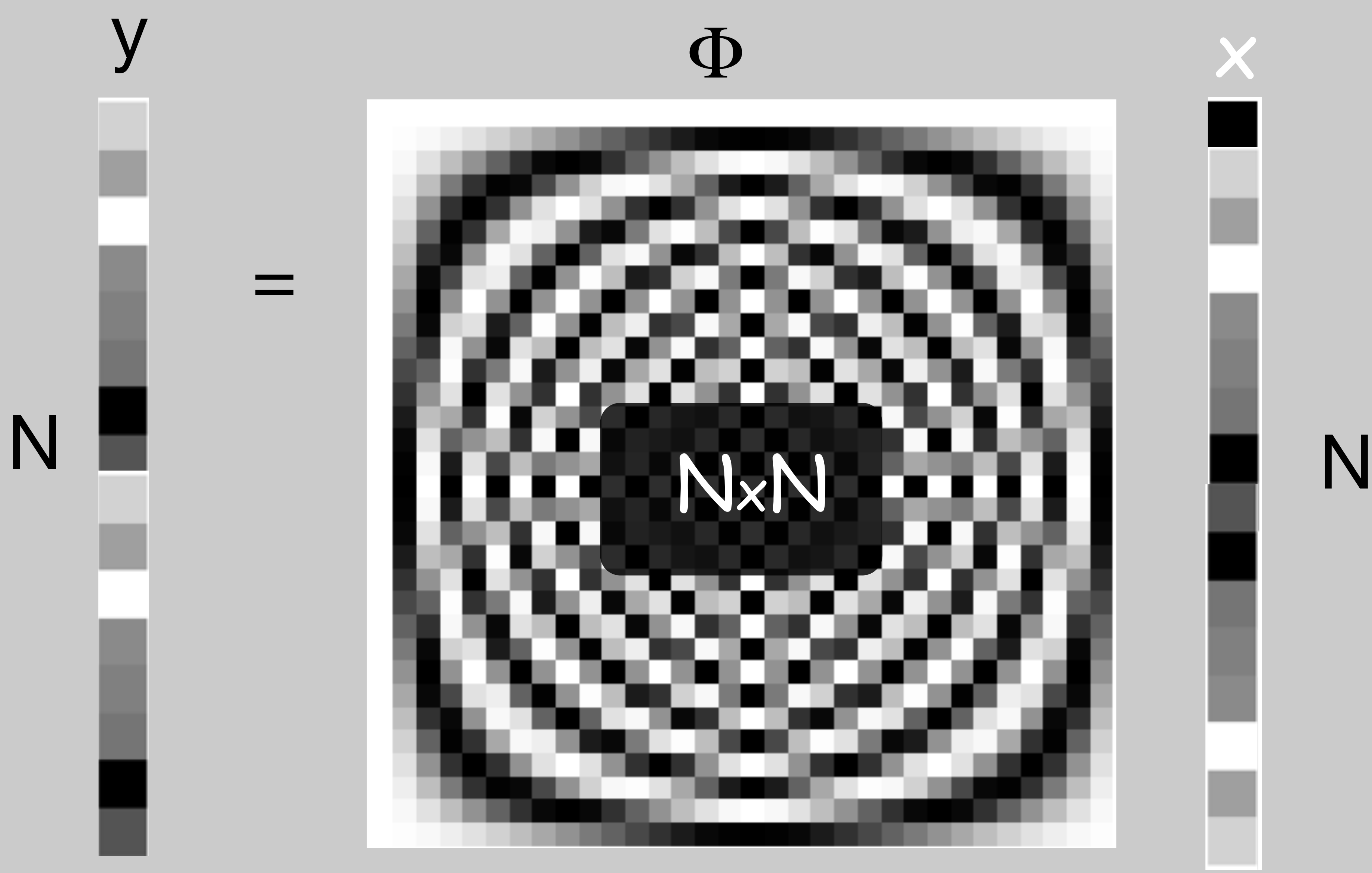


sensing matrix

Traditional Sensing

MRI Fourier Imaging

- $x \in \Re^N$ is a signal
  - Make N linear measurements

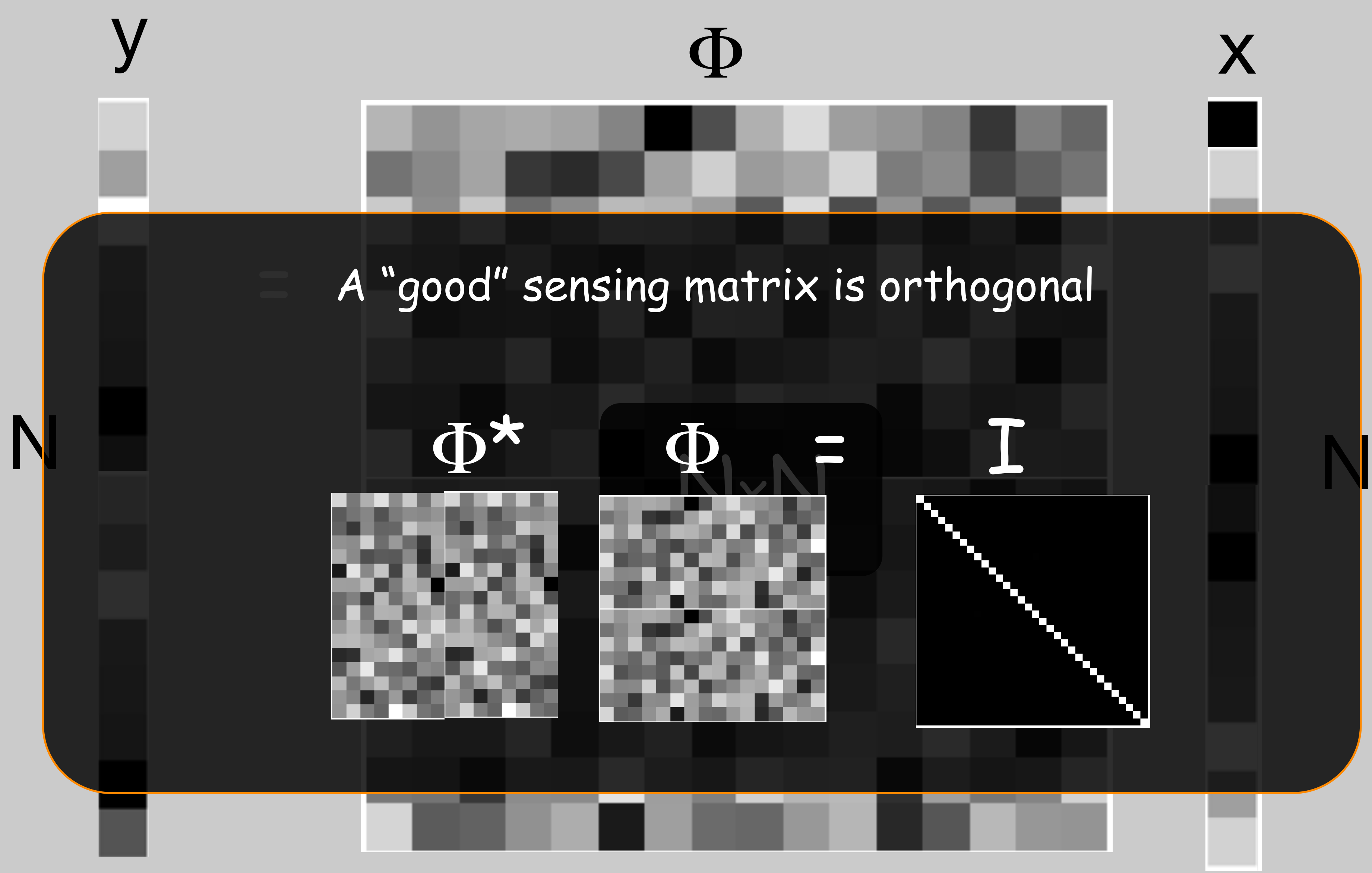


sensing matrix

Traditional Sensing

Arbitrary sensing

- $x \in \Re^N$ is a signal
  - Make N linear measurements

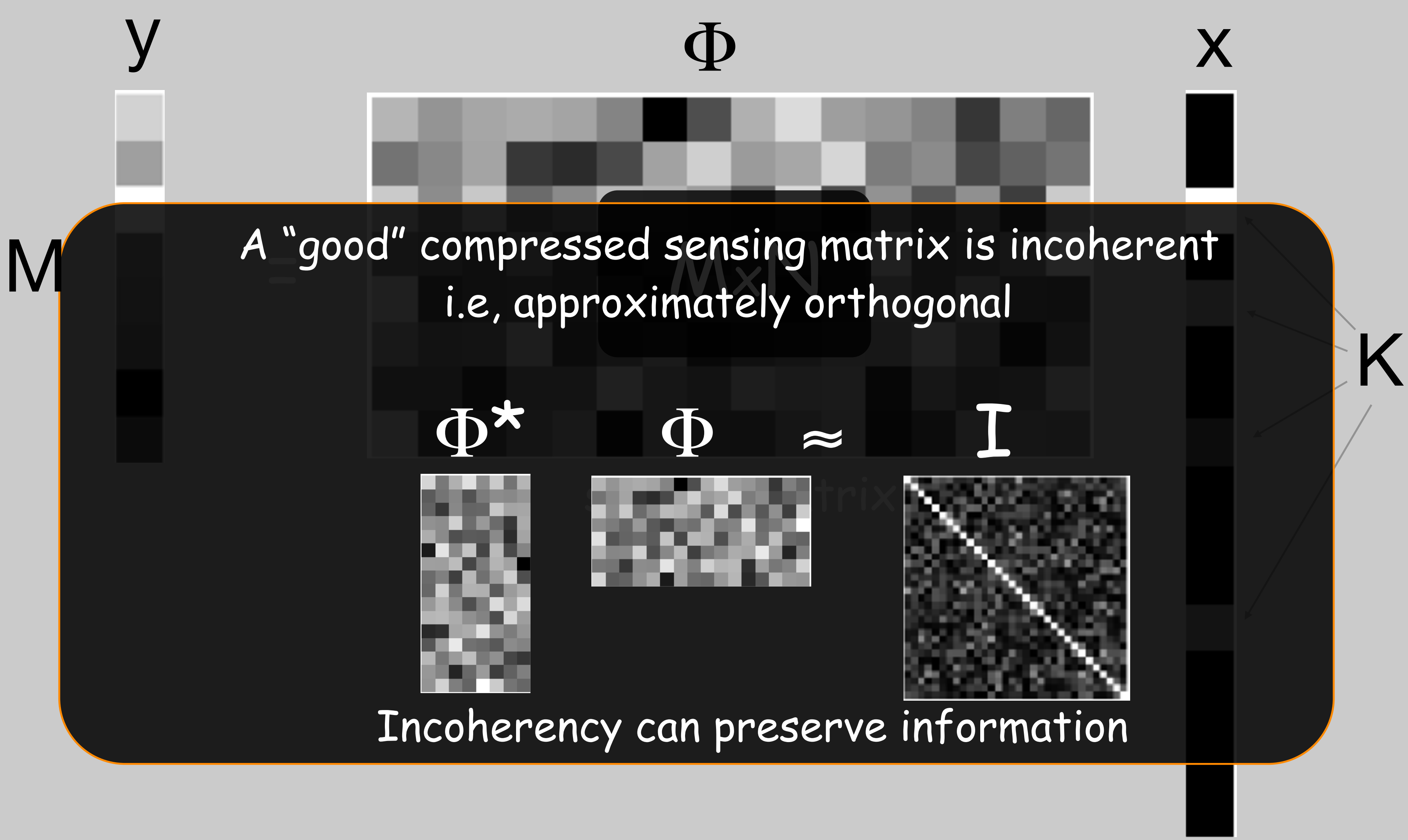


sensing matrix

Compressed Sensing

(Candes, Romber, Tao 2006; Donoho 2006)

- $x \in \Re^N$ is a K-sparse signal (K<<N)
  - Make M (K<M<<N) incoherent linear projections

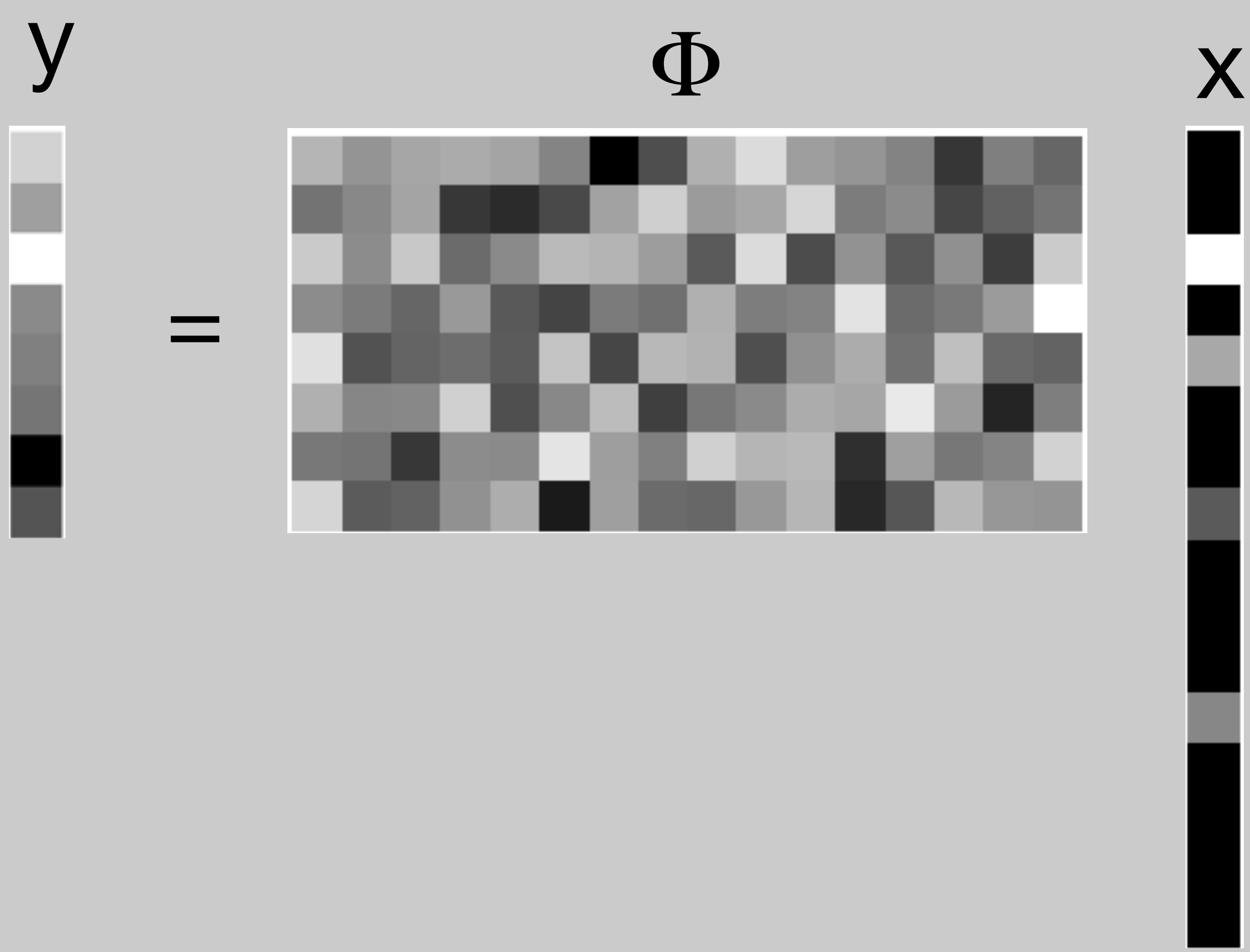


CS recovery

- Given $y = \Phi x$
  find x

Under-determined

- But there's hope, x is sparse!



CS recovery

- Given $y = \Phi x$
  find x

Under-determined

- But there's hope, x is sparse!

CS recovery

- Given $y = \Phi x$
  find x

$$\left.\begin{matrix} \text{Given } y = \Phi x \\ \text{find } x \end{matrix}\right\} \text{Under-determined}$$

- But there's hope, x is sparse!

minimize $||x||_2$

s.t. $y = \Phi x$

WRONG!

CS recovery

- Given $y = \Phi x$
  find x — Under-determined
- But there's hope, x is sparse!

$$\text{minimize } \|x\|_0$$
$$\text{s.t. } y = \Phi x$$

HARD!

CS recovery

- Given $y = \Phi x$
  find x

Under-determined

- But there's hope, x is sparse!

$$\text{minimize } \|x\|_1$$

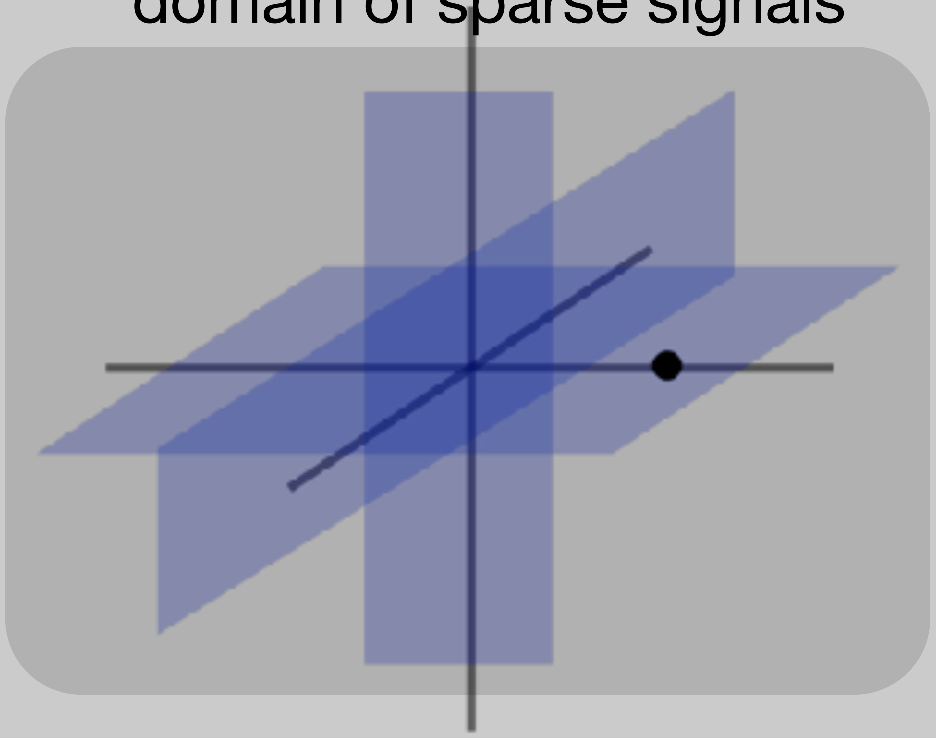
$$\text{s.t. } y = \Phi x$$

need $M \approx K \log(N) << N$

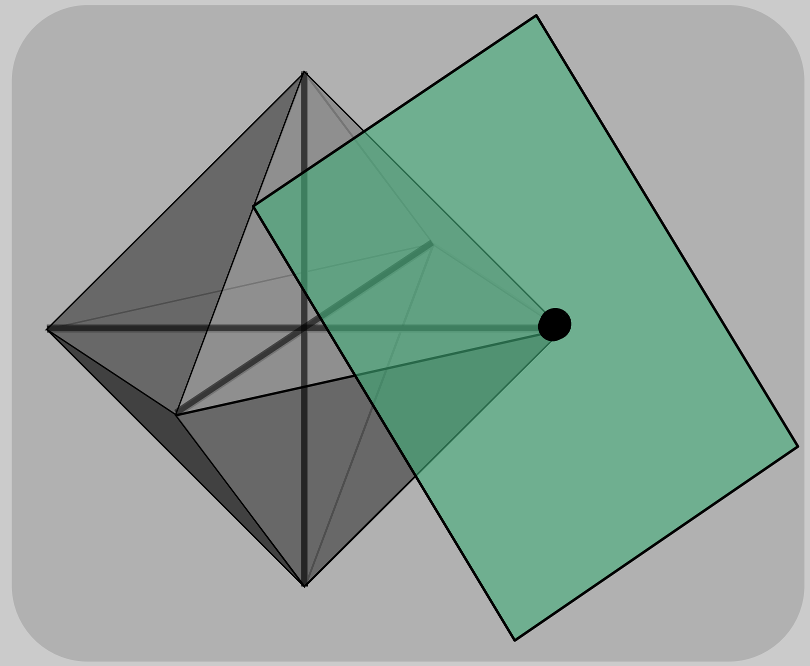
Solved by linear-programming

Geometric Interpretation

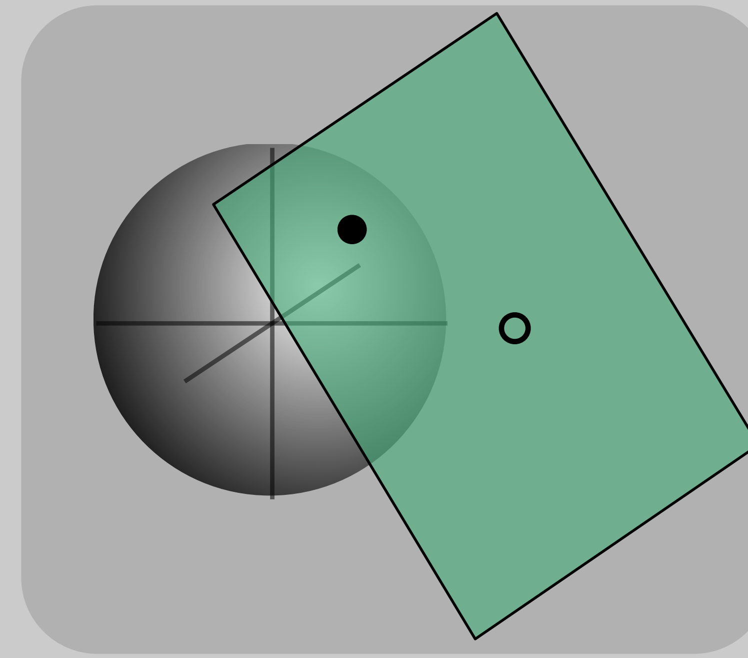
domain of sparse signals



minimum $||x||_1$



minimum $||x||_2$



$$\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [y_1]$$

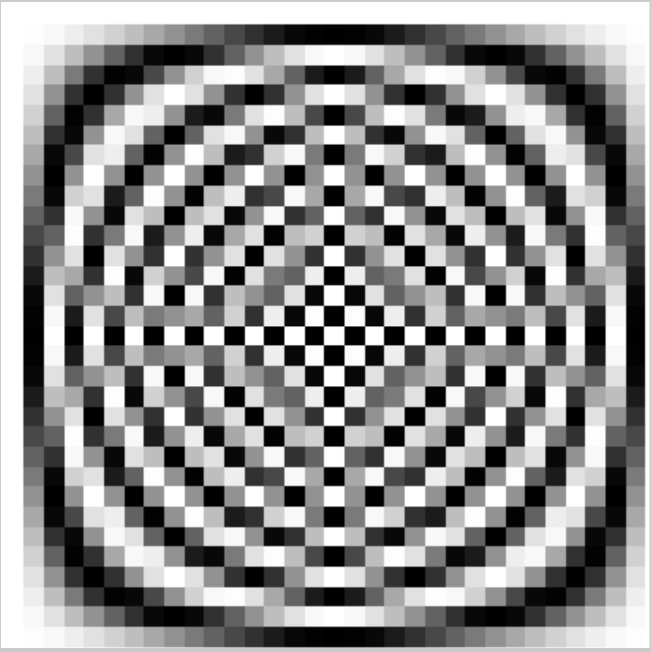
A non-linear sampling theorem

- $f \in C^N$ supported on a set Ω in Fourier
- Shannon:
  - –Ω is known connected set, size B
  - –Exact recovery from B equispaced time samples
  - –Linear reconstruction by sinc interpolation
- Non-linear sampling theorem
  - –Ω is an arbitrary, unknown set of size B
  - –Exact recovery from ~ B logN (almost) arbitrary placed samples
  - –Nonlinear reconstruction by convex programming

Practicality of CS

- Can such sensing system exist in practice?

Fourier matrix



Practicality of CS

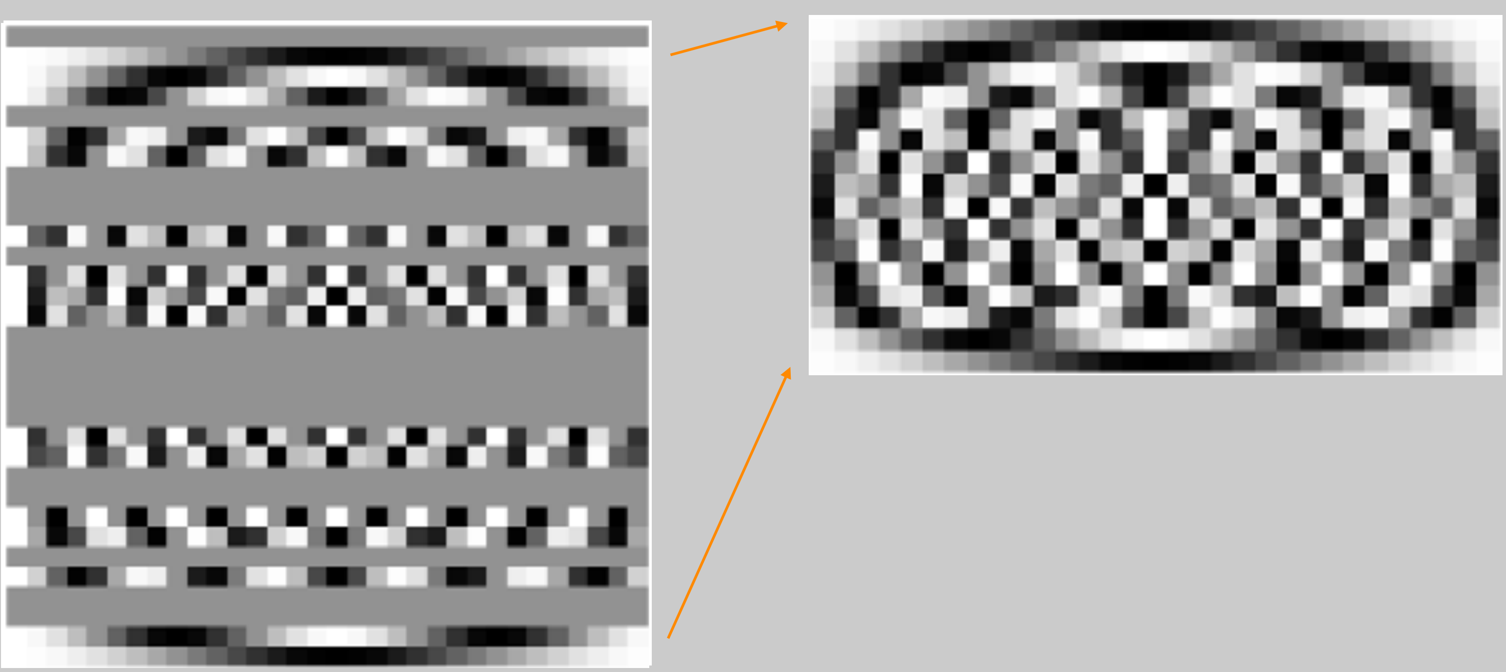
- Can such sensing system exist in practice?

Fourier matrix

Practicality of CS

- Can such sensing system exist in practice?

Fourier matrix



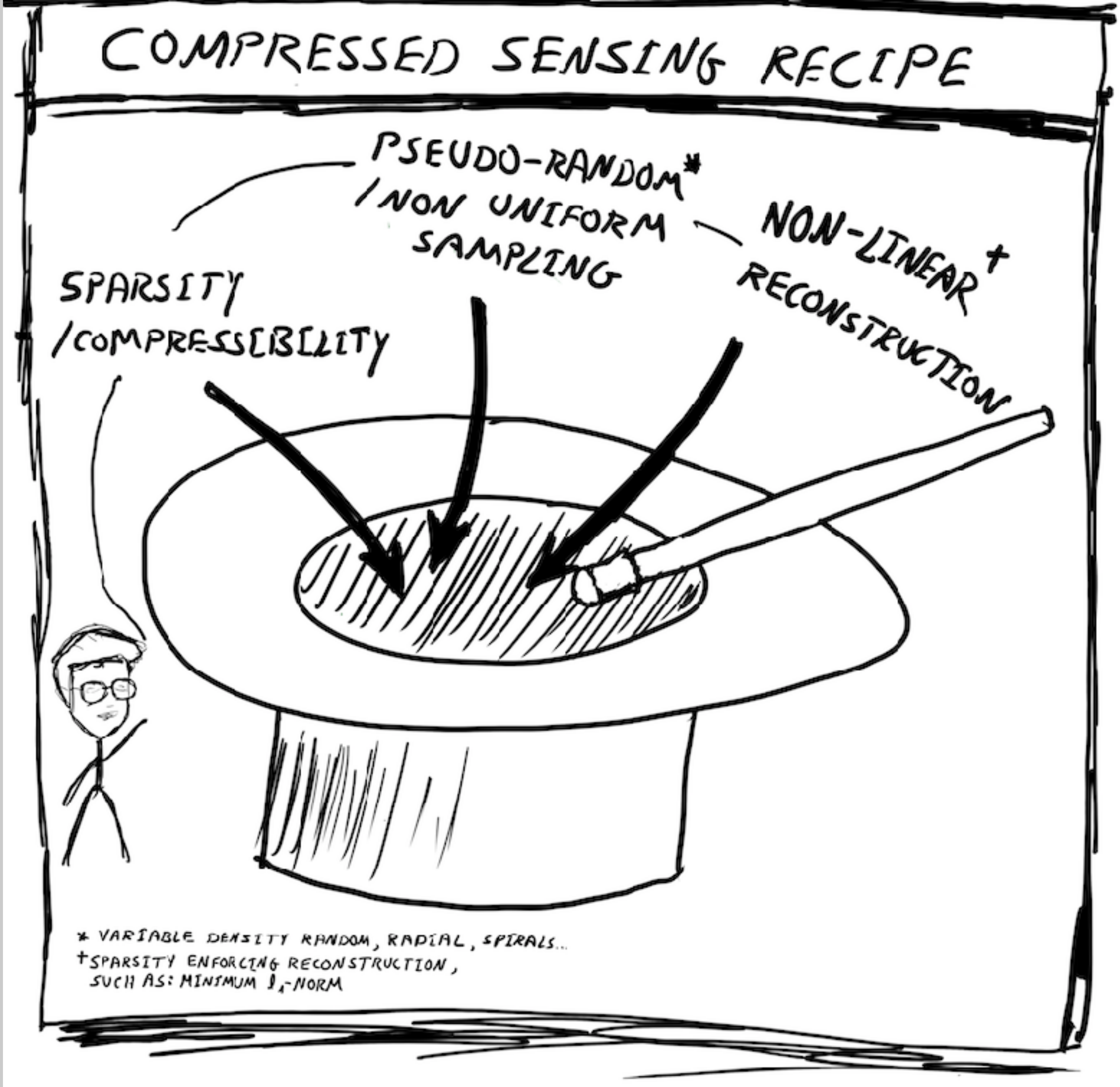
Practicality of CS

- Can such sensing system exist in practice?

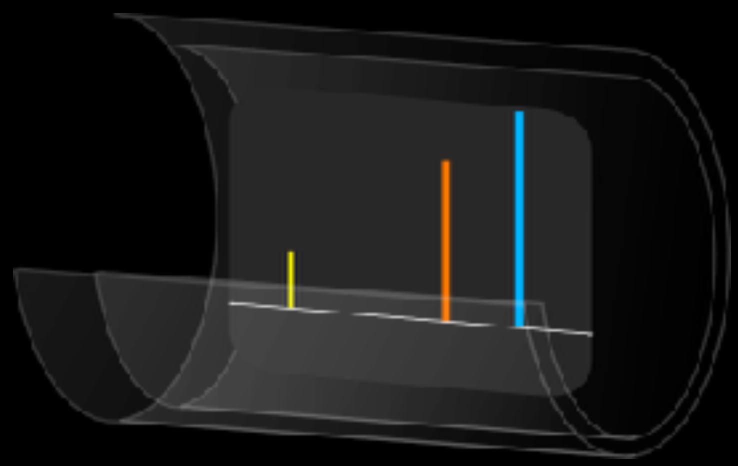
- Randomly undersampled Fourier is incoherent

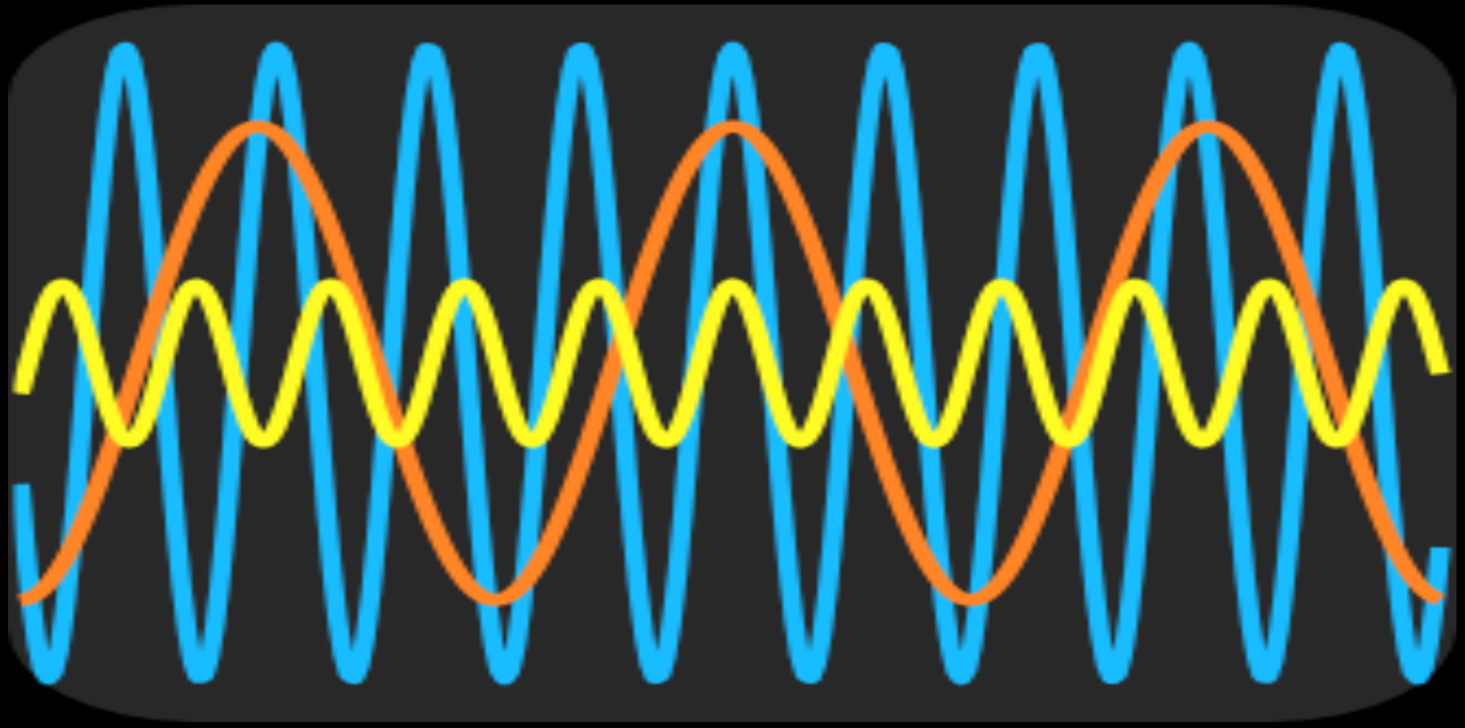
- MRI samples in the Fourier domain!

$$\Phi^* \qquad \Phi \qquad \approx \qquad I$$

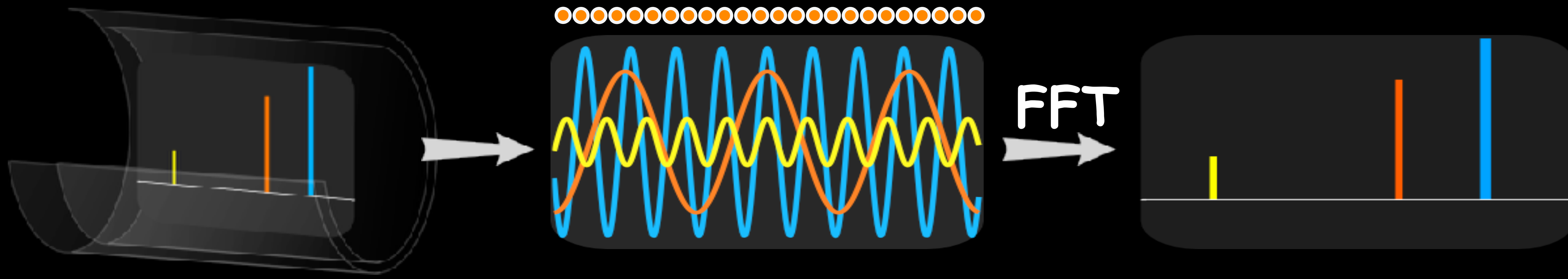

COMPRESSED SENSING RECIPE
PSEUDO-RANDOM*
/NON UNIFORM
SAMPLING
SPARSITY
/COMPRESSIBILITY
NON-LINEAR+
RECONSTRUCTION
* VARIABLE DENSITY RANDOM, RADIAL, SPIRALS...
+SPARSITY ENFORCING RECONSTRUCTION,
SUCH AS: MINIMUM l_1-NORM

Intuitive example of CS

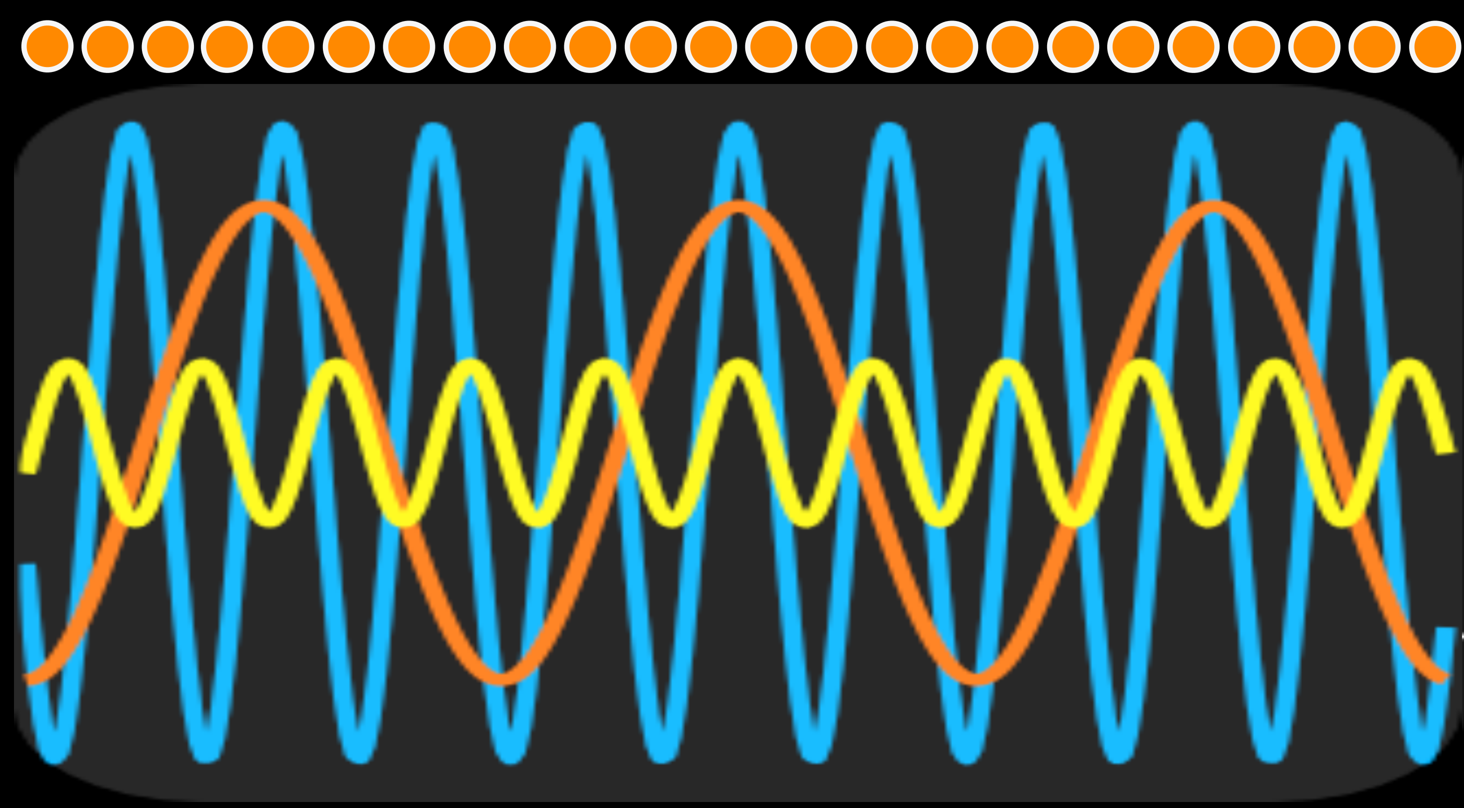




Intuitive example of CS

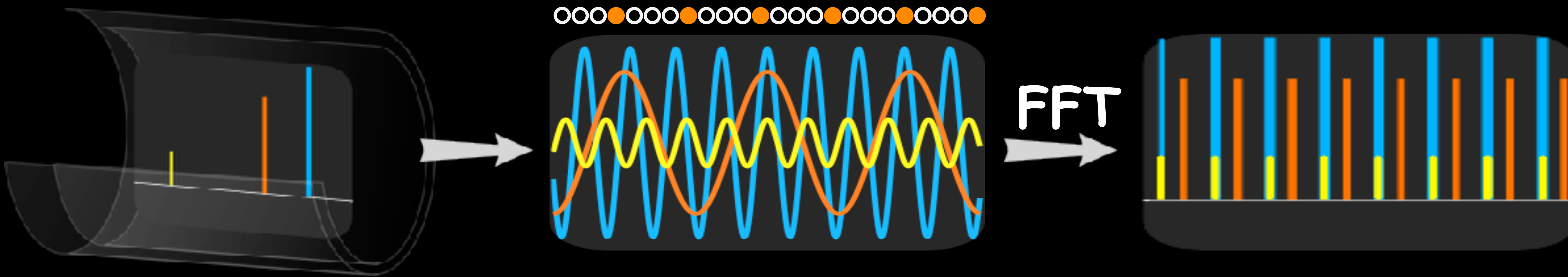


sampling

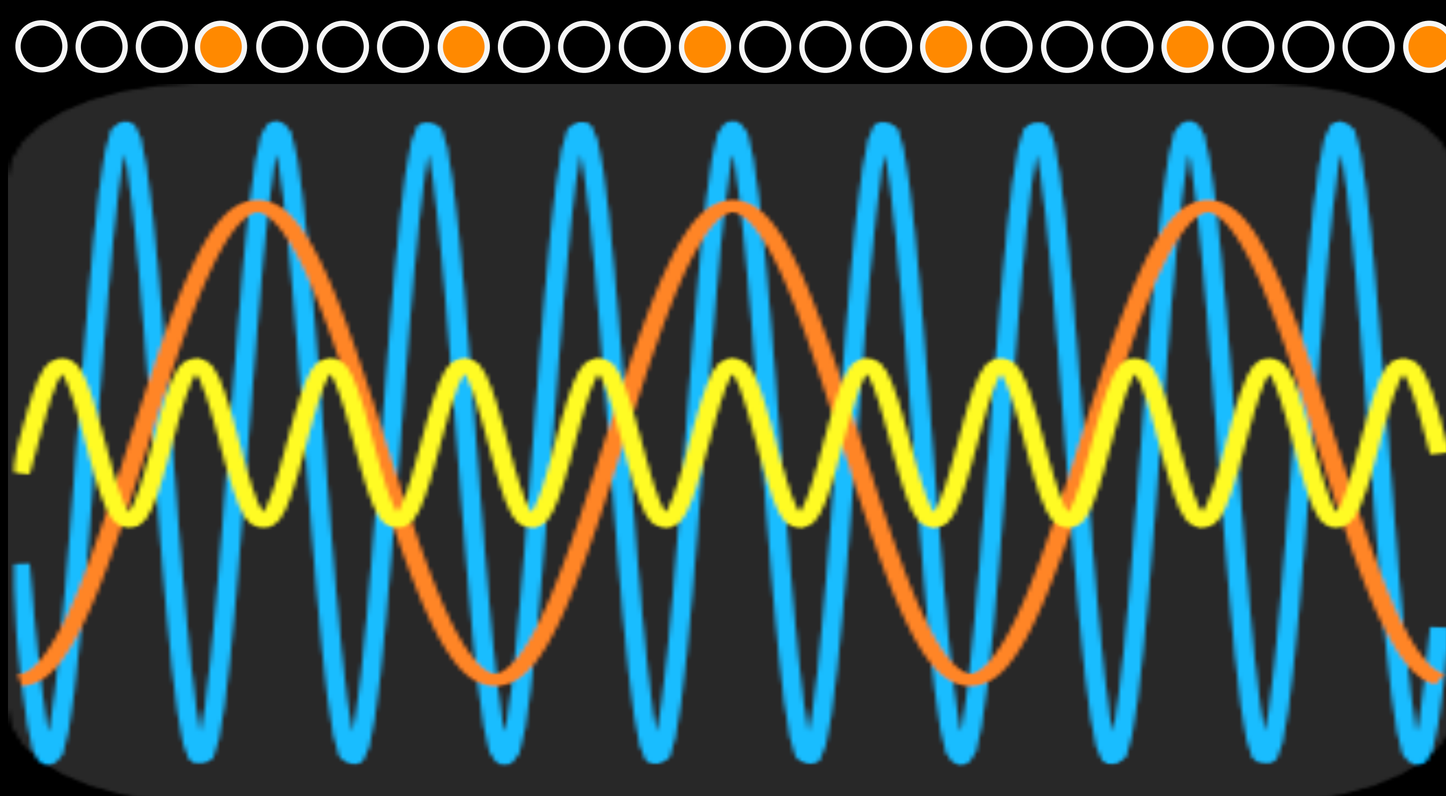


Nyquist

Intuitive example of CS

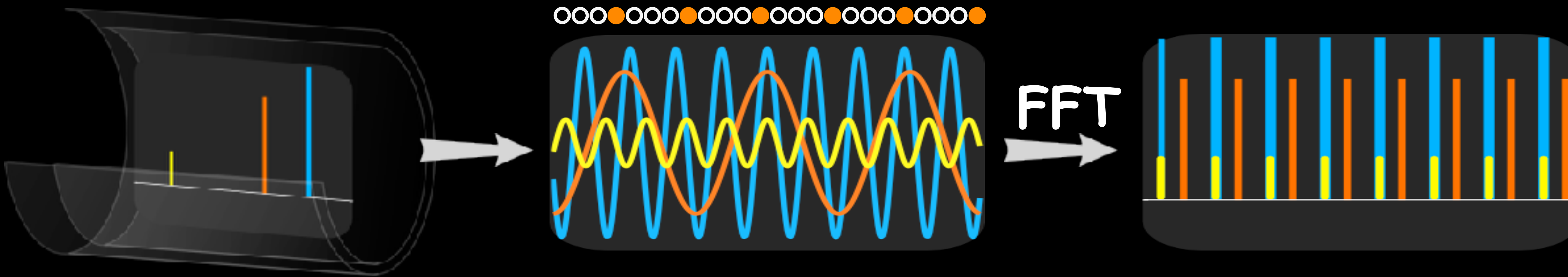


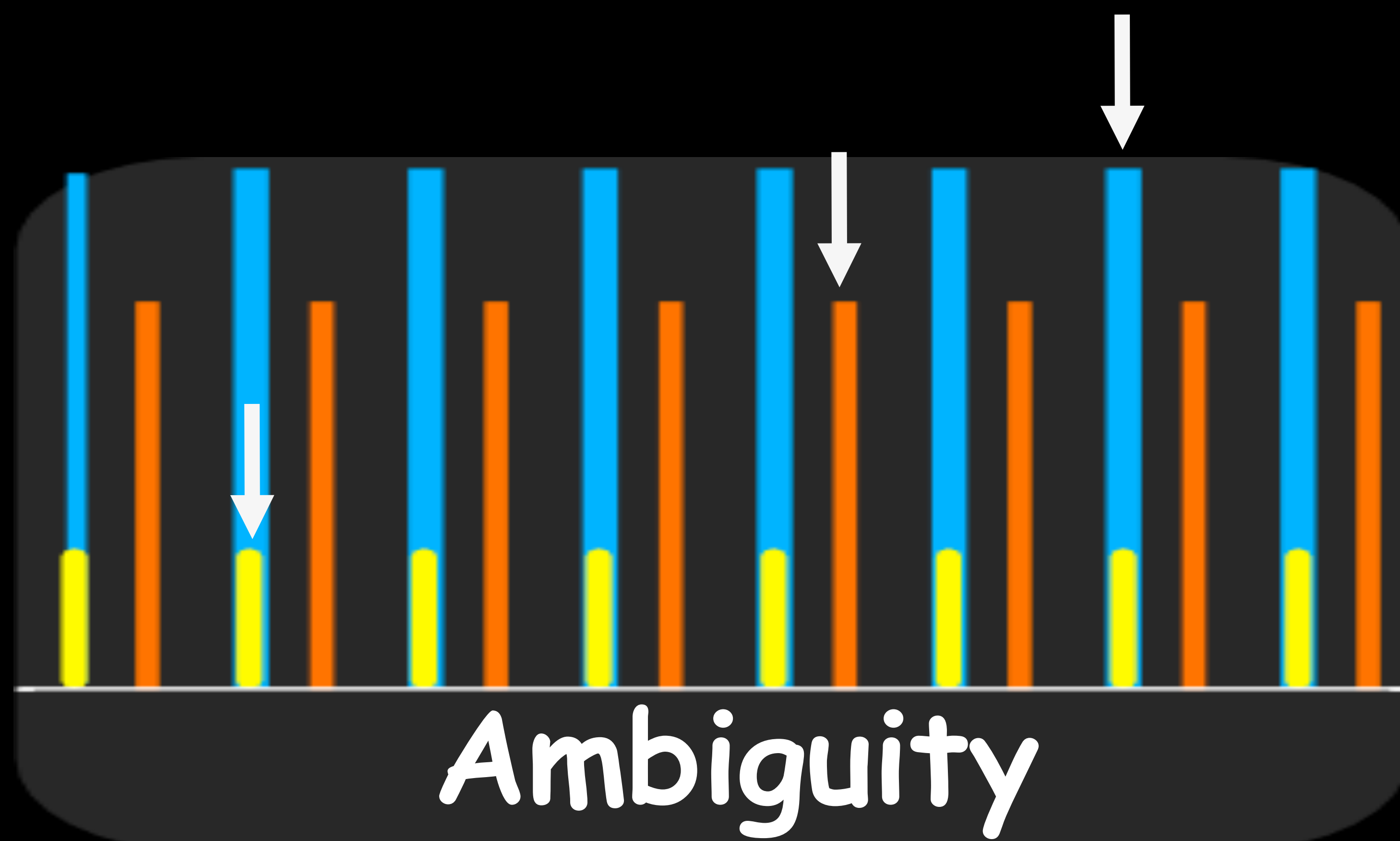
equispaced



sub-Nyquist

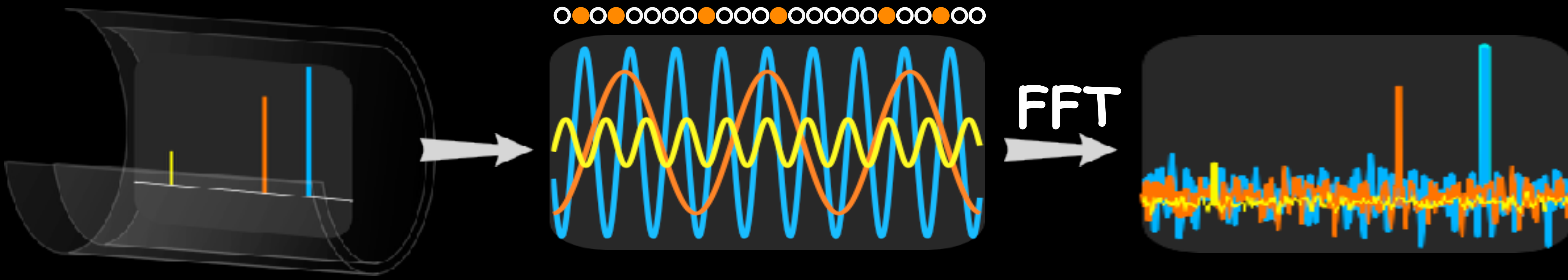
Intuitive example of CS

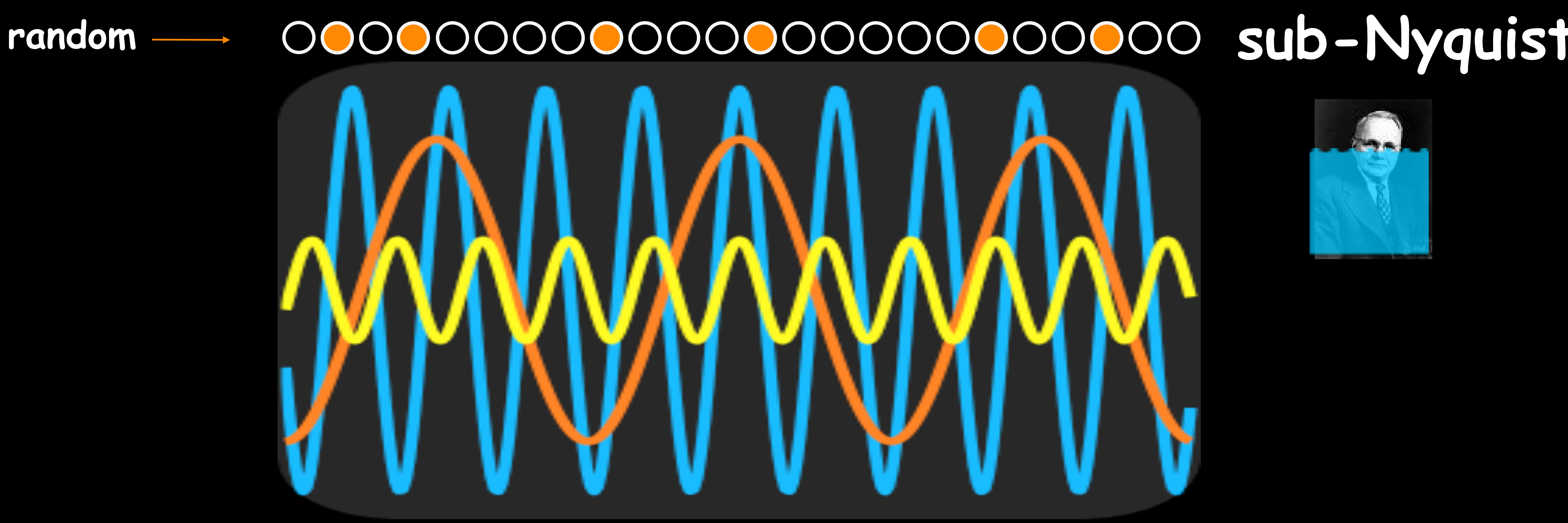


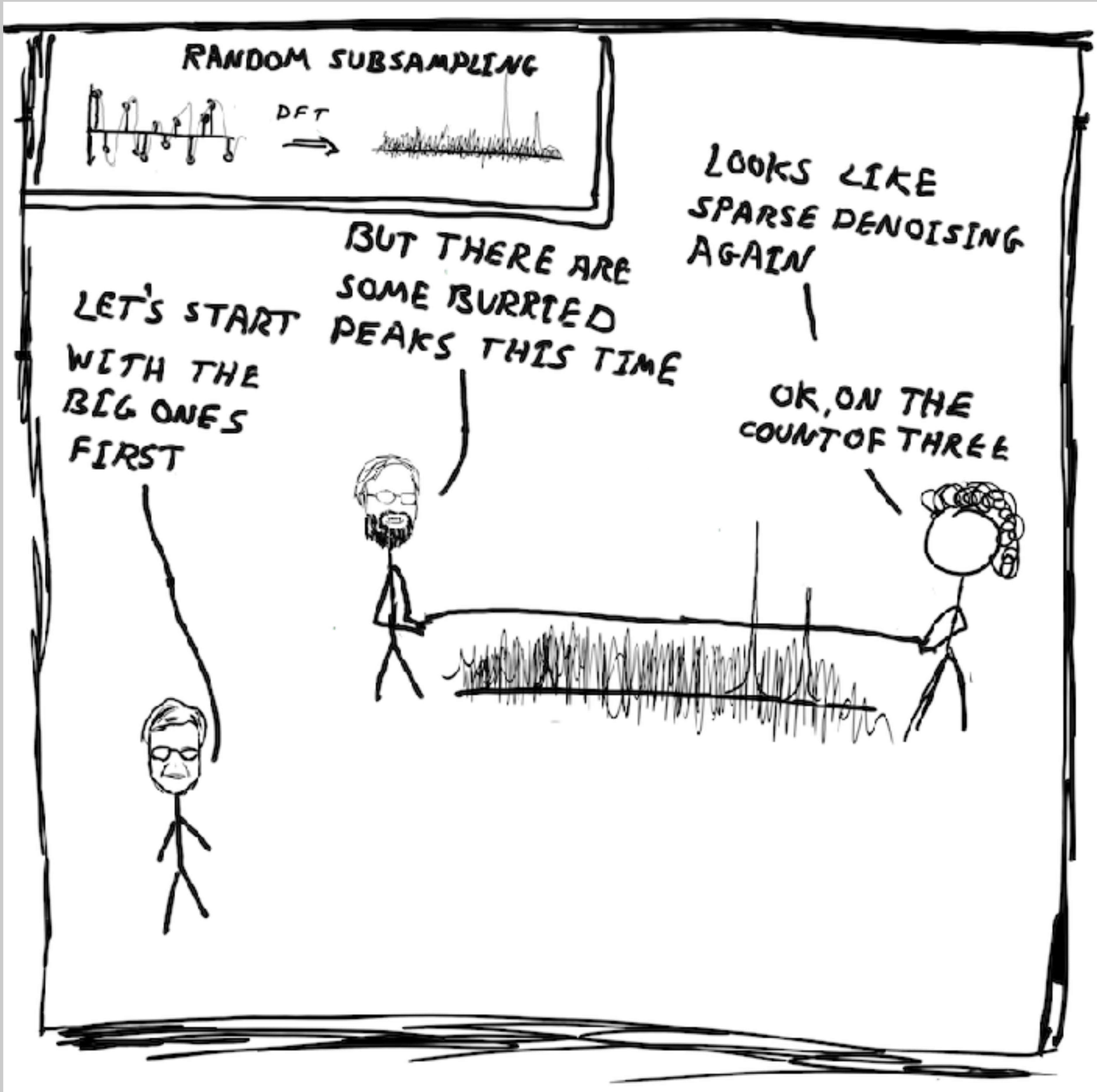


sub-Nyquist

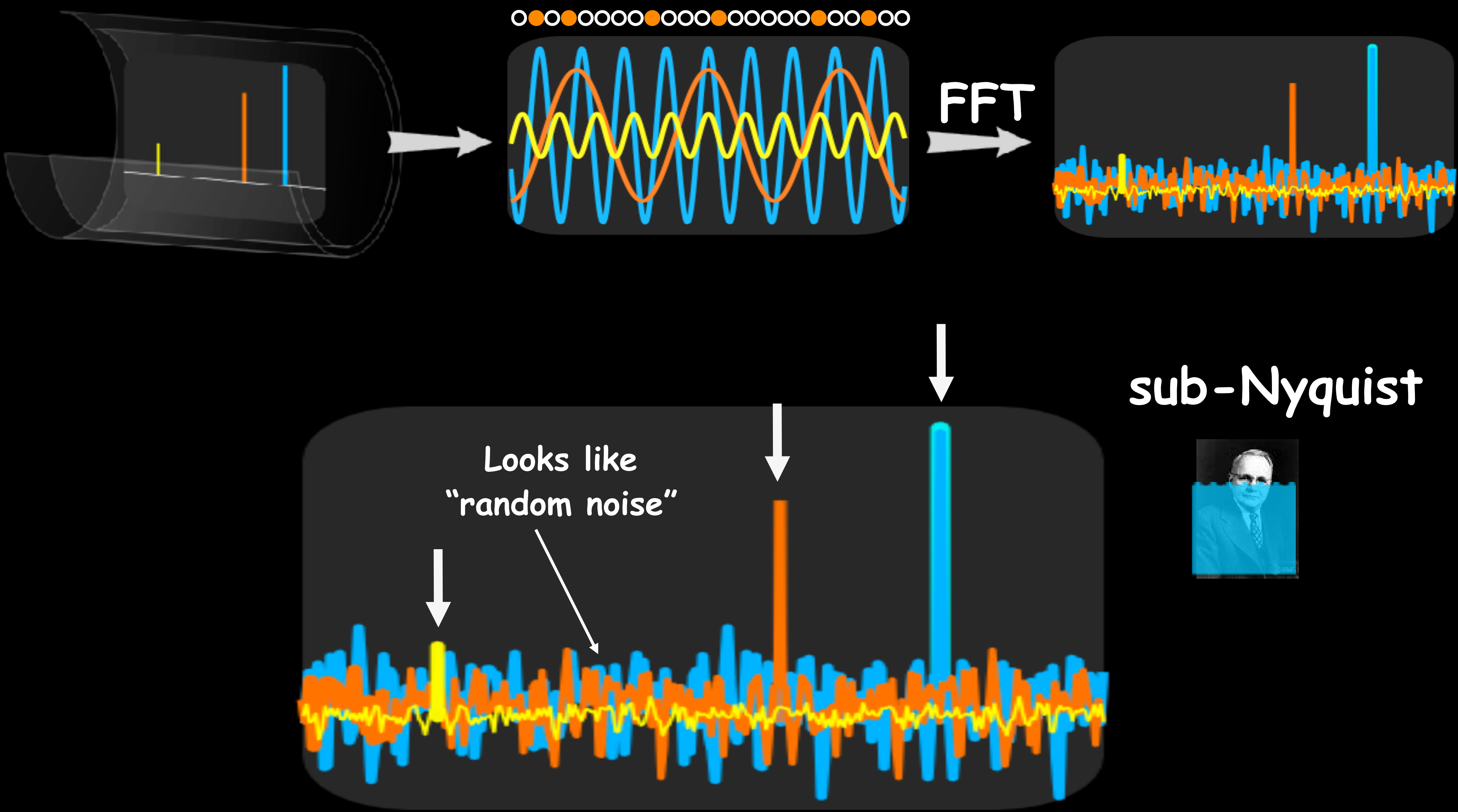
Intuitive example of CS



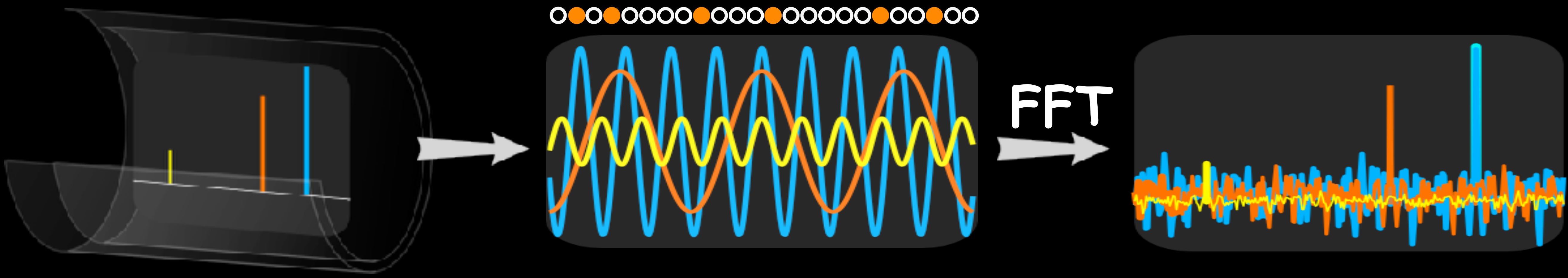


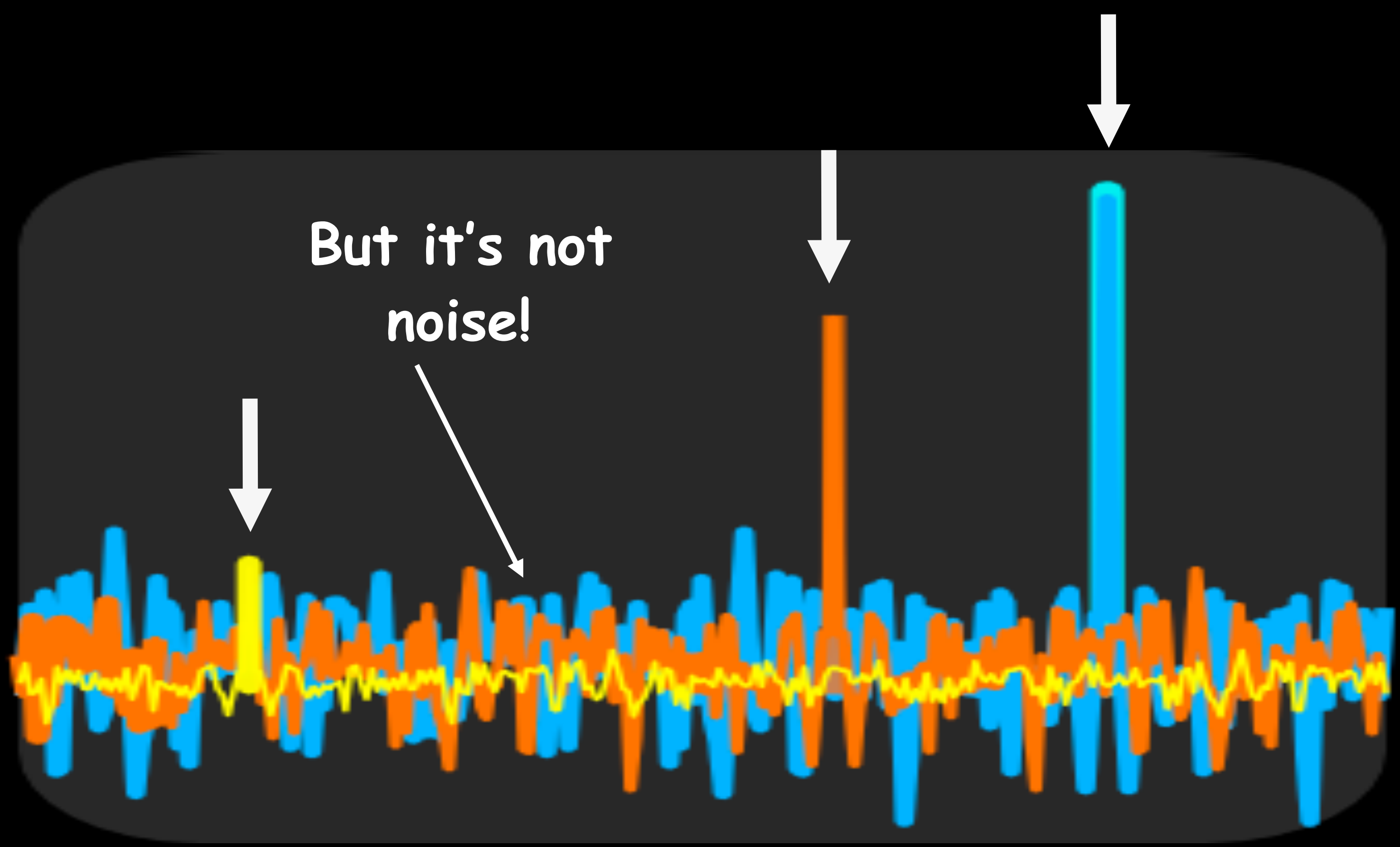

RANDOM SUBSAMPLING
DFT
LET'S START WITH THE BIG ONES FIRST
BUT THERE ARE SOME BURRIED PEAKS THIS TIME
LOOKS LIKE SPARSE DENOISING AGAIN
OK, ON THE COUNT OF THREE

Intuitive example of CS



Intuitive example of CS

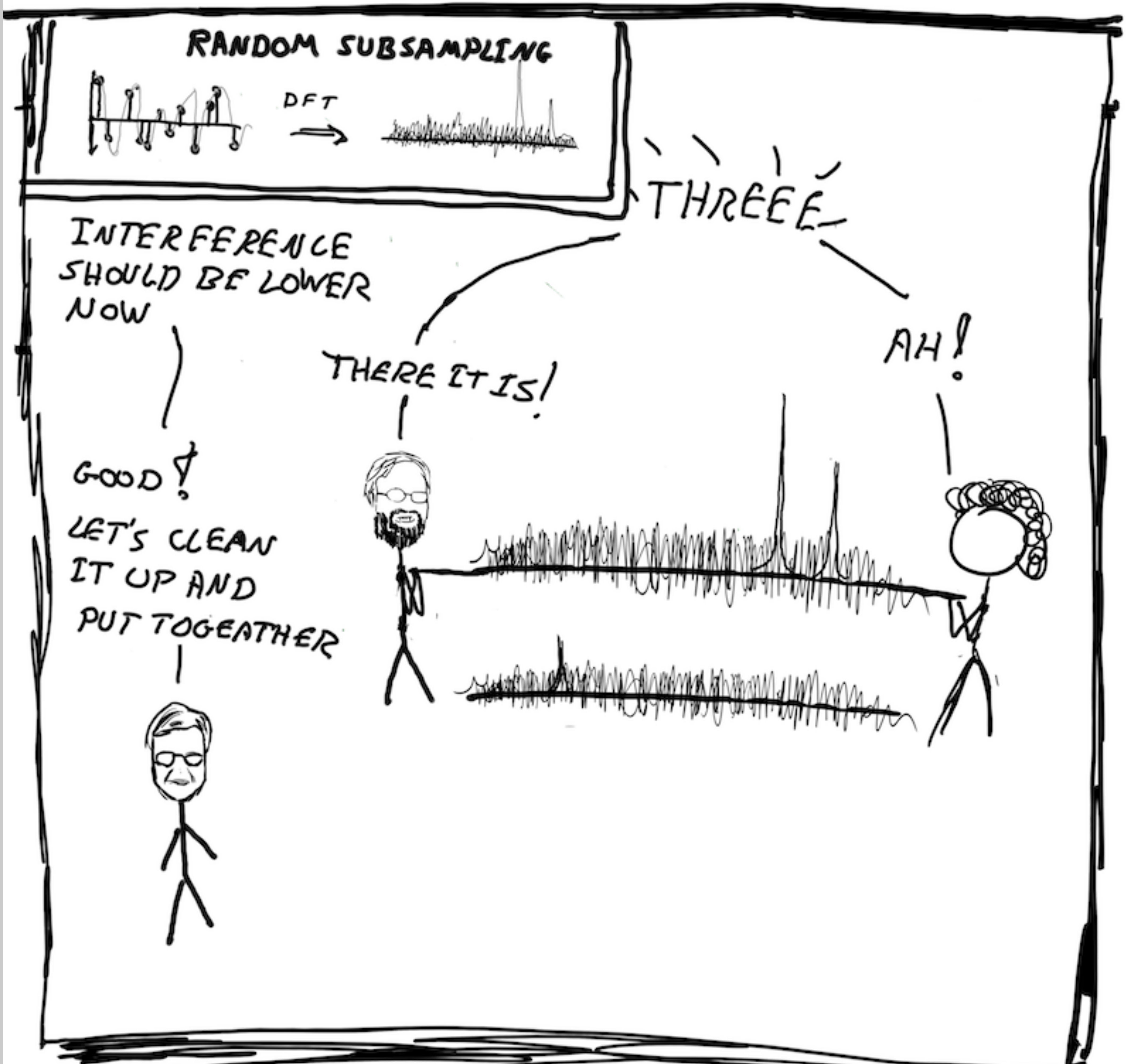




sub-Nyquist

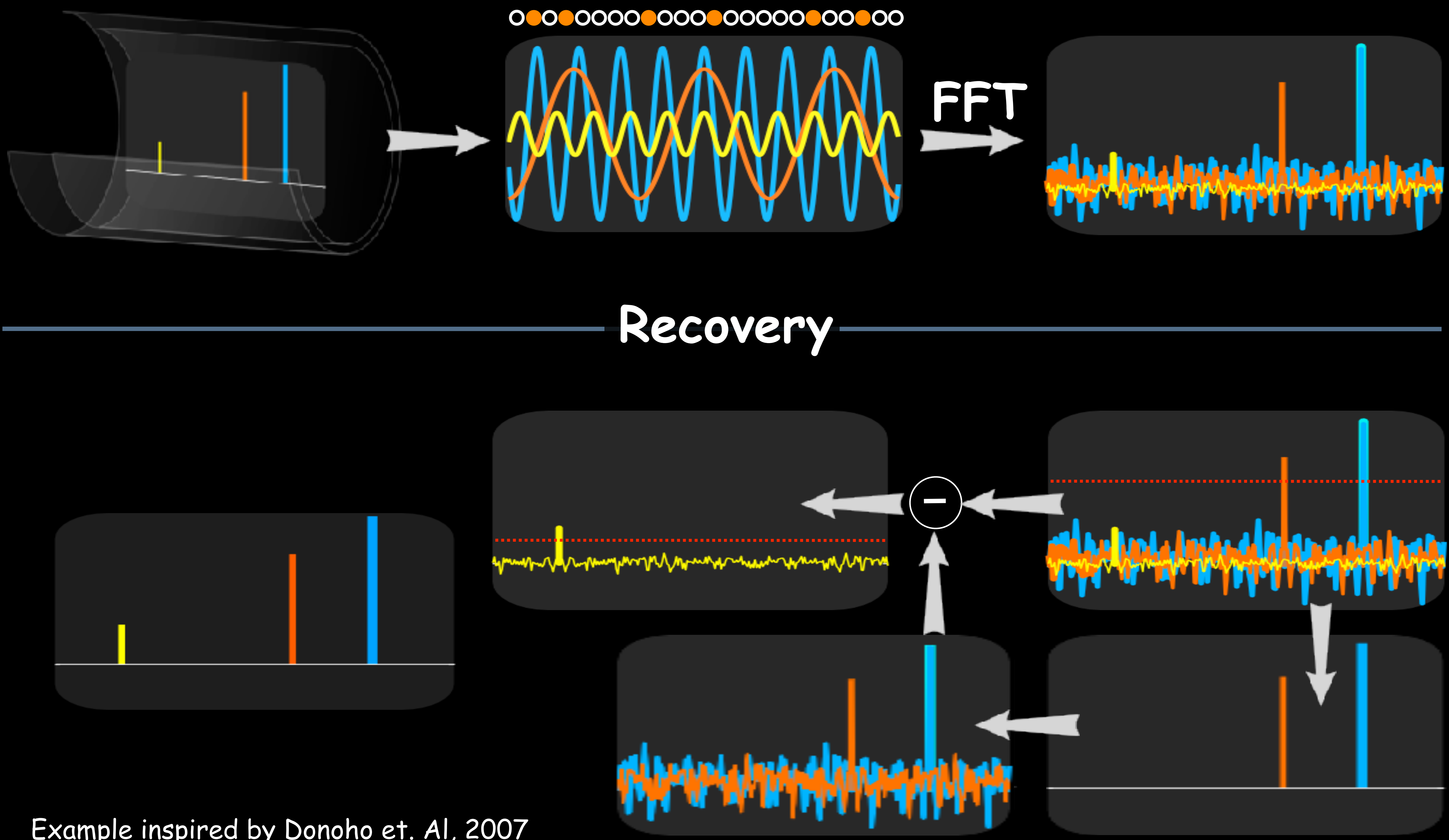

RANDOM SUBSAMPLING
DFT
WE CAN CALCULATE THE INTERFERENCE THEY CREATE AND REMOVE IT
TWO
ONE

C Berkeley

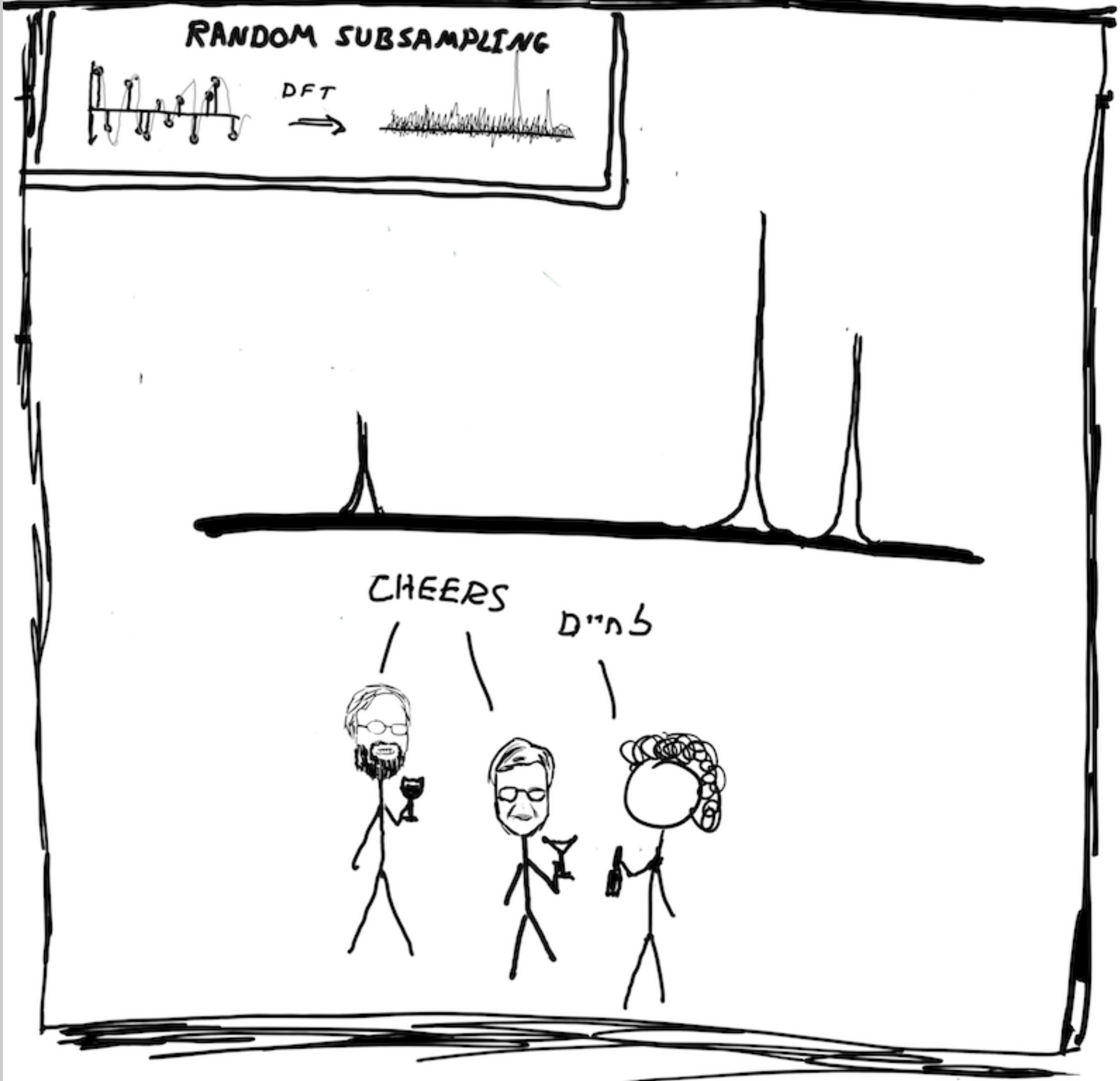

RANDOM SUBSAMPLING
DFT
THREEE
INTERFERENCE SHOULD BE LOWER NOW
THERE IT IS!
AH!
GOOD!
LET'S CLEAN IT UP AND PUT TOGEATHER

C Berkeley

Intuitive example of CS

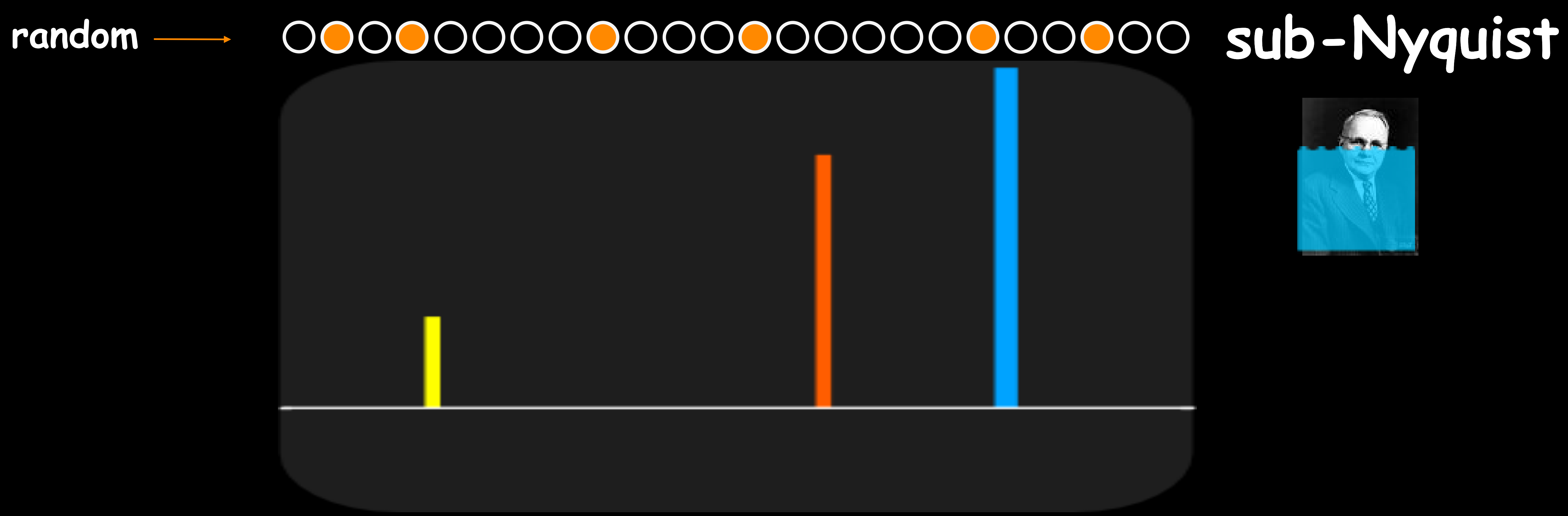


Example inspired by Donoho et. Al, 2007

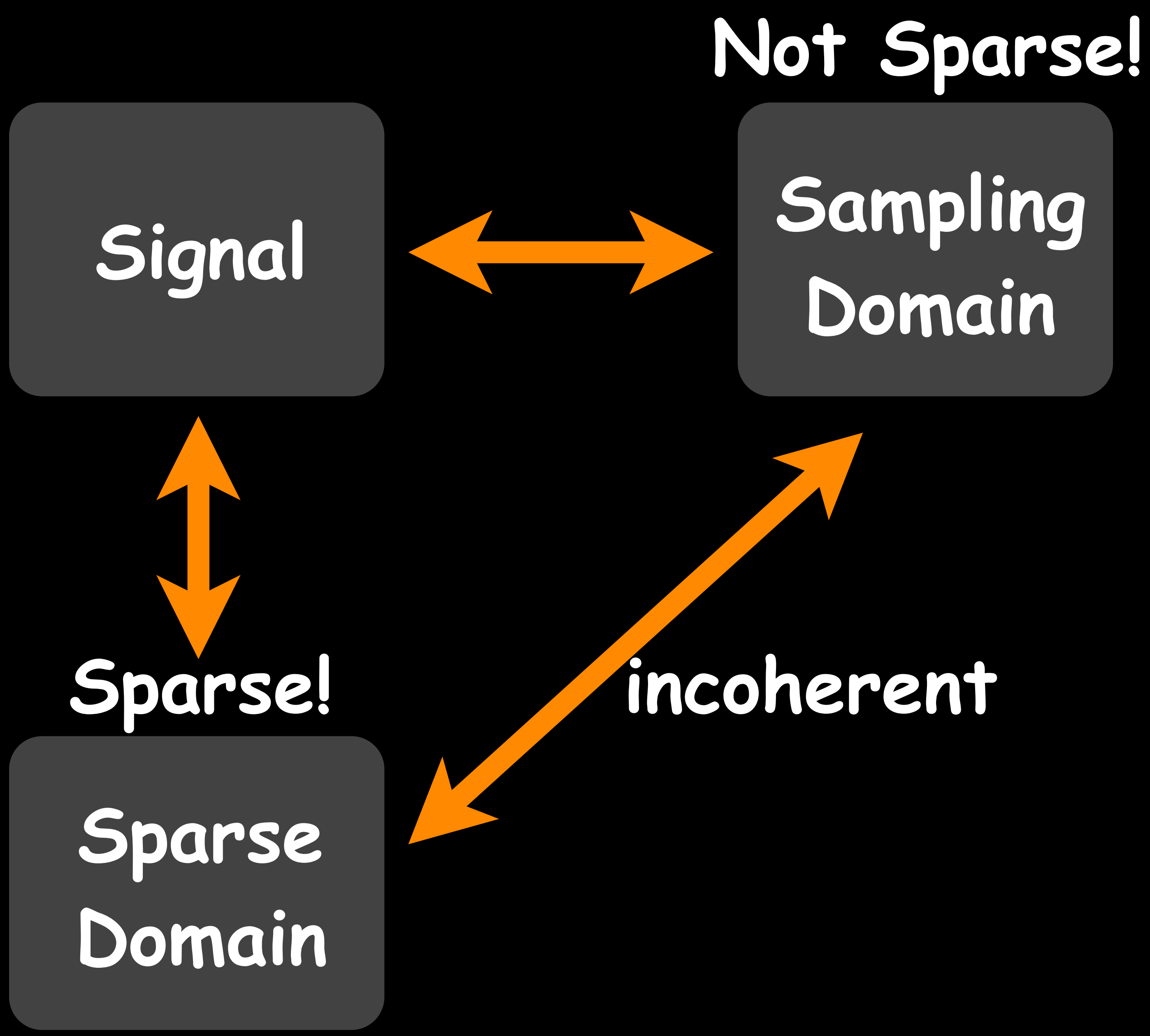

RANDOM SUBSAMPLING
DFT
CHEERS
לח״ם

Question!

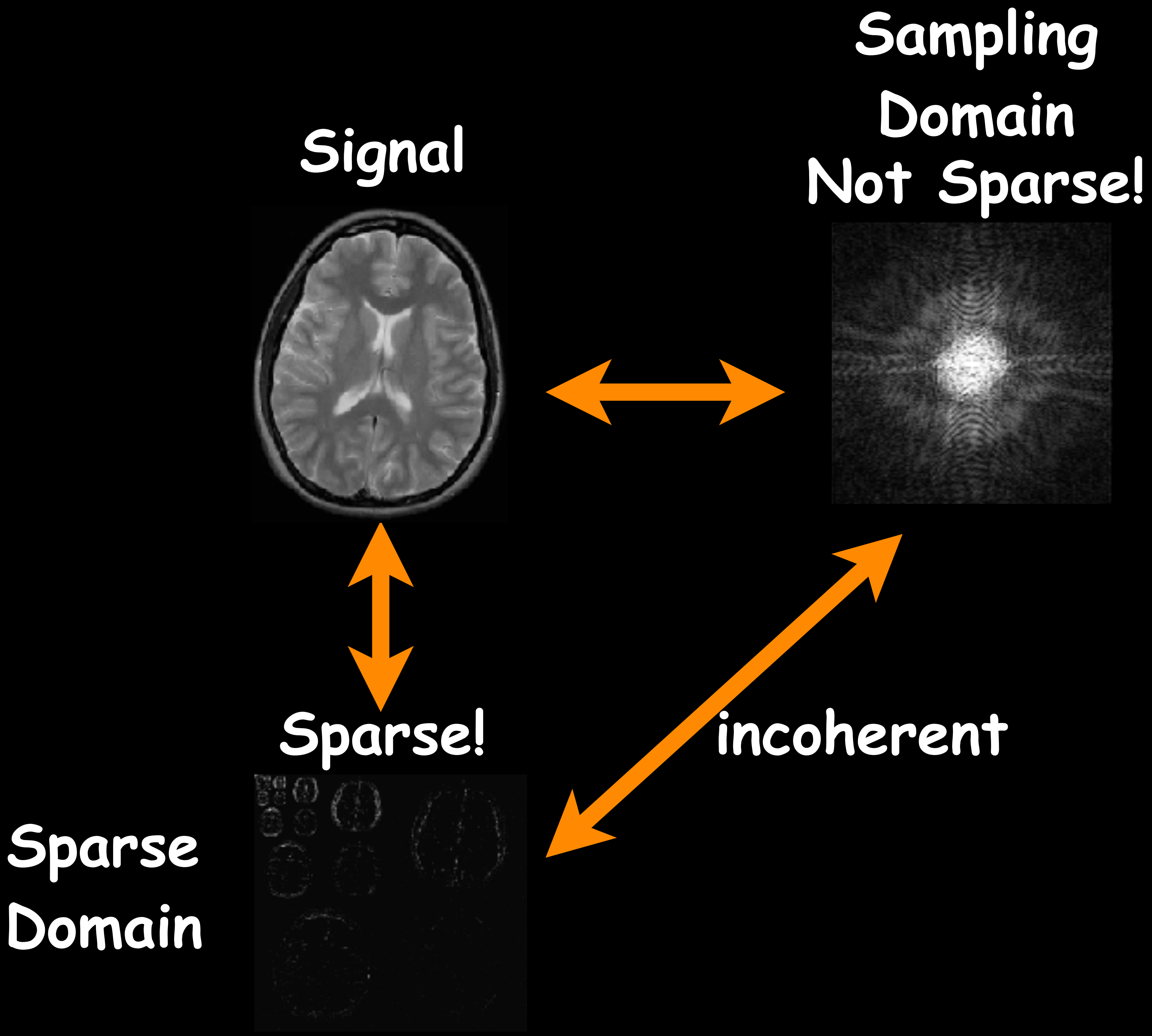
- What if this was the signal?
- Would CS still work?



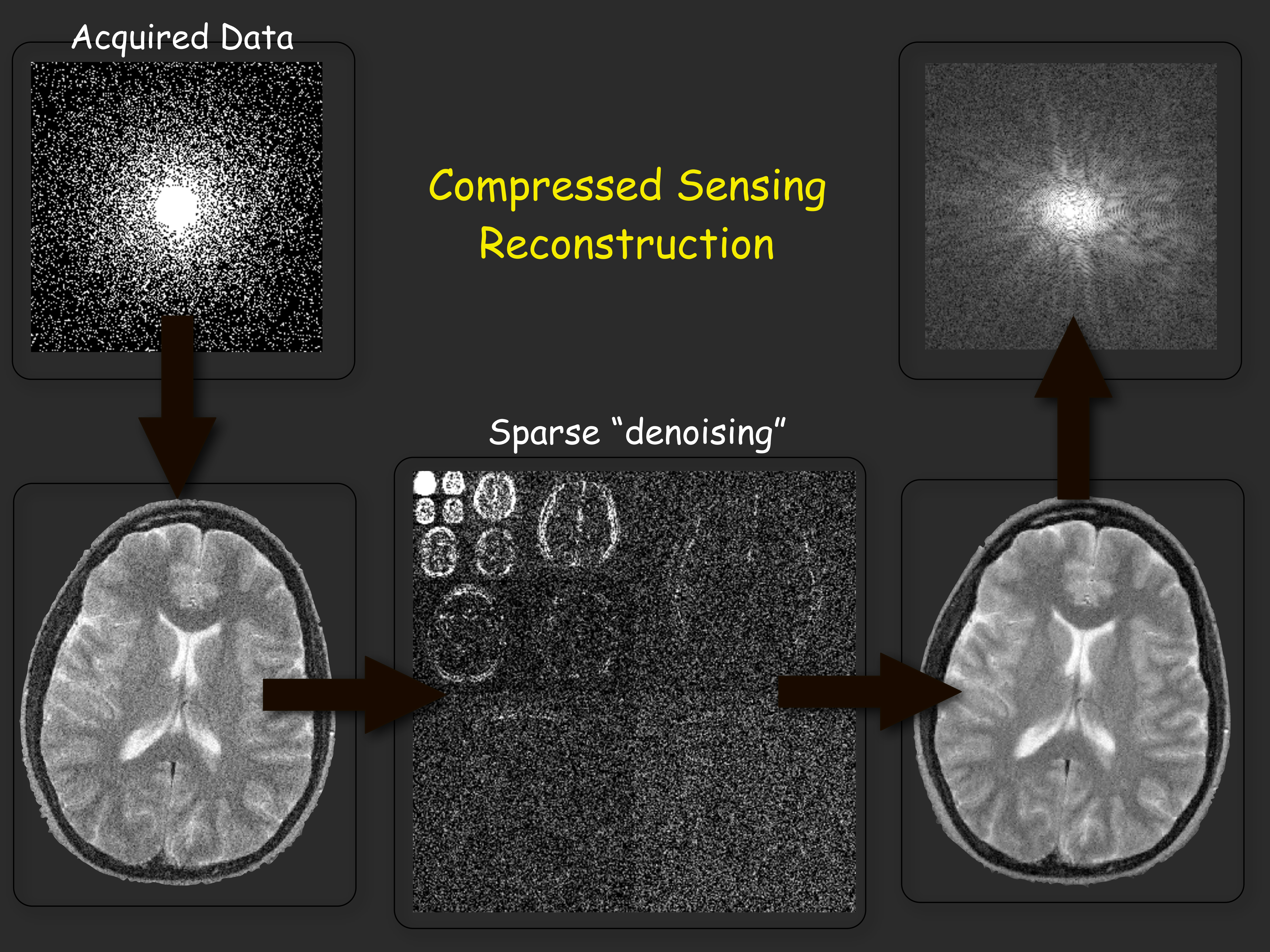
Domains in Compressed Sensing

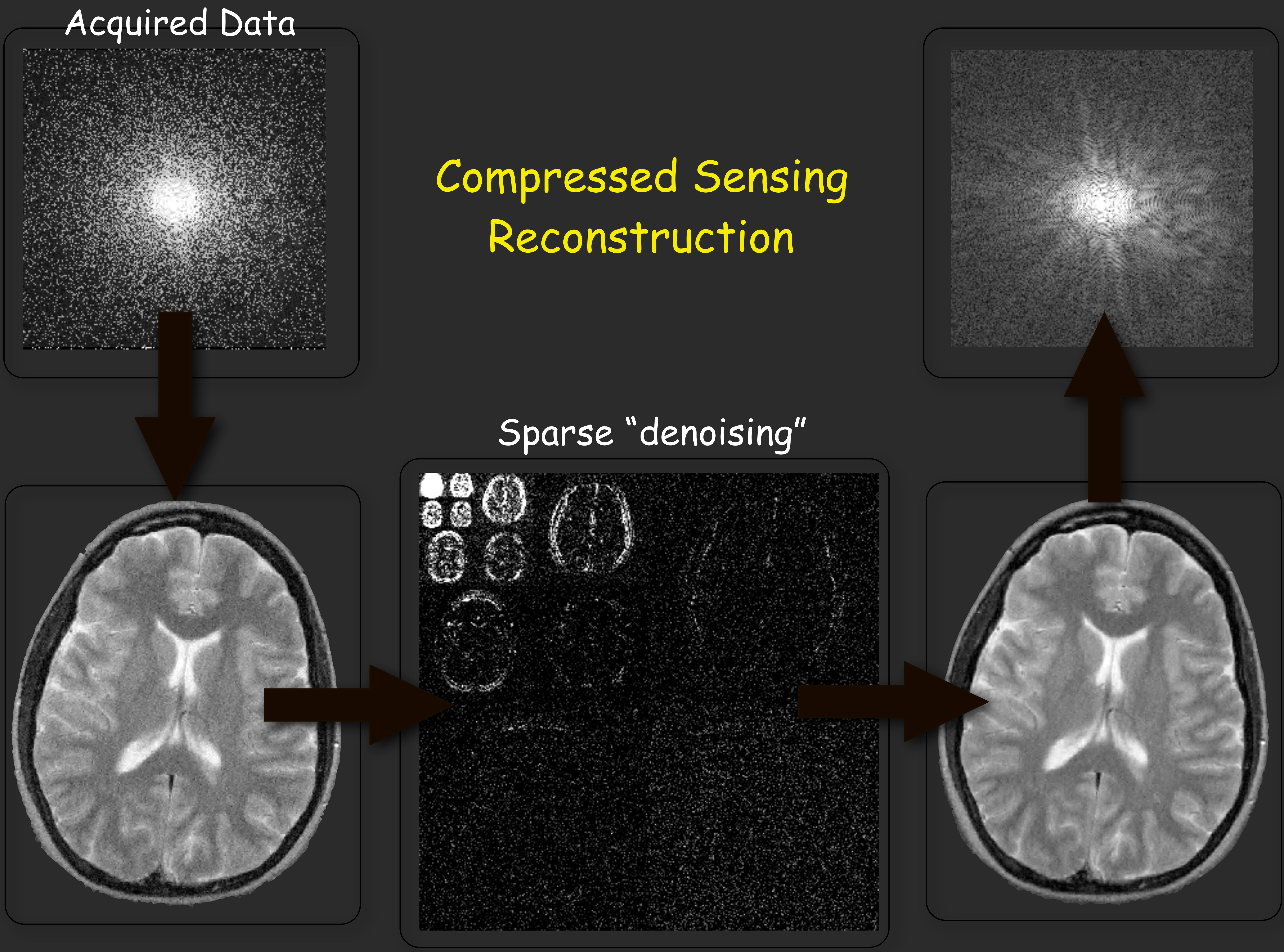


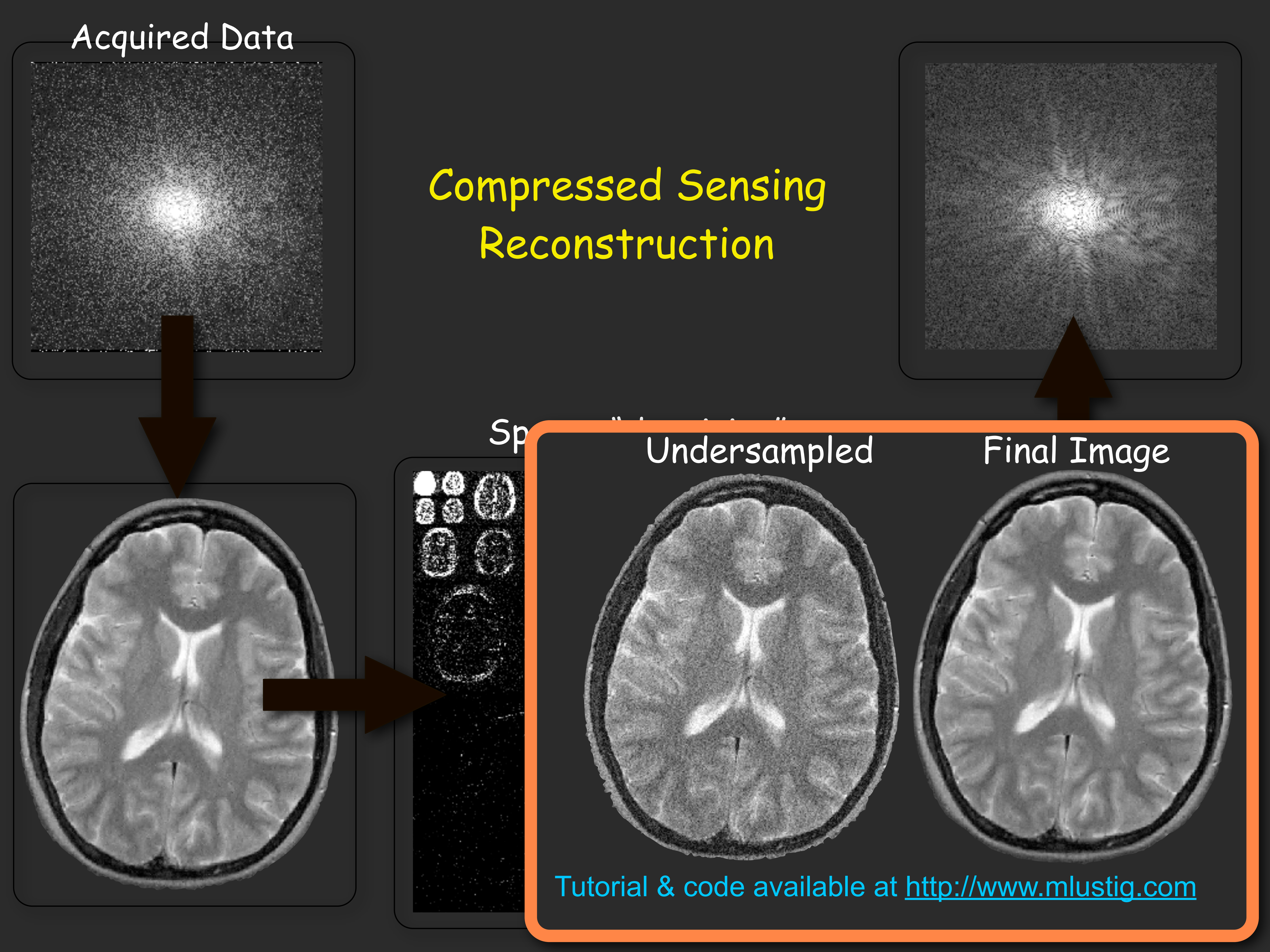
MRI



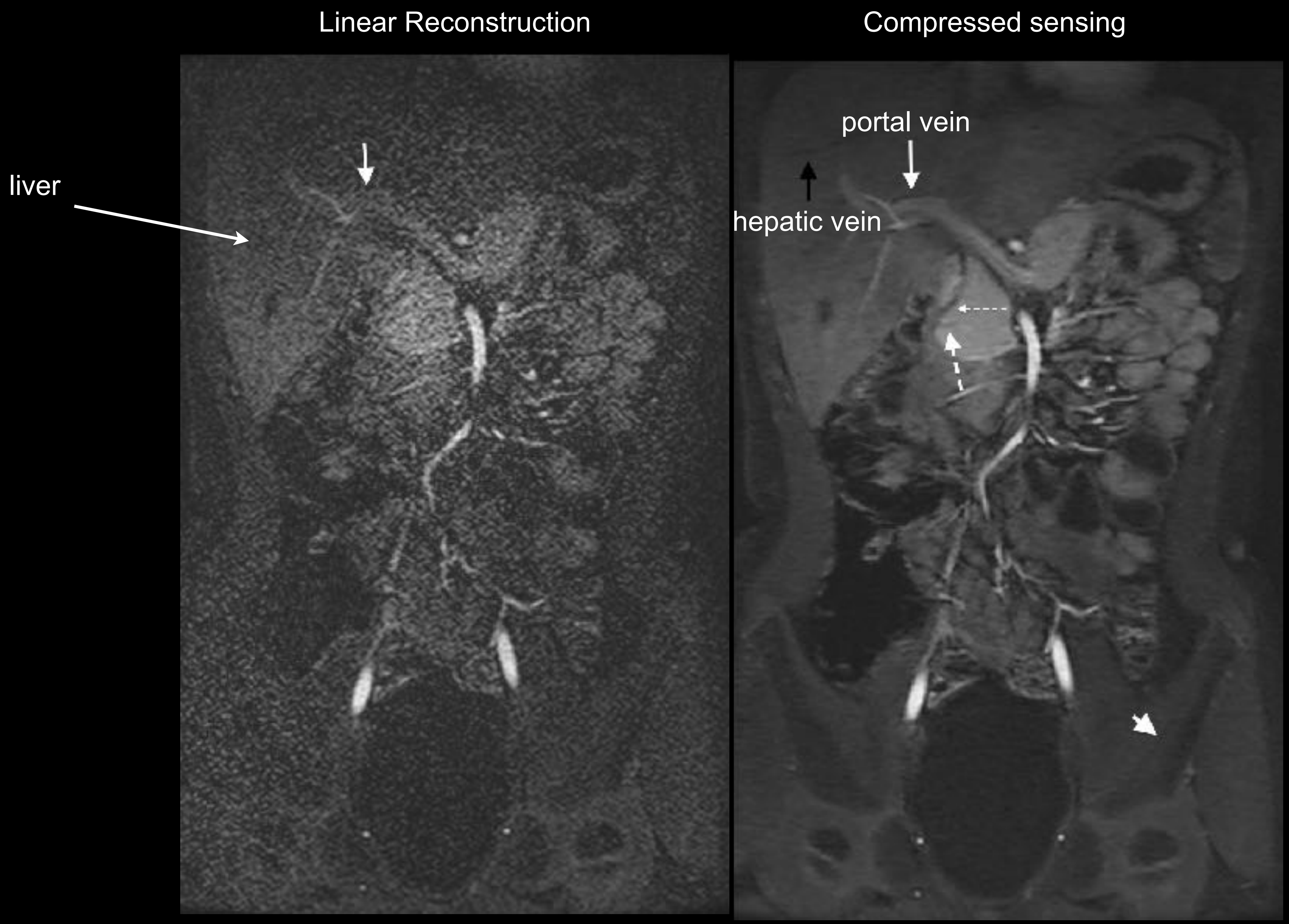
Sampling
Domain
Not Sparse!
Signal
Sparse!
incoherent
Sparse
Domain


Acquired Data
Compressed Sensing
Reconstruction
Sparse "denoising"

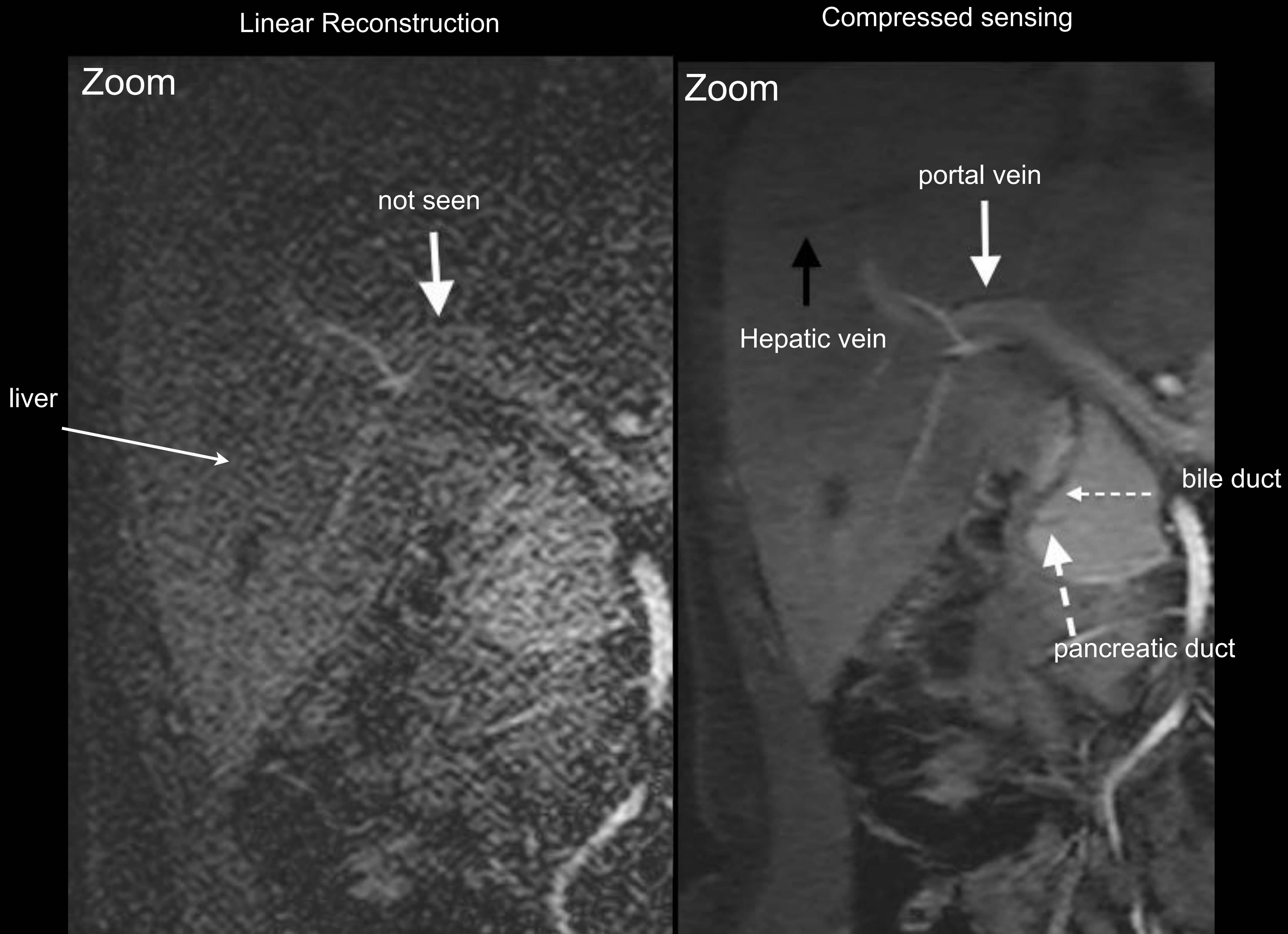

Acquired Data
Compressed Sensing
Reconstruction
Sparse "denoising"


Acquired Data
Compressed Sensing
Reconstruction
Undersampled
Final Image
Tutorial & code available at http://www.mlustig.com

6 year old male abdomen. Fine structures (arrows) are buried in noise (artifactual + noise amplification) and are recovered by CS with L1-wavelets. x8 acceleration

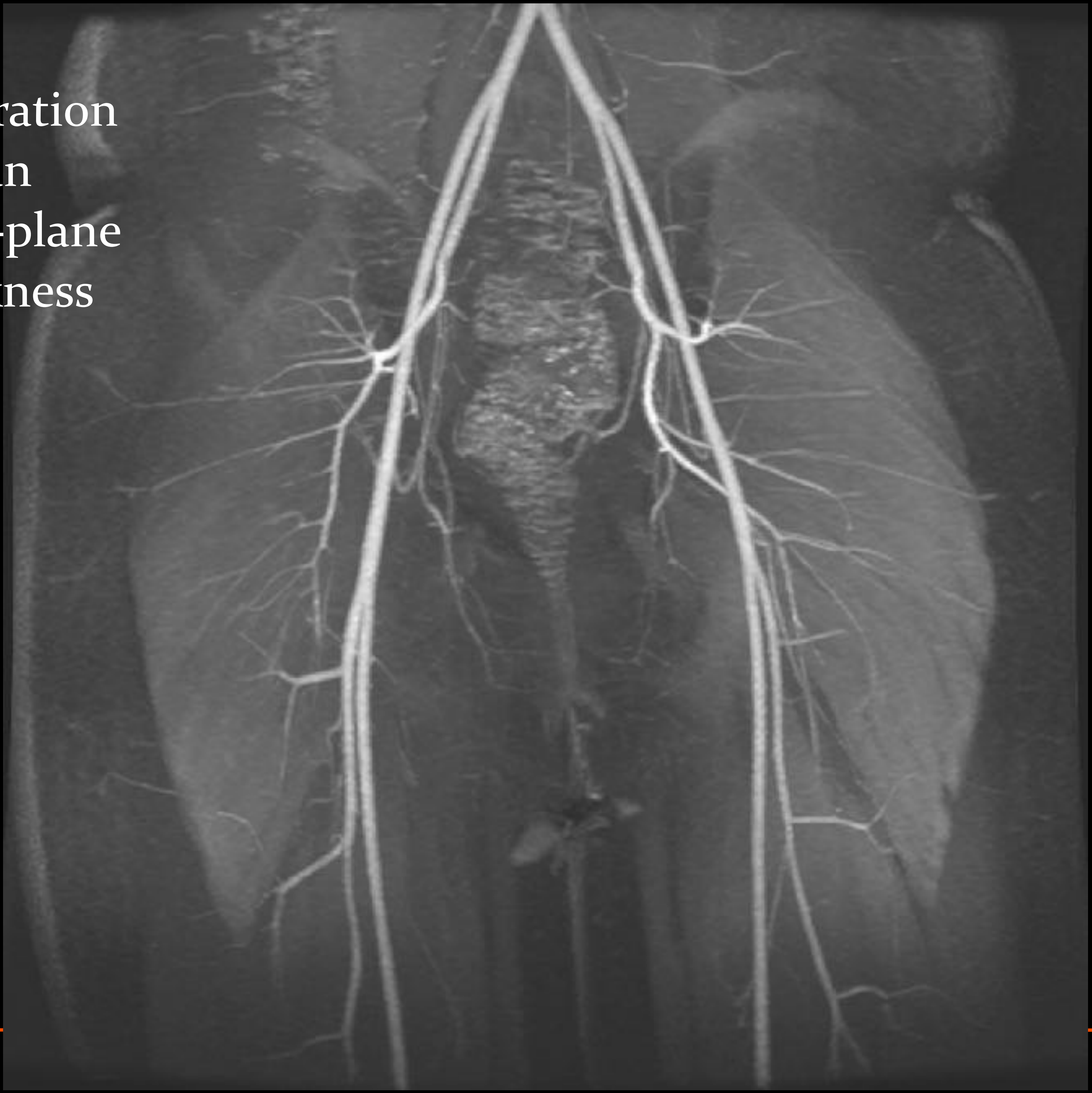


6 year old male abdomen. Fine structures (arrows) are buried in noise (artifactual + noise amplification) and are recovered by CS with L1-wavelets.



Back to Results

6 year old
8-fold acceleration
16 second scan
0.875 mm in-plane
1.6 slice thickness



Dynamic Compressed Sensing DCE - Free Breathing

Free Breathing

6 years old female

1x1x1mm

4.3 sec/frame

Cardiac-Resolved Volumetric Phase-Contrast Imaging (4D Flow)

<u>12 month male (114 bpm)</u>

Scan time: 11:05 min; *Total acceleration:* 15.0

Resolution: 0.9 x 0.9 x 1.4 mm^3

VENC: (150, 150, 150) cm/s

Contrast: ferumoxytol

Other Applications

- Compressive Imaging
- Medical Imaging
- Analog to information conversion
- Biosensing
- Geophysical Data Analysis
- Compressive Radar
- Astronomy
- Communications
- More ……

Resources

- CS + parallel imaging matlab code, examples

  http://www.eecs.berkeley.edu/~mlustig/software/

- Rice University CS page: papers, tutorials, codes, ….
  http://www.dsp.ece.rice.edu/cs/

- IEEE Signal Processing Magazine, special issue on compressive sampling 2008;25(2)

- March 2010 Issue Wired Magazine: "Filling the Blanks"

- Igor Caron Blog: http://nuit-blanche.blogspot.com/

Thank you!
תודה רבה