EE123
Digital Signal Processing

Lecture 14A
Compressed Sensing

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Lab + Frequency challenge

- Lab 4
  - – Make sure you get good signal -- like the one I recorded
  - – Think of detecting bursts -- a robust method will lead to good results in the last part
- Frequency challenge
  - – Beacon in 5th floor, around 144.280MHz using 1ppb accurate GPSDO. Accurate up to 1/100 Hz.
  - – Transmits my callsign in morse code 5 times then 2 minutes break.
  - – Submit frequency on bcourses by Thursday 04/07
  - – You can only use the rtl-sdr to participate -- no cheating!
  - – Closest submission will win a radio!

Radios

- https://inst.eecs.berkeley.edu/~ee123/sp16/radio.html

- Jon Tamir will take over my office hours today starting 4:15-5:15

Compressive Sampling

Q: What is the rate you need to sample at?

A: At least Nyquist!

Compressive Sampling

Q: What is the rate you need to sample at?

A: Maybe less than Nyquist....

Image Compression

Images are compressible

Standard approach: First collect, then compress

1001101001101
0001001110101
0100110100010
0010101101010
1010101100101
1101110111010
1010110110110
10100111111

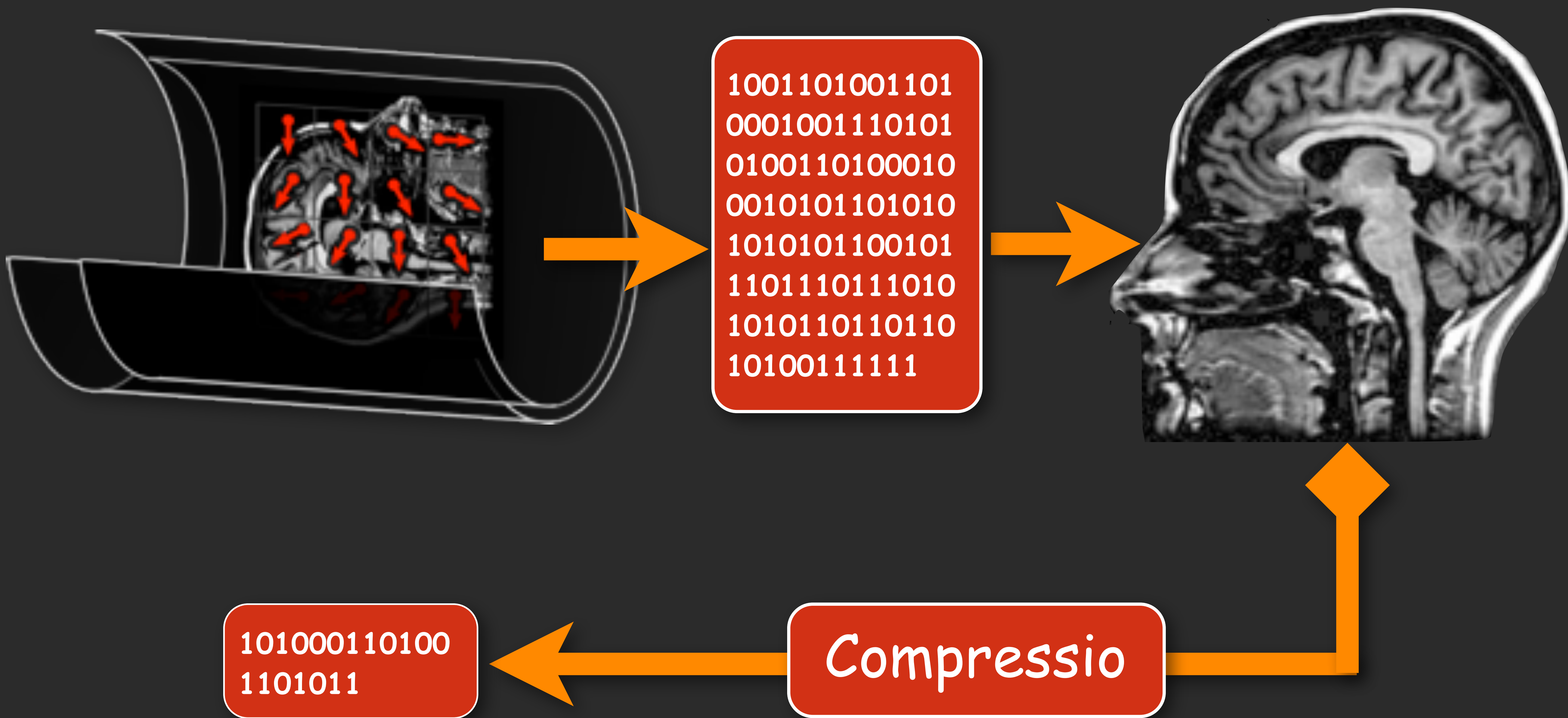
Compressio

101000110100
1101011

Image Compression

Medical images are compressible

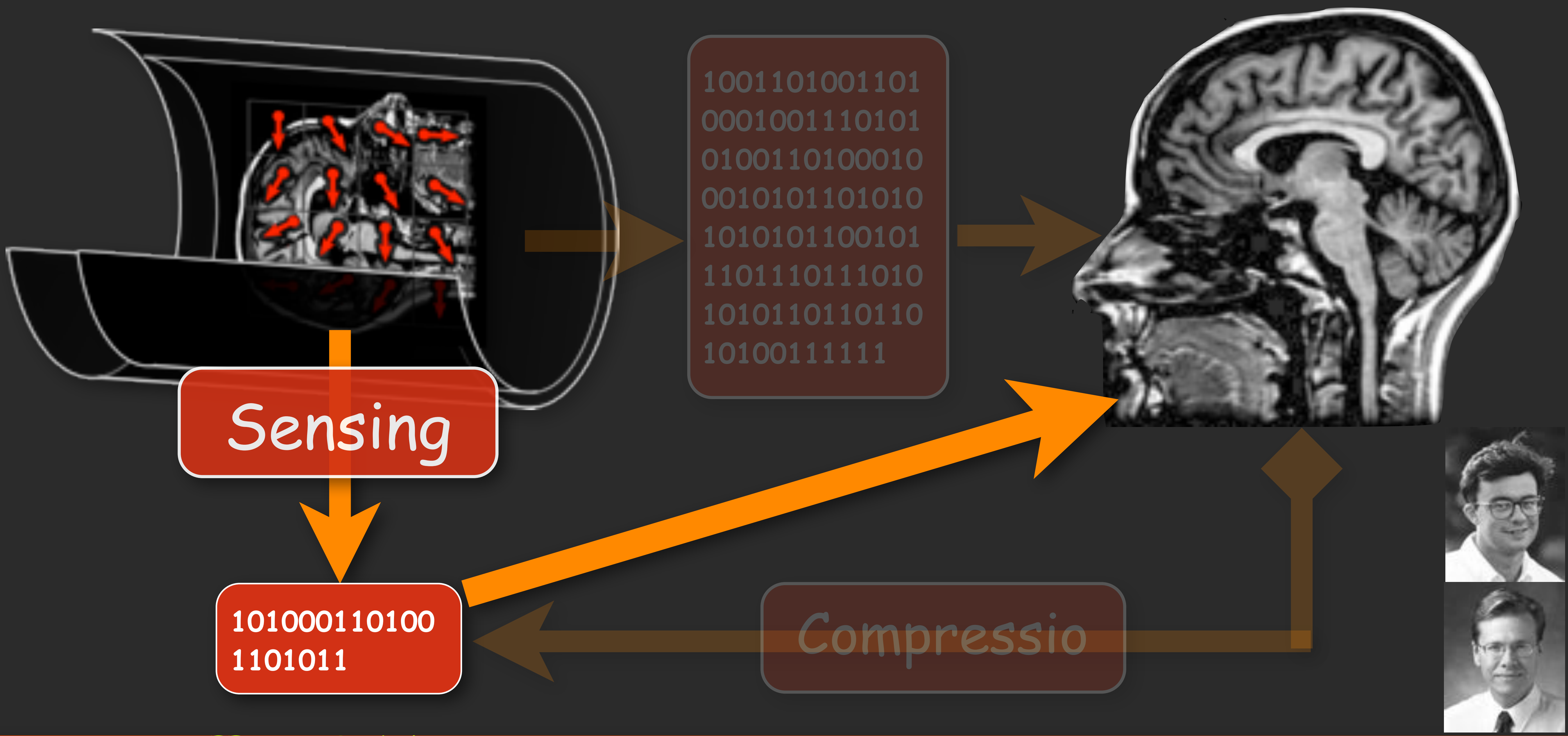
- Standard approach: First collect, then compress



Compressed Sensing

Medical images are compressible

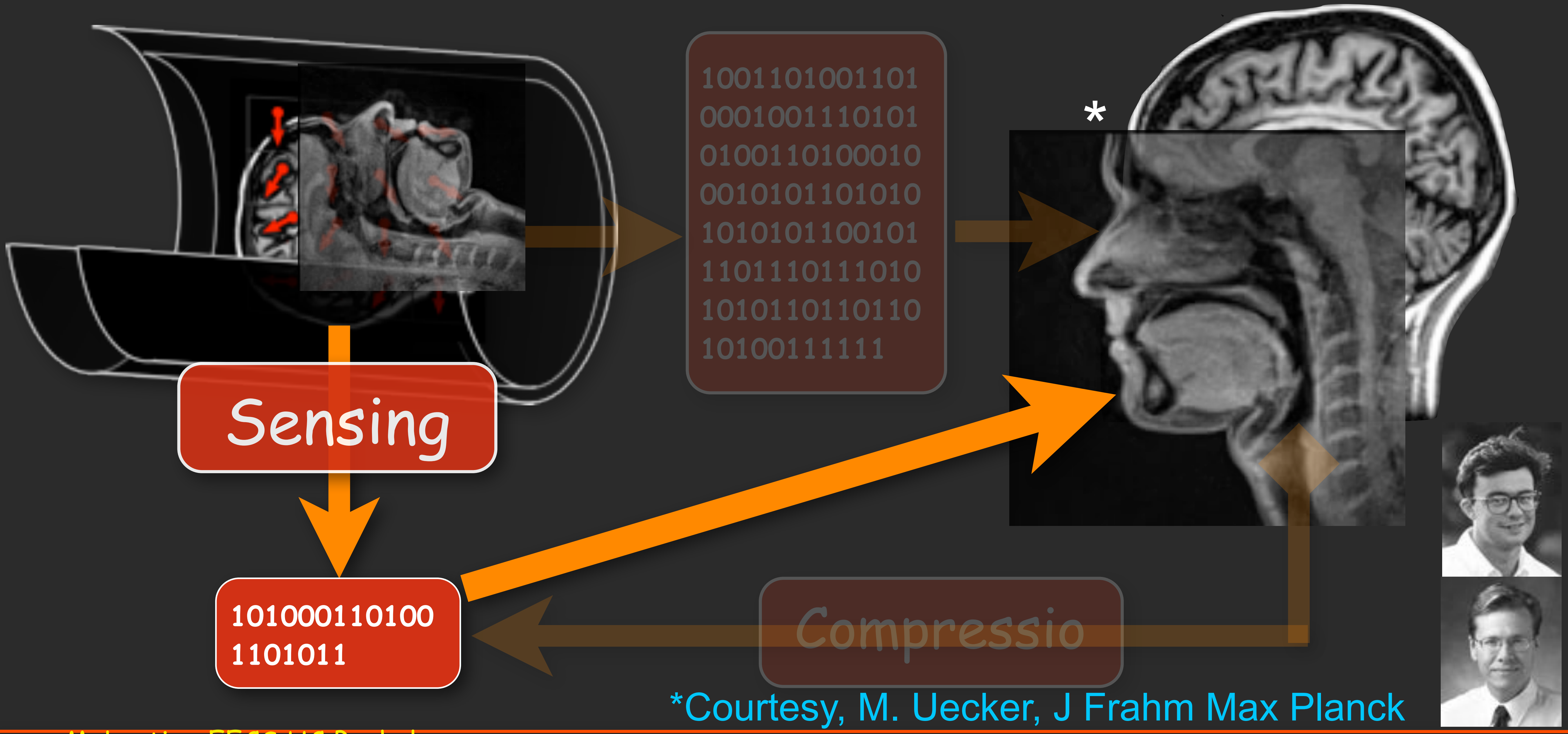
Standard approach: First collect, then compress



Compressed Sensing

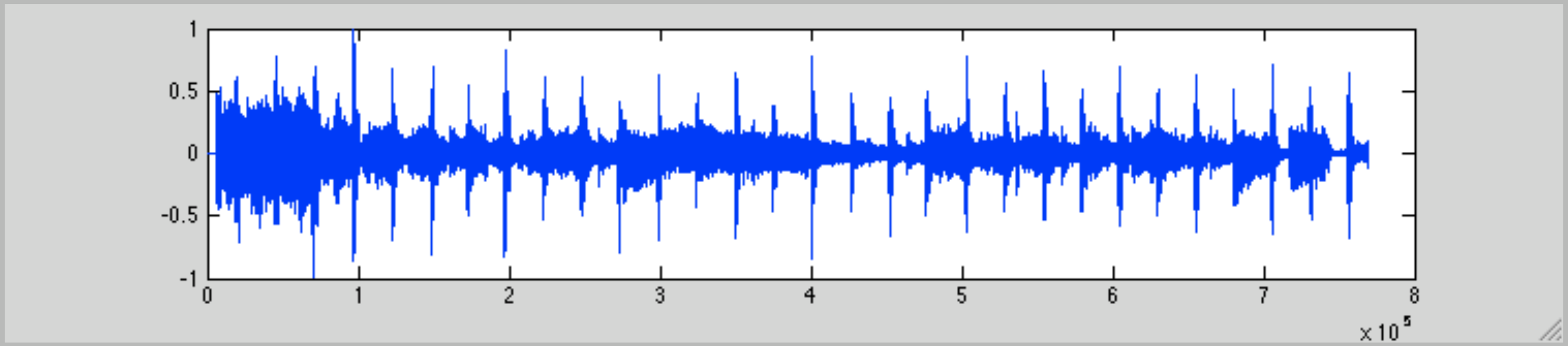
Medical images are compressible

Standard approach: First collect, then compress



*Courtesy, M. Uecker, J Frahm Max Planck

Example I: Audio



Raw audio: 44.1Khz, 16bit, stereo = 1378 Kbit/sec

MP3: 44.1Khz, 16bit, stereo = 128 Kbit/sec

10.76 fold!

Example II: Images

Raw image (RGB): 24 bit/pixel

JPEG : 1280x960, normal = 1.09 bit/pixel

22 fold!

Example III: Videos

Raw Video: (480x360)p x 24b/p x 24fps + 44.1Khz x 16b x 2 = 98,578 Kb/s

MPEG4 : 1300 Kb/s

75 fold!

Compression

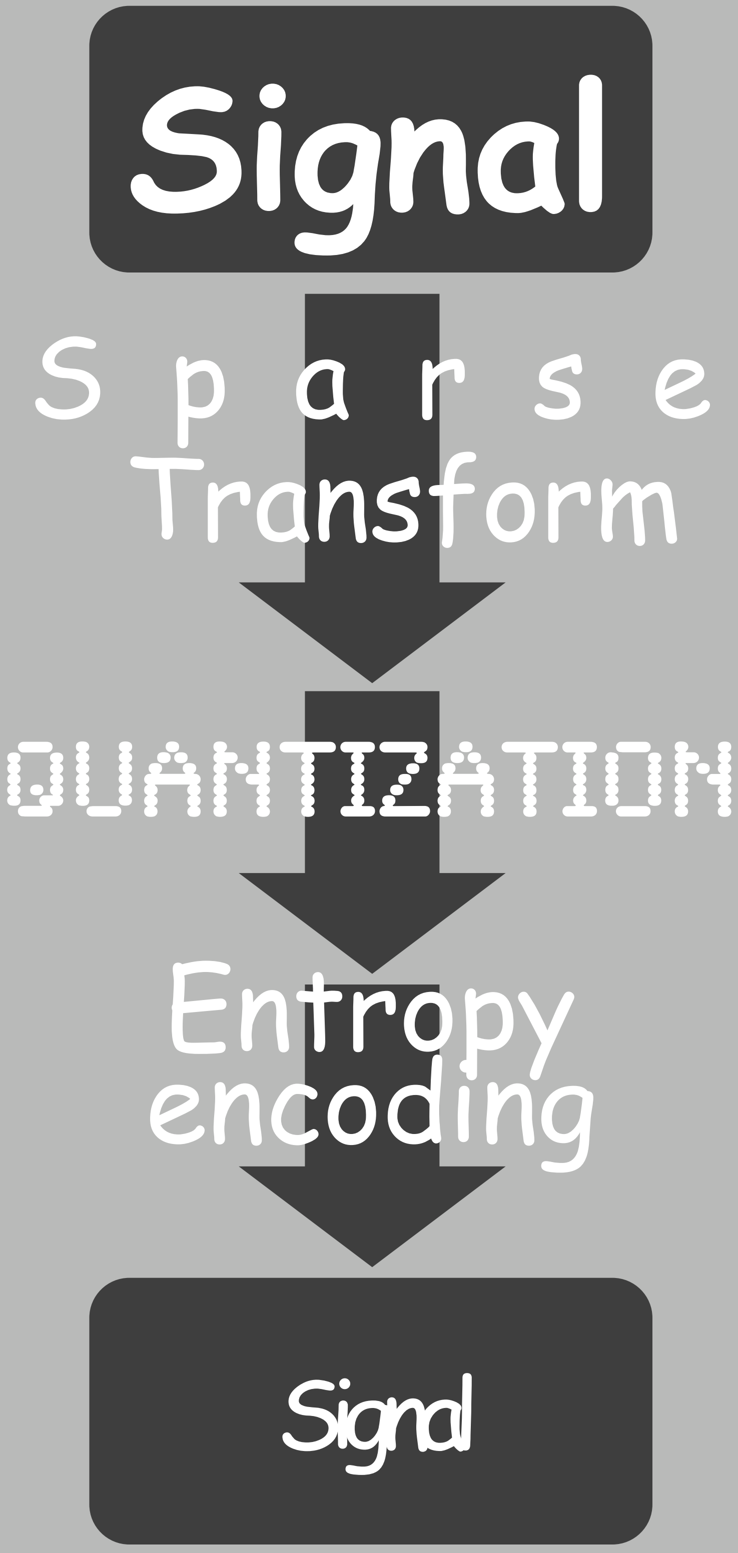
Almost all compression algorithm use transform coding

mp3: DCT

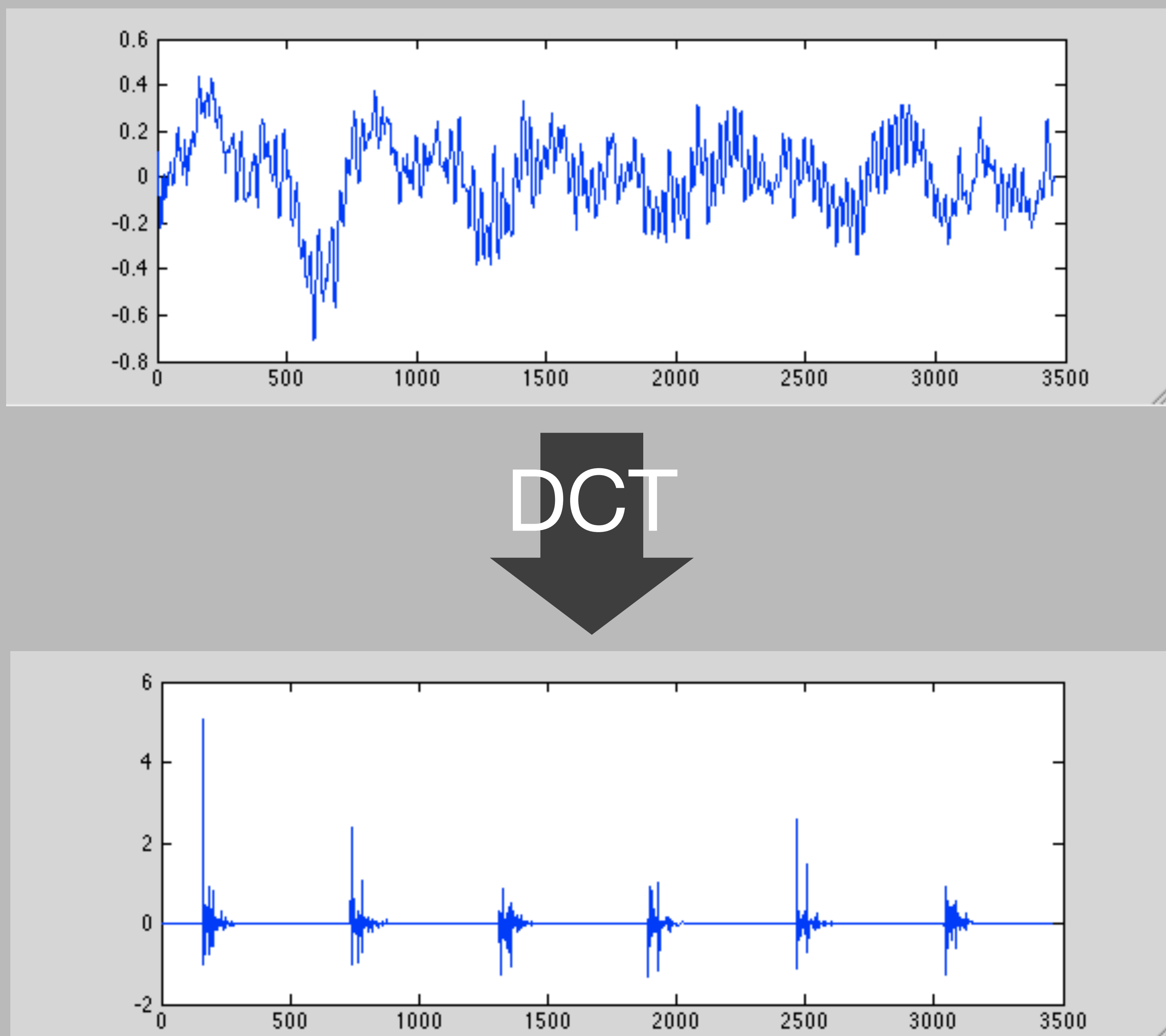
JPEG: DCT

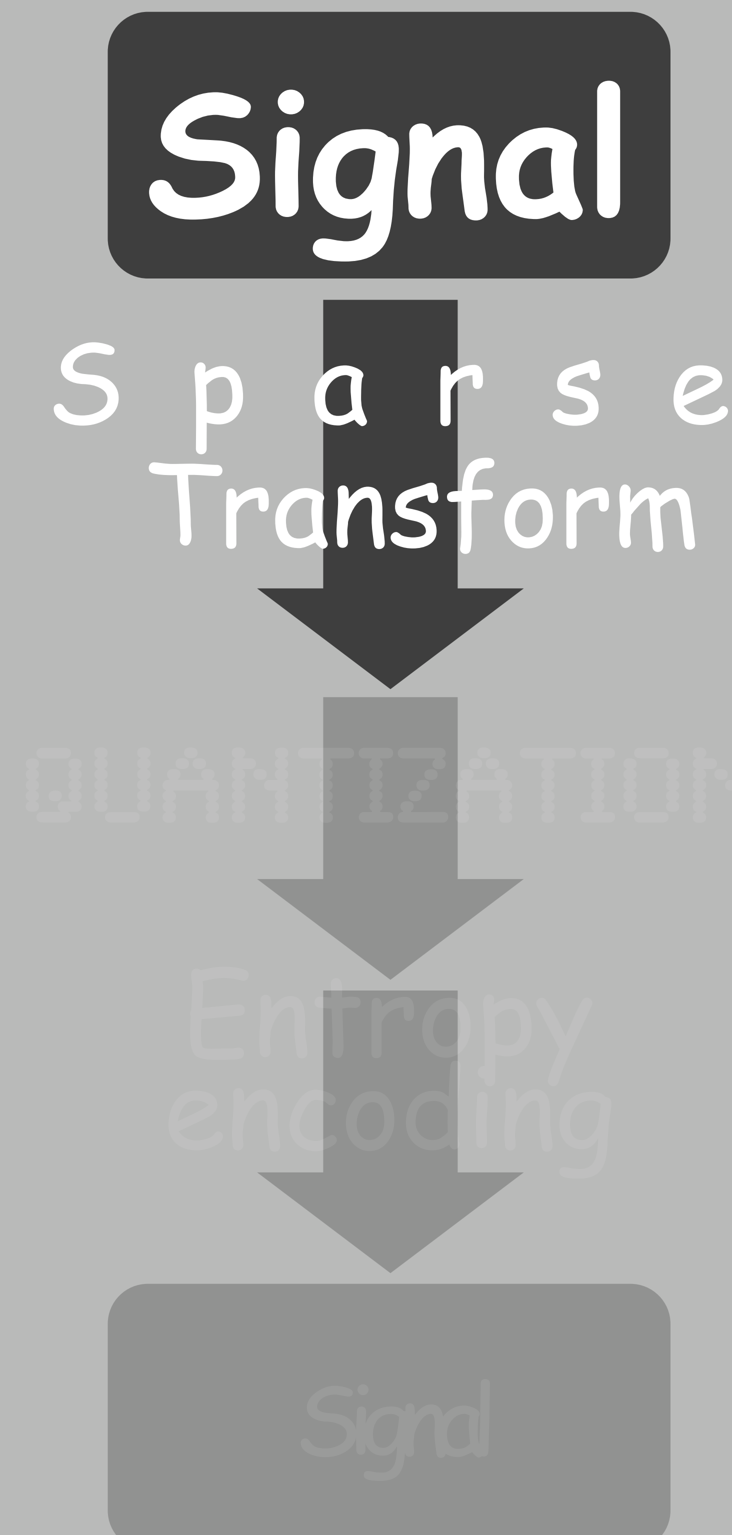
JPEG2000: Wavelet

MPEG: DCT & time-difference

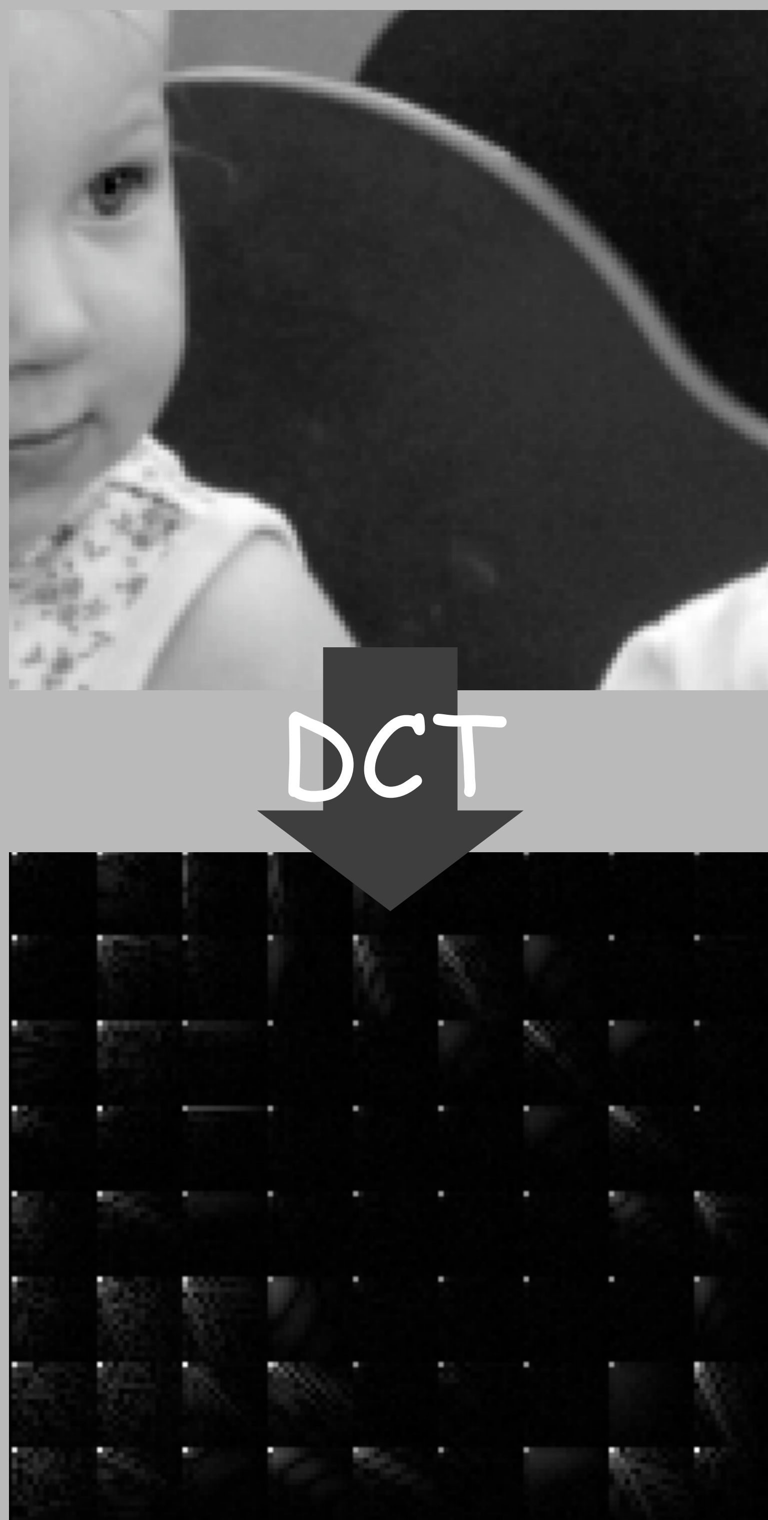


Sparse Transform

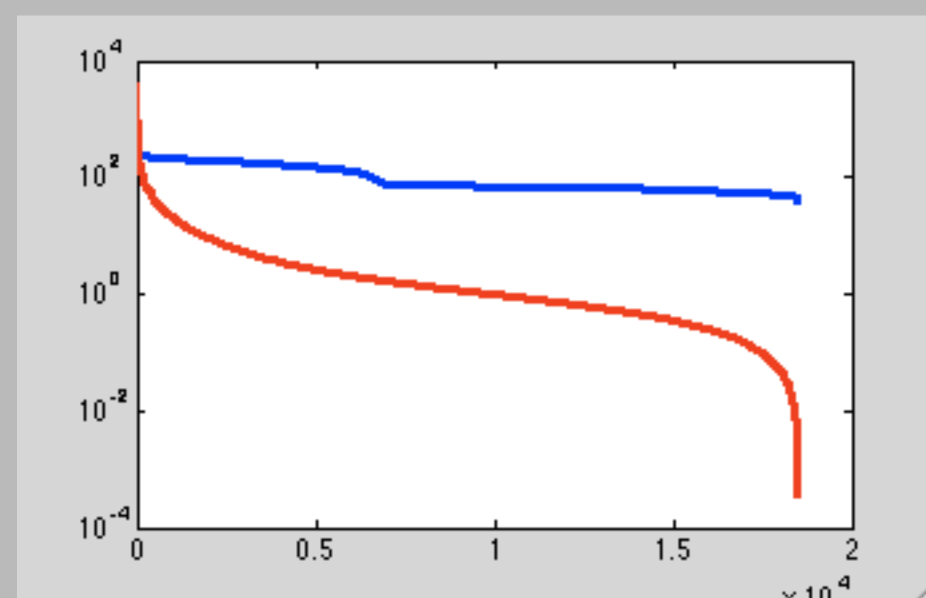




Sparse Transform

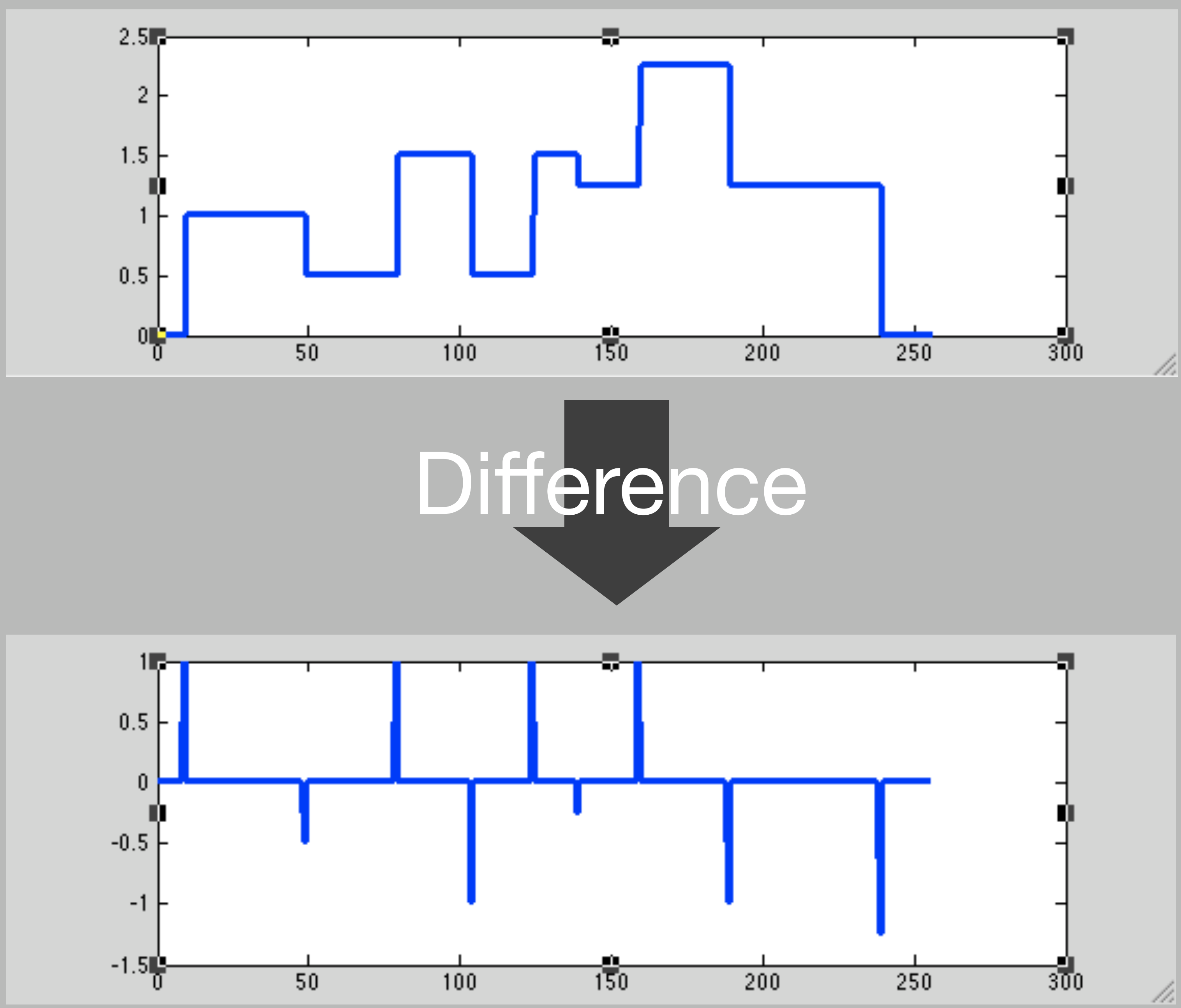


sorted coefficients



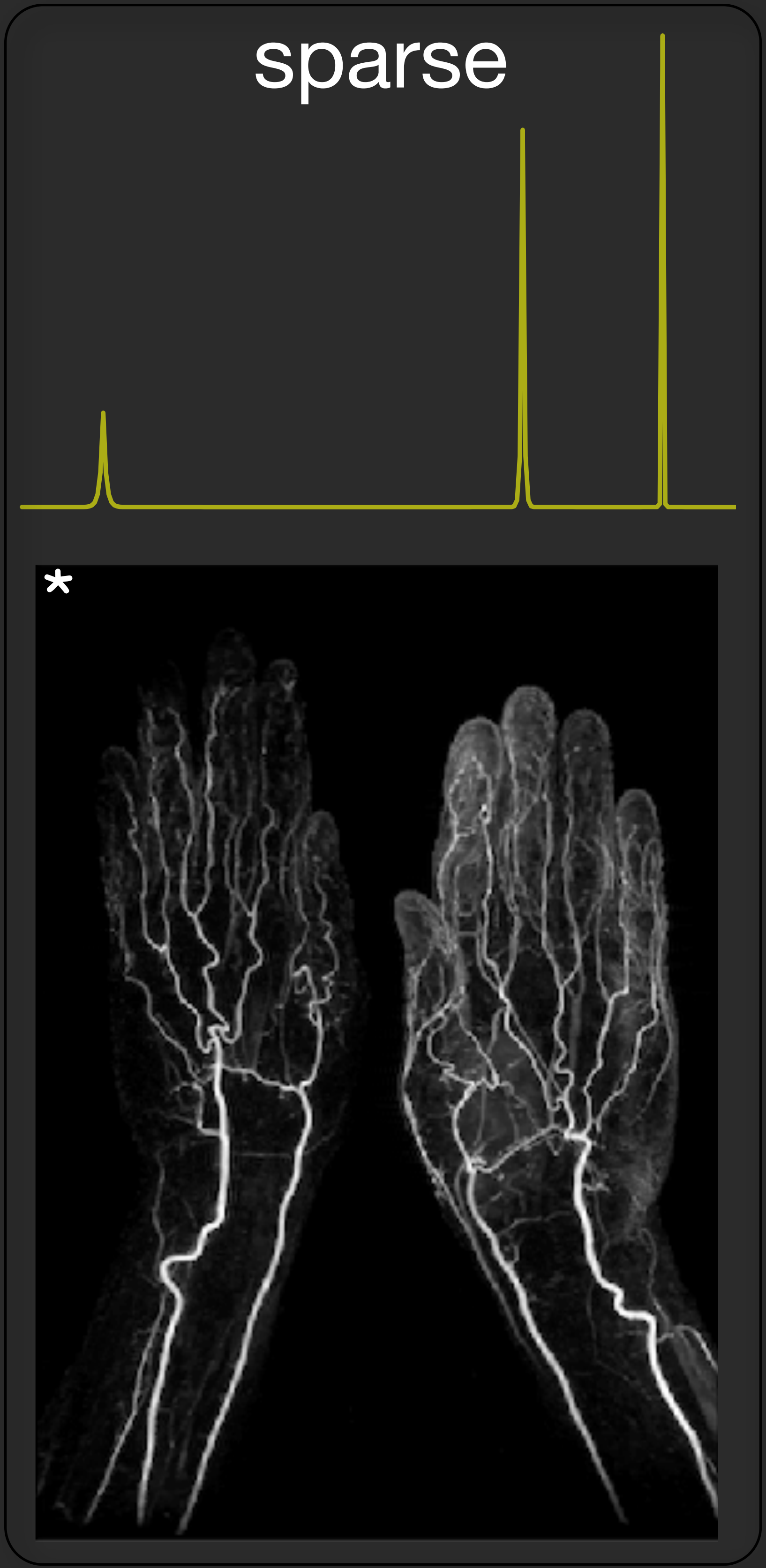
Sparse Transform

What sparsifying transform would you use here?



S p a r s i t y
&
Compressibility

Sparsity and Noise



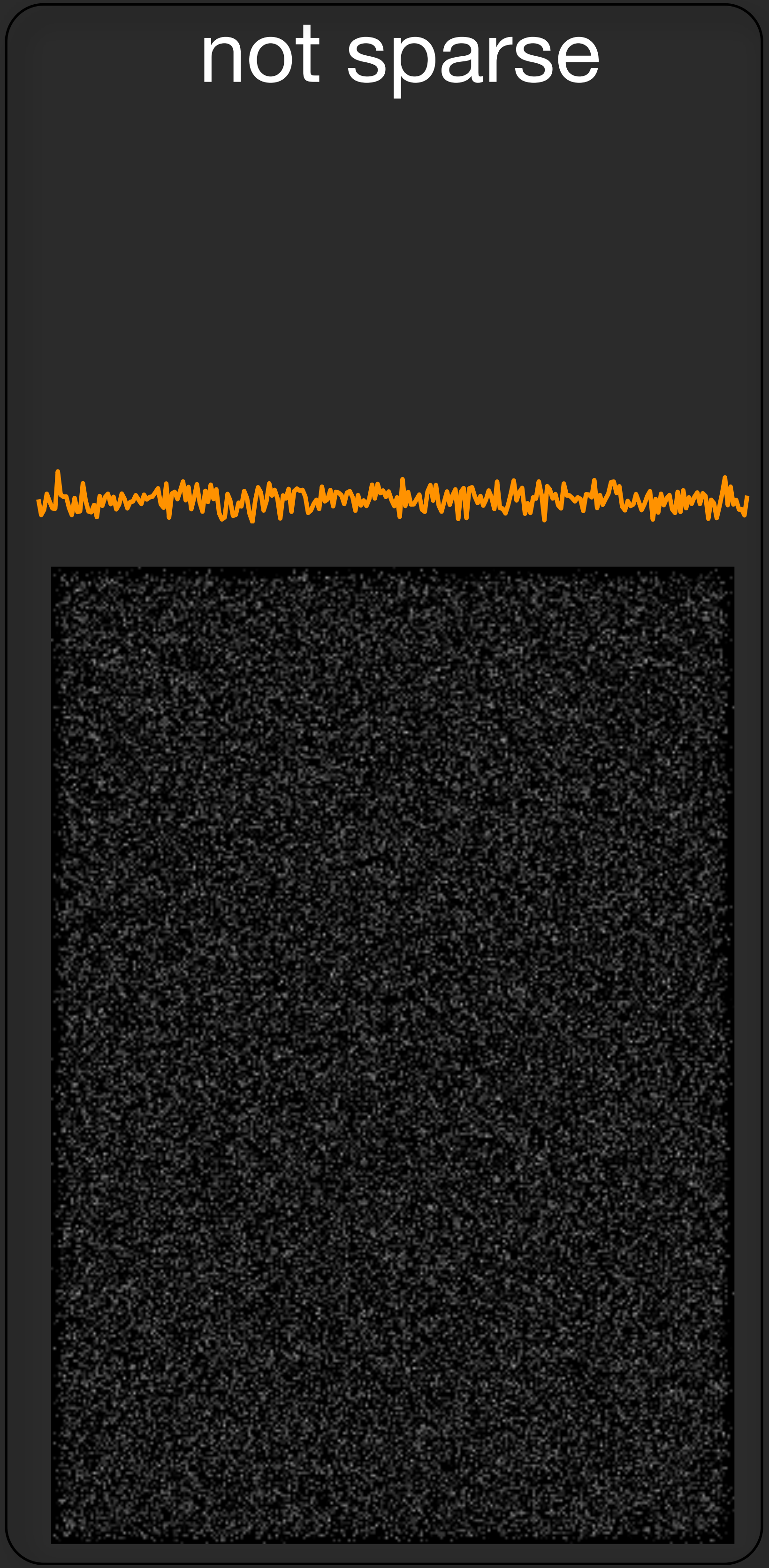


*image courtesy of J. Trzasko

Sparsity and Noise

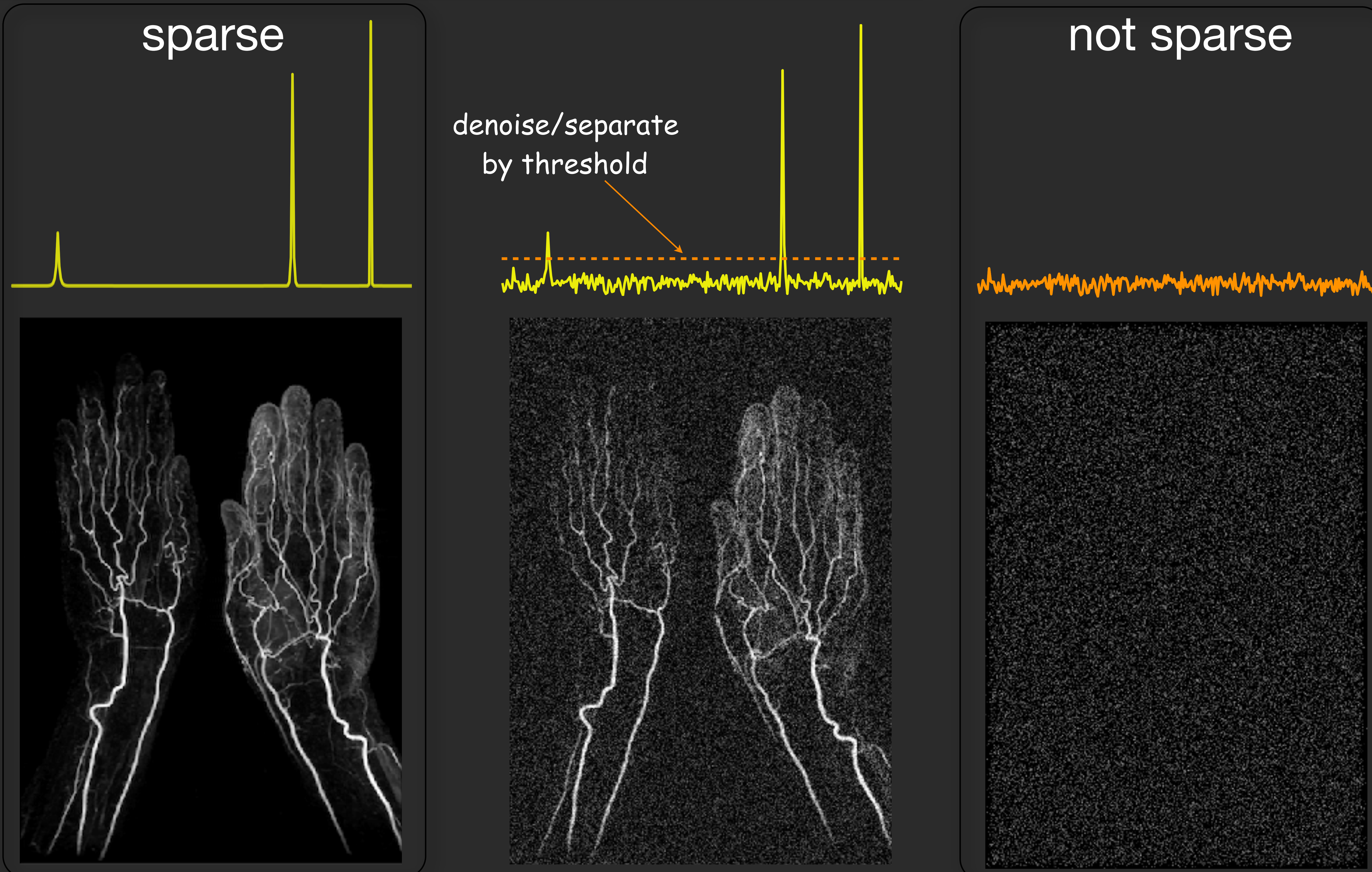
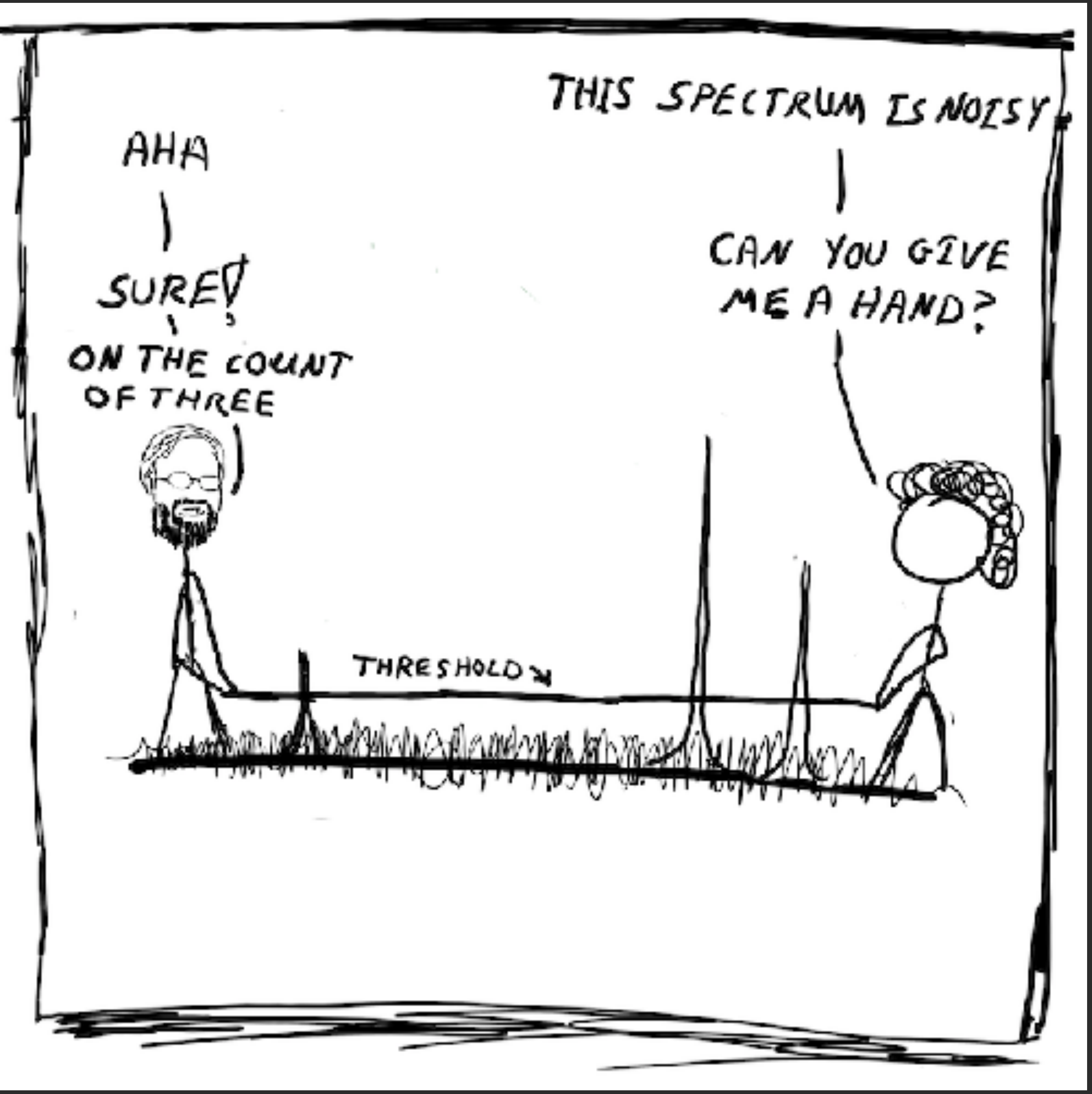
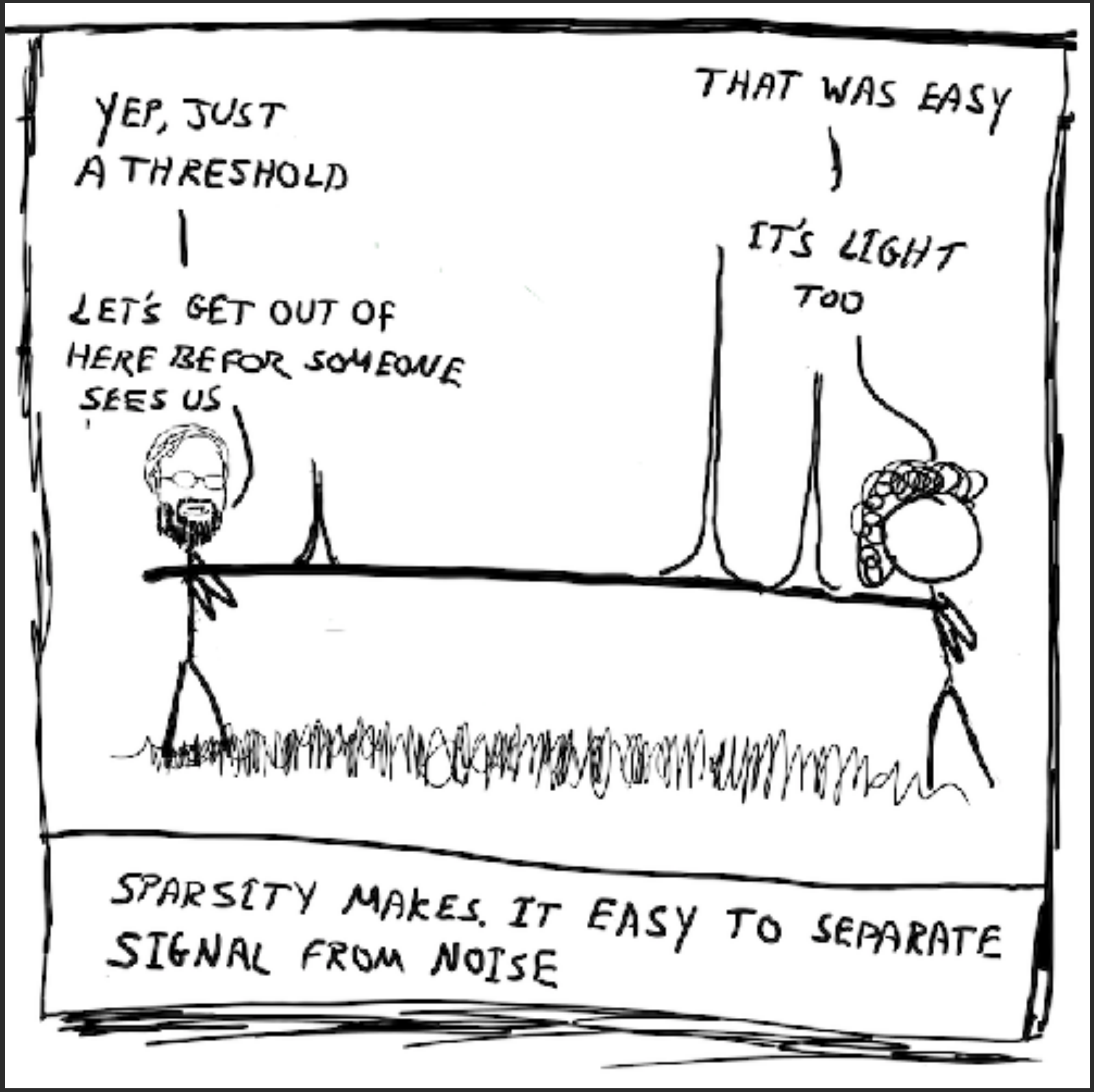


*image courtesy of J. Trzasko

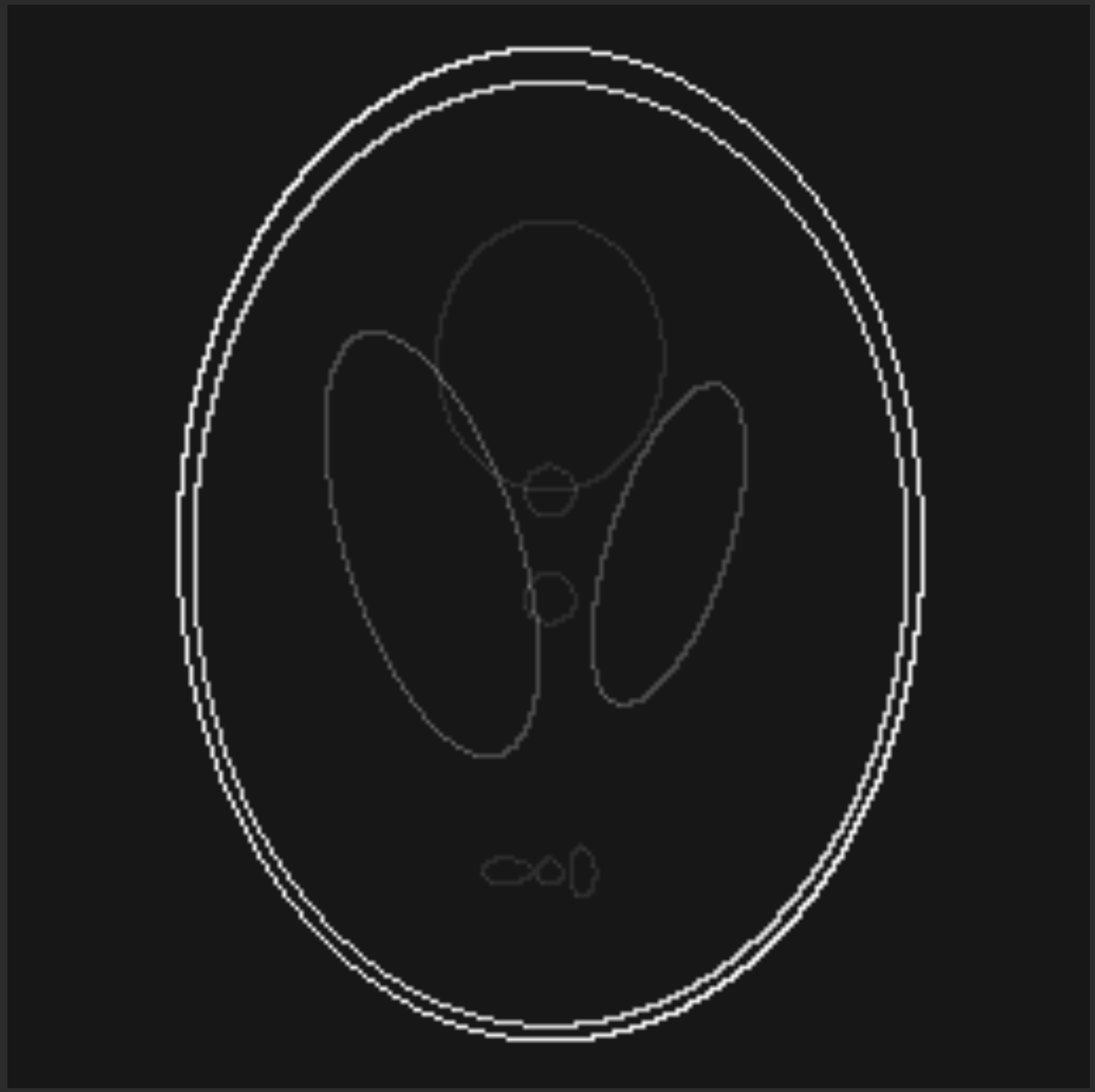

THIS SPECTRUM IS NOISY
CAN YOU GIVE ME A HAND?
AHA
SURE!
ON THE COUNT OF THREE
THRESHOLD


YEP, JUST A THRESHOLD
THAT WAS EASY
IT'S LIGHT TOO
LET'S GET OUT OF HERE BEFOR SOMEONE SEES US
SPARSITY MAKES IT EASY TO SEPARATE SIGNAL FROM NOISE

Transform Sparsity

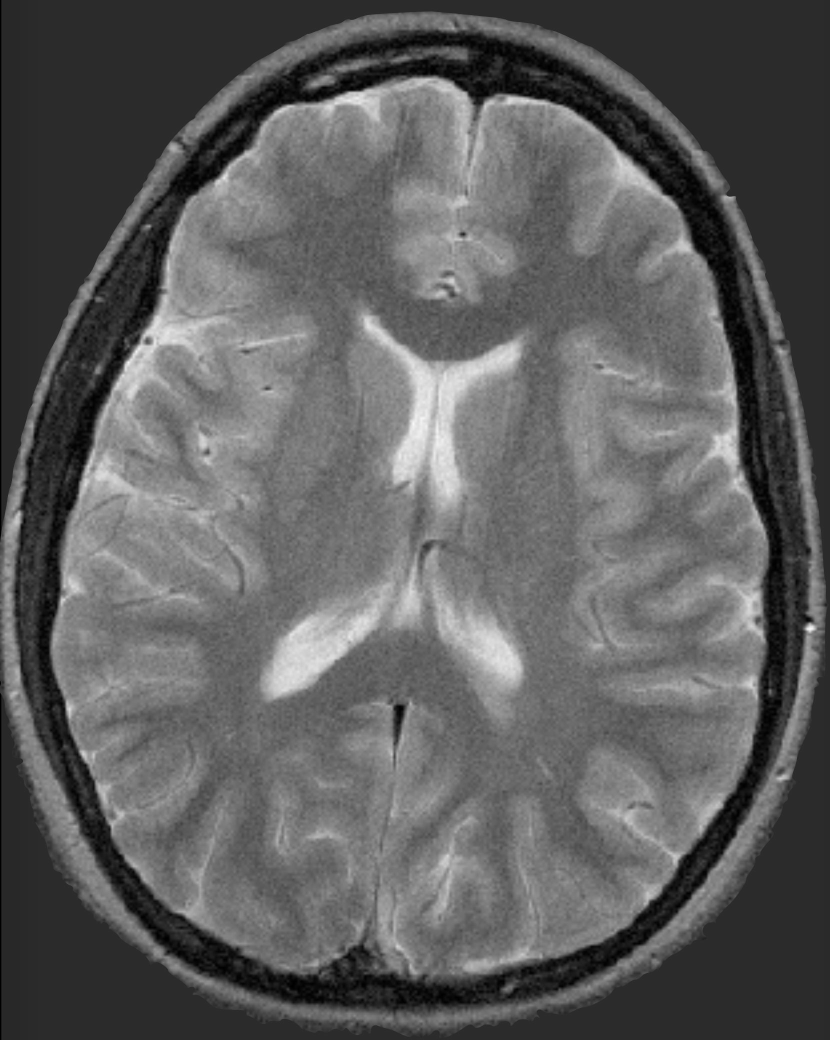
not sparse

Sparse Edges



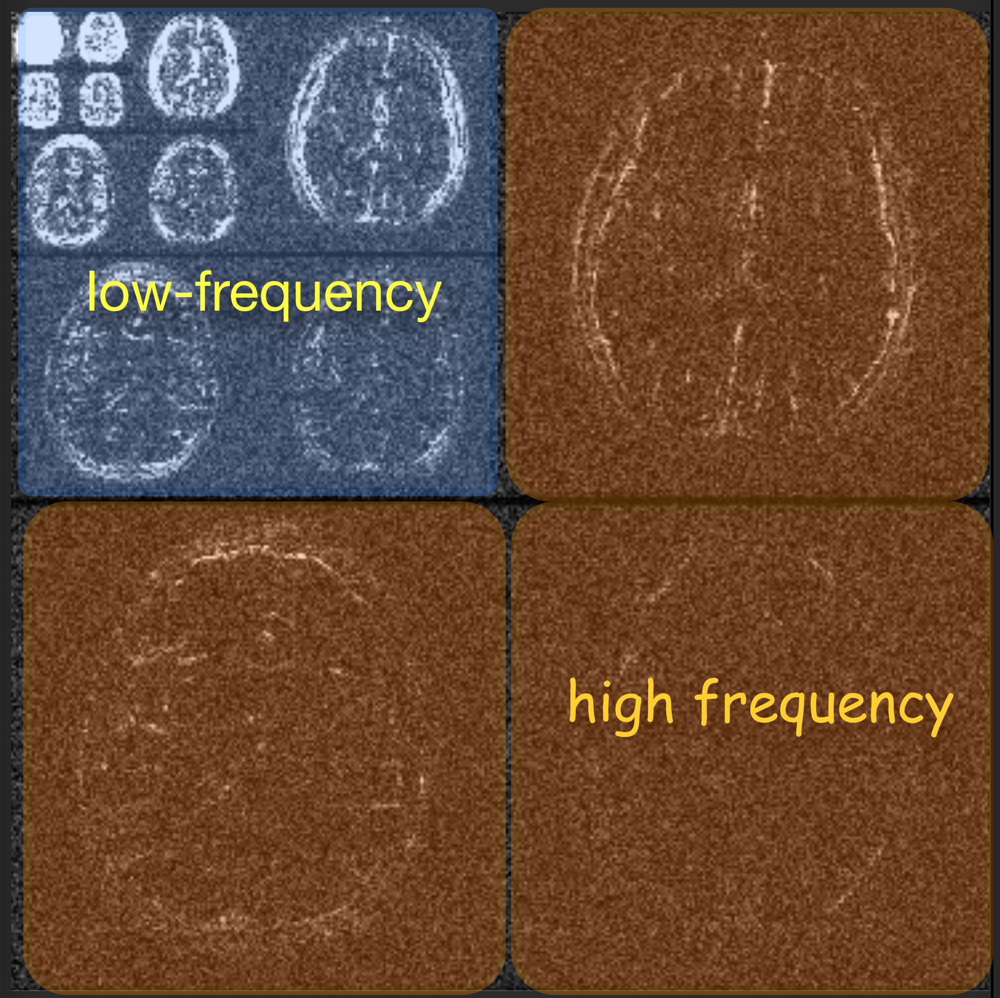
Transform Sparsity and Denoising

not sparse



sparse

wavelet transform

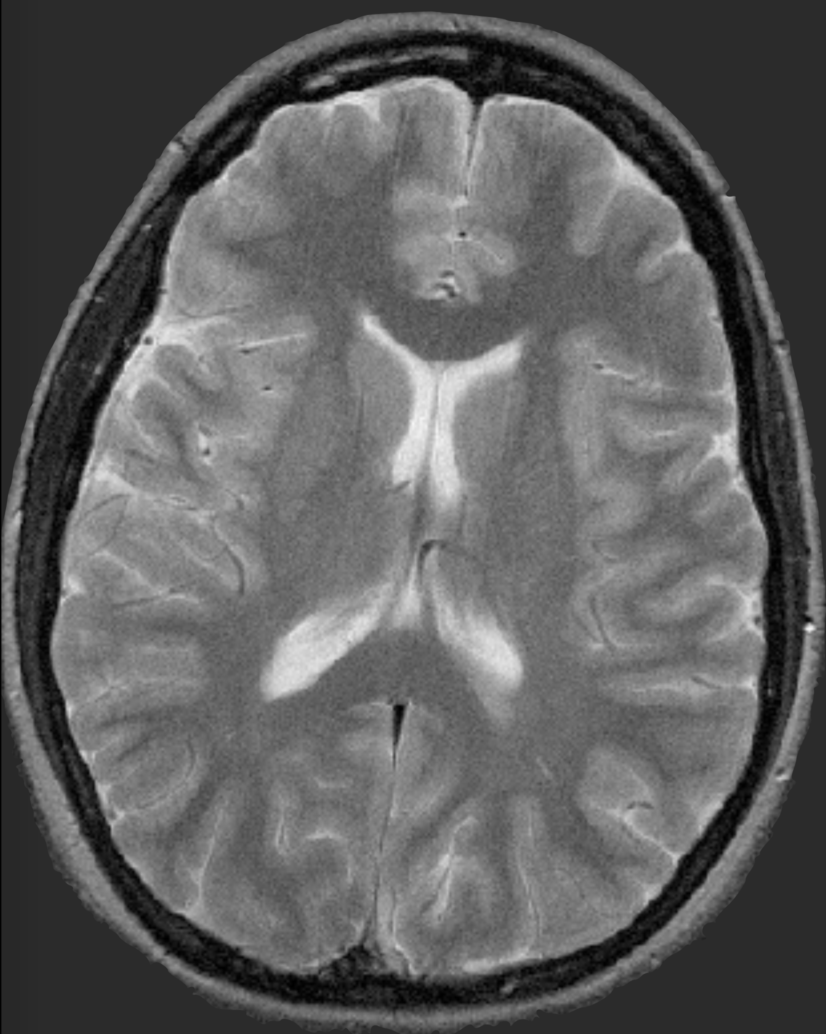


denoised

DL Donoho, I Johnstone Biometrika 1994;81(3):425-55

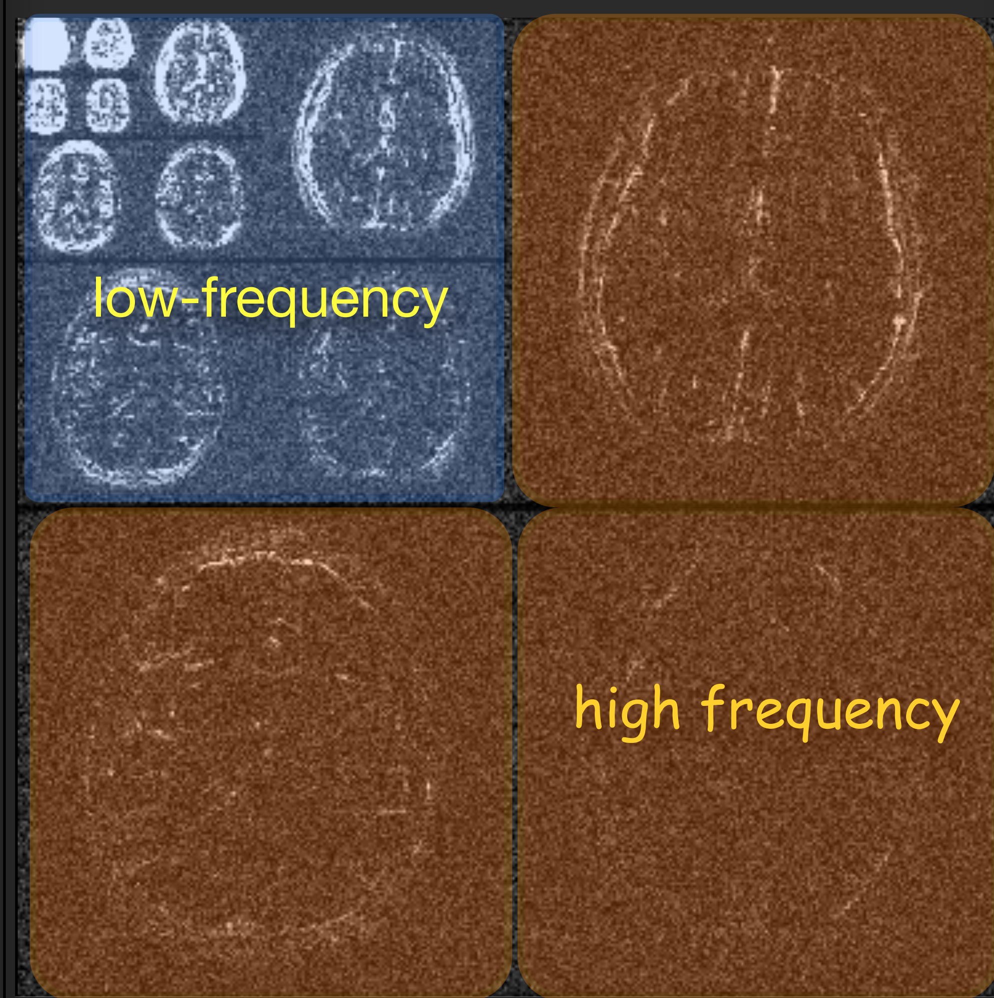
Transform Sparsity and Denoising

not sparse



sparse

wavelet transform

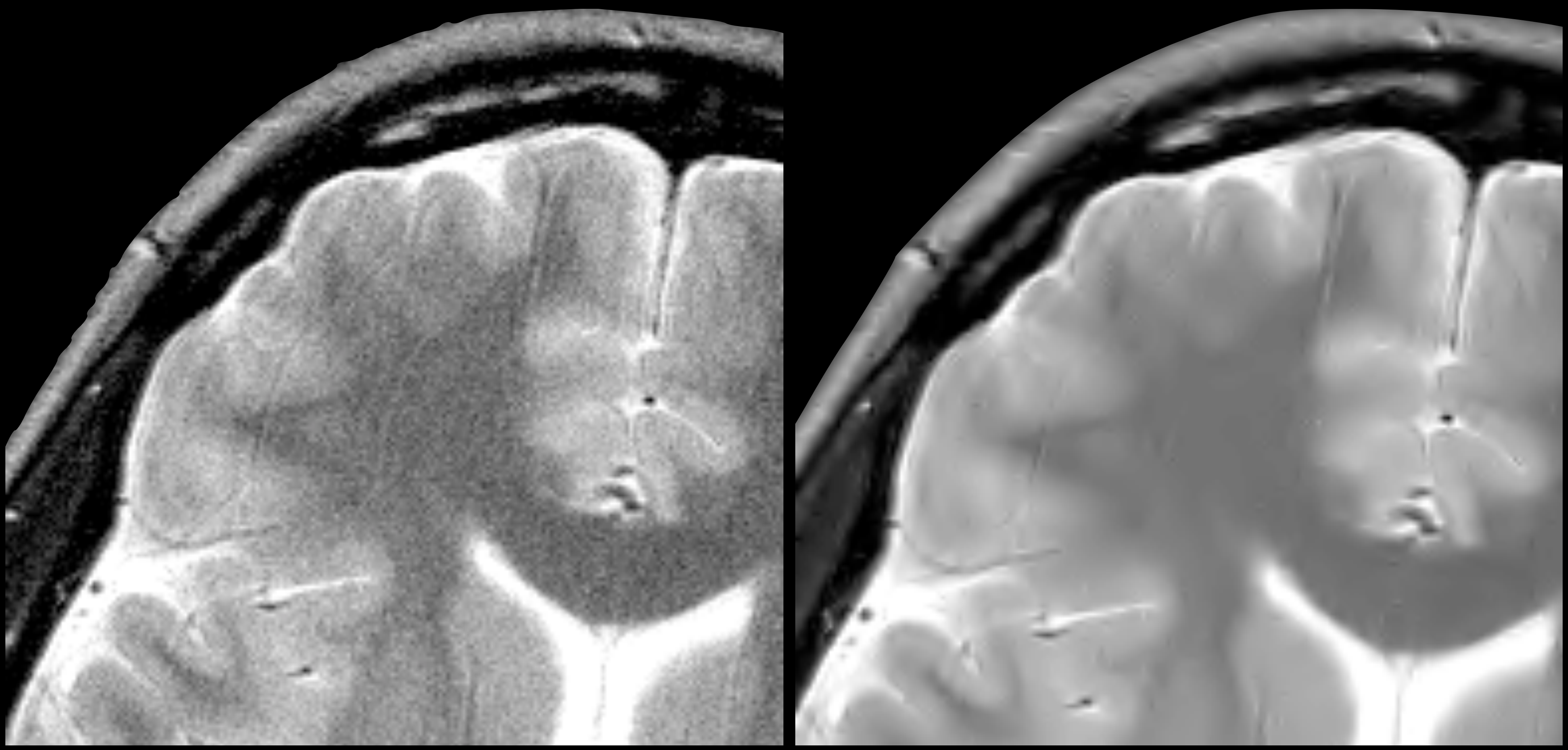


denoised

DL Donoho, I Johnstone Biometrika 1994;81(3):425-55

Transform Sparsity and Denoising

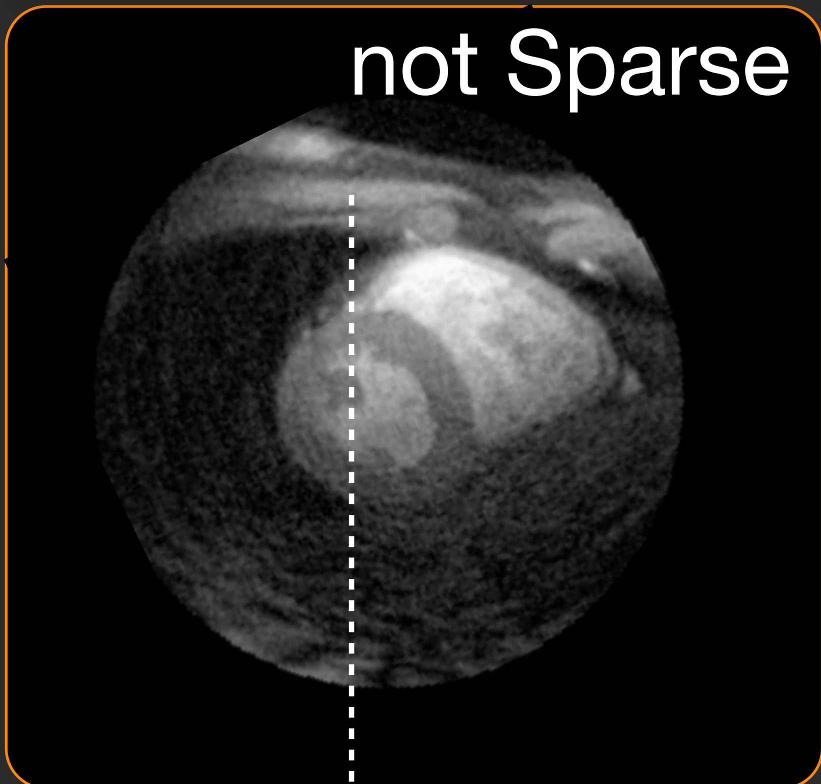
wavelet denoising

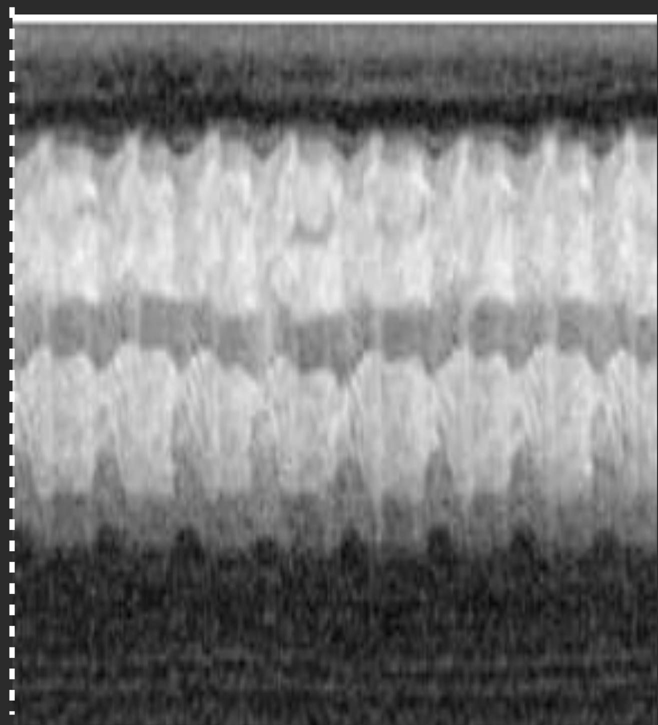


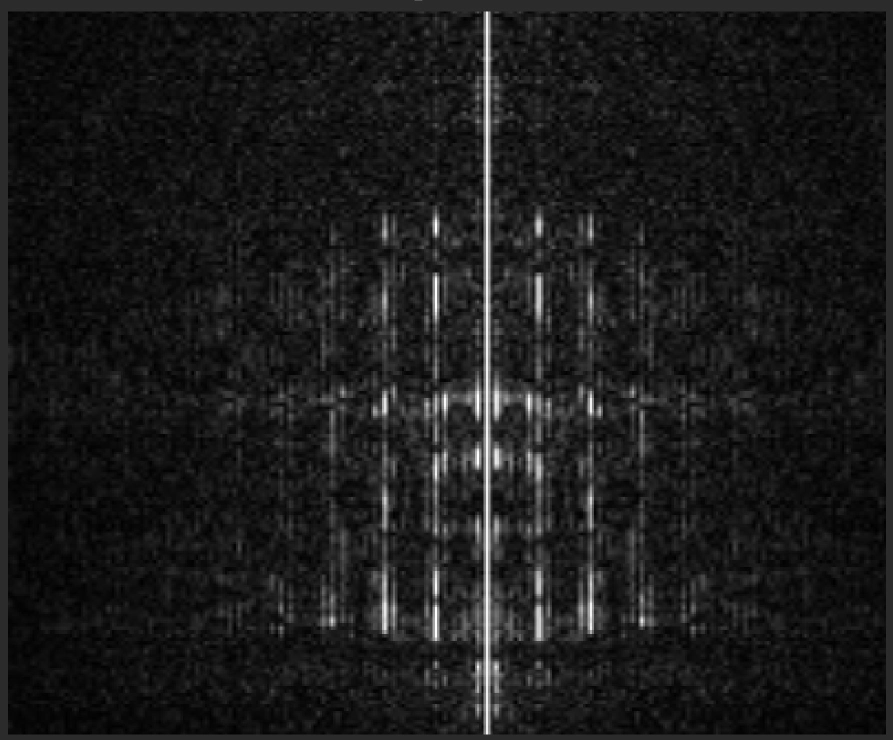
DL Donoho, I Johnstone Biometrika 1994;81(3):425-55

More Sparse Transforms

*Video courtesy of Juan Santos, Heart Vista

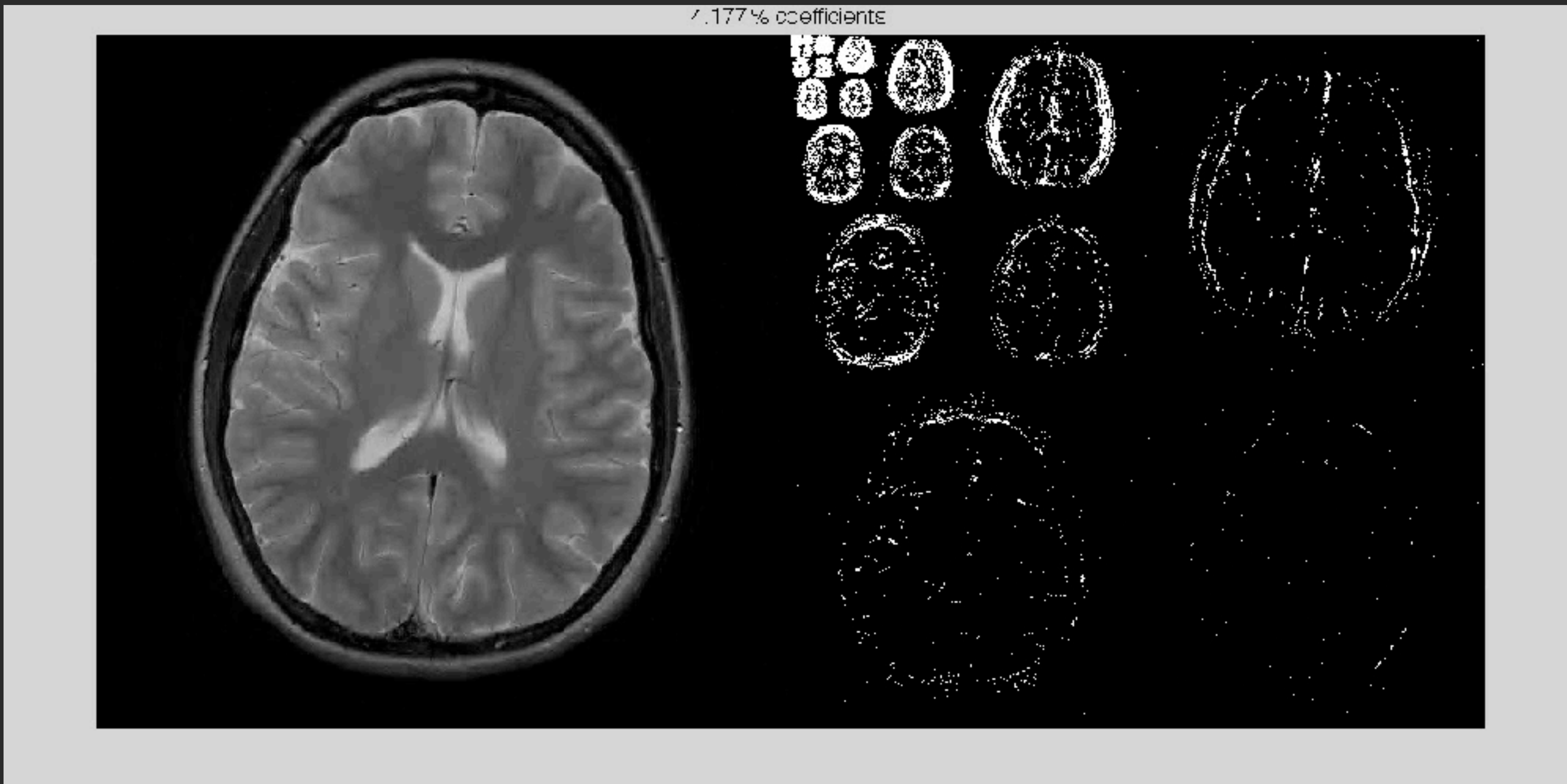






Sparsity and Compression

- Only need to store non-zeros



From Samples to Measurements

- Shanon-Nyquist sampling
  – Worst case for ANY bandlimited data

- Compressive sampling (CS)

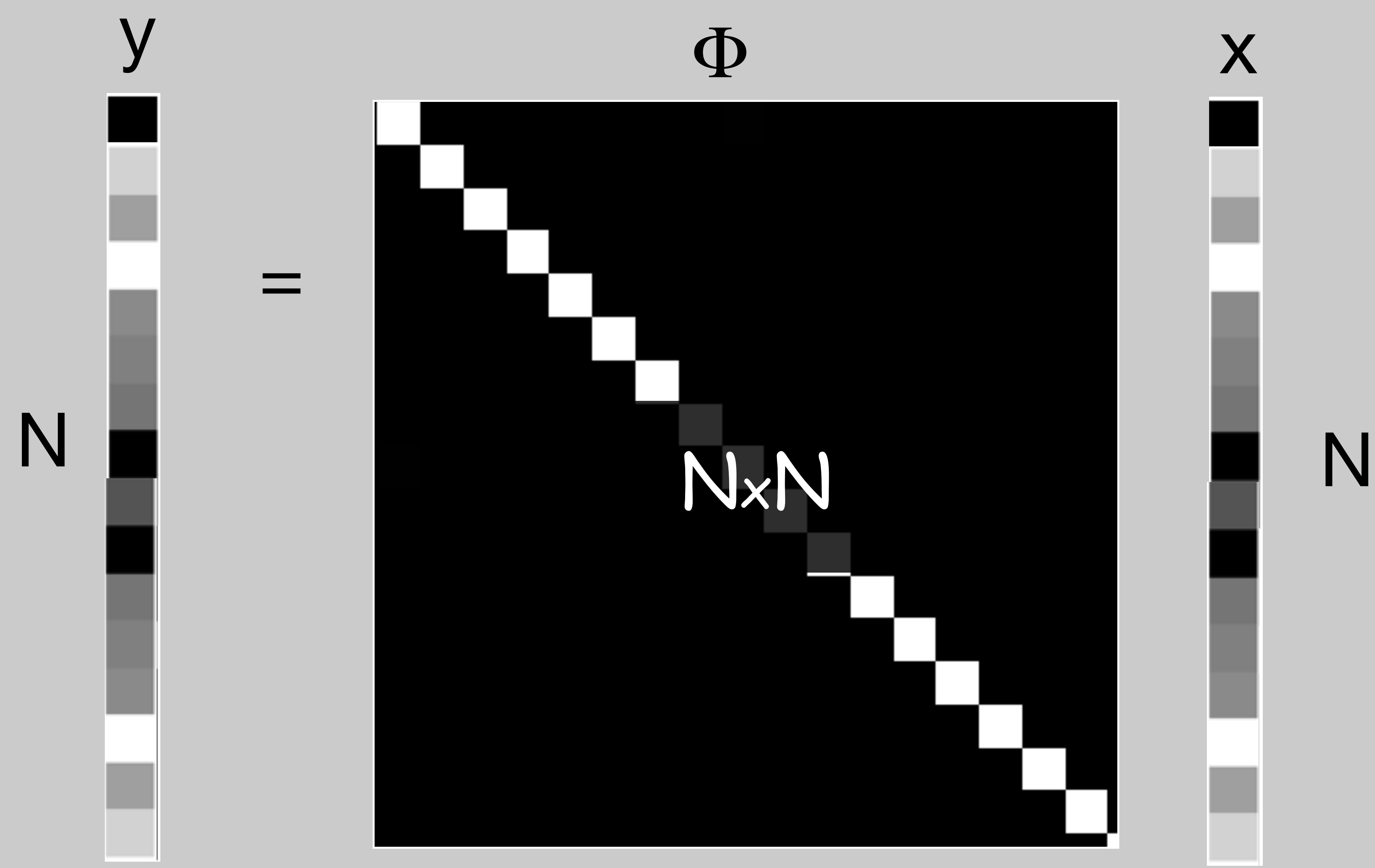
  "Sparse signals statistics can be recovered from a small number of non-adaptive linear measurements"

  – Integrated sensing, compression and processing.
  – Based on concepts of incoherency between signal and measurements

Traditional Sensing

- $x \in \Re^N$ is a signal
  - Make N linear measurements

Desktop scanner/ digital camera sensing

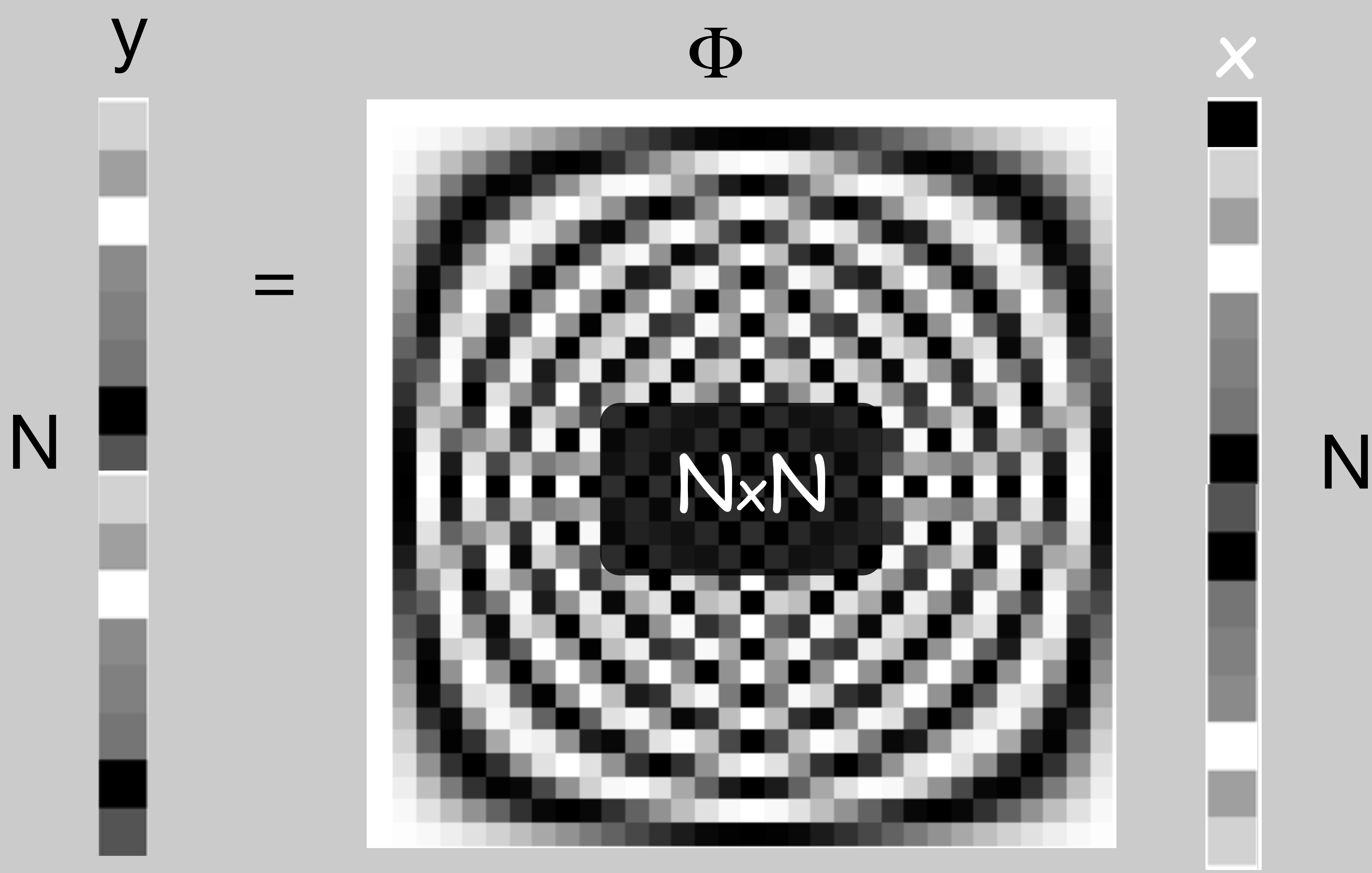


sensing matrix

Traditional Sensing

MRI Fourier Imaging

- $x \in \mathfrak{R}^N$ is a signal
  - Make N linear measurements

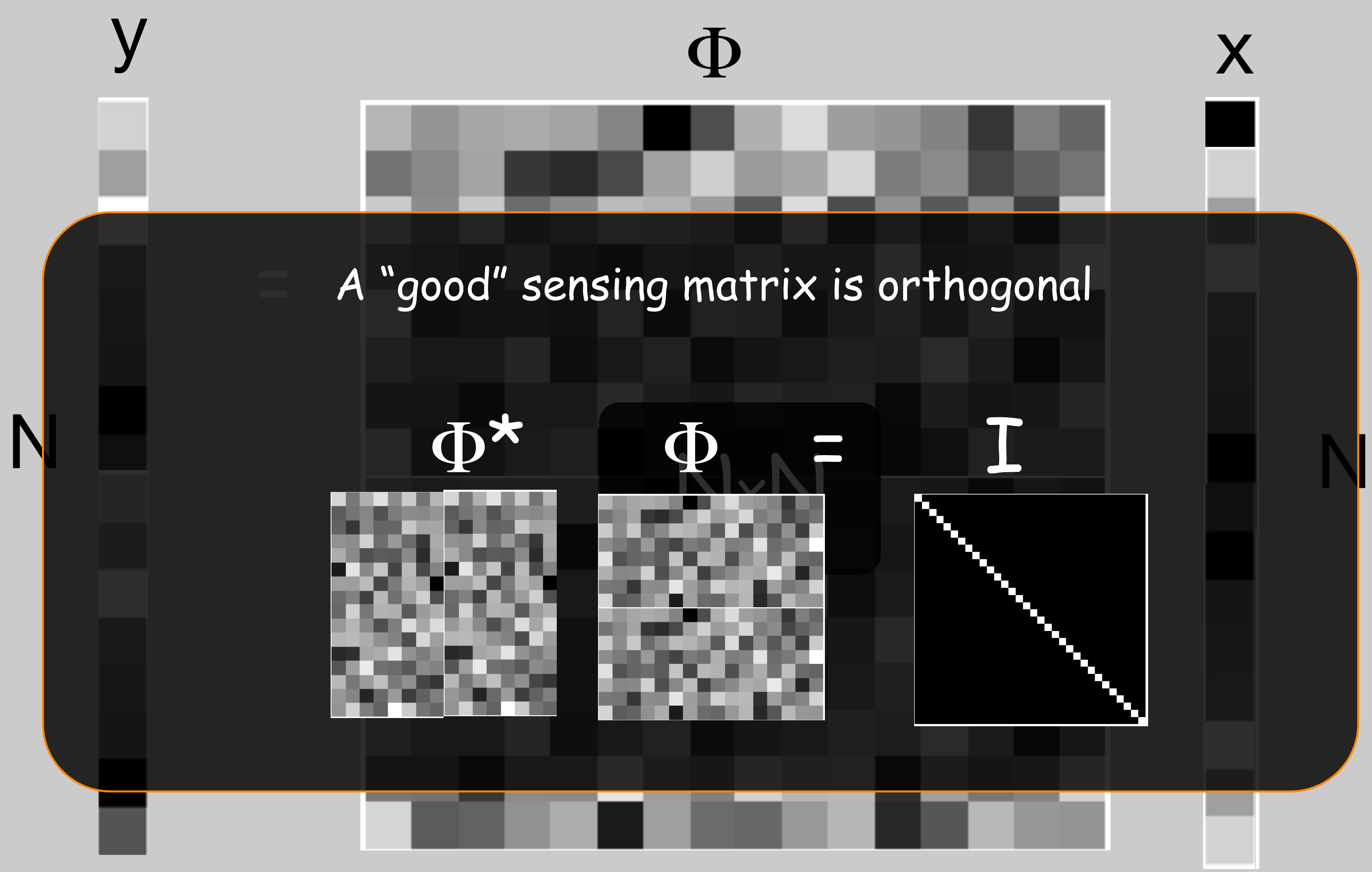


sensing matrix

Traditional Sensing

- $x \in \Re^N$ is a signal
  - Make N linear measurements

Arbitrary sensing

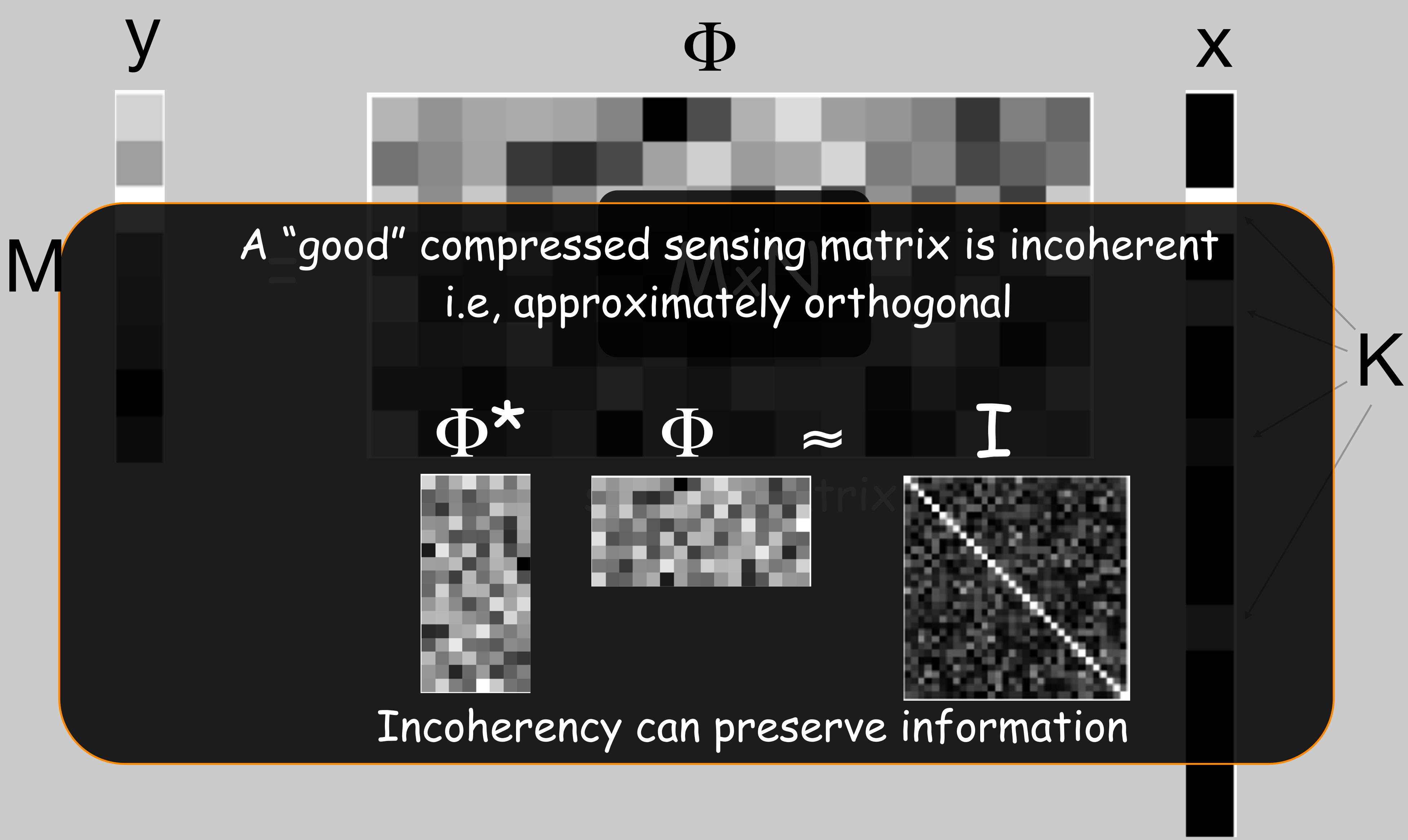


sensing matrix

Compressed Sensing

(Candes, Romber, Tao 2006; Donoho 2006)

- $x \in \Re^N$ is a K-sparse signal (K<<N)
  - Make M (K<M<<N) incoherent linear projections

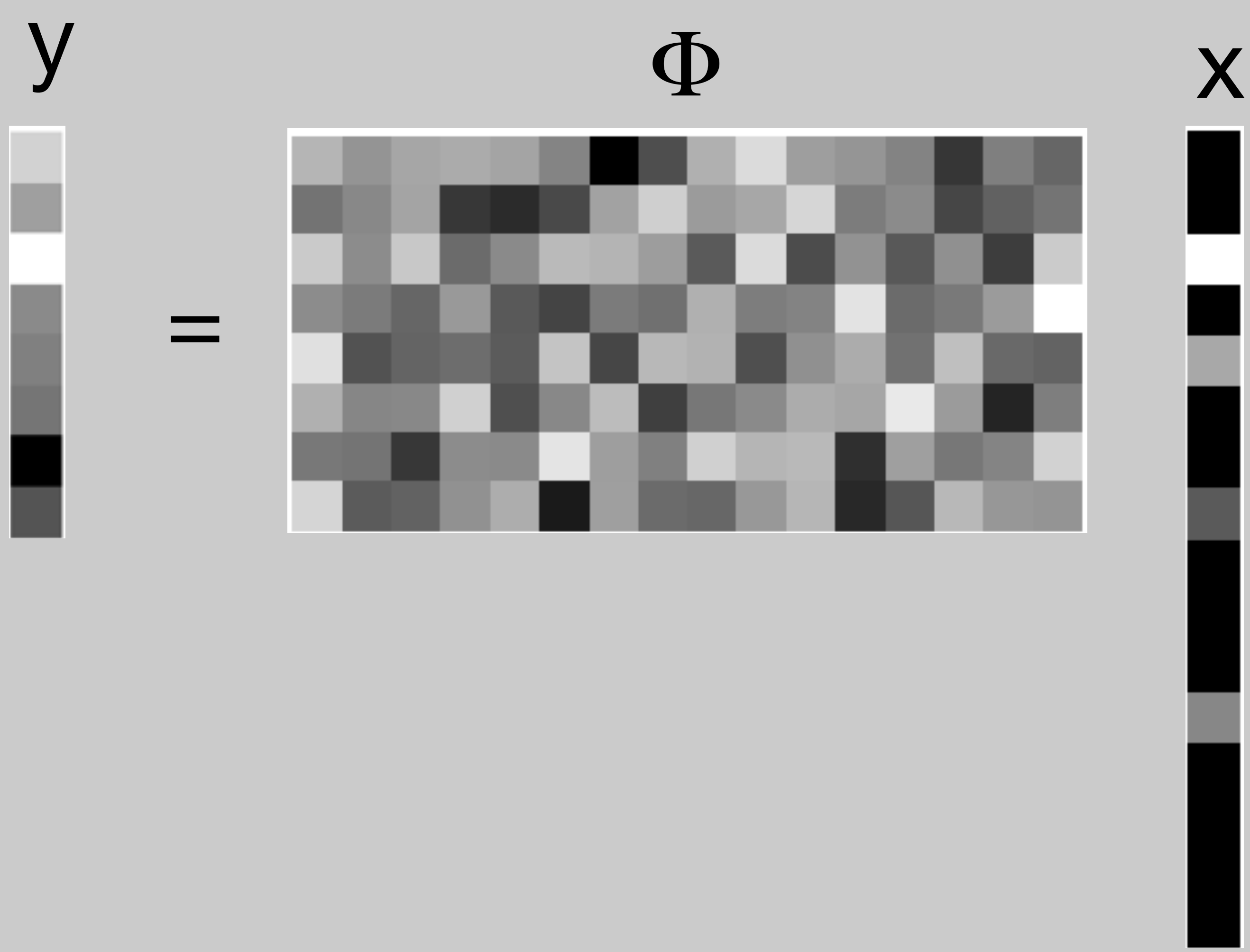


CS recovery

- Given $y = \Phi x$
  find x

Under-determined

- But there's hope, x is sparse!



CS recovery

- Given $y = \Phi x$
  find x

Under-determined

- But there's hope, x is sparse!

CS recovery

- Given $y = \Phi x$
  find x

  } Under-determined

- But there's hope, x is sparse!

$$\text{minimize } \|x\|_2$$

$$\text{s.t. } y = \Phi x$$

WRONG!

CS recovery

- Given $y = \Phi x$
  find x

Under-determined

- But there's hope, x is sparse!

$$\text{minimize } \|x\|_0$$

$$\text{s.t. } y = \Phi x$$

HARD!

CS recovery

- Given $y = \Phi x$
  find x

  Under-determined

- But there's hope, x is sparse!

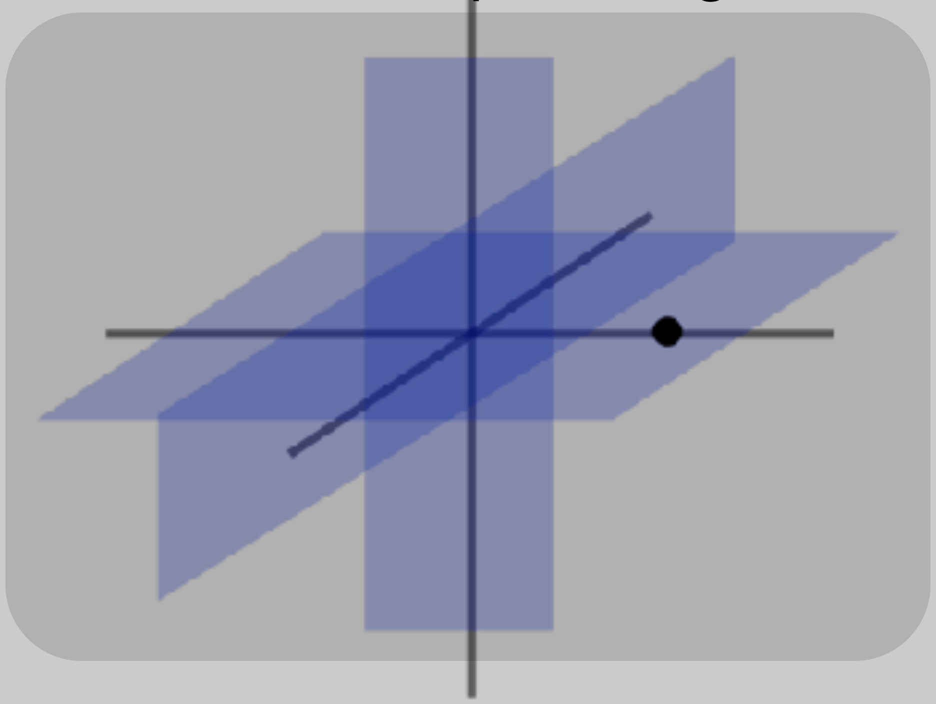
$$\text{minimize } \|x\|_1$$
$$\text{s.t. } y = \Phi x$$

need $M \approx K \log(N) << N$

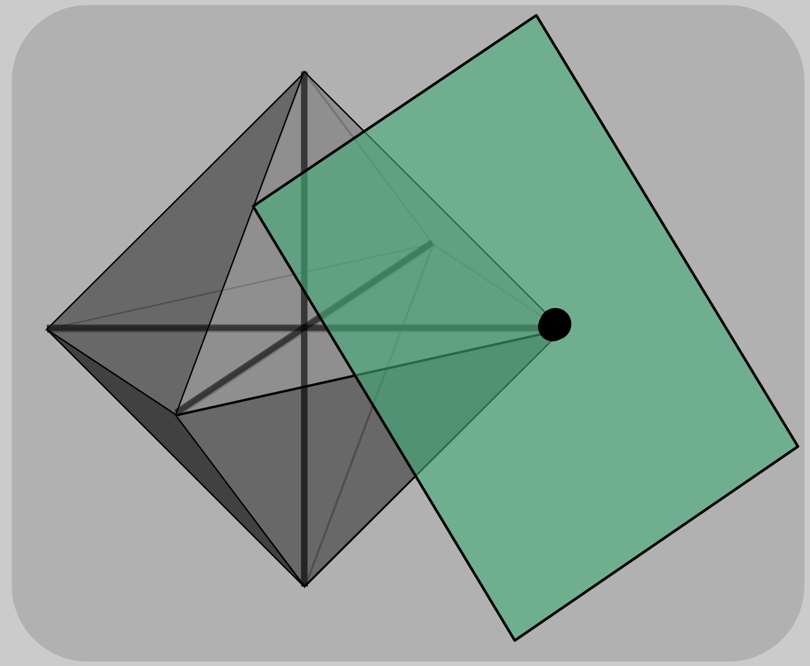
Solved by linear-programming

Geometric Interpretation

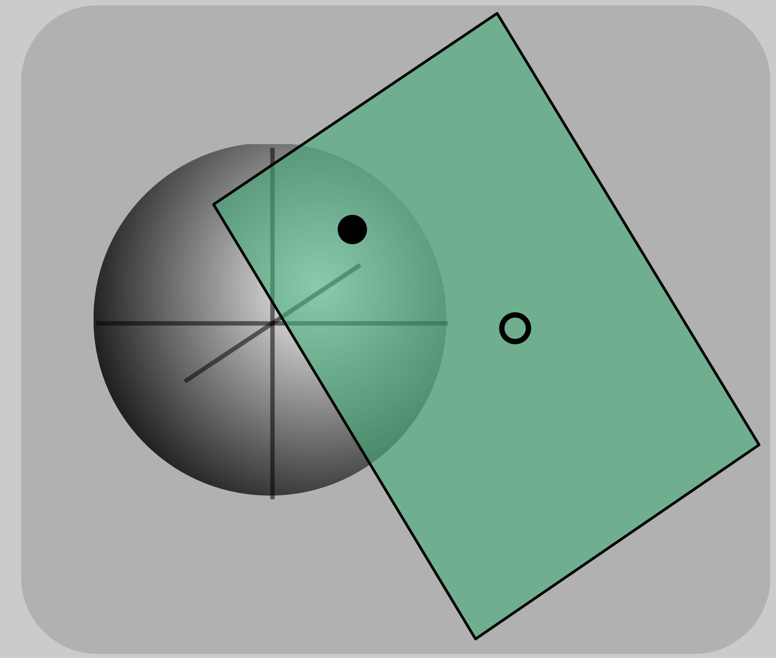
domain of sparse signals



minimum $\|x\|_1$



minimum $\|x\|_2$



$$\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [y_1]$$