Exercises FYS4480, Third set

Exercise 5

Starting with the Slater determinant

$$\Phi_0 = \prod_{i=1}^{n} a_{\alpha_i}^\dagger |0\rangle ,$$

use Wick's theorem to compute the normalization integral $< \Phi_0|\Phi_0 >$.

Exercise 6

Write the two-particle operator

$$G = \frac{1}{4}\sum_{\alpha\beta\gamma\delta} \langle\alpha\beta| g |\gamma\delta\rangle a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma$$

in the quasi-particle representation for particles and holes

$$b_\alpha^\dagger = \left\{ \begin{array}{l} a_\alpha^\dagger \\ a_\alpha \end{array} \right. \qquad\qquad b_\alpha = \left\{ \begin{array}{ll} a_\alpha & \alpha > \alpha_F \\ a_\alpha^\dagger & \alpha \leq \alpha_F \end{array} \right.$$

The two-body matrix elements are antisymmetric.

Exercise 7

Use the results from exercise 6 and Wick's theorem to calculate

$$\left\langle \beta_1\gamma_1^{-1} \right| G \left| \beta_2\gamma_2^{-1} \right\rangle$$

You need to consider that case that β_1 be equal β_2 and that γ_1 be equal γ_2.

Exercise 8

Show that the onebody part of the Hamiltonian

$$\hat{H}_0 = \sum_{pq} \langle p| \hat{h}_0 |q\rangle a_p^\dagger a_q$$

can be written, using standard annihilation and creation operators, in normal-ordered form as

$$\begin{aligned} \hat{H}_0 &= \sum_{pq} \langle p| \hat{h}_0 |q\rangle a_p^\dagger a_q \\ &= \sum_{pq} \langle p| \hat{h}_0 |q\rangle \left\{ a_p^\dagger a_q \right\} + \delta_{pq\in i} \sum_{pq} \langle p| \hat{h}_0 |q\rangle \\ &= \sum_{pq} \langle p| \hat{h}_0 |q\rangle \left\{ a_p^\dagger a_q \right\} + \sum_{i} \langle i| \hat{h}_0 |i\rangle \end{aligned}$$

Explain the meaning of the various symbols. Which reference vacuum has been used?

Exercise 9

Show that the twobody part of the Hamiltonian

$$\hat{H}_I = \frac{1}{4}\sum_{pqrs} \langle pq|\,\hat{v}\,|rs\rangle\, a_p^\dagger a_q^\dagger a_s a_r$$

can be written, using standard annihilation and creation operators, in normal-ordered form as

$$\begin{aligned}
\hat{H}_I &= \frac{1}{4}\sum_{pqrs} \langle pq|\,\hat{v}\,|rs\rangle\, a_p^\dagger a_q^\dagger a_s a_r \\
&= \frac{1}{4}\sum_{pqrs} \langle pq|\,\hat{v}\,|rs\rangle \left\{a_p^\dagger a_q^\dagger a_s a_r\right\} + \sum_{pqi} \langle pi|\,\hat{v}\,|qi\rangle \left\{a_p^\dagger a_q\right\} + \frac{1}{2}\sum_{ij} \langle ij|\,\hat{v}\,|ij\rangle
\end{aligned}$$

Explain again the meaning of the various symbols.

Derive the normal-ordered form of the threebody part of the Hamiltonian.

$$\hat{H}_3 = \frac{1}{36}\sum_{\substack{pqr\\stu}} \langle pqr|\,\hat{v}_3\,|stu\rangle\, a_p^\dagger a_q^\dagger a_r^\dagger a_u a_t a_s$$

and specify the contributions to the twobody, onebody and the scalar part.