Volume 3: List of Multi-run Quadratizations

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PRODUCT OF POLYNOMIALS

$$f_1 f_2 \dots f_\kappa = \min\left(f_1,f_2,...,f_\kappa\right), \qquad f_i\left(b_{k_i}, b_{k_i+1}, \dots, b_{k_{i+1}-1}\right) \geq 0 \quad (1)$$

$$f_1 f_2 \dots f_\kappa = \min\left(f_1 f_2 \dots f_{\kappa-1} \max f_\kappa, f_\kappa - \min f_\kappa + f_1 f_2 \dots f_{\kappa-1}\right), \min f_\kappa < 0, \ f_{i<\kappa}\left(b_{k_i}, b_{k_i+1}, \dots, b_{k_{i+1}-1}\right) \geq 0 \quad (2)$$

$b_1 b_2 b_3 b_4 + b_2 b_3 b_4 - b_3 b_4 b_5$: (Example of Eq. 2). (3)

$\longrightarrow 2 b_3 b_4$ 25/32 (78%) (4)

$\longrightarrow b_1 b_2 + b_2 - b_5 - b_3 b_4 + 1$ 32/32(100%) (5)

MONOMIALS

$$b_1 b_2 b_3 \dots b_k = \min\left(b_1 b_2 \dots b_{k_1}, b_{k_1+1} b_{k_1+2} \dots b_{k_2}, b_{k_2+1} b_{k_2+2} \dots b_{k_3}, ..., b_{k_n+1} b_{k_n+2} ... b_k\right) \quad \text{(Example of Eq. 1).} \quad (6)$$

$$b_1 b_2 b_3 \dots b_k = \min\left(b_1, b_2, b_3, \dots, b_k\right) \quad \text{(Example of Eq. 6: Linearization of a degree-}k\text{ monomial).} \quad (7)$$

$$b_1 b_2 b_3 b_4 = \min\left(b_1 b_2, b_3 b_4\right) \quad \text{(Example of Eq. 6: Quadratization of a degree-4 monomial).} \quad (8)$$

$b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8$: (9)

$\longrightarrow 3 b_a + b_1 b_2 + b_1 b_3 + b_1 b_4 + b_2 b_3 + b_2 b_4 + b_3 b_4 - 2 b_a (b_1 + b_2 + b_3 + b_4)$ (10)

$\longrightarrow 3 b_a + b_5 b_6 + b_5 b_7 + b_5 b_8 + b_6 b_7 + b_6 b_8 + b_7 b_8 - 2 b_a (b_5 + b_6 + b_7 + b_8)$ (11)

$$s_1 s_2 \dots s_k = \min\left(1 + s_1 s_2 - s_3 s_4 \dots s_k, 1 - s_1 s_2 + s_3 s_4 \dots s_k\right), s_i \in \{x, y, z\} \quad (12)$$

$$s_1 s_2 \dots s_k = \min\left(1 + s_1 s_2 ... s_j - s_{j+1} s_{j+2} \dots s_k, 1 - s_1 s_2 ... s_j + s_{j+1} s_{j+2} \dots s_k\right), s_i \in \{x, y, z\} \quad (13)$$

$x_1 z_2 x_3 z_4 y_5 x_6$ (Example of Eq. 12). (14)

$\longrightarrow + x_1 z_2 - x_3 z_4 y_5 x_6 + 1$ 48/64 (75%) (15)

$\longrightarrow - x_1 z_2 + x_3 z_4 y_5 x_6 + 1$ 64/64(100%) (16)

$x_1 z_2 x_3 z_4 y_5 x_6 x_7 y_8 y_9 z_{10}$ (Example of Eq. 13). (17)

$\longrightarrow + x_1 z_2 x_3 z_4 y_5 - x_6 x_7 y_8 y_9 z_{10} + 1$ 768/1024 (75%) (18)

$\longrightarrow - x_1 z_2 x_3 z_4 y_5 + x_6 x_7 y_8 y_9 z_{10} + 1$ 1024/1024(100%) (19)

$z_1z_2x_3 + z_1x_2z_3 :$ (20)
$\longrightarrow \ +2z1 - x2z3 - z2x3 + 2$ 5/8 (63%) (21)
$\longrightarrow \ -2z1 + x2z3 + z2x3 + 2$ 8/8(100%) (22)

$z_1z_2x_3 + 2z_1x_2z_3 :$ (23)
$\longrightarrow \ +3z1 - 2x2z3 - z2x3 + 3$ 5/8 (63%) (24)
$\longrightarrow \ -3z1 + 2x2z3 + z2x3 + 3$ 8/8(100%) (25)

$z_1z_2x_3 + 3z_1x_2z_3 :$ (26)
$\longrightarrow \ +4z1 - 3x2z3 - z2x3 + 4$ 5/8 (63%) (27)
$\longrightarrow \ -4z1 + 3x2z3 + z2x3 + 4$ 8/8(100%) (28)

$z_1z_2x_3 - z_1x_2z_3 :$ (29)
$\longrightarrow \ +2z1 + x2z3 - z2x3 + 2$ 5/8 (63%) (30)
$\longrightarrow \ -2z1 - x2z3 + z2x3 + 2$ 8/8(100%) (31)

$z_1z_2x_3 - 2z_1x_2z_3 :$ (32)
$\longrightarrow \ +3z1 + 2x2z3 - z2x3 + 3$ 5/8 (63%) (33)
$\longrightarrow \ -3z1 - 2x2z3 + z2x3 + 3$ 8/8(100%) (34)

$z_1(Az_2x_3 + Bx_2z_3) :$ (35)
$\longrightarrow \ +(|A| + |B|)z_1 - (Az_2x_3 + Bx_2z_3) + |A| + |B|$ (36)
$\longrightarrow \ -(|A| + |B|)z_1 + (Az_2x_3 + Bx_2z_3) + |A| + |B|$ (37)

$z_1x_2z_3 + z_1z_2x_3 + z_1x_2x_3 :$ (38)
$\longrightarrow \ +3z1 - x2z3 - z2x3 - x2x3 + 3$ 4/8 (50%) (39)
$\longrightarrow \ -3z1 + x2z3 + z2x3 + x2x3 + 3$ 8/8(100%) (40)

$z_1x_2z_3 + z_1z_2x_3 + z_1x_2x_3 + z_1z_2z_3 :$ (41)
$\longrightarrow \ +4z1 - x2z3 - z2x3 - x2x3 - z2z3 + 4$ 4/8 (50%) (42)
$\longrightarrow \ -4z1 + x2z3 + z2x3 + x2x3 + z2z3 + 4$ 8/8(100%) (43)

$z_1x_2z_3 + z_1z_2x_3 + z_1x_2x_3 + z_1z_2z_3 :$ (44)
$\longrightarrow \ +2z1 - x2z3 - z2x3 - x2x3 - z2z3 + 2$ 6/8 (75%) (45)
$\longrightarrow \ -2z1 + x2z3 + z2x3 + x2x3 + z2z3 + 2$ 8/8(100%) (46)

$z_1x_2z_3 + z_1z_2x_3 + z_1x_2x_3 + z_1z_2z_3 + z_1x_2y_3 :$ (47)
$\longrightarrow \ +3z1 - x2x3 - x2z3 - z2x3 - z2z3 - x2y3 + 3$ 4/8 (50%) (48)
$\longrightarrow \ -3z1 + x2x3 + x2z3 + z2x3 + z2z3 + x2y3 + 3$ 8/8(100%) (49)

$z_1x_2z_3 + z_1z_2x_3 + z_1x_2x_3 + z_1z_2z_3 + z_1x_2y_3 + z_1y_2x_3 + z_1y_2y_3 :$ (50)

$\longrightarrow \ -3z1 + x2y3 + x2z3 + y2x3 + z2x3 + x2x3 + y2y3 + z2z3 + 3$ 5/8 (63%) (51)

$\longrightarrow \ +4z1 - x2y3 - x2z3 - y2x3 - z2x3 - x2x3 - y2y3 - z2z3 + 4$ 8/8(100%) (52)

$(A_1x_1z_2 + A_2y_1x_2 + A_3x_1x_2 + A_4y_1y_2)z_3y_4 :$ (53)

$\longrightarrow \ -(A_1x_1z_2 + A_2y_1x_2 + A_3x_1x_2 + A_4y_1y_2) + \sum_i |A_i|\, z_3y_4 + \sum_i |A_i|$ (54)

$\longrightarrow \ +(A_1x_1z_2 + A_2y_1x_2 + A_3x_1x_2 + A_4y_1y_2) - \sum_i |A_i|\, z_3y_4 + \sum_i |A_i|$ (55)

$z_1z_2x_3 :$ (56)

$\longrightarrow \ z_1z_2 + z_1x_3 + z_2x_3 - z_1 - z_2 - x_3 + 1$ 7/8 (88%) (57)

$\longrightarrow \ z_1z_2 - z_1x_3 - z_2x_3 + z_1 + z_2 - x_3 + 1$ 8/8(100%) (58)

$z_1z_2x_3 :$ (59)

$\longrightarrow \ +z_1z_2 + z_1x_3 + z_2x_3 - z_1 - z_2 - x_3 + 1$ 7/8 (88%) (60)

$\longrightarrow \ -z_1z_2 + z_1x_3 - z_2x_3 + z_1 - z_2 + x_3 + 1$ 8/8(100%) (61)

$z_1z_2x_3 :$ (62)

$\longrightarrow \ +z_1z_2 - z_1x_3 - z_2x_3 + z_1 + z_2 - x_3 + 1$ 7/8 (88%) (63)

$\longrightarrow \ -z_1z_2 + z_1x_3 - z_2x_3 + z_1 - z_2 + x_3 + 1$ 8/8(100%) (64)

$z_1z_2x_3 :$ (65)

$\longrightarrow \ -z_1z_2 - z_1x_3 + z_2x_3 - z_1 + z_2 + x_3 + 1$ 7/8 (88%) (66)

$\longrightarrow \ -z_1z_2 + z_1x_3 - z_2x_3 + z_1 - z_2 + x_3 + 1$ 8/8(100%) (67)

$z_1z_2x_3 :$ (68)

$\longrightarrow \ z_1z_2 + z_1x_3 + z_2x_3 - z_1 - z_2 - x_3 + 1$ 7/8 (88%) (69)

$\longrightarrow \ z_1 - z_2x_3 + 1$ 8/8(100%) (70)

$z_1z_2x_3 :$ (71)

$\longrightarrow \ z_1z_2 + z_1x_3 + z_2x_3 - z_1 - z_2 - x_3 + 1$ 7/8 (88%) (72)

$\longrightarrow \ x_3 - z_1z_2 + 1$ 8/8(100%) (73)

$z_1z_2x_3 :$ (74)

$\longrightarrow \ z_1z_2 - z_1x_3 - z_2x_3 + z_1 + z_2 - x_3 + 1$ 7/8 (88%) (75)

$\longrightarrow \ -z_1 + z_2x_3 + 1$ 8/8(100%) (76)

$z_1z_2x_3:$ (77)
$\longrightarrow\ z_1z_2 - z_1x_3 - z_2x_3 + z_1 + z_2 - x_3 + 1$ 7/8 (88%) (78)
$\longrightarrow\ x_3 - z_1z_2 + 1$ 8/8(100%) (79)

$z_1z_2x_3:$ (80)
$\longrightarrow\ -z_1z_2 + z_1x_3 - z_2x_3 + z_1 - z_2 + x_3 + 1$ 7/8 (88%) (81)
$\longrightarrow\ -z_1 + z_2x_3 + 1$ 8/8(100%) (82)

$z_1z_2x_3:$ (83)
$\longrightarrow\ -z_1z_2 + z_1x_3 - z_2x_3 + z_1 - z_2 + x_3 + 1$ 7/8 (88%) (84)
$\longrightarrow\ z_2 - z_1x_3 + 1$ 8/8(100%) (85)

$z_1z_2x_3:$ (86)
$\longrightarrow\ -z_1z_2 + z_1x_3 - z_2x_3 + z_1 - z_2 + x_3 + 1$ 7/8 (88%) (87)
$\longrightarrow\ -x_3 + z_1z_2 + 1$ 8/8(100%) (88)

$z_1z_2x_3:$ (89)
$\longrightarrow\ +z_1z_2 - x_3 + 1$ 6/8 (75%) (90)
$\longrightarrow\ -z_1z_2 + x_3 + 1$ 8/8(100%) (91)

$z_1z_2x_3:$ (92)
$\longrightarrow\ +z_1x_3 - z_2 + 1$ 6/8 (75%) (93)
$\longrightarrow\ -z_1x_3 + z_2 + 1$ 8/8(100%) (94)

BINOMIALS OF DEGREE-k TERMS

$b_1b_2b_3b_4 + b_3b_4b_5b_6 = \min(2b_3b_4, b_1b_2 + b_5b_6)$ $(k,n) = (4,6).$ (95)

$b_1b_2b_3b_4 + b_3b_4b_5b_6 = \min_{b_a}(b_2b_3 + b_a(1 - b_2 - b_3 + 2b_4) + b_3b_4, b_1b_2 + b_5b_6 + b_5b_a)$ $(k,n) = (4,6).$ (96)

$b_1b_2b_3b_4 + b_4b_5b_6b_7:$ $(k,n) = (4,7).$ (97)
$\longrightarrow\ b_3b_4 + b_4b_6 + b_a(b_5 + b_7)$ 89/128 (70%) (98)
$\longrightarrow\ b_1b_2 + b_5b_7 + b_a(1 - b_5 + b_6 - b_7)$ 125/128 (98%) (99)
$\longrightarrow\ b_5b_7 + b_3$ 128/128(100%) (100)

$b_1b_2b_3b_4 + b_4b_5b_6b_7:$ $(k,n) = (4,7).$ (101)
$\longrightarrow\ b_3b_4 + b_4b_6$ 89/128 (70%) (102)
$\longrightarrow\ b_1b_2 + b_6b_7$ 118/128 (92%) (103)
$\longrightarrow\ b_2b_3 - b_5b_6 + b_5b_7 + b_5$ 127/128 (99%) (104)
$\longrightarrow\ b_1b_4 + 2b_5 - b_7 + 1$ 128/128(100%) (105)

$b_1b_2b_3b_4b_5 + b_3b_4b_5b_6b_7$: $(k,n) = (5,7)$. (106)
$\longrightarrow\ b_1b_5 + b_5b_6 + b_5b_7 + b_6b_7 + b_a(1 - b_5 - 2b_6 - b_7) + b_6$ 188/256 (73%) (107)
$\longrightarrow\ b_3b_4 + b_a(b_4 - b_6) + b_6$ 236/256 (92%) (108)
$\longrightarrow\ b_2b_3 + b_3b_6 - b_4b_6 + b_6b_a + b_6$ 254/256 (99%) (109)
$\longrightarrow\ b_2b_5 + b_5b_7$ 256/256(100%) (110)

$b_1b_2b_3b_4b_5 + b_3b_4b_5b_6b_7$: $(k,n) = (5,7)$. (111)
$\longrightarrow\ b_2b_3 + b_3b_7$ 85/128 (66%) (112)
$\longrightarrow\ 2b_4b_5$ 121/128 (95%) (113)
$\longrightarrow\ b_1b_2 + b_6b_7 - b_5 + 1$ 128/128(100%) (114)

$b_1b_2b_3b_4b_5b_6 + b_2b_3b_4b_5b_6b_7$: $(k,n) = (6,7)$. (115)
$\longrightarrow\ b_5b_6 + b_6b_7 + b_a(1 + b_5 - b_6 - b_7)$ 196/256 (77%) (116)
$\longrightarrow\ b_1b_4 + b_2b_4 + b_7b_a$ 238/256 (93%) (117)
$\longrightarrow\ b_1b_3 + b_3b_7 - b_4b_6 + 2b_5b_a - b_6b_7 - b_5 + b_6 + b_7 + b_a + 1$ 252/256 (98%) (118)
$\longrightarrow\ b_2b_6 + b_2 - b_6 + 1$ 256/256(100%) (119)

$b_1b_2b_3b_4b_5b_6 + b_2b_3b_4b_5b_6b_7$: $(k,n) = (6,7)$. (120)
$\longrightarrow\ 2b_5b_6$ 97/128 (76%) (121)
$\longrightarrow\ b_1b_4 + b_4b_7$ 119/128 (93%) (122)
$\longrightarrow\ b_1b_3 + b_1b_7 + b_2b_3 - b_3b_6 + b_3b_7 - b_4b_5 - b_1 - b_7 + 2$ 127/128 (99%) (123)
$\longrightarrow\ b_1b_2 + b_2b_6$ 128/128(100%) (124)

$b_1b_2b_3b_4b_5 + b_4b_5b_6b_7b_8$: $(k,n) = (5,8)$. (125)
$\longrightarrow\ b_3b_5 + b_7b_8 + b_a(-1 - b_6 + b_7 + b_8) + b_6 - b_7 - b_8 + 1$ 360/512 (70%) (126)
$\longrightarrow\ b_1b_4 + b_4b_8 + b_a(b_4 + b_6)$ 468/512 (91%) (127)
$\longrightarrow\ b_1b_2 + b_7b_8 + b_a(1 + b_6 - b_7 - b_8)$ 496/512 (97%) (128)
$\longrightarrow\ b_3b_5 + b_5$ 512/512(100%) (129)

$b_1b_2b_3b_4b_5 + b_4b_5b_6b_7b_8$: $(k,n) = (5,8)$. (130)
$\longrightarrow\ b_2b_5 + b_5b_8$ 169/256 (66%) (131)
$\longrightarrow\ b_1b_4 + b_4b_7 - b_5b_8 + b_8$ 233/256 (91%) (132)
$\longrightarrow\ b_1b_3 + b_6b_7 + b_6b_8 + b_7b_8 - b_6 - b_7 - b_8 + 1$ 252/256 (98%) (133)
$\longrightarrow\ b_2b_3 + b_6b_7$ 256/256(100%) (134)

$b_1b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7b_8$: $(k,n) = (6,8)$. (135)
$\longrightarrow\ b_1b_6 + b_7b_8 + b_a(1 + b_6 - b_7 - b_8)$ 364/512 (71%) (136)
$\longrightarrow\ b_2b_3 + b_5b_8 - b_6b_8 + b_7b_a - b_7 + b_8 - b_a + 1$ 450/512 (88%) (137)
$\longrightarrow\ b_1b_4 + b_4$ 488/512 (95%) (138)
$\longrightarrow\ b_2b_3 + b_3b_7 - b_6b_8 + b_8 - b_a + 1$ 502/512 (98%) (139)
$\longrightarrow\ b_2b_5 + b_5$ 512/512(100%) (140)

$b_1b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7b_8$: $(k,n) = (6,8)$. (141)
$\longrightarrow\ 2b_5b_6$ 193/256 (75%) (142)
$\longrightarrow\ b_1b_4 + b_4b_8$ 237/256 (93%) (143)
$\longrightarrow\ b_2b_3 + b_3b_7 - b_4b_6 + b_4b_8 - b_5b_7 - b_5b_8 + b_6b_8 - b_6 + b_7 - b_8 + 2$ 254/256 (99%) (144)
$\longrightarrow\ b_1b_2 + b_7b_8$ 256/256(100%) (145)

$b_1b_2b_3b_4b_5b_6b_7 + b_2b_3b_4b_5b_6b_7b_8$: $(k,n) = (7,8)$. (146)
$\longrightarrow\ b_6b_7 + b_6b_8 + b_a(1 - b_6 + b_7 - b_8)$ 388/512 (76%) (147)
$\longrightarrow\ b_1b_3 + b_3b_8 + b_a(1 + b_8)$ 470/512 (92%) (148)
$\longrightarrow\ b_2b_4 - b_3b_8 + b_4b_5 + b_a(1 - b_7) + b_8$ 500/512 (98%) (149)
$\longrightarrow\ b_2b_5 + b_2b_8 - b_4b_8 - b_6b_7 + b_6b_8 + b_a(-1 - b_4 - b_7 + b_8) - b_3 + b_7 - b_8 + 4$ 508/512 (99%) (150)
$\longrightarrow\ b_2b_5 - b_7b_8 + b_5 + 1$ 512/512(100%) (151)

$b_1b_2b_3b_4b_5b_6b_7 + b_2b_3b_4b_5b_6b_7b_8$: $(k,n) = (7,8)$. (152)
$\longrightarrow\ 2b_5b_6$ 193/256 (75%) (153)
$\longrightarrow\ b_1b_4 + b_4b_8$ 235/256 (92%) (154)
$\longrightarrow\ b_2b_3 + b_2b_7 - b_5b_6 + b_6b_8 + b_5 - b_6 - b_8 + 1$ 250/256 (98%) (155)
$\longrightarrow\ b_3b_7 + b_7b_8$ 254/256 (99%) (156)
$\longrightarrow\ b_3b_8 + b_3$ 256/256(100%) (157)

$b_1b_2b_3b_4 + b_5b_6b_7b_8$: $(k,n) = (4,8)$. (158)
$\longrightarrow\ b_2b_3 + b_6b_8 + b_a(1 - b_6 + b_7 - b_8)$ 390/512 (76%) (159)
$\longrightarrow\ b_1b_4 + b_6b_8 + b_a(1 - b_6 + b_7 - b_8)$ 480/512 (94%) (160)
$\longrightarrow\ b_2b_4 + b_5 - b_a + 1$ 506/512 (99%) (161)
$\longrightarrow\ b_1b_3 - b_6b_a + b_5 + 1$ 512/512(100%) (162)

$b_1b_2b_3b_4 + b_5b_6b_7b_8$: $(k,n) = (4,8)$. (163)
$\longrightarrow\ b_1b_2 + b_6b_7$ 169/256 (66%) (164)
$\longrightarrow\ b_3b_4 + b_5b_8$ 238/256 (93%) (165)
$\longrightarrow\ b_1b_4 + b_5b_6 + b_5b_7 + b_6b_7 - b_5 - b_6 - b_7 + 1$ 248/256 (97%) (166)
$\longrightarrow\ b_2b_3 + b_6b_7 + b_6b_8 + b_7b_8 - b_6 - b_7 - b_8 + 1$ 254/256 (99%) (167)
$\longrightarrow\ b_1b_2 + b_5b_8$ 256/256(100%) (168)

$b_1b_2b_3b_4b_5 + b_6b_7b_8b_9b_{10}$: $(k,n) = (5,10)$. (169)
$\longrightarrow\ b_1b_4 + b_7b_9$ 625/1024 (61%) (170)
$\longrightarrow\ b_3b_5 + b_6b_8$ 889/1024 (87%) (171)
$\longrightarrow\ b_2b_5 + b_7b_{10}$ 972/1024 (95%) (172)
$\longrightarrow\ b_2b_4 + b_6b_8$ 999/1024 (98%) (173)
$\longrightarrow\ b_1b_3 + b_9b_{10} + b_9b_a$ 1016/1024 (99%) (174)
$\longrightarrow\ b_1b_5 + b_6b_9$ 1020/1024 (99%) (175)
$\longrightarrow\ b_1b_4 + b_8b_{10}$ 1022/1024 (99%) (176)
$\longrightarrow\ b_2b_3 - b_4b_{10} + b_7b_9 + b_9b_a + 1$ 1024/1024(100%) (177)

$b_1b_2b_3b_4b_5 + b_6b_7b_8b_9b_{10}$: $(k,n) = (5,10)$. (178)

$\longrightarrow\ b_1b_3 + b_9b_{10}$ 625/1024 (61%) (179)

$\longrightarrow\ b_2b_4 + b_7b_{10}$ 851/1024 (83%) (180)

$\longrightarrow\ b_3b_5 + b_5b_{10} + b_8b_9$ 924/1024 (90%) (181)

$\longrightarrow\ b_1b_2 + b_6$ 972/1024 (95%) (182)

$\longrightarrow\ b_3b_4 + b_8b_9$ 997/1024 (97%) (183)

$\longrightarrow\ b_1b_5 + b_7b_{10}$ 1010/1024 (99%) (184)

$\longrightarrow\ b_2b_3 - b_1b_7 - b_1b_{10} - b_2b_8 - b_2b_{10} + b_3b_5 + b_6b_9 + b_7b_{10} - b_8b_9 + b_9b_{10} - b_3 - b_7 + b_8 + 3$ 1016/1024 (99%) (185)

$\longrightarrow\ b_1b_3 + b_7b_8$ 1020/1024 (99%) (186)

$\longrightarrow\ b_2b_4 + b_2b_6 - b_2b_9 - b_3b_{10} - b_5b_7 + b_7b_{10} + b_9b_{10} - b_{10} + 2$ 1023/1024 (99%) (187)

$\longrightarrow\ b_2b_5 + b_2b_9 + b_6b_8$ 1024/1024(100%) (188)

$b_1b_2b_3b_4b_5b_6 + b_5b_6b_7b_8b_9b_{10}$: $(k,n) = (6,10)$. (189)

$\longrightarrow\ b_4b_5 + b_5b_9$ 657/1024 (64%) (190)

$\longrightarrow\ b_2b_6 + b_6b_8$ 905/1024 (88%) (191)

$\longrightarrow\ b_1b_3 + b_7b_8$ 982/1024 (96%) (192)

$\longrightarrow\ b_2b_3 + b_a(b_{10} - b_9) + b_9$ 1011/1024 (99%) (193)

$\longrightarrow\ b_2b_4 + b_7b_{10}$ 1020/1024 (99%) (194)

$\longrightarrow\ b_9b_{10} + b_1$ 1023/1024 (99%) (195)

$\longrightarrow\ b_7b_8 + b_4$ 1024/1024(100%) (196)

$b_1b_2b_3b_4b_5b_6 + b_5b_6b_7b_8b_9b_{10}$: $(k,n) = (6,10)$. (197)

$\longrightarrow\ 2b_5b_6$ 769/1024 (75%) (198)

$\longrightarrow\ b_1b_3 + b_8b_9$ 934/1024 (92%) (199)

$\longrightarrow\ b_2b_4 + b_7b_{10} + b_8b_9 - b_8 - b_9 + 1$ 997/1024 (97%) (200)

$\longrightarrow\ -b_1b_3 + b_1b_9 + b_2b_4 + b_4b_9 + b_5b_8 + b_8b_9 - b_5 - b_8 - b_9 + 2$ 769/1024 (99%) (201)

$\longrightarrow\ b_1b_3 + b_7b_{10} - b_8 - b_9 + 2$ 1014/1024 (99%) (202)

$\longrightarrow\ b_2b_3 + b_8b_9$ 1024/1024(100%) (203)

$b_1b_2b_3b_4b_5b_6b_7 + b_4b_5b_6b_7b_8b_9b_{10}$: $(k,n) = (7,10)$. (204)

$\longrightarrow\ b_3b_5 + b_5b_8$ 649/1024 (63%) (205)

$\longrightarrow\ b_2b_4 + b_4b_9$ 893/1024 (87%) (206)

$\longrightarrow\ b_1b_7 + b_7b_{10}$ 985/1024 (96%) (207)

$\longrightarrow\ b_1b_6 + b_6b_9 + b_a$ 1015/1024 (99%) (208)

$\longrightarrow\ b_2b_3 + b_8b_{10} + b_a$ 1022/1024 (99%) (209)

$\longrightarrow\ b_1b_3 + b_8b_9$ 1024/1024(100%) (210)

$b_1b_2b_3b_4b_5b_6b_7 + b_4b_5b_6b_7b_8b_9b_{10}$: $(k,n) = (7,10)$. (211)

$\longrightarrow\ b_3b_7 + b_7b_{10}$ 649/1024 (63%) (212)

$\longrightarrow\ 2b_4b_6$ 937/1024 (92%) (213)

$\longrightarrow\ b_1b_5 + b_5b_8$ 1001/1024 (98%) (214)

$\longrightarrow\ b_1b_2 + b_9b_{10}$ 1019/1024 (99%) (215)

$\longrightarrow\ b_2b_3 + b_8$ 1023/1024 (99%) (216)

$\longrightarrow\ b_3b_7 + b_9b_{10}$ 1024/1024(100%) (217)

$b_1b_2b_3b_4b_5b_6b_7b_8 + b_3b_4b_5b_6b_7b_8b_9b_{10}$: $(k,n) = (8,10)$. (218)
$\longrightarrow\ b_2b_8 + b_8b_9$ 645/1024 (63%) (219)
$\longrightarrow\ b_1b_3 + b_3b_{10} + b_9b_a$ 887/1024 (87%) (220)
$\longrightarrow\ b_4b_6 + b_5b_6$ 977/1024 (95%) (221)
$\longrightarrow\ b_2b_7 + b_7b_{10}$ 1007/1024 (98%) (222)
$\longrightarrow\ b_1b_4 + b_4b_5 + 2b_9b_a$ 1018/1024 (99%) (223)
$\longrightarrow\ b_1b_5 + b_5b_9$ 1024/1024(100%) (224)

$b_1b_2b_3b_4b_5b_6b_7b_8 + b_3b_4b_5b_6b_7b_8b_9b_{10}$: $(k,n) = (8,10)$. (225)
$\longrightarrow\ 4b_3b_7$ 768/1024 (75%) (226)
$\longrightarrow\ b_2b_8 + b_8b_9$ 933/1024 (91%) (227)
$\longrightarrow\ 2b_4b_6 + b_8b_9 - b_8 - b_9 + 1$ 1005/1024 (98%) (228)
$\longrightarrow\ b_1b_5 + b_5b_{10} + b_8b_9 - b_8 - b_9 + 1$ 1022/1024 (99%) (229)
$\longrightarrow\ b_1b_2 + b_8b_9 + b_9b_{10} - b_8 - b_9 + 1$ 1024/1024(100%) (230)

$b_1b_2b_3b_4b_5b_6b_7b_8b_9 + b_2b_3b_4b_5b_6b_7b_8b_9b_{10}$: $(k,n) = (9,10)$. (231)
$\longrightarrow\ b_1b_9 + b_9b_{10} + b_{10}b_a$ 643/1024 (63%) (232)
$\longrightarrow\ b_2b_4 + b_4b_5$ 883/1024 (86%) (233)
$\longrightarrow\ b_3b_7 + b_3b_8$ 973/1024 (95%) (234)
$\longrightarrow\ b_2b_6 + b_6b_8$ 1003/1024 (98%) (235)
$\longrightarrow\ b_2b_5 + b_5b_7 - b_{10}b_a + b_{10}$ 1015/1024 (99%) (236)
$\longrightarrow\ b_1b_8 + b_7b_8$ 1019/1024 (99%) (237)
$\longrightarrow\ b_2b_7 + b_2b_{10} - b_4b_5 - b_{10}b_a + b_{10} + 1$ 1023/1024 (99%) (238)
$\longrightarrow\ b_4b_7 + b_7$ 1024/1024(100%) (239)

$b_1b_2b_3b_4b_5b_6b_7b_8b_9 + b_2b_3b_4b_5b_6b_7b_8b_9b_{10}$: $(k,n) = (9,10)$. (240)
$\longrightarrow\ 2b_2b_3 - b_8b_9 + b_9$ 577/1024 (56%) (241)
$\longrightarrow\ 3b_8b_9$ 961/1024 (94%) (242)
$\longrightarrow\ 2b_4b_6 - b_8b_9 - b_8b_{10} + b_{10} + 1$ 1009/1024 (99%) (243)
$\longrightarrow\ 2b_5b_7 - b_8b_{10} + b_{10}$ 1021/1024 (99%) (244)
$\longrightarrow\ b_1b_6 + b_{10}$ 1024/1024(100%) (245)

DEGREE-k, EXACT-k-OF-n TRINOMIALS

$b_1b_2b_3b_4 + b_2b_3b_4b_5 + b_3b_4b_5b_6$: $(k,n) = (4,6)$. (246)
$\longrightarrow\ b_1b_4 + 2b_4b_5 + b_7$ 44/64 (69%) (247)
$\longrightarrow\ b_1b_3 + b_2b_3 + b_3b_6 + b_6b_7$ 60/64 (94%) (248)
$\longrightarrow\ b_2b_4 + b_5b_6 + b_2$ 64/64(100%) (249)

$b_1b_2b_3b_4 + b_2b_3b_4b_5 + b_3b_4b_5b_6$: $(k,n) = (4,6)$. (250)
$\longrightarrow\ b_2b_4 + 2b_4b_5$ 43/64 (67%) (251)
$\longrightarrow\ b_1b_3 + b_2b_3 + b_2b_5 + b_3b_6 - b_4b_5 - b_2 + 1$ 60/64 (94%) (252)
$\longrightarrow\ b_1b_2 + b_2b_5 + b_5b_6$ 64/64(100%) (253)

$b_1b_2b_3b_4 + b_3b_4b_5b_6 + b_5b_6b_7b_8$: $(k,n) = (4,8)$. (254)
$\longrightarrow\ b_1b_4 + 2b_5b_6$ 159/256 (62%) (255)
$\longrightarrow\ b_2b_3 + b_3b_5 + b_7b_8$ 225/256 (88%) (256)
$\longrightarrow\ b_1b_4 + b_3b_4 - b_5b_7 + b_6b_7 + b_7b_8 - b_6 + 1$ 244/256 (95.3%) (257)
$\longrightarrow\ b_2b_3 + b_6b_8 + b_6$ 253/256 (98.8%) (258)
$\longrightarrow\ b_2b_3 + b_5b_7 + b_5$ 256/256 (100%) (259)

$b_1b_2b_3b_4 + b_3b_4b_5b_6 + b_5b_6b_7b_8$: $(k,n) = (4,8)$. (260)
$\longrightarrow\ b_2b_4 + 2b_5b_6$ 159/256 (62%) (261)
$\longrightarrow\ b_3b_6 + b_7b_8 + b_3$ 212/256 (83%) (262)
$\longrightarrow\ b_2b_4 - b_5b_7 + b_7b_8 + b_4 + b_7$ 234/256 (91%) (263)
$\longrightarrow\ b_1b_3 + 2b_5b_6$ 253/256 (99%) (264)
$\longrightarrow\ b_7b_8 + b_1 + b_6$ 256/256(100%) (265)

$b_1b_2b_3b_4b_5 + b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7$: $(k,n) = (5,7)$. (266)
$\longrightarrow\ b_1b_5 + b_5b_6 + b_6b_7 + b_a(-2 - b_5 + 2b_6 + b_7) + b_5 - 2b_6 - b_7 + 2$ 86/128 (67%) (267)
$\longrightarrow\ b_1b_3 + b_3b_4 - b_3b_6 + b_3b_7 + b_a(b_5 + b_7) + b_3$ 112/128 (88%) (268)
$\longrightarrow\ b_1b_4 + b_2b_4 + b_4b_7 + b_5b_7 + b_a(-1 - b_6 - b_7) - b_5 + b_6 + 2$ 124/128 (97%) (269)
$\longrightarrow\ b_2b_4 - 2b_5b_a + b_6b_7 + b_2 + b_5 + 1$ 128/128(100%) (270)

$b_1b_2b_3b_4b_5 + b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7$: $(k,n) = (5,7)$. (271)
$\longrightarrow\ 2b_4b_5 + b_4b_6$ 81/128 (63%) (272)
$\longrightarrow\ b_1b_3 + b_3b_6 + b_3b_7 - b_4b_5 + b_5$ 111/128 (87%) (273)
$\longrightarrow\ b_1b_2 + b_2b_6 - b_4b_5 + b_6b_7 + b_5$ 122/128 (95%) (274)
$\longrightarrow\ 2b_4b_5 + b_5$ 128/128(100%) (275)

$b_1b_2b_3b_4b_5b_6 + b_2b_3b_4b_5b_6b_7 + b_3b_4b_5b_6b_7b_8$: $(k,n) = (6,8)$. (276)
$\longrightarrow\ b_1b_6 + 2b_6b_7$ 164/256 (64%) (277)
$\longrightarrow\ b_1b_5 + b_2b_5 - b_3b_6 + b_5b_8 + b_3$ 219/256 (86%) (278)
$\longrightarrow\ b_2b_4 + b_4b_7 + b_4b_8 - b_6 + 1$ 243/256 (95%) (279)
$\longrightarrow\ b_2b_3 + b_3b_8 - b_5b_6 + b_3 + b_6$ 253/256 (99%) (280)
$\longrightarrow\ b_1b_2 + b_2b_6 + b_5b_7 - b_6b_7 + b_7b_8 - b_5 + 1$ 256/256(100%) (281)

$b_1b_2b_3b_4b_5b_6b_7b_8 + b_2b_3b_4b_5b_6b_7b_8b_9 + b_3b_4b_5b_6b_7b_8b_9b_{10}$: $(k,n) = (8,10)$. (282)
$\longrightarrow\ 3b_5b_8$ 769/1024 (75%) (283)
$\longrightarrow\ 2b_2b_6 + b_4b_6$ 931/1024 (91%) (284)
$\longrightarrow\ b_1b_7 - b_5b_{10} + b_7b_9 + b_9b_{10} - b_6 + b_{10} + 1$ 984/1024 (96%) (285)
$\longrightarrow\ 3b_2b_3 + b_3b_{10} - b_6b_8 + 1$ 1011/1024 (99%) (286)
$\longrightarrow\ b_4b_7 + b_4b_8 - b_3 + b_4 - b_8 + 2$ 1019/1024 (99%) (287)
$\longrightarrow\ b_2b_3 - b_2b_4 - b_3b_4 - b_3b_8 - b_5b_{10} - b_6b_9 + b_7b_8 + b_7b_9 + b_8b_9 + b_7 + 3$ 1023/1024 (99%) (288)
$\longrightarrow\ b_2b_8 + 2b_8b_9$ 1024/1024(100%) (289)

$b_1b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7b_8 + b_5b_6b_7b_8b_9b_{10}$: $(k,n) = (6,10)$. (290)
$\longrightarrow\ b_5b_6 + b_5b_8 + b_5b_9 + b_8b_{11} + b_9b_{11} + b_{10}b_{11}$ 583/1024 (57%) (291)
$\longrightarrow\ b_1b_2 + b_4b_7 + b_7b_{10} + b_9b_{11} - b_9 - b_{11} + 1$ 815/1024 (80%) (292)
$\longrightarrow\ b_1b_6 + b_5b_6 + b_6 - b_{11} + 1$ 917/1024 (90%) (293)
$\longrightarrow\ b_3b_4 + b_3b_7 + b_8b_9 + b_9b_{11}$ 979/1024 (96%) (294)
$\longrightarrow\ b_2b_4 + b_4b_8 + b_8b_9 - b_9b_{11} + b_9 - b_{11} + 1$ 1007/1024 (98%) (295)
$\longrightarrow\ b_1b_3 + b_7b_{10} + b_{10}b_{11} + b_3$ 1016/1024 (99%) (296)
$\longrightarrow\ b_1b_4 + b_4b_8 + b_7b_{10} + b_9b_{11} + b_{10}b_{11} - b_9 - b_{11} + 1$ 1021/1024 (99%) (297)
$\longrightarrow\ b_1b_3 - b_2b_{11} + b_7b_8 + b_8b_9 - b_{10}b_{11} - b_{11} + 3$ 1024/1024(100%) (298)

$b_1b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7b_8 + b_5b_6b_7b_8b_9b_{10}$: $(k,n) = (6,10)$. (299)
$\longrightarrow\ 2b_3b_4 + b_7b_{10}$ 591/1024 (58%) (300)
$\longrightarrow\ 2b_3b_5 + b_5b_6$ 847/1024 (83%) (301)
$\longrightarrow\ b_1b_2 + b_7b_8 + b_8b_9$ 951/1024 (93%) (302)
$\longrightarrow\ 3b_5b_6$ 995/1024 (97%) (303)
$\longrightarrow\ b_1b_3 + b_3b_4 + b_9b_{10}$ 1009/1024 (99%) (304)
$\longrightarrow\ b_1b_2 + b_5b_7 + b_7b_{10}$ 1018/1024 (99%) (305)
$\longrightarrow\ 2b_1b_4 - b_1b_{10} + b_2b_4 + b_4b_5 + b_4b_{10} + b_5b_8 - b_6b_8 + b_8b_9 + b_7(b_{10} - b_6 - b_5 - b_1) + 3$ 1023/1024 (99%) (306)
$\longrightarrow\ b_2b_8 + b_3b_6 + b_6b_8$ 1024/1024(100%) (307)

$b_1b_2b_3b_4 + b_4b_5b_6b_7 + b_7b_8b_9b_{10}$: $(k,n) = (4,10)$. (308)
$\longrightarrow\ b_2b_3 + b_6b_7 + b_7b_9 + 2b_9b_a$ 581/1024 (57%) (309)
$\longrightarrow\ b_2b_4 + b_4b_6 + b_a(b_9 - b_{10}) + b_{10}$ 823/1024 (80%) (310)
$\longrightarrow\ b_1b_3 + b_5b_6 + b_8b_9 + b_a(b_9 - b_{10}) - b_9 + 1$ 930/1024 (91%) (311)
$\longrightarrow\ b_1b_4 + b_4b_5 + b_4b_{10} + b_8b_{10} + b_a(1 - b_7 + b_9)$ 978/1024 (96%) (312)
$\longrightarrow\ b_1b_4 + b_7b_8 + b_a(1 + b_9) + b_7$ 1000/1024 (98%) (313)
$\longrightarrow\ b_2b_3 + b_a(b_9 - b_{10}) + b_5 + b_{10}$ 1015/1024 (99%) (314)
$\longrightarrow\ b_1b_3 + b_6 + b_{10}$ 1020/1024 (99%) (315)
$\longrightarrow\ b_5b_6 + b_2 + b_8 + b_a$ 1024/1024(100%) (316)

$b_1b_2b_3b_4 + b_4b_5b_6b_7 + b_7b_8b_9b_{10}$: $(k,n) = (4,10)$. (317)
$\longrightarrow\ b_3b_4 + b_4b_6 + b_9b_{10}$ 581/1024 (57%) (318)
$\longrightarrow\ b_1b_2 + b_5b_7 - b_8b_9 + b_9b_{10} + b_9$ 759/1024 (74%) (319)
$\longrightarrow\ b_5b_6 + b_8b_9 + b_1 + b_8$ 842/1024 (82%) (320)
$\longrightarrow\ b_2b_4 + b_7b_{10} - b_8b_9 + b_7 + b_8$ 935/1024 (91%) (321)
$\longrightarrow\ b_2b_4 + b_4b_6 - b_8b_9 + b_8b_{10} - b_7 + b_8 + 1$ 969/1024 (95%) (322)
$\longrightarrow\ b_1b_3 + b_3b_4 + b_5b_7 + b_7b_9 - b_8b_9 + b_9$ 992/1024 (97%) (323)
$\longrightarrow\ b_2b_3 + b_3b_5 + b_3b_{10} + b_4b_8 + b_5b_6 - b_4 + 1$ 1004/1024 (98%) (324)
$\longrightarrow\ b_1b_3 + b_6b_7 + b_9b_{10}$ 1013/1024 (99%) (325)
$\longrightarrow\ b_1b_9 + b_7b_8 - b_8b_9 - b_9b_{10} + b_1 + b_7 + b_8 + b_9$ 1019/1024 (99%) (326)
$\longrightarrow\ b_2b_3 + b_5b_6 - b_8b_9 + b_9b_{10} + b_9$ 1022/1024 (99%) (327)
$\longrightarrow\ -b_1b_5 + b_1b_8 + b_3b_7 + b_3 + b_7 + 1$ 1023/1024 (99%) (328)
$\longrightarrow\ b_1b_5 - b_1b_{10} + b_2 + b_8 - b_{10} + 2$ 1024/1024(100%) (329)

$b_1b_2b_3b_4b_5 + b_3b_4b_5b_6 + b_4b_5b_6b_7b_8$: $(k,n) = (5,8)$. (330)
$\longrightarrow\ b_1b_3 + b_3b_4 + b_6b_8 - b_6b_9 + b_7b_9 - b_8b_9 + b_9$ 156/256 (61%) (331)
$\longrightarrow\ b_1b_5 + b_5b_7 + b_7b_9 + b_8b_9 + b_5 - b_7 - b_9 + 1$ 202/256 (79%) (332)
$\longrightarrow\ b_2b_4 + b_6b_8 + b_6b_9 - b_7b_9 + b_8b_9 + b_7 - b_8 - b_9 + 1$ 230/256 (90%) (333)
$\longrightarrow\ b_2b_4 + b_4b_8 + b_4 - b_9 + 1$ 246/256 (96%) (334)
$\longrightarrow\ b_1b_5 + 2b_6$ 252/256 (98%) (335)
$\longrightarrow\ b_2b_5 + b_7b_8 + b_5$ 256/256(100%) (336)

$b_1b_2b_3b_4b_5 + b_3b_4b_5b_6 + b_4b_5b_6b_7b_8$: $(k,n) = (5,8)$. (337)
$\longrightarrow\ b_4b_5 + 2b_5b_6$ 165/256 (64%) (338)
$\longrightarrow\ b_2b_4 + b_3b_4 + b_4b_8 - b_5b_7 + b_7$ 215/256 (84%) (339)
$\longrightarrow\ b_2b_3 + b_3b_6 - b_4b_5 - b_5b_7 + b_7b_8 + b_5 + b_7$ 242/256 (95%) (340)
$\longrightarrow\ b_1b_3 + b_5b_6 + b_6b_7$ 254/256 (99%) (341)
$\longrightarrow\ b_1b_2 + b_5b_6 + b_6b_8$ 256/256(100%) (342)

DEGREE-k, EXACT-k-OF-n QUADRINOMIALS

$b_1b_2b_3 + b_1b_2b_4 + b_1b_3b_4 + b_2b_3b_4$: $(k,n) = (3,4)$. (343)
$\longrightarrow\ 2b_1b_2 + b_1b_3 + 2b_1b_4 + b_2b_3 + 2b_2b_4 + b_3b_4 - 2b_1 - 2b_2 - b_3 - 2b_4 + 3$ 13/16 (81%) (344)
$\longrightarrow\ 2b_1b_3 + b_2b_3 + b_2$ 16/16(100%) (345)

$b_1b_2b_3b_4b_5b_6b_7 + b_2b_3b_4b_5b_6b_7b_8 + b_3b_4b_5b_6b_7b_8b_9 + b_4b_5b_6b_7b_8b_9b_{10}$: $(k,n) = (7,10)$. (346)
$\longrightarrow\ 4b_4b_5$ 769/1024 (75%) (347)
$\longrightarrow\ b_2b_6 + 2b_3b_6 + b_6b_9$ 915/1024 (89%) (348)
$\longrightarrow\ b_1b_7 + b_5b_7 + b_6b_7 + b_7b_{10}$ 974/1024 (95%) (349)
$\longrightarrow\ b_1b_2 + b_2b_8 + b_7b_8 + b_9b_{10}$ 995/1024 (97%) (350)
$\longrightarrow\ b_2b_3 + b_3b_4 + b_3b_6 + b_9b_{10}$ 1008/1024 (98%) (351)
$\longrightarrow\ b_1b_2 + b_2b_4 + b_9b_{10} + b_9$ 1016/1024 (99%) (352)
$\longrightarrow\ b_1b_3 - b_2b_8 + b_7b_8 + b_8b_9 + b_8b_{10} + 2b_8$ 1023/1024 (99%) (353)
$\longrightarrow\ b_1b_8 + b_2b_7 - b_5b_{10} + b_7b_8 + b_8b_9 - b_5 + 2$ 1024/1024(100%) (354)

DEGREE-k, NOT EXACT-k-OF-n MULTINOMIALS

$b_1b_2b_3b_4 + 2b_1b_2b_3 + b_1b_2b_4 + b_1b_3b_4 + b_2b_3b_4$: $(k,n) = (4,4)$. (355)
$\longrightarrow\ b_1b_2 + 4b_1b_3 + b_1b_4 + b_2b_3 + b_2b_4 + b_3b_4 - b_1 - b_2 - b_3 - b_4 + 1$ 12/16 (75%) (356)
$\longrightarrow\ b_1b_2 + b_1b_3 + 4b_1b_4 + b_2b_4$ 16/16(100%) (357)

$b_1b_2b_3b_4 + 2b_1b_2b_3 + b_1b_2b_4 + b_1b_3b_4 + b_2b_3b_4$: $(k,n) = (4,4)$. (358)
$\longrightarrow\ b_1b_2 + 4b_1b_3 + b_1b_4 + b_2b_3 + b_2b_4 + b_3b_4 - b_1 - b_2 - b_3 - b_4 + 1$ 12/16 (75%) (359)
$\longrightarrow\ 2b_2b_3 + 3b_2b_4 + b_3b_4$ 16/16(100%) (360)

$b_1b_2b_3b_4 + 2b_1b_2b_3 + b_1b_2b_4 + 3b_1b_3b_4 + b_2b_3b_4 :$ $(k,n) = (4,4).$ (361)
$\longrightarrow\ 2b_1b_2 + 5b_1b_4 + b_3b_4$ $11/16\ (69\%)$ (362)
$\longrightarrow\ -b_1b_2 + 3b_1b_3 + 4b_2b_3 + 2b_2b_4 - 4b_3b_4 + 4b_3 - b_4 + 1$ $16/16(100\%)$ (363)

$b_1b_2b_3b_4 + 2b_1b_2b_3 + b_1b_3b_4 :$ $(k,n) = (4,4).$ (364)
$\longrightarrow\ 4b_1b_3$ $13/16\ (81\%)$ (365)
$\longrightarrow\ 2b_1b_2 + b_1b_4 + b_2b_4$ $16/16(100\%)$ (366)

$b_1b_2b_3b_4 + 2b_1b_2b_3 + b_1b_3b_4 :$ $(k,n) = (4,4).$ (367)
$\longrightarrow\ 2b_1b_3 + 2b_3b_4$ $12/16\ (75\%)$ (368)
$\longrightarrow\ 3b_1b_2 + b_1b_4$ $16/16(100\%)$ (369)