

Volume 3: List of Multi-run Quadratizations

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PRODUCT OF POLYNOMIALS

$$f_1 f_2 \dots f_\kappa = \min(f_1, f_2, \dots, f_\kappa), \quad f_i(b_{k_i}, b_{k_i+1}, \dots, b_{k_{i+1}-1}) \geq 0 \quad (1)$$

$$f_1 f_2 \dots f_\kappa = \min(f_1 f_2 \dots f_{\kappa-1} \max f_\kappa, f_\kappa - \min f_\kappa + f_1 f_2 \dots f_{\kappa-1}), \min f_\kappa < 0, f_{i < \kappa}(b_{k_i}, b_{k_i+1}, \dots, b_{k_{i+1}-1}) \geq 0 \quad (2)$$

$$b_1 b_2 b_3 b_4 + b_2 b_3 b_4 - b_3 b_4 b_5 : \quad (\text{Example of Eq. 2}). \quad (3)$$

$$\longrightarrow 2b_3 b_4 \quad 25/32 \text{ (78\%)} \quad (4)$$

$$\longrightarrow b_1 b_2 + b_2 - b_5 - b_3 b_4 + 1 \quad 32/32(100\%) \quad (5)$$

MONOMIALS

$$b_1 b_2 b_3 \dots b_k = \min(b_1 b_2 \dots b_{k_1}, b_{k_1+1} b_{k_1+2} \dots b_{k_2}, b_{k_2+1} b_{k_2+2} \dots b_{k_3}, \dots, b_{k_n+1} b_{k_n+2} \dots b_k) \quad (\text{Example of Eq. 1}). \quad (6)$$

$$b_1 b_2 b_3 \dots b_k = \min(b_1, b_2, b_3, \dots, b_k) \quad (\text{Example of Eq. 6: Linearization of a degree-}k \text{ monomial}). \quad (7)$$

$$b_1 b_2 b_3 b_4 = \min(b_1 b_2, b_3 b_4) \quad (\text{Example of Eq. 6: Quadratization of a degree-4 monomial}). \quad (8)$$

$$b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 : \quad (9)$$

$$\longrightarrow 3b_a + b_1 b_2 + b_1 b_3 + b_1 b_4 + b_2 b_3 + b_2 b_4 + b_3 b_4 - 2b_a(b_1 + b_2 + b_3 + b_4) \quad (10)$$

$$\longrightarrow 3b_a + b_5 b_6 + b_5 b_7 + b_5 b_8 + b_6 b_7 + b_6 b_8 + b_7 b_8 - 2b_a(b_5 + b_6 + b_7 + b_8) \quad (11)$$

$$s_1 s_2 \dots s_k = \min(1 + s_1 s_2 - s_3 s_4 \dots s_k, 1 - s_1 s_2 + s_3 s_4 \dots s_k), s_i \in \{x, y, z\} \quad (12)$$

$$x_1 z_2 x_3 z_4 y_5 x_6 \quad (\text{Example of Eq. 12}). \quad (13)$$

$$\longrightarrow +x_1 z_2 - x_3 z_4 y_5 x_6 + 1 \quad 48/64 \text{ (75\%)} \quad (14)$$

$$\longrightarrow -x_1 z_2 + x_3 z_4 y_5 x_6 + 1 \quad 64/64(100\%) \quad (15)$$

$$z_1 z_2 x_3 + z_1 x_2 z_3 : \quad (16)$$

$$\longrightarrow +2z_1 - x_2 z_3 - z_2 x_3 + 2 \quad 5/8 \text{ (63\%)} \quad (17)$$

$$\longrightarrow -2z_1 + x_2 z_3 + z_2 x_3 + 2 \quad 8/8(100\%) \quad (18)$$

$$z_1 z_2 x_3 + 2z_1 x_2 z_3 : \quad (19)$$

$$\longrightarrow +3z_1 - 2x_2 z_3 - z_2 x_3 + 3 \quad 5/8 \text{ (63\%)} \quad (20)$$

$$\longrightarrow -3z_1 + 2x_2 z_3 + z_2 x_3 + 3 \quad 8/8(100\%) \quad (21)$$

$$z_1 z_2 x_3 + 3z_1 x_2 z_3 : \quad (22)$$

$$\longrightarrow +4z_1 - 3x_2 z_3 - z_2 x_3 + 4 \quad 5/8 \text{ (63\%)} \quad (23)$$

$$\longrightarrow -4z_1 + 3x_2 z_3 + z_2 x_3 + 4 \quad 8/8(100\%) \quad (24)$$

$$z_1 z_2 x_3 - z_1 x_2 z_3 : \quad (25)$$

$$\longrightarrow +2z_1 + x_2 z_3 - z_2 x_3 + 2 \quad 5/8 \text{ (63\%)} \quad (26)$$

$$\longrightarrow -2z_1 - x_2 z_3 + z_2 x_3 + 2 \quad 8/8(100\%) \quad (27)$$

$$z_1 z_2 x_3 - 2z_1 x_2 z_3 : \quad (28)$$

$$\longrightarrow +3z_1 + 2x_2 z_3 - z_2 x_3 + 3 \quad 5/8 \text{ (63\%)} \quad (29)$$

$$\longrightarrow -3z_1 - 2x_2 z_3 + z_2 x_3 + 3 \quad 8/8(100\%) \quad (30)$$

$$z_1(Az_2 x_3 + Bx_2 z_3) : \quad (31)$$

$$\longrightarrow +(|A| + |B|)z_1 - (Az_2 x_3 + Bx_2 z_3) + |A| + |B| \quad (32)$$

$$\longrightarrow -(|A| + |B|)z_1 + (Az_2 x_3 + Bx_2 z_3) + |A| + |B| \quad (33)$$

$$z_1 x_2 z_3 + z_1 z_2 x_3 + z_1 x_2 x_3 : \quad (34)$$

$$\longrightarrow +3z_1 - x_2 z_3 - z_2 x_3 - x_2 x_3 + 3 \quad 4/8 \text{ (50\%)} \quad (35)$$

$$\longrightarrow -3z_1 + x_2 z_3 + z_2 x_3 + x_2 x_3 + 3 \quad 8/8(100\%) \quad (36)$$

$$z_1 x_2 z_3 + z_1 z_2 x_3 + z_1 x_2 x_3 + z_1 z_2 z_3 : \quad (37)$$

$$\longrightarrow +4z_1 - x_2 z_3 - z_2 x_3 - x_2 x_3 - z_2 z_3 + 4 \quad 4/8 \text{ (50\%)} \quad (38)$$

$$\longrightarrow -4z_1 + x_2 z_3 + z_2 x_3 + x_2 x_3 + z_2 z_3 + 4 \quad 8/8(100\%) \quad (39)$$

$$z_1 x_2 z_3 + z_1 z_2 x_3 + z_1 x_2 x_3 + z_1 z_2 z_3 : \quad (40)$$

$$\longrightarrow +2z_1 - x_2 z_3 - z_2 x_3 - x_2 x_3 - z_2 z_3 + 2 \quad 6/8 \text{ (75\%)} \quad (41)$$

$$\longrightarrow -2z_1 + x_2 z_3 + z_2 x_3 + x_2 x_3 + z_2 z_3 + 2 \quad 8/8(100\%) \quad (42)$$

$$z_1 x_2 z_3 + z_1 z_2 x_3 + z_1 x_2 x_3 + z_1 z_2 z_3 + z_1 x_2 y_3 : \quad (43)$$

$$\longrightarrow +3z_1 - x_2 x_3 - x_2 z_3 - z_2 x_3 - z_2 z_3 - x_2 y_3 + 3 \quad 4/8 \text{ (50\%)} \quad (44)$$

$$\longrightarrow -3z_1 + x_2 x_3 + x_2 z_3 + z_2 x_3 + z_2 z_3 + x_2 y_3 + 3 \quad 8/8(100\%) \quad (45)$$

$$z_1 x_2 z_3 + z_1 z_2 x_3 + z_1 x_2 x_3 + z_1 z_2 z_3 + z_1 x_2 y_3 + z_1 y_2 x_3 + z_1 y_2 y_3 : \quad (46)$$

$$\longrightarrow -3z_1 + x_2 y_3 + x_2 z_3 + y_2 x_3 + z_2 x_3 + x_2 x_3 + y_2 y_3 + z_2 z_3 + 3 \quad 5/8 \text{ (63\%)} \quad (47)$$

$$\longrightarrow +4z_1 - x_2 y_3 - x_2 z_3 - y_2 x_3 - z_2 x_3 - x_2 x_3 - y_2 y_3 - z_2 z_3 + 4 \quad 8/8(100\%) \quad (48)$$

$$(A_1x_1z_2 + A_2y_1x_2 + A_3x_1x_2 + A_4y_1y_2)z_3y_4 : \quad (49)$$

$$\longrightarrow - (A_1x_1z_2 + A_2y_1x_2 + A_3x_1x_2 + A_4y_1y_2) + \sum_i |A_i| z_3y_4 + \sum_i |A_i| \quad (50)$$

$$\longrightarrow + (A_1x_1z_2 + A_2y_1x_2 + A_3x_1x_2 + A_4y_1y_2) - \sum_i |A_i| z_3y_4 + \sum_i |A_i| \quad (51)$$

$$z_1z_2x_3 : \quad (52)$$

$$\longrightarrow z_1z_2 + z_1x_3 + z_2x_3 - z_1 - z_2 - x_3 + 1 \quad 7/8 \text{ (88\%)} \quad (53)$$

$$\longrightarrow z_1z_2 - z_1x_3 - z_2x_3 + z_1 + z_2 - x_3 + 1 \quad 8/8(100\%) \quad (54)$$

$$z_1z_2x_3 : \quad (55)$$

$$\longrightarrow + z_1z_2 + z_1x_3 + z_2x_3 - z_1 - z_2 - x_3 + 1 \quad 7/8 \text{ (88\%)} \quad (56)$$

$$\longrightarrow - z_1z_2 + z_1x_3 - z_2x_3 + z_1 - z_2 + x_3 + 1 \quad 8/8(100\%) \quad (57)$$

$$z_1z_2x_3 : \quad (58)$$

$$\longrightarrow + z_1z_2 - z_1x_3 - z_2x_3 + z_1 + z_2 - x_3 + 1 \quad 7/8 \text{ (88\%)} \quad (59)$$

$$\longrightarrow - z_1z_2 + z_1x_3 - z_2x_3 + z_1 - z_2 + x_3 + 1 \quad 8/8(100\%) \quad (60)$$

$$z_1z_2x_3 : \quad (61)$$

$$\longrightarrow - z_1z_2 - z_1x_3 + z_2x_3 - z_1 + z_2 + x_3 + 1 \quad 7/8 \text{ (88\%)} \quad (62)$$

$$\longrightarrow - z_1z_2 + z_1x_3 - z_2x_3 + z_1 - z_2 + x_3 + 1 \quad 8/8(100\%) \quad (63)$$

$$z_1z_2x_3 : \quad (64)$$

$$\longrightarrow z_1z_2 + z_1x_3 + z_2x_3 - z_1 - z_2 - x_3 + 1 \quad 7/8 \text{ (88\%)} \quad (65)$$

$$\longrightarrow z_1 - z_2x_3 + 1 \quad 8/8(100\%) \quad (66)$$

$$z_1z_2x_3 : \quad (67)$$

$$\longrightarrow z_1z_2 + z_1x_3 + z_2x_3 - z_1 - z_2 - x_3 + 1 \quad 7/8 \text{ (88\%)} \quad (68)$$

$$\longrightarrow x_3 - z_1z_2 + 1 \quad 8/8(100\%) \quad (69)$$

$$z_1z_2x_3 : \quad (70)$$

$$\longrightarrow z_1z_2 - z_1x_3 - z_2x_3 + z_1 + z_2 - x_3 + 1 \quad 7/8 \text{ (88\%)} \quad (71)$$

$$\longrightarrow - z_1 + z_2x_3 + 1 \quad 8/8(100\%) \quad (72)$$

$$z_1z_2x_3 : \quad (73)$$

$$\longrightarrow z_1z_2 - z_1x_3 - z_2x_3 + z_1 + z_2 - x_3 + 1 \quad 7/8 \text{ (88\%)} \quad (74)$$

$$\longrightarrow x_3 - z_1z_2 + 1 \quad 8/8(100\%) \quad (75)$$

$$z_1 z_2 x_3 : \quad (76)$$

$$\longrightarrow -z_1 z_2 + z_1 x_3 - z_2 x_3 + z_1 - z_2 + x_3 + 1 \quad 7/8 \text{ (88\%)} \quad (77)$$

$$\longrightarrow -z_1 + z_2 x_3 + 1 \quad 8/8(100\%) \quad (78)$$

$$z_1 z_2 x_3 : \quad (79)$$

$$\longrightarrow -z_1 z_2 + z_1 x_3 - z_2 x_3 + z_1 - z_2 + x_3 + 1 \quad 7/8 \text{ (88\%)} \quad (80)$$

$$\longrightarrow z_2 - z_1 x_3 + 1 \quad 8/8(100\%) \quad (81)$$

$$z_1 z_2 x_3 : \quad (82)$$

$$\longrightarrow -z_1 z_2 + z_1 x_3 - z_2 x_3 + z_1 - z_2 + x_3 + 1 \quad 7/8 \text{ (88\%)} \quad (83)$$

$$\longrightarrow -x_3 + z_1 z_2 + 1 \quad 8/8(100\%) \quad (84)$$

$$z_1 z_2 x_3 : \quad (85)$$

$$\longrightarrow +z_1 z_2 - x_3 + 1 \quad 6/8 \text{ (75\%)} \quad (86)$$

$$\longrightarrow -z_1 z_2 + x_3 + 1 \quad 8/8(100\%) \quad (87)$$

$$z_1 z_2 x_3 : \quad (88)$$

$$\longrightarrow +z_1 x_3 - z_2 + 1 \quad 6/8 \text{ (75\%)} \quad (89)$$

$$\longrightarrow -z_1 x_3 + z_2 + 1 \quad 8/8(100\%) \quad (90)$$

BINOMIALS OF DEGREE- k TERMS

$$b_1 b_2 b_3 b_4 + b_3 b_4 b_5 b_6 = \min(2b_3 b_4, b_1 b_2 + b_5 b_6) \quad (k, n) = (4, 6). \quad (91)$$

$$b_1 b_2 b_3 b_4 + b_3 b_4 b_5 b_6 = \min_{b_a} (b_2 b_3 + b_a(1 - b_2 - b_3 + 2b_4) + b_3 b_4, b_1 b_2 + b_5 b_6 + b_5 b_a) \quad (k, n) = (4, 6). \quad (92)$$

$$b_1 b_2 b_3 b_4 + b_4 b_5 b_6 b_7 : \quad (k, n) = (4, 7). \quad (93)$$

$$\longrightarrow b_3 b_4 + b_4 b_6 + b_a(b_5 + b_7) \quad 89/128 \text{ (70\%)} \quad (94)$$

$$\longrightarrow b_1 b_2 + b_5 b_7 + b_a(1 - b_5 + b_6 - b_7) \quad 125/128 \text{ (98\%)} \quad (95)$$

$$\longrightarrow b_5 b_7 + b_3 \quad 128/128(100\%) \quad (96)$$

$$b_1 b_2 b_3 b_4 + b_4 b_5 b_6 b_7 : \quad (k, n) = (4, 7). \quad (97)$$

$$\longrightarrow b_3 b_4 + b_4 b_6 \quad 89/128 \text{ (70\%)} \quad (98)$$

$$\longrightarrow b_1 b_2 + b_6 b_7 \quad 118/128 \text{ (92\%)} \quad (99)$$

$$\longrightarrow b_2 b_3 - b_5 b_6 + b_5 b_7 + b_5 \quad 127/128 \text{ (99\%)} \quad (100)$$

$$\longrightarrow b_1 b_4 + 2b_5 - b_7 + 1 \quad 128/128(100\%) \quad (101)$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 + b_3 b_4 b_5 b_6 b_7 : & (k, n) = (5, 7). \quad (102) \\
\longrightarrow & b_1 b_5 + b_5 b_6 + b_5 b_7 + b_6 b_7 + b_a(1 - b_5 - 2b_6 - b_7) + b_6 & 188/256 \quad (73\%) \quad (103) \\
\longrightarrow & b_3 b_4 + b_a(b_4 - b_6) + b_6 & 236/256 \quad (92\%) \quad (104) \\
\longrightarrow & b_2 b_3 + b_3 b_6 - b_4 b_6 + b_6 b_a + b_6 & 254/256 \quad (99\%) \quad (105) \\
\longrightarrow & b_2 b_5 + b_5 b_7 & 256/256(100\%) \quad (106)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 + b_3 b_4 b_5 b_6 b_7 : & (k, n) = (5, 7). \quad (107) \\
\longrightarrow & b_2 b_3 + b_3 b_7 & 85/128 \quad (66\%) \quad (108) \\
\longrightarrow & 2b_4 b_5 & 121/128 \quad (95\%) \quad (109) \\
\longrightarrow & b_1 b_2 + b_6 b_7 - b_5 + 1 & 128/128(100\%) \quad (110)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_2 b_3 b_4 b_5 b_6 b_7 : & (k, n) = (6, 7). \quad (111) \\
\longrightarrow & b_5 b_6 + b_6 b_7 + b_a(1 + b_5 - b_6 - b_7) & 196/256 \quad (77\%) \quad (112) \\
\longrightarrow & b_1 b_4 + b_2 b_4 + b_7 b_a & 238/256 \quad (93\%) \quad (113) \\
\longrightarrow & b_1 b_3 + b_3 b_7 - b_4 b_6 + 2b_5 b_a - b_6 b_7 - b_5 + b_6 + b_7 + b_a + 1 & 252/256 \quad (98\%) \quad (114) \\
\longrightarrow & b_2 b_6 + b_2 - b_6 + 1 & 256/256(100\%) \quad (115)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_2 b_3 b_4 b_5 b_6 b_7 : & (k, n) = (6, 7). \quad (116) \\
\longrightarrow & 2b_5 b_6 & 97/128 \quad (76\%) \quad (117) \\
\longrightarrow & b_1 b_4 + b_4 b_7 & 119/128 \quad (93\%) \quad (118) \\
\longrightarrow & b_1 b_3 + b_1 b_7 + b_2 b_3 - b_3 b_6 + b_3 b_7 - b_4 b_5 - b_1 - b_7 + 2 & 127/128 \quad (99\%) \quad (119) \\
\longrightarrow & b_1 b_2 + b_2 b_6 & 128/128(100\%) \quad (120)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 + b_4 b_5 b_6 b_7 b_8 : & (k, n) = (5, 8). \quad (121) \\
\longrightarrow & b_3 b_5 + b_7 b_8 + b_a(-1 - b_6 + b_7 + b_8) + b_6 - b_7 - b_8 + 1 & 360/512 \quad (70\%) \quad (122) \\
\longrightarrow & b_1 b_4 + b_4 b_8 + b_a(b_4 + b_6) & 468/512 \quad (91\%) \quad (123) \\
\longrightarrow & b_1 b_2 + b_7 b_8 + b_a(1 + b_6 - b_7 - b_8) & 496/512 \quad (97\%) \quad (124) \\
\longrightarrow & b_3 b_5 + b_5 & 512/512(100\%) \quad (125)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 + b_4 b_5 b_6 b_7 b_8 : & (k, n) = (5, 8). \quad (126) \\
\longrightarrow & b_2 b_5 + b_5 b_8 & 169/256 \quad (66\%) \quad (127) \\
\longrightarrow & b_1 b_4 + b_4 b_7 - b_5 b_8 + b_8 & 233/256 \quad (91\%) \quad (128) \\
\longrightarrow & b_1 b_3 + b_6 b_7 + b_6 b_8 + b_7 b_8 - b_6 - b_7 - b_8 + 1 & 252/256 \quad (98\%) \quad (129) \\
\longrightarrow & b_2 b_3 + b_6 b_7 & 256/256(100\%) \quad (130)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_3 b_4 b_5 b_6 b_7 b_8 : & (k, n) = (6, 8). \quad (131) \\
\longrightarrow & b_1 b_6 + b_7 b_8 + b_a(1 + b_6 - b_7 - b_8) & 364/512 \quad (71\%) \quad (132) \\
\longrightarrow & b_2 b_3 + b_5 b_8 - b_6 b_8 + b_7 b_a - b_7 + b_8 - b_a + 1 & 450/512 \quad (88\%) \quad (133) \\
\longrightarrow & b_1 b_4 + b_4 & 488/512 \quad (95\%) \quad (134) \\
\longrightarrow & b_2 b_3 + b_3 b_7 - b_6 b_8 + b_8 - b_a + 1 & 502/512 \quad (98\%) \quad (135) \\
\longrightarrow & b_2 b_5 + b_5 & 512/512(100\%) \quad (136)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_3 b_4 b_5 b_6 b_7 b_8 : & (k, n) = (6, 8). \quad (137) \\
& \longrightarrow 2b_5 b_6 & 193/256 \quad (75\%) \quad (138) \\
& \longrightarrow b_1 b_4 + b_4 b_8 & 237/256 \quad (93\%) \quad (139) \\
& \longrightarrow b_2 b_3 + b_3 b_7 - b_4 b_6 + b_4 b_8 - b_5 b_7 - b_5 b_8 + b_6 b_8 - b_6 + b_7 - b_8 + 2 & 254/256 \quad (99\%) \quad (140) \\
& \longrightarrow b_1 b_2 + b_7 b_8 & 256/256(100\%) \quad (141)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 + b_2 b_3 b_4 b_5 b_6 b_7 b_8 : & (k, n) = (7, 8). \quad (142) \\
& \longrightarrow b_6 b_7 + b_6 b_8 + b_a(1 - b_6 + b_7 - b_8) & 388/512 \quad (76\%) \quad (143) \\
& \longrightarrow b_1 b_3 + b_3 b_8 + b_a(1 + b_8) & 470/512 \quad (92\%) \quad (144) \\
& \longrightarrow b_2 b_4 - b_3 b_8 + b_4 b_5 + b_a(1 - b_7) + b_8 & 500/512 \quad (98\%) \quad (145) \\
& \longrightarrow b_2 b_5 + b_2 b_8 - b_4 b_8 - b_6 b_7 + b_6 b_8 + b_a(-1 - b_4 - b_7 + b_8) - b_3 + b_7 - b_8 + 4 & 508/512 \quad (99\%) \quad (146) \\
& \longrightarrow b_2 b_5 - b_7 b_8 + b_5 + 1 & 512/512(100\%) \quad (147)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 + b_2 b_3 b_4 b_5 b_6 b_7 b_8 : & (k, n) = (7, 8). \quad (148) \\
& \longrightarrow 2b_5 b_6 & 193/256 \quad (75\%) \quad (149) \\
& \longrightarrow b_1 b_4 + b_4 b_8 & 235/256 \quad (92\%) \quad (150) \\
& \longrightarrow b_2 b_3 + b_2 b_7 - b_5 b_6 + b_6 b_8 + b_5 - b_6 - b_8 + 1 & 250/256 \quad (98\%) \quad (151) \\
& \longrightarrow b_3 b_7 + b_7 b_8 & 254/256 \quad (99\%) \quad (152) \\
& \longrightarrow b_3 b_8 + b_3 & 256/256(100\%) \quad (153)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + b_5 b_6 b_7 b_8 : & (k, n) = (4, 8). \quad (154) \\
& \longrightarrow b_2 b_3 + b_6 b_8 + b_a(1 - b_6 + b_7 - b_8) & 390/512 \quad (76\%) \quad (155) \\
& \longrightarrow b_1 b_4 + b_6 b_8 + b_a(1 - b_6 + b_7 - b_8) & 480/512 \quad (94\%) \quad (156) \\
& \longrightarrow b_2 b_4 + b_5 - b_a + 1 & 506/512 \quad (99\%) \quad (157) \\
& \longrightarrow b_1 b_3 - b_6 b_a + b_5 + 1 & 512/512(100\%) \quad (158)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + b_5 b_6 b_7 b_8 : & (k, n) = (4, 8). \quad (159) \\
& \longrightarrow b_1 b_2 + b_6 b_7 & 169/256 \quad (66\%) \quad (160) \\
& \longrightarrow b_3 b_4 + b_5 b_8 & 238/256 \quad (93\%) \quad (161) \\
& \longrightarrow b_1 b_4 + b_5 b_6 + b_5 b_7 + b_6 b_7 - b_5 - b_6 - b_7 + 1 & 248/256 \quad (97\%) \quad (162) \\
& \longrightarrow b_2 b_3 + b_6 b_7 + b_6 b_8 + b_7 b_8 - b_6 - b_7 - b_8 + 1 & 254/256 \quad (99\%) \quad (163) \\
& \longrightarrow b_1 b_2 + b_5 b_8 & 256/256(100\%) \quad (164)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 + b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (5, 10). \quad (165) \\
& \longrightarrow b_1 b_4 + b_7 b_9 & 625/1024 \quad (61\%) \quad (166) \\
& \longrightarrow b_3 b_5 + b_6 b_8 & 889/1024 \quad (87\%) \quad (167) \\
& \longrightarrow b_2 b_5 + b_7 b_{10} & 972/1024 \quad (95\%) \quad (168) \\
& \longrightarrow b_2 b_4 + b_6 b_8 & 999/1024 \quad (98\%) \quad (169) \\
& \longrightarrow b_1 b_3 + b_9 b_{10} + b_9 b_a & 1016/1024 \quad (99\%) \quad (170) \\
& \longrightarrow b_1 b_5 + b_6 b_9 & 1020/1024 \quad (99\%) \quad (171) \\
& \longrightarrow b_1 b_4 + b_8 b_{10} & 1022/1024 \quad (99\%) \quad (172) \\
& \longrightarrow b_2 b_3 - b_4 b_{10} + b_7 b_9 + b_9 b_a + 1 & 1024/1024(100\%) \quad (173)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 + b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (5, 10). \quad (174) \\
\longrightarrow & b_1 b_3 + b_9 b_{10} & 625/1024 \quad (61\%) \quad (175) \\
\longrightarrow & b_2 b_4 + b_7 b_{10} & 851/1024 \quad (83\%) \quad (176) \\
\longrightarrow & b_3 b_5 + b_5 b_{10} + b_8 b_9 & 924/1024 \quad (90\%) \quad (177) \\
\longrightarrow & b_1 b_2 + b_6 & 972/1024 \quad (95\%) \quad (178) \\
\longrightarrow & b_3 b_4 + b_8 b_9 & 997/1024 \quad (97\%) \quad (179) \\
\longrightarrow & b_1 b_5 + b_7 b_{10} & 1010/1024 \quad (99\%) \quad (180) \\
\longrightarrow & b_2 b_3 - b_1 b_7 - b_1 b_{10} - b_2 b_8 - b_2 b_{10} + b_3 b_5 + b_6 b_9 + b_7 b_{10} - b_8 b_9 + b_9 b_{10} - b_3 - b_7 + b_8 + 3 & 1016/1024 \quad (99\%) \quad (181) \\
\longrightarrow & b_1 b_3 + b_7 b_8 & 1020/1024 \quad (99\%) \quad (182) \\
\longrightarrow & b_2 b_4 + b_2 b_6 - b_2 b_9 - b_3 b_{10} - b_5 b_7 + b_7 b_{10} + b_9 b_{10} - b_{10} + 2 & 1023/1024 \quad (99\%) \quad (183) \\
\longrightarrow & b_2 b_5 + b_2 b_9 + b_6 b_8 & 1024/1024(100\%) \quad (184)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (6, 10). \quad (185) \\
\longrightarrow & b_4 b_5 + b_5 b_9 & 657/1024 \quad (64\%) \quad (186) \\
\longrightarrow & b_2 b_6 + b_6 b_8 & 905/1024 \quad (88\%) \quad (187) \\
\longrightarrow & b_1 b_3 + b_7 b_8 & 982/1024 \quad (96\%) \quad (188) \\
\longrightarrow & b_2 b_3 + b_a(b_{10} - b_9) + b_9 & 1011/1024 \quad (99\%) \quad (189) \\
\longrightarrow & b_2 b_4 + b_7 b_{10} & 1020/1024 \quad (99\%) \quad (190) \\
\longrightarrow & b_9 b_{10} + b_1 & 1023/1024 \quad (99\%) \quad (191) \\
\longrightarrow & b_7 b_8 + b_4 & 1024/1024(100\%) \quad (192)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (6, 10). \quad (193) \\
\longrightarrow & 2b_5 b_6 & 769/1024 \quad (75\%) \quad (194) \\
\longrightarrow & b_1 b_3 + b_8 b_9 & 934/1024 \quad (92\%) \quad (195) \\
\longrightarrow & b_2 b_4 + b_7 b_{10} + b_8 b_9 - b_8 - b_9 + 1 & 997/1024 \quad (97\%) \quad (196) \\
\longrightarrow & -b_1 b_3 + b_1 b_9 + b_2 b_4 + b_4 b_9 + b_5 b_8 + b_8 b_9 - b_5 - b_8 - b_9 + 2 & 769/1024 \quad (99\%) \quad (197) \\
\longrightarrow & b_1 b_3 + b_7 b_{10} - b_8 - b_9 + 2 & 1014/1024 \quad (99\%) \quad (198) \\
\longrightarrow & b_2 b_3 + b_8 b_9 & 1024/1024(100\%) \quad (199)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 + b_4 b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (7, 10). \quad (200) \\
\longrightarrow & b_3 b_5 + b_5 b_8 & 649/1024 \quad (63\%) \quad (201) \\
\longrightarrow & b_2 b_4 + b_4 b_9 & 893/1024 \quad (87\%) \quad (202) \\
\longrightarrow & b_1 b_7 + b_7 b_{10} & 985/1024 \quad (96\%) \quad (203) \\
\longrightarrow & b_1 b_6 + b_6 b_9 + b_a & 1015/1024 \quad (99\%) \quad (204) \\
\longrightarrow & b_2 b_3 + b_8 b_{10} + b_a & 1022/1024 \quad (99\%) \quad (205) \\
\longrightarrow & b_1 b_3 + b_8 b_9 & 1024/1024(100\%) \quad (206)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 + b_4 b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (7, 10). \quad (207) \\
\longrightarrow & b_3 b_7 + b_7 b_{10} & 649/1024 \quad (63\%) \quad (208) \\
\longrightarrow & 2b_4 b_6 & 937/1024 \quad (92\%) \quad (209) \\
\longrightarrow & b_1 b_5 + b_5 b_8 & 1001/1024 \quad (98\%) \quad (210) \\
\longrightarrow & b_1 b_2 + b_9 b_{10} & 1019/1024 \quad (99\%) \quad (211) \\
\longrightarrow & b_2 b_3 + b_8 & 1023/1024 \quad (99\%) \quad (212) \\
\longrightarrow & b_3 b_7 + b_9 b_{10} & 1024/1024(100\%) \quad (213)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5b_6b_7b_8 + b_3b_4b_5b_6b_7b_8b_9b_{10} : & (k, n) = (8, 10). \quad (214) \\
& \longrightarrow b_2b_8 + b_8b_9 & 645/1024 \quad (63\%) \quad (215) \\
& \longrightarrow b_1b_3 + b_3b_{10} + b_9b_a & 887/1024 \quad (87\%) \quad (216) \\
& \longrightarrow b_4b_6 + b_5b_6 & 977/1024 \quad (95\%) \quad (217) \\
& \longrightarrow b_2b_7 + b_7b_{10} & 1007/1024 \quad (98\%) \quad (218) \\
& \longrightarrow b_1b_4 + b_4b_5 + 2b_9b_a & 1018/1024 \quad (99\%) \quad (219) \\
& \longrightarrow b_1b_5 + b_5b_9 & 1024/1024(100\%) \quad (220)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5b_6b_7b_8 + b_3b_4b_5b_6b_7b_8b_9b_{10} : & (k, n) = (8, 10). \quad (221) \\
& \longrightarrow 4b_3b_7 & 768/1024 \quad (75\%) \quad (222) \\
& \longrightarrow b_2b_8 + b_8b_9 & 933/1024 \quad (91\%) \quad (223) \\
& \longrightarrow 2b_4b_6 + b_8b_9 - b_8 - b_9 + 1 & 1005/1024 \quad (98\%) \quad (224) \\
& \longrightarrow b_1b_5 + b_5b_{10} + b_8b_9 - b_8 - b_9 + 1 & 1022/1024 \quad (99\%) \quad (225) \\
& \longrightarrow b_1b_2 + b_8b_9 + b_9b_{10} - b_8 - b_9 + 1 & 1024/1024(100\%) \quad (226)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5b_6b_7b_8b_9 + b_2b_3b_4b_5b_6b_7b_8b_9b_{10} : & (k, n) = (9, 10). \quad (227) \\
& \longrightarrow b_1b_9 + b_9b_{10} + b_{10}b_a & 643/1024 \quad (63\%) \quad (228) \\
& \longrightarrow b_2b_4 + b_4b_5 & 883/1024 \quad (86\%) \quad (229) \\
& \longrightarrow b_3b_7 + b_3b_8 & 973/1024 \quad (95\%) \quad (230) \\
& \longrightarrow b_2b_6 + b_6b_8 & 1003/1024 \quad (98\%) \quad (231) \\
& \longrightarrow b_2b_5 + b_5b_7 - b_{10}b_a + b_{10} & 1015/1024 \quad (99\%) \quad (232) \\
& \longrightarrow b_1b_8 + b_7b_8 & 1019/1024 \quad (99\%) \quad (233) \\
& \longrightarrow b_2b_7 + b_2b_{10} - b_4b_5 - b_{10}b_a + b_{10} + 1 & 1023/1024 \quad (99\%) \quad (234) \\
& \longrightarrow b_4b_7 + b_7 & 1024/1024(100\%) \quad (235)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5b_6b_7b_8b_9 + b_2b_3b_4b_5b_6b_7b_8b_9b_{10} : & (k, n) = (9, 10). \quad (236) \\
& \longrightarrow 2b_2b_3 - b_8b_9 + b_9 & 577/1024 \quad (56\%) \quad (237) \\
& \longrightarrow 3b_8b_9 & 961/1024 \quad (94\%) \quad (238) \\
& \longrightarrow 2b_4b_6 - b_8b_9 - b_8b_{10} + b_{10} + 1 & 1009/1024 \quad (99\%) \quad (239) \\
& \longrightarrow 2b_5b_7 - b_8b_{10} + b_{10} & 1021/1024 \quad (99\%) \quad (240) \\
& \longrightarrow b_1b_6 + b_{10} & 1024/1024(100\%) \quad (241)
\end{aligned}$$

DEGREE- k , EXACT- k -OF- n TRINOMIALS

$$\begin{aligned}
& b_1b_2b_3b_4 + b_2b_3b_4b_5 + b_3b_4b_5b_6 : & (k, n) = (4, 6). \quad (242) \\
& \longrightarrow b_1b_4 + 2b_4b_5 + b_7 & 44/64 \quad (69\%) \quad (243) \\
& \longrightarrow b_1b_3 + b_2b_3 + b_3b_6 + b_6b_7 & 60/64 \quad (94\%) \quad (244) \\
& \longrightarrow b_2b_4 + b_5b_6 + b_2 & 64/64(100\%) \quad (245)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4 + b_2b_3b_4b_5 + b_3b_4b_5b_6 : & (k, n) = (4, 6). \quad (246) \\
& \longrightarrow b_2b_4 + 2b_4b_5 & 43/64 \quad (67\%) \quad (247) \\
& \longrightarrow b_1b_3 + b_2b_3 + b_2b_5 + b_3b_6 - b_4b_5 - b_2 + 1 & 60/64 \quad (94\%) \quad (248) \\
& \longrightarrow b_1b_2 + b_2b_5 + b_5b_6 & 64/64(100\%) \quad (249)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4 + b_3b_4b_5b_6 + b_5b_6b_7b_8 : & (k, n) = (4, 8). \quad (250) \\
\longrightarrow & b_1b_4 + 2b_5b_6 & 159/256 \text{ (62\%)} \quad (251) \\
\longrightarrow & b_2b_3 + b_3b_5 + b_7b_8 & 225/256 \text{ (88\%)} \quad (252) \\
\longrightarrow & b_1b_4 + b_3b_4 - b_5b_7 + b_6b_7 + b_7b_8 - b_6 + 1 & 244/256 \text{ (95.3\%)} \quad (253) \\
\longrightarrow & b_2b_3 + b_6b_8 + b_6 & 253/256 \text{ (98.8\%)} \quad (254) \\
\longrightarrow & b_2b_3 + b_5b_7 + b_5 & 256/256 \text{ (100\%)} \quad (255)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4 + b_3b_4b_5b_6 + b_5b_6b_7b_8 : & (k, n) = (4, 8). \quad (256) \\
\longrightarrow & b_2b_4 + 2b_5b_6 & 159/256 \text{ (62\%)} \quad (257) \\
\longrightarrow & b_3b_6 + b_7b_8 + b_3 & 212/256 \text{ (83\%)} \quad (258) \\
\longrightarrow & b_2b_4 - b_5b_7 + b_7b_8 + b_4 + b_7 & 234/256 \text{ (91\%)} \quad (259) \\
\longrightarrow & b_1b_3 + 2b_5b_6 & 253/256 \text{ (99\%)} \quad (260) \\
\longrightarrow & b_7b_8 + b_1 + b_6 & 256/256 \text{ (100\%)} \quad (261)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5 + b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7 : & (k, n) = (5, 7). \quad (262) \\
\longrightarrow & b_1b_5 + b_5b_6 + b_6b_7 + b_a(-2 - b_5 + 2b_6 + b_7) + b_5 - 2b_6 - b_7 + 2 & 86/128 \text{ (67\%)} \quad (263) \\
\longrightarrow & b_1b_3 + b_3b_4 - b_3b_6 + b_3b_7 + b_a(b_5 + b_7) + b_3 & 112/128 \text{ (88\%)} \quad (264) \\
\longrightarrow & b_1b_4 + b_2b_4 + b_4b_7 + b_5b_7 + b_a(-1 - b_6 - b_7) - b_5 + b_6 + 2 & 124/128 \text{ (97\%)} \quad (265) \\
\longrightarrow & b_2b_4 - 2b_5b_a + b_6b_7 + b_2 + b_5 + 1 & 128/128 \text{ (100\%)} \quad (266)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5 + b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7 : & (k, n) = (5, 7). \quad (267) \\
\longrightarrow & 2b_4b_5 + b_4b_6 & 81/128 \text{ (63\%)} \quad (268) \\
\longrightarrow & b_1b_3 + b_3b_6 + b_3b_7 - b_4b_5 + b_5 & 111/128 \text{ (87\%)} \quad (269) \\
\longrightarrow & b_1b_2 + b_2b_6 - b_4b_5 + b_6b_7 + b_5 & 122/128 \text{ (95\%)} \quad (270) \\
\longrightarrow & 2b_4b_5 + b_5 & 128/128 \text{ (100\%)} \quad (271)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5b_6 + b_2b_3b_4b_5b_6b_7 + b_3b_4b_5b_6b_7b_8 : & (k, n) = (6, 8). \quad (272) \\
\longrightarrow & b_1b_6 + 2b_6b_7 & 164/256 \text{ (64\%)} \quad (273) \\
\longrightarrow & b_1b_5 + b_2b_5 - b_3b_6 + b_5b_8 + b_3 & 219/256 \text{ (86\%)} \quad (274) \\
\longrightarrow & b_2b_4 + b_4b_7 + b_4b_8 - b_6 + 1 & 243/256 \text{ (95\%)} \quad (275) \\
\longrightarrow & b_2b_3 + b_3b_8 - b_5b_6 + b_3 + b_6 & 253/256 \text{ (99\%)} \quad (276) \\
\longrightarrow & b_1b_2 + b_2b_6 + b_5b_7 - b_6b_7 + b_7b_8 - b_5 + 1 & 256/256 \text{ (100\%)} \quad (277)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5b_6b_7b_8 + b_2b_3b_4b_5b_6b_7b_8b_9 + b_3b_4b_5b_6b_7b_8b_9b_{10} : & (k, n) = (8, 10). \quad (278) \\
\longrightarrow & 3b_5b_8 & 769/1024 \text{ (75\%)} \quad (279) \\
\longrightarrow & 2b_2b_6 + b_4b_6 & 931/1024 \text{ (91\%)} \quad (280) \\
\longrightarrow & b_1b_7 - b_5b_{10} + b_7b_9 + b_9b_{10} - b_6 + b_{10} + 1 & 984/1024 \text{ (96\%)} \quad (281) \\
\longrightarrow & 3b_2b_3 + b_3b_{10} - b_6b_8 + 1 & 1011/1024 \text{ (99\%)} \quad (282) \\
\longrightarrow & b_4b_7 + b_4b_8 - b_3 + b_4 - b_8 + 2 & 1019/1024 \text{ (99\%)} \quad (283) \\
\longrightarrow & b_2b_3 - b_2b_4 - b_3b_4 - b_3b_8 - b_5b_{10} - b_6b_9 + b_7b_8 + b_7b_9 + b_8b_9 + b_7 + 3 & 1023/1024 \text{ (99\%)} \quad (284) \\
\longrightarrow & b_2b_8 + 2b_8b_9 & 1024/1024 \text{ (100\%)} \quad (285)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_3 b_4 b_5 b_6 b_7 b_8 + b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (6, 10). \quad (286) \\
\longrightarrow & b_5 b_6 + b_5 b_8 + b_5 b_9 + b_8 b_{11} + b_9 b_{11} + b_{10} b_{11} & 583/1024 \quad (57\%) \quad (287) \\
\longrightarrow & b_1 b_2 + b_4 b_7 + b_7 b_{10} + b_9 b_{11} - b_9 - b_{11} + 1 & 815/1024 \quad (80\%) \quad (288) \\
\longrightarrow & b_1 b_6 + b_5 b_6 + b_6 - b_{11} + 1 & 917/1024 \quad (90\%) \quad (289) \\
\longrightarrow & b_3 b_4 + b_3 b_7 + b_8 b_9 + b_9 b_{11} & 979/1024 \quad (96\%) \quad (290) \\
\longrightarrow & b_2 b_4 + b_4 b_8 + b_8 b_9 - b_9 b_{11} + b_9 - b_{11} + 1 & 1007/1024 \quad (98\%) \quad (291) \\
\longrightarrow & b_1 b_3 + b_7 b_{10} + b_{10} b_{11} + b_3 & 1016/1024 \quad (99\%) \quad (292) \\
\longrightarrow & b_1 b_4 + b_4 b_8 + b_7 b_{10} + b_9 b_{11} + b_{10} b_{11} - b_9 - b_{11} + 1 & 1021/1024 \quad (99\%) \quad (293) \\
\longrightarrow & b_1 b_3 - b_2 b_{11} + b_7 b_8 + b_8 b_9 - b_{10} b_{11} - b_{11} + 3 & 1024/1024(100\%) \quad (294)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_3 b_4 b_5 b_6 b_7 b_8 + b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (6, 10). \quad (295) \\
\longrightarrow & 2b_3 b_4 + b_7 b_{10} & 591/1024 \quad (58\%) \quad (296) \\
\longrightarrow & 2b_3 b_5 + b_5 b_6 & 847/1024 \quad (83\%) \quad (297) \\
\longrightarrow & b_1 b_2 + b_7 b_8 + b_8 b_9 & 951/1024 \quad (93\%) \quad (298) \\
\longrightarrow & 3b_5 b_6 & 995/1024 \quad (97\%) \quad (299) \\
\longrightarrow & b_1 b_3 + b_3 b_4 + b_9 b_{10} & 1009/1024 \quad (99\%) \quad (300) \\
\longrightarrow & b_1 b_2 + b_5 b_7 + b_7 b_{10} & 1018/1024 \quad (99\%) \quad (301) \\
\longrightarrow & 2b_1 b_4 - b_1 b_{10} + b_2 b_4 + b_4 b_5 + b_4 b_{10} + b_5 b_8 - b_6 b_8 + b_8 b_9 + b_7(b_{10} - b_6 - b_5 - b_1) + 3 & 1023/1024 \quad (99\%) \quad (302) \\
\longrightarrow & b_2 b_8 + b_3 b_6 + b_6 b_8 & 1024/1024(100\%) \quad (303)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + b_4 b_5 b_6 b_7 + b_7 b_8 b_9 b_{10} : & (k, n) = (4, 10). \quad (304) \\
\longrightarrow & b_2 b_3 + b_6 b_7 + b_7 b_9 + 2b_9 b_a & 581/1024 \quad (57\%) \quad (305) \\
\longrightarrow & b_2 b_4 + b_4 b_6 + b_a(b_9 - b_{10}) + b_{10} & 823/1024 \quad (80\%) \quad (306) \\
\longrightarrow & b_1 b_3 + b_5 b_6 + b_8 b_9 + b_a(b_9 - b_{10}) - b_9 + 1 & 930/1024 \quad (91\%) \quad (307) \\
\longrightarrow & b_1 b_4 + b_4 b_5 + b_4 b_{10} + b_8 b_{10} + b_a(1 - b_7 + b_9) & 978/1024 \quad (96\%) \quad (308) \\
\longrightarrow & b_1 b_4 + b_7 b_8 + b_a(1 + b_9) + b_7 & 1000/1024 \quad (98\%) \quad (309) \\
\longrightarrow & b_2 b_3 + b_a(b_9 - b_{10}) + b_5 + b_{10} & 1015/1024 \quad (99\%) \quad (310) \\
\longrightarrow & b_1 b_3 + b_6 + b_{10} & 1020/1024 \quad (99\%) \quad (311) \\
\longrightarrow & b_5 b_6 + b_2 + b_8 + b_a & 1024/1024(100\%) \quad (312)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + b_4 b_5 b_6 b_7 + b_7 b_8 b_9 b_{10} : & (k, n) = (4, 10). \quad (313) \\
\longrightarrow & b_3 b_4 + b_4 b_6 + b_9 b_{10} & 581/1024 \quad (57\%) \quad (314) \\
\longrightarrow & b_1 b_2 + b_5 b_7 - b_8 b_9 + b_9 b_{10} + b_9 & 759/1024 \quad (74\%) \quad (315) \\
\longrightarrow & b_5 b_6 + b_8 b_9 + b_1 + b_8 & 842/1024 \quad (82\%) \quad (316) \\
\longrightarrow & b_2 b_4 + b_7 b_{10} - b_8 b_9 + b_7 + b_8 & 935/1024 \quad (91\%) \quad (317) \\
\longrightarrow & b_2 b_4 + b_4 b_6 - b_8 b_9 + b_8 b_{10} - b_7 + b_8 + 1 & 969/1024 \quad (95\%) \quad (318) \\
\longrightarrow & b_1 b_3 + b_3 b_4 + b_5 b_7 + b_7 b_9 - b_8 b_9 + b_9 & 992/1024 \quad (97\%) \quad (319) \\
\longrightarrow & b_2 b_3 + b_3 b_5 + b_3 b_{10} + b_4 b_8 + b_5 b_6 - b_4 + 1 & 1004/1024 \quad (98\%) \quad (320) \\
\longrightarrow & b_1 b_3 + b_6 b_7 + b_9 b_{10} & 1013/1024 \quad (99\%) \quad (321) \\
\longrightarrow & b_1 b_9 + b_7 b_8 - b_8 b_9 - b_9 b_{10} + b_1 + b_7 + b_8 + b_9 & 1019/1024 \quad (99\%) \quad (322) \\
\longrightarrow & b_2 b_3 + b_5 b_6 - b_8 b_9 + b_9 b_{10} + b_9 & 1022/1024 \quad (99\%) \quad (323) \\
\longrightarrow & -b_1 b_5 + b_1 b_8 + b_3 b_7 + b_3 + b_7 + 1 & 1023/1024 \quad (99\%) \quad (324) \\
\longrightarrow & b_1 b_5 - b_1 b_{10} + b_2 + b_8 - b_{10} + 2 & 1024/1024(100\%) \quad (325)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5 + b_3b_4b_5b_6 + b_4b_5b_6b_7b_8 : & (k, n) = (5, 8). \quad (326) \\
\longrightarrow & b_1b_3 + b_3b_4 + b_6b_8 - b_6b_9 + b_7b_9 - b_8b_9 + b_9 & 156/256 \text{ (61\%)} \quad (327) \\
\longrightarrow & b_1b_5 + b_5b_7 + b_7b_9 + b_8b_9 + b_5 - b_7 - b_9 + 1 & 202/256 \text{ (79\%)} \quad (328) \\
\longrightarrow & b_2b_4 + b_6b_8 + b_6b_9 - b_7b_9 + b_8b_9 + b_7 - b_8 - b_9 + 1 & 230/256 \text{ (90\%)} \quad (329) \\
\longrightarrow & b_2b_4 + b_4b_8 + b_4 - b_9 + 1 & 246/256 \text{ (96\%)} \quad (330) \\
\longrightarrow & b_1b_5 + 2b_6 & 252/256 \text{ (98\%)} \quad (331) \\
\longrightarrow & b_2b_5 + b_7b_8 + b_5 & 256/256(100\%) \quad (332)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5 + b_3b_4b_5b_6 + b_4b_5b_6b_7b_8 : & (k, n) = (5, 8). \quad (333) \\
\longrightarrow & b_4b_5 + 2b_5b_6 & 165/256 \text{ (64\%)} \quad (334) \\
\longrightarrow & b_2b_4 + b_3b_4 + b_4b_8 - b_5b_7 + b_7 & 215/256 \text{ (84\%)} \quad (335) \\
\longrightarrow & b_2b_3 + b_3b_6 - b_4b_5 - b_5b_7 + b_7b_8 + b_5 + b_7 & 242/256 \text{ (95\%)} \quad (336) \\
\longrightarrow & b_1b_3 + b_5b_6 + b_6b_7 & 254/256 \text{ (99\%)} \quad (337) \\
\longrightarrow & b_1b_2 + b_5b_6 + b_6b_8 & 256/256(100\%) \quad (338)
\end{aligned}$$

DEGREE- k , EXACT- k -OF- n QUADRINOMIALS

$$\begin{aligned}
& b_1b_2b_3 + b_1b_2b_4 + b_1b_3b_4 + b_2b_3b_4 : & (k, n) = (3, 4). \quad (339) \\
\longrightarrow & 2b_1b_2 + b_1b_3 + 2b_1b_4 + b_2b_3 + 2b_2b_4 + b_3b_4 - 2b_1 - 2b_2 - b_3 - 2b_4 + 3 & 13/16 \text{ (81\%)} \quad (340) \\
\longrightarrow & 2b_1b_3 + b_2b_3 + b_2 & 16/16(100\%) \quad (341)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5b_6b_7 + b_2b_3b_4b_5b_6b_7b_8 + b_3b_4b_5b_6b_7b_8b_9 + b_4b_5b_6b_7b_8b_9b_{10} : & (k, n) = (7, 10). \quad (342) \\
\longrightarrow & 4b_4b_5 & 769/1024 \text{ (75\%)} \quad (343) \\
\longrightarrow & b_2b_6 + 2b_3b_6 + b_6b_9 & 915/1024 \text{ (89\%)} \quad (344) \\
\longrightarrow & b_1b_7 + b_5b_7 + b_6b_7 + b_7b_{10} & 974/1024 \text{ (95\%)} \quad (345) \\
\longrightarrow & b_1b_2 + b_2b_8 + b_7b_8 + b_9b_{10} & 995/1024 \text{ (97\%)} \quad (346) \\
\longrightarrow & b_2b_3 + b_3b_4 + b_3b_6 + b_9b_{10} & 1008/1024 \text{ (98\%)} \quad (347) \\
\longrightarrow & b_1b_2 + b_2b_4 + b_9b_{10} + b_9 & 1016/1024 \text{ (99\%)} \quad (348) \\
\longrightarrow & b_1b_3 - b_2b_8 + b_7b_8 + b_8b_9 + b_8b_{10} + 2b_8 & 1023/1024 \text{ (99\%)} \quad (349) \\
\longrightarrow & b_1b_8 + b_2b_7 - b_5b_{10} + b_7b_8 + b_8b_9 - b_5 + 2 & 1024/1024(100\%) \quad (350)
\end{aligned}$$

DEGREE- k , NOT EXACT- k -OF- n MULTINOMIALS

$$\begin{aligned}
& b_1b_2b_3b_4 + 2b_1b_2b_3 + b_1b_2b_4 + b_1b_3b_4 + b_2b_3b_4 : & (k, n) = (4, 4). \quad (351) \\
\longrightarrow & b_1b_2 + 4b_1b_3 + b_1b_4 + b_2b_3 + b_2b_4 + b_3b_4 - b_1 - b_2 - b_3 - b_4 + 1 & 12/16 \text{ (75\%)} \quad (352) \\
\longrightarrow & b_1b_2 + b_1b_3 + 4b_1b_4 + b_2b_4 & 16/16(100\%) \quad (353)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4 + 2b_1b_2b_3 + b_1b_2b_4 + b_1b_3b_4 + b_2b_3b_4 : & (k, n) = (4, 4). \quad (354) \\
\longrightarrow & b_1b_2 + 4b_1b_3 + b_1b_4 + b_2b_3 + b_2b_4 + b_3b_4 - b_1 - b_2 - b_3 - b_4 + 1 & 12/16 \text{ (75\%)} \quad (355) \\
\longrightarrow & 2b_2b_3 + 3b_2b_4 + b_3b_4 & 16/16(100\%) \quad (356)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + 2b_1 b_2 b_3 + b_1 b_2 b_4 + 3b_1 b_3 b_4 + b_2 b_3 b_4 : & (k, n) = (4, 4). \quad (357) \\
\longrightarrow & 2b_1 b_2 + 5b_1 b_4 + b_3 b_4 & 11/16 \quad (69\%) \quad (358) \\
\longrightarrow & -b_1 b_2 + 3b_1 b_3 + 4b_2 b_3 + 2b_2 b_4 - 4b_3 b_4 + 4b_3 - b_4 + 1 & 16/16(100\%) \quad (359)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + 2b_1 b_2 b_3 + b_1 b_3 b_4 : & (k, n) = (4, 4). \quad (360) \\
\longrightarrow & 4b_1 b_3 & 13/16 \quad (81\%) \quad (361) \\
\longrightarrow & 2b_1 b_2 + b_1 b_4 + b_2 b_4 & 16/16(100\%) \quad (362)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + 2b_1 b_2 b_3 + b_1 b_3 b_4 : & (k, n) = (4, 4). \quad (363) \\
\longrightarrow & 2b_1 b_3 + 2b_3 b_4 & 12/16 \quad (75\%) \quad (364) \\
\longrightarrow & 3b_1 b_2 + b_1 b_4 & 16/16(100\%) \quad (365)
\end{aligned}$$