

Volume 3: List of Multi-run Quadratizations

Nike Dattani and Andreas Soteriou
(Dated: 22nd March 2020)

DECOMPOSITION OF A MONOMIAL

$$b_1 b_2 b_3 \dots b_k = \min (b_1 b_2 \dots b_{k_1}, b_{k_1+1} b_{k_1+2} \dots b_{k_2}, b_{k_2+1} b_{k_2+2} \dots b_{k_3}, \dots, b_{k_n+1} b_{k_n+2} \dots b_k) \quad (1)$$

$$b_1 b_2 b_3 \dots b_k = \min (b_1, b_2, b_3, \dots, b_k) \quad (\text{Example of Eq. 1: Linearization of a degree-}k \text{ monomial}). \quad (2)$$

$$b_1 b_2 b_3 b_4 = \min (b_1 b_2, b_3 b_4) \quad (\text{Example of Eq. 1: Quadratzation of a degree-4 monomial}). \quad (3)$$

$$b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 : \quad (4)$$

$$\longrightarrow 3b_a + b_1 b_2 + b_1 b_3 + b_1 b_4 + b_2 b_3 + b_2 b_4 + b_3 b_4 - 2b_a(b_1 + b_2 + b_3 + b_4) \quad (5)$$

$$\longrightarrow 3b_a + b_5 b_6 + b_5 b_7 + b_5 b_8 + b_6 b_7 + b_6 b_8 + b_7 b_8 - 2b_a(b_5 + b_6 + b_7 + b_8) \quad (6)$$

Factorization Envelope:

Let F be a quadratic function with $E_0 \leq F \leq E_1$ for some energies E_0, E_1 . If $E_0 \geq 0$, we have:

$$b_1 b_2 \dots b_k F : \quad (7)$$

$$\longrightarrow E_1 b_1 b_2 \dots b_k \quad (8)$$

$$\longrightarrow F \quad (9)$$

where Eq. (8) can be quadratized using Eq. (1). If $E_0 < 0$, we have:

$$b_1 b_2 \dots b_k F : \quad (10)$$

$$\longrightarrow E_1 b_1 b_2 \dots b_k \quad (11)$$

$$\longrightarrow F - E_0(1 - b_1 b_2 \dots b_k) \quad (12)$$

where the deg- k monomials can again be quadratized using Eq. (1).

An example of this factorization envelope is:

$$b_1 b_2 b_3 b_4 + b_2 b_3 b_4 - b_3 b_4 b_5 : \quad (13)$$

$$\longrightarrow 2b_3 b_4 \quad 25/32 \text{ (78\%)} \quad (14)$$

$$\longrightarrow b_1 b_2 + b_2 - b_5 - b_3 b_4 + 1 \quad 32/32(100\%) \quad (15)$$

Quantum envelopes:

Cubic:

$$A_1 B_2 C_3 : \quad (16)$$

$$\longrightarrow 1 + (A_1 - B_2 C_3) \quad 6/8 \text{ (75\%)} \quad (17)$$

$$\longrightarrow 1 - (A_1 - B_2 C_3) \quad 8/8(100\%) \quad (18)$$

where A, B and C can be any of the Pauli matrices X, Y or Z . For example:

$$Z_1 Y_2 Y_3 : \quad (19)$$

$$\longrightarrow 1 + (Z_1 - Y_2 Y_3) \quad 6/8 \text{ (75\%)} \quad (20)$$

$$\longrightarrow 1 - (Z_1 - Y_2 Y_3) \quad 8/8(100\%) \quad (21)$$

or

$$X_1 Y_2 Z_3 : \quad (22)$$

$$\longrightarrow 1 + (X_1 - Y_2 Z_3) \quad 6/8 \text{ (75\%)} \quad (23)$$

$$\longrightarrow 1 - (X_1 - Y_2 Z_3) \quad 8/8(100\%) \quad (24)$$

Quartic:

$$A_1 B_2 C_3 D_4 : \quad (25)$$

$$\longrightarrow 1 + (A_1 B_2 - C_3 D_4) \quad 12/16 \text{ (75\%)} \quad (26)$$

$$\longrightarrow 1 - (A_1 B_2 - C_3 D_4) \quad 16/16(100\%) \quad (27)$$

where A, B, C and D can be any of the Pauli matrices X, Y or Z . For example:

$$Z_1 X_2 Y_3 X_4 : \quad (28)$$

$$\longrightarrow 1 + (Z_1 X_2 - Y_3 X_4) \quad 12/16 \text{ (75\%)} \quad (29)$$

$$\longrightarrow 1 - (Z_1 X_2 - Y_3 X_4) \quad 16/16(100\%) \quad (30)$$

Degree-k:

$$A_1 B_2 C_3 \dots D_k : \quad (31)$$

$$\longrightarrow 1 + (A_1 B_2 - C_3 \dots D_k) \quad (75\%) \quad (32)$$

$$\longrightarrow 1 - (A_1 B_2 - C_3 \dots D_k) \quad (100\%) \quad (33)$$

where A, B, C, \dots, D can be any of the Pauli matrices X, Y or Z . For example:

$$X_1 Y_2 X_3 Z_4 Y_5 Z_6 : \quad (34)$$

$$\longrightarrow 1 + (X_1 Y_2 - X_3 Z_4 Y_5 Z_6) \quad 48/64 \text{ (75\%)} \quad (35)$$

$$\longrightarrow 1 - (X_1 Y_2 - X_3 Z_4 Y_5 Z_6) \quad 64/64(100\%) \quad (36)$$

DECOMPOSITION OF BINOMIALS OF DEGREE- k TERMS

$$b_1 b_2 b_3 b_4 + b_3 b_4 b_5 b_6 = \min(2b_3 b_4, b_1 b_2 + b_5 b_6) \quad (k, n) = (4, 6). \quad (37)$$

$$b_1 b_2 b_3 b_4 + b_3 b_4 b_5 b_6 = \min_{b_a} (b_2 b_3 + b_a(1 - b_2 - b_3 + 2b_4) + b_3 b_4, b_1 b_2 + b_5 b_6 + b_5 b_a) \quad (k, n) = (4, 6). \quad (38)$$

$$b_1 b_2 b_3 b_4 + b_4 b_5 b_6 b_7 : \quad (k, n) = (4, 7). \quad (39)$$

$$\longrightarrow b_3 b_4 + b_4 b_6 + b_a(b_5 + b_7) \quad 89/128 \text{ (70\%)} \quad (40)$$

$$\longrightarrow b_1 b_2 + b_5 b_7 + b_a(1 - b_5 + b_6 - b_7) \quad 125/128 \text{ (98\%)} \quad (41)$$

$$\longrightarrow b_5 b_7 + b_3 \quad 128/128(100\%) \quad (42)$$

$$b_1b_2b_3b_4 + b_4b_5b_6b_7 : \quad (k, n) = (4, 7). \quad (43)$$

$$\longrightarrow b_3b_4 + b_4b_6 \quad 89/128 \quad (70\%) \quad (44)$$

$$\longrightarrow b_1b_2 + b_6b_7 \quad 118/128 \quad (92\%) \quad (45)$$

$$\longrightarrow b_2b_3 - b_5b_6 + b_5b_7 + b_5 \quad 127/128 \quad (99\%) \quad (46)$$

$$\longrightarrow b_1b_4 + 2b_5 - b_7 + 1 \quad 128/128(100\%) \quad (47)$$

$$b_1b_2b_3b_4b_5 + b_3b_4b_5b_6b_7 : \quad (k, n) = (5, 7). \quad (48)$$

$$\longrightarrow b_1b_5 + b_5b_6 + b_5b_7 + b_6b_7 + b_a(1 - b_5 - 2b_6 - b_7) + b_6 \quad 188/256 \quad (73\%) \quad (49)$$

$$\longrightarrow b_3b_4 + b_a(b_4 - b_6) + b_6 \quad 236/256 \quad (92\%) \quad (50)$$

$$\longrightarrow b_2b_3 + b_3b_6 - b_4b_6 + b_6b_a + b_6 \quad 254/256 \quad (99\%) \quad (51)$$

$$\longrightarrow b_2b_5 + b_5b_7 \quad 256/256(100\%) \quad (52)$$

$$b_1b_2b_3b_4b_5 + b_3b_4b_5b_6b_7 : \quad (k, n) = (5, 7). \quad (53)$$

$$\longrightarrow b_2b_3 + b_3b_7 \quad 85/128 \quad (66\%) \quad (54)$$

$$\longrightarrow 2b_4b_5 \quad 121/128 \quad (95\%) \quad (55)$$

$$\longrightarrow b_1b_2 + b_6b_7 - b_5 + 1 \quad 128/128(100\%) \quad (56)$$

$$b_1b_2b_3b_4b_5b_6 + b_2b_3b_4b_5b_6b_7 : \quad (k, n) = (6, 7). \quad (57)$$

$$\longrightarrow b_5b_6 + b_6b_7 + b_a(1 + b_5 - b_6 - b_7) \quad 196/256 \quad (77\%) \quad (58)$$

$$\longrightarrow b_1b_4 + b_2b_4 + b_7b_a \quad 238/256 \quad (93\%) \quad (59)$$

$$\longrightarrow b_1b_3 + b_3b_7 - b_4b_6 + 2b_5b_a - b_6b_7 - b_5 + b_6 + b_7 + b_a + 1 \quad 252/256 \quad (98\%) \quad (60)$$

$$\longrightarrow b_2b_6 + b_2 - b_6 + 1 \quad 256/256(100\%) \quad (61)$$

$$b_1b_2b_3b_4b_5b_6 + b_2b_3b_4b_5b_6b_7 : \quad (k, n) = (6, 7). \quad (62)$$

$$\longrightarrow 2b_5b_6 \quad 97/128 \quad (76\%) \quad (63)$$

$$\longrightarrow b_1b_4 + b_4b_7 \quad 119/128 \quad (93\%) \quad (64)$$

$$\longrightarrow b_1b_3 + b_1b_7 + b_2b_3 - b_3b_6 + b_3b_7 - b_4b_5 - b_1 - b_7 + 2 \quad 127/128 \quad (99\%) \quad (65)$$

$$\longrightarrow b_1b_2 + b_2b_6 \quad 128/128(100\%) \quad (66)$$

$$b_1b_2b_3b_4b_5 + b_4b_5b_6b_7b_8 : \quad (k, n) = (5, 8). \quad (67)$$

$$\longrightarrow b_3b_5 + b_7b_8 + b_a(-1 - b_6 + b_7 + b_8) + b_6 - b_7 - b_8 + 1 \quad 360/512 \quad (70\%) \quad (68)$$

$$\longrightarrow b_1b_4 + b_4b_8 + b_a(b_4 + b_6) \quad 468/512 \quad (91\%) \quad (69)$$

$$\longrightarrow b_1b_2 + b_7b_8 + b_a(1 + b_6 - b_7 - b_8) \quad 496/512 \quad (97\%) \quad (70)$$

$$\longrightarrow b_3b_5 + b_5 \quad 512/512(100\%) \quad (71)$$

$$b_1b_2b_3b_4b_5 + b_4b_5b_6b_7b_8 : \quad (k, n) = (5, 8). \quad (72)$$

$$\longrightarrow b_2b_5 + b_5b_8 \quad 169/256 \quad (66\%) \quad (73)$$

$$\longrightarrow b_1b_4 + b_4b_7 - b_5b_8 + b_8 \quad 233/256 \quad (91\%) \quad (74)$$

$$\longrightarrow b_1b_3 + b_6b_7 + b_6b_8 + b_7b_8 - b_6 - b_7 - b_8 + 1 \quad 252/256 \quad (98\%) \quad (75)$$

$$\longrightarrow b_2b_3 + b_6b_7 \quad 256/256(100\%) \quad (76)$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_3 b_4 b_5 b_6 b_7 b_8 : & (k, n) = (6, 8). \quad (77) \\
& \longrightarrow b_1 b_6 + b_7 b_8 + b_a(1 + b_6 - b_7 - b_8) & 364/512 \quad (71\%) \quad (78) \\
& \longrightarrow b_2 b_3 + b_5 b_8 - b_6 b_8 + b_7 b_a - b_7 + b_8 - b_a + 1 & 450/512 \quad (88\%) \quad (79) \\
& \longrightarrow b_1 b_4 + b_4 & 488/512 \quad (95\%) \quad (80) \\
& \longrightarrow b_2 b_3 + b_3 b_7 - b_6 b_8 + b_8 - b_a + 1 & 502/512 \quad (98\%) \quad (81) \\
& \longrightarrow b_2 b_5 + b_5 & 512/512(100\%) \quad (82)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_3 b_4 b_5 b_6 b_7 b_8 : & (k, n) = (6, 8). \quad (83) \\
& \longrightarrow 2b_5 b_6 & 193/256 \quad (75\%) \quad (84) \\
& \longrightarrow b_1 b_4 + b_4 b_8 & 237/256 \quad (93\%) \quad (85) \\
& \longrightarrow b_2 b_3 + b_3 b_7 - b_4 b_6 + b_4 b_8 - b_5 b_7 - b_5 b_8 + b_6 b_8 - b_6 + b_7 - b_8 + 2 & 254/256 \quad (99\%) \quad (86) \\
& \longrightarrow b_1 b_2 + b_7 b_8 & 256/256(100\%) \quad (87)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 + b_2 b_3 b_4 b_5 b_6 b_7 b_8 : & (k, n) = (7, 8). \quad (88) \\
& \longrightarrow b_6 b_7 + b_6 b_8 + b_a(1 - b_6 + b_7 - b_8) & 388/512 \quad (76\%) \quad (89) \\
& \longrightarrow b_1 b_3 + b_3 b_8 + b_a(1 + b_8) & 470/512 \quad (92\%) \quad (90) \\
& \longrightarrow b_2 b_4 - b_3 b_8 + b_4 b_5 + b_a(1 - b_7) + b_8 & 500/512 \quad (98\%) \quad (91) \\
& \longrightarrow b_2 b_5 + b_2 b_8 - b_4 b_8 - b_6 b_7 + b_6 b_8 + b_a(-1 - b_4 - b_7 + b_8) - b_3 + b_7 - b_8 + 4 & 508/512 \quad (99\%) \quad (92) \\
& \longrightarrow b_2 b_5 - b_7 b_8 + b_5 + 1 & 512/512(100\%) \quad (93)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 + b_2 b_3 b_4 b_5 b_6 b_7 b_8 : & (k, n) = (7, 8). \quad (94) \\
& \longrightarrow 2b_5 b_6 & 193/256 \quad (75\%) \quad (95) \\
& \longrightarrow b_1 b_4 + b_4 b_8 & 235/256 \quad (92\%) \quad (96) \\
& \longrightarrow b_2 b_3 + b_2 b_7 - b_5 b_6 + b_6 b_8 + b_5 - b_6 - b_8 + 1 & 250/256 \quad (98\%) \quad (97) \\
& \longrightarrow b_3 b_7 + b_7 b_8 & 254/256 \quad (99\%) \quad (98) \\
& \longrightarrow b_3 b_8 + b_3 & 256/256(100\%) \quad (99)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + b_5 b_6 b_7 b_8 : & (k, n) = (4, 8). \quad (100) \\
& \longrightarrow b_2 b_3 + b_6 b_8 + b_a(1 - b_6 + b_7 - b_8) & 390/512 \quad (76\%) \quad (101) \\
& \longrightarrow b_1 b_4 + b_6 b_8 + b_a(1 - b_6 + b_7 - b_8) & 480/512 \quad (94\%) \quad (102) \\
& \longrightarrow b_2 b_4 + b_5 - b_a + 1 & 506/512 \quad (99\%) \quad (103) \\
& \longrightarrow b_1 b_3 - b_6 b_a + b_5 + 1 & 512/512(100\%) \quad (104)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + b_5 b_6 b_7 b_8 : & (k, n) = (4, 8). \quad (105) \\
& \longrightarrow b_1 b_2 + b_6 b_7 & 169/256 \quad (66\%) \quad (106) \\
& \longrightarrow b_3 b_4 + b_5 b_8 & 238/256 \quad (93\%) \quad (107) \\
& \longrightarrow b_1 b_4 + b_5 b_6 + b_5 b_7 + b_6 b_7 - b_5 - b_6 - b_7 + 1 & 248/256 \quad (97\%) \quad (108) \\
& \longrightarrow b_2 b_3 + b_6 b_7 + b_6 b_8 + b_7 b_8 - b_6 - b_7 - b_8 + 1 & 254/256 \quad (99\%) \quad (109) \\
& \longrightarrow b_1 b_2 + b_5 b_8 & 256/256(100\%) \quad (110)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 + b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (5, 10). \quad (111) \\
\longrightarrow & b_1 b_4 + b_7 b_9 & 625/1024 \quad (61\%) \quad (112) \\
\longrightarrow & b_3 b_5 + b_6 b_8 & 889/1024 \quad (87\%) \quad (113) \\
\longrightarrow & b_2 b_5 + b_7 b_{10} & 972/1024 \quad (95\%) \quad (114) \\
\longrightarrow & b_2 b_4 + b_6 b_8 & 999/1024 \quad (98\%) \quad (115) \\
\longrightarrow & b_1 b_3 + b_9 b_{10} + b_9 b_a & 1016/1024 \quad (99\%) \quad (116) \\
\longrightarrow & b_1 b_5 + b_6 b_9 & 1020/1024 \quad (99\%) \quad (117) \\
\longrightarrow & b_1 b_4 + b_8 b_{10} & 1022/1024 \quad (99\%) \quad (118) \\
\longrightarrow & b_2 b_3 - b_4 b_{10} + b_7 b_9 + b_9 b_a + 1 & 1024/1024(100\%) \quad (119)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 + b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (5, 10). \quad (120) \\
\longrightarrow & b_1 b_3 + b_9 b_{10} & 625/1024 \quad (61\%) \quad (121) \\
\longrightarrow & b_2 b_4 + b_7 b_{10} & 851/1024 \quad (83\%) \quad (122) \\
\longrightarrow & b_3 b_5 + b_5 b_{10} + b_8 b_9 & 924/1024 \quad (90\%) \quad (123) \\
\longrightarrow & b_1 b_2 + b_6 & 972/1024 \quad (95\%) \quad (124) \\
\longrightarrow & b_3 b_4 + b_8 b_9 & 997/1024 \quad (97\%) \quad (125) \\
\longrightarrow & b_1 b_5 + b_7 b_{10} & 1010/1024 \quad (99\%) \quad (126) \\
\longrightarrow & -b_1 b_7 - b_1 b_{10} + b_2 b_3 - b_2 b_8 - b_2 b_{10} + b_3 b_5 & (127) \\
& + b_6 b_9 + b_7 b_{10} - b_8 b_9 + b_9 b_{10} - b_3 - b_7 + b_8 + 3 & 1016/1024 \quad (99\%) \quad (128) \\
\longrightarrow & b_1 b_3 + b_7 b_8 & 1020/1024 \quad (99\%) \quad (129) \\
\longrightarrow & b_2 b_4 + b_2 b_6 - b_2 b_9 - b_3 b_{10} - b_5 b_7 + b_7 b_{10} + b_9 b_{10} - b_{10} + 2 & 1023/1024 \quad (99\%) \quad (130) \\
\longrightarrow & b_2 b_5 + b_2 b_9 + b_6 b_8 & 1024/1024(100\%) \quad (131)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (6, 10). \quad (132) \\
\longrightarrow & b_4 b_5 + b_5 b_9 & 657/1024 \quad (64\%) \quad (133) \\
\longrightarrow & b_2 b_6 + b_6 b_8 & 905/1024 \quad (88\%) \quad (134) \\
\longrightarrow & b_1 b_3 + b_7 b_8 & 982/1024 \quad (96\%) \quad (135) \\
\longrightarrow & b_2 b_3 + b_a(b_{10} - b_9) + b_9 & 1011/1024 \quad (99\%) \quad (136) \\
\longrightarrow & b_2 b_4 + b_7 b_{10} & 1020/1024 \quad (99\%) \quad (137) \\
\longrightarrow & b_9 b_{10} + b_1 & 1023/1024 \quad (99\%) \quad (138) \\
\longrightarrow & b_7 b_8 + b_4 & 1024/1024(100\%) \quad (139)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (6, 10). \quad (140) \\
\longrightarrow & 2b_5 b_6 & 769/1024 \quad (75\%) \quad (141) \\
\longrightarrow & b_1 b_3 + b_8 b_9 & 934/1024 \quad (92\%) \quad (142) \\
\longrightarrow & b_2 b_4 + b_7 b_{10} + b_8 b_9 - b_8 - b_9 + 1 & 997/1024 \quad (97\%) \quad (143) \\
\longrightarrow & -b_1 b_3 + b_1 b_9 + b_2 b_4 + b_4 b_9 + b_5 b_8 + b_8 b_9 - b_5 - b_8 - b_9 + 2 & 769/1024 \quad (99\%) \quad (144) \\
\longrightarrow & b_1 b_3 + b_7 b_{10} - b_8 - b_9 + 2 & 1014/1024 \quad (99\%) \quad (145) \\
\longrightarrow & b_2 b_3 + b_8 b_9 & 1024/1024(100\%) \quad (146)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 + b_4 b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (7, 10). \quad (147) \\
& \longrightarrow b_3 b_5 + b_5 b_8 & 649/1024 \quad (63\%) \quad (148) \\
& \longrightarrow b_2 b_4 + b_4 b_9 & 893/1024 \quad (87\%) \quad (149) \\
& \longrightarrow b_1 b_7 + b_7 b_{10} & 985/1024 \quad (96\%) \quad (150) \\
& \longrightarrow b_1 b_6 + b_6 b_9 + b_a & 1015/1024 \quad (99\%) \quad (151) \\
& \longrightarrow b_2 b_3 + b_8 b_{10} + b_a & 1022/1024 \quad (99\%) \quad (152) \\
& \longrightarrow b_1 b_3 + b_8 b_9 & 1024/1024(100\%) \quad (153)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 + b_4 b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (7, 10). \quad (154) \\
& \longrightarrow b_3 b_7 + b_7 b_{10} & 649/1024 \quad (63\%) \quad (155) \\
& \longrightarrow 2b_4 b_6 & 937/1024 \quad (92\%) \quad (156) \\
& \longrightarrow b_1 b_5 + b_5 b_8 & 1001/1024 \quad (98\%) \quad (157) \\
& \longrightarrow b_1 b_2 + b_9 b_{10} & 1019/1024 \quad (99\%) \quad (158) \\
& \longrightarrow b_2 b_3 + b_8 & 1023/1024 \quad (99\%) \quad (159) \\
& \longrightarrow b_3 b_7 + b_9 b_{10} & 1024/1024(100\%) \quad (160)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 + b_3 b_4 b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (8, 10). \quad (161) \\
& \longrightarrow b_2 b_8 + b_8 b_9 & 645/1024 \quad (63\%) \quad (162) \\
& \longrightarrow b_1 b_3 + b_3 b_{10} + b_9 b_a & 887/1024 \quad (87\%) \quad (163) \\
& \longrightarrow b_4 b_6 + b_5 b_6 & 977/1024 \quad (95\%) \quad (164) \\
& \longrightarrow b_2 b_7 + b_7 b_{10} & 1007/1024 \quad (98\%) \quad (165) \\
& \longrightarrow b_1 b_4 + b_4 b_5 + 2b_9 b_a & 1018/1024 \quad (99\%) \quad (166) \\
& \longrightarrow b_1 b_5 + b_5 b_9 & 1024/1024(100\%) \quad (167)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 + b_3 b_4 b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (8, 10). \quad (168) \\
& \longrightarrow 4b_3 b_7 & 768/1024 \quad (75\%) \quad (169) \\
& \longrightarrow b_2 b_8 + b_8 b_9 & 933/1024 \quad (91\%) \quad (170) \\
& \longrightarrow 2b_4 b_6 + b_8 b_9 - b_8 - b_9 + 1 & 1005/1024 \quad (98\%) \quad (171) \\
& \longrightarrow b_1 b_5 + b_5 b_{10} + b_8 b_9 - b_8 - b_9 + 1 & 1022/1024 \quad (99\%) \quad (172) \\
& \longrightarrow b_1 b_2 + b_8 b_9 + b_9 b_{10} - b_8 - b_9 + 1 & 1024/1024(100\%) \quad (173)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 b_9 + b_2 b_3 b_4 b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (9, 10). \quad (174) \\
& \longrightarrow b_1 b_9 + b_9 b_{10} + b_{10} b_a & 643/1024 \quad (63\%) \quad (175) \\
& \longrightarrow b_2 b_4 + b_4 b_5 & 883/1024 \quad (86\%) \quad (176) \\
& \longrightarrow b_3 b_7 + b_3 b_8 & 973/1024 \quad (95\%) \quad (177) \\
& \longrightarrow b_2 b_6 + b_6 b_8 & 1003/1024 \quad (98\%) \quad (178) \\
& \longrightarrow b_2 b_5 + b_5 b_7 - b_{10} b_a + b_{10} & 1015/1024 \quad (99\%) \quad (179) \\
& \longrightarrow b_1 b_8 + b_7 b_8 & 1019/1024 \quad (99\%) \quad (180) \\
& \longrightarrow b_2 b_7 + b_2 b_{10} - b_4 b_5 - b_{10} b_a + b_{10} + 1 & 1023/1024 \quad (99\%) \quad (181) \\
& \longrightarrow b_4 b_7 + b_7 & 1024/1024(100\%) \quad (182)
\end{aligned}$$

$b_1b_2b_3b_4b_5b_6b_7b_8b_9 + b_2b_3b_4b_5b_6b_7b_8b_9b_{10} :$	$(k, n) = (9, 10). \quad (183)$
$\longrightarrow 2b_2b_3 - b_8b_9 + b_9$	$577/1024 \quad (56\%) \quad (184)$
$\longrightarrow 3b_8b_9$	$961/1024 \quad (94\%) \quad (185)$
$\longrightarrow 2b_4b_6 - b_8b_9 - b_8b_{10} + b_{10} + 1$	$1009/1024 \quad (99\%) \quad (186)$
$\longrightarrow 2b_5b_7 - b_8b_{10} + b_{10}$	$1021/1024 \quad (99\%) \quad (187)$
$\longrightarrow b_1b_6 + b_{10}$	$1024/1024(100\%) \quad (188)$

DECOMPOSITION OF DEGREE- k , EXACT- k -OF- n TRINOMIALS

$b_1b_2b_3b_4 + b_2b_3b_4b_5 + b_3b_4b_5b_6 :$	$(k, n) = (4, 6). \quad (189)$
$\longrightarrow b_1b_4 + 2b_4b_5 + b_7$	$44/64 \quad (69\%) \quad (190)$
$\longrightarrow b_1b_3 + b_2b_3 + b_3b_6 + b_6b_7$	$60/64 \quad (94\%) \quad (191)$
$\longrightarrow b_2b_4 + b_5b_6 + b_2$	$64/64(100\%) \quad (192)$

$b_1b_2b_3b_4 + b_2b_3b_4b_5 + b_3b_4b_5b_6 :$	$(k, n) = (4, 6). \quad (193)$
$\longrightarrow b_2b_4 + 2b_4b_5$	$43/64 \quad (67\%) \quad (194)$
$\longrightarrow b_1b_3 + b_2b_3 + b_2b_5 + b_3b_6 - b_4b_5 - b_2 + 1$	$60/64 \quad (94\%) \quad (195)$
$\longrightarrow b_1b_2 + b_2b_5 + b_5b_6$	$64/64(100\%) \quad (196)$

$b_1b_2b_3b_4 + b_3b_4b_5b_6 + b_5b_6b_7b_8 :$	$(k, n) = (4, 8). \quad (197)$
$\longrightarrow b_1b_4 + 2b_5b_6$	$159/256 \quad (62\%) \quad (198)$
$\longrightarrow b_2b_3 + b_3b_5 + b_7b_8$	$225/256 \quad (88\%) \quad (199)$
$\longrightarrow b_1b_4 + b_3b_4 - b_5b_7 + b_6b_7 + b_7b_8 - b_6 + 1$	$244/256 \quad (95.3\%) \quad (200)$
$\longrightarrow b_2b_3 + b_6b_8 + b_6$	$253/256 \quad (98.8\%) \quad (201)$
$\longrightarrow b_2b_3 + b_5b_7 + b_5$	$256/256 \quad (100\%) \quad (202)$

$b_1b_2b_3b_4 + b_3b_4b_5b_6 + b_5b_6b_7b_8 :$	$(k, n) = (4, 8). \quad (203)$
$\longrightarrow b_2b_4 + 2b_5b_6$	$159/256 \quad (62\%) \quad (204)$
$\longrightarrow b_3b_6 + b_7b_8 + b_3$	$212/256 \quad (83\%) \quad (205)$
$\longrightarrow b_2b_4 - b_5b_7 + b_7b_8 + b_4 + b_7$	$234/256 \quad (91\%) \quad (206)$
$\longrightarrow b_1b_3 + 2b_5b_6$	$253/256 \quad (99\%) \quad (207)$
$\longrightarrow b_7b_8 + b_1 + b_6$	$256/256(100\%) \quad (208)$

$b_1b_2b_3b_4b_5 + b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7 :$	$(k, n) = (5, 7). \quad (209)$
$\longrightarrow b_1b_5 + b_5b_6 + b_6b_7 + b_a(-2 - b_5 + 2b_6 + b_7) + b_5 - 2b_6 - b_7 + 2$	$86/128 \quad (67\%) \quad (210)$
$\longrightarrow b_1b_3 + b_3b_4 - b_3b_6 + b_3b_7 + b_a(b_5 + b_7) + b_3$	$112/128 \quad (88\%) \quad (211)$
$\longrightarrow b_1b_4 + b_2b_4 + b_4b_7 + b_5b_7 + b_a(-1 - b_6 - b_7) - b_5 + b_6 + 2$	$124/128 \quad (97\%) \quad (212)$
$\longrightarrow b_2b_4 - 2b_5b_a + b_6b_7 + b_2 + b_5 + 1$	$128/128(100\%) \quad (213)$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 + b_2 b_3 b_4 b_5 b_6 + b_3 b_4 b_5 b_6 b_7 : & (k, n) = (5, 7). \quad (214) \\
& \longrightarrow 2b_4 b_5 + b_4 b_6 & 81/128 \quad (63\%) \quad (215) \\
& \longrightarrow b_1 b_3 + b_3 b_6 + b_3 b_7 - b_4 b_5 + b_5 & 111/128 \quad (87\%) \quad (216) \\
& \longrightarrow b_1 b_2 + b_2 b_6 - b_4 b_5 + b_6 b_7 + b_5 & 122/128 \quad (95\%) \quad (217) \\
& \longrightarrow 2b_4 b_5 + b_5 & 128/128(100\%) \quad (218)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_2 b_3 b_4 b_5 b_6 b_7 + b_3 b_4 b_5 b_6 b_7 b_8 : & (k, n) = (6, 8). \quad (219) \\
& \longrightarrow b_1 b_6 + 2b_6 b_7 & 164/256 \quad (64\%) \quad (220) \\
& \longrightarrow b_1 b_5 + b_2 b_5 - b_3 b_6 + b_5 b_8 + b_3 & 219/256 \quad (86\%) \quad (221) \\
& \longrightarrow b_2 b_4 + b_4 b_7 + b_4 b_8 - b_6 + 1 & 243/256 \quad (95\%) \quad (222) \\
& \longrightarrow b_2 b_3 + b_3 b_8 - b_5 b_6 + b_3 + b_6 & 253/256 \quad (99\%) \quad (223) \\
& \longrightarrow b_1 b_2 + b_2 b_6 + b_5 b_7 - b_6 b_7 + b_7 b_8 - b_5 + 1 & 256/256(100\%) \quad (224)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 + b_2 b_3 b_4 b_5 b_6 b_7 b_8 b_9 + b_3 b_4 b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (8, 10). \quad (225) \\
& \longrightarrow 3b_5 b_8 & 769/1024 \quad (75\%) \quad (226) \\
& \longrightarrow 2b_2 b_6 + b_4 b_6 & 931/1024 \quad (91\%) \quad (227) \\
& \longrightarrow b_1 b_7 - b_5 b_{10} + b_7 b_9 + b_9 b_{10} - b_6 + b_{10} + 1 & 984/1024 \quad (96\%) \quad (228) \\
& \longrightarrow 3b_2 b_3 + b_3 b_{10} - b_6 b_8 + 1 & 1011/1024 \quad (99\%) \quad (229) \\
& \longrightarrow b_4 b_7 + b_4 b_8 - b_3 + b_4 - b_8 + 2 & 1019/1024 \quad (99\%) \quad (230) \\
& \longrightarrow b_2 b_3 - b_2 b_4 - b_3 b_4 - b_3 b_8 - b_5 b_{10} - b_6 b_9 + b_7 b_8 + b_7 b_9 + b_8 b_9 + b_7 + 3 & 1023/1024 \quad (99\%) \quad (231) \\
& \longrightarrow b_2 b_8 + 2b_8 b_9 & 1024/1024(100\%) \quad (232)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_3 b_4 b_5 b_6 b_7 b_8 + b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (6, 10). \quad (233) \\
& \longrightarrow b_5 b_6 + b_5 b_8 + b_5 b_9 + b_8 b_{11} + b_9 b_{11} + b_{10} b_{11} & 583/1024 \quad (57\%) \quad (234) \\
& \longrightarrow b_1 b_2 + b_4 b_7 + b_7 b_{10} + b_9 b_{11} - b_9 - b_{11} + 1 & 815/1024 \quad (80\%) \quad (235) \\
& \longrightarrow b_1 b_6 + b_5 b_6 + b_6 - b_{11} + 1 & 917/1024 \quad (90\%) \quad (236) \\
& \longrightarrow b_3 b_4 + b_3 b_7 + b_8 b_9 + b_9 b_{11} & 979/1024 \quad (96\%) \quad (237) \\
& \longrightarrow b_2 b_4 + b_4 b_8 + b_8 b_9 - b_9 b_{11} + b_9 - b_{11} + 1 & 1007/1024 \quad (98\%) \quad (238) \\
& \longrightarrow b_1 b_3 + b_7 b_{10} + b_{10} b_{11} + b_3 & 1016/1024 \quad (99\%) \quad (239) \\
& \longrightarrow b_1 b_4 + b_4 b_8 + b_7 b_{10} + b_9 b_{11} + b_{10} b_{11} - b_9 - b_{11} + 1 & 1021/1024 \quad (99\%) \quad (240) \\
& \longrightarrow b_1 b_3 - b_2 b_{11} + b_7 b_8 + b_8 b_9 - b_{10} b_{11} - b_{11} + 3 & 1024/1024(100\%) \quad (241)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_3 b_4 b_5 b_6 b_7 b_8 + b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (6, 10). \quad (242) \\
& \longrightarrow 2b_3 b_4 + b_7 b_{10} & 591/1024 \quad (58\%) \quad (243) \\
& \longrightarrow 2b_3 b_5 + b_5 b_6 & 847/1024 \quad (83\%) \quad (244) \\
& \longrightarrow b_1 b_2 + b_7 b_8 + b_8 b_9 & 951/1024 \quad (93\%) \quad (245) \\
& \longrightarrow 3b_5 b_6 & 995/1024 \quad (97\%) \quad (246) \\
& \longrightarrow b_1 b_3 + b_3 b_4 + b_9 b_{10} & 1009/1024 \quad (99\%) \quad (247) \\
& \longrightarrow b_1 b_2 + b_5 b_7 + b_7 b_{10} & 1018/1024 \quad (99\%) \quad (248) \\
& \longrightarrow 2b_1 b_4 - b_1 b_{10} + b_2 b_4 + b_4 b_5 + b_4 b_{10} + b_5 b_8 - b_6 b_8 + b_8 b_9 + b_7(b_{10} - b_6 - b_5 - b_1) + 3 & 1023/1024 \quad (99\%) \quad (249) \\
& \longrightarrow b_2 b_8 + b_3 b_6 + b_6 b_8 & 1024/1024(100\%) \quad (250)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4 + b_4b_5b_6b_7 + b_7b_8b_9b_{10} : & (k, n) = (4, 10). \quad (251) \\
& \longrightarrow b_2b_3 + b_6b_7 + b_7b_9 + 2b_9b_a & 581/1024 \quad (57\%) \quad (252) \\
& \longrightarrow b_2b_4 + b_4b_6 + b_a(b_9 - b_{10}) + b_{10} & 823/1024 \quad (80\%) \quad (253) \\
& \longrightarrow b_1b_3 + b_5b_6 + b_8b_9 + b_a(b_9 - b_{10}) - b_9 + 1 & 930/1024 \quad (91\%) \quad (254) \\
& \longrightarrow b_1b_4 + b_4b_5 + b_4b_{10} + b_8b_{10} + b_a(1 - b_7 + b_9) & 978/1024 \quad (96\%) \quad (255) \\
& \longrightarrow b_1b_4 + b_7b_8 + b_a(1 + b_9) + b_7 & 1000/1024 \quad (98\%) \quad (256) \\
& \longrightarrow b_2b_3 + b_a(b_9 - b_{10}) + b_5 + b_{10} & 1015/1024 \quad (99\%) \quad (257) \\
& \longrightarrow b_1b_3 + b_6 + b_{10} & 1020/1024 \quad (99\%) \quad (258) \\
& \longrightarrow b_5b_6 + b_2 + b_8 + b_a & 1024/1024(100\%) \quad (259)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4 + b_4b_5b_6b_7 + b_7b_8b_9b_{10} : & (k, n) = (4, 10). \quad (260) \\
& \longrightarrow b_3b_4 + b_4b_6 + b_9b_{10} & 581/1024 \quad (57\%) \quad (261) \\
& \longrightarrow b_1b_2 + b_5b_7 - b_8b_9 + b_9b_{10} + b_9 & 759/1024 \quad (74\%) \quad (262) \\
& \longrightarrow b_5b_6 + b_8b_9 + b_1 + b_8 & 842/1024 \quad (82\%) \quad (263) \\
& \longrightarrow b_2b_4 + b_7b_{10} - b_8b_9 + b_7 + b_8 & 935/1024 \quad (91\%) \quad (264) \\
& \longrightarrow b_2b_4 + b_4b_6 - b_8b_9 + b_8b_{10} - b_7 + b_8 + 1 & 969/1024 \quad (95\%) \quad (265) \\
& \longrightarrow b_1b_3 + b_3b_4 + b_5b_7 + b_7b_9 - b_8b_9 + b_9 & 992/1024 \quad (97\%) \quad (266) \\
& \longrightarrow b_2b_3 + b_3b_5 + b_3b_{10} + b_4b_8 + b_5b_6 - b_4 + 1 & 1004/1024 \quad (98\%) \quad (267) \\
& \longrightarrow b_1b_3 + b_6b_7 + b_9b_{10} & 1013/1024 \quad (99\%) \quad (268) \\
& \longrightarrow b_1b_9 + b_7b_8 - b_8b_9 - b_9b_{10} + b_1 + b_7 + b_8 + b_9 & 1019/1024 \quad (99\%) \quad (269) \\
& \longrightarrow b_2b_3 + b_5b_6 - b_8b_9 + b_9b_{10} + b_9 & 1022/1024 \quad (99\%) \quad (270) \\
& \longrightarrow -b_1b_5 + b_1b_8 + b_3b_7 + b_3 + b_7 + 1 & 1023/1024 \quad (99\%) \quad (271) \\
& \longrightarrow b_1b_5 - b_1b_{10} + b_2 + b_8 - b_{10} + 2 & 1024/1024(100\%) \quad (272)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5 + b_3b_4b_5b_6 + b_4b_5b_6b_7b_8 : & (k, n) = (5, 8). \quad (273) \\
& \longrightarrow b_1b_3 + b_3b_4 + b_6b_8 - b_6b_9 + b_7b_9 - b_8b_9 + b_9 & 156/256 \quad (61\%) \quad (274) \\
& \longrightarrow b_1b_5 + b_5b_7 + b_7b_9 + b_8b_9 + b_5 - b_7 - b_9 + 1 & 202/256 \quad (79\%) \quad (275) \\
& \longrightarrow b_2b_4 + b_6b_8 + b_6b_9 - b_7b_9 + b_8b_9 + b_7 - b_8 - b_9 + 1 & 230/256 \quad (90\%) \quad (276) \\
& \longrightarrow b_2b_4 + b_4b_8 + b_4 - b_9 + 1 & 246/256 \quad (96\%) \quad (277) \\
& \longrightarrow b_1b_5 + 2b_6 & 252/256 \quad (98\%) \quad (278) \\
& \longrightarrow b_2b_5 + b_7b_8 + b_5 & 256/256(100\%) \quad (279)
\end{aligned}$$

$$\begin{aligned}
& b_1b_2b_3b_4b_5 + b_3b_4b_5b_6 + b_4b_5b_6b_7b_8 : & (k, n) = (5, 8). \quad (280) \\
& \longrightarrow b_4b_5 + 2b_5b_6 & 165/256 \quad (64\%) \quad (281) \\
& \longrightarrow b_2b_4 + b_3b_4 + b_4b_8 - b_5b_7 + b_7 & 215/256 \quad (84\%) \quad (282) \\
& \longrightarrow b_2b_3 + b_3b_6 - b_4b_5 - b_5b_7 + b_7b_8 + b_5 + b_7 & 242/256 \quad (95\%) \quad (283) \\
& \longrightarrow b_1b_3 + b_5b_6 + b_6b_7 & 254/256 \quad (99\%) \quad (284) \\
& \longrightarrow b_1b_2 + b_5b_6 + b_6b_8 & 256/256(100\%) \quad (285)
\end{aligned}$$

DECOMPOSITION OF DEGREE- k , EXACT- k -OF- n QUADRINOMIALS

$$\begin{aligned}
& b_1b_2b_3 + b_1b_2b_4 + b_1b_3b_4 + b_2b_3b_4 : & (k, n) = (3, 4). \quad (286) \\
& \longrightarrow 2b_1b_2 + b_1b_3 + 2b_1b_4 + b_2b_3 + 2b_2b_4 + b_3b_4 - 2b_1 - 2b_2 - b_3 - 2b_4 + 3 & 13/16 \quad (81\%) \quad (287) \\
& \longrightarrow 2b_1b_3 + b_2b_3 + b_2 & 16/16(100\%) \quad (288)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 + b_2 b_3 b_4 b_5 b_6 b_7 b_8 + b_3 b_4 b_5 b_6 b_7 b_8 b_9 + b_4 b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (7, 10). \quad (289) \\
\longrightarrow & 4b_4 b_5 & 769/1024 \quad (75\%) \quad (290) \\
\longrightarrow & b_2 b_6 + 2b_3 b_6 + b_6 b_9 & 915/1024 \quad (89\%) \quad (291) \\
\longrightarrow & b_1 b_7 + b_5 b_7 + b_6 b_7 + b_7 b_{10} & 974/1024 \quad (95\%) \quad (292) \\
\longrightarrow & b_1 b_2 + b_2 b_8 + b_7 b_8 + b_9 b_{10} & 995/1024 \quad (97\%) \quad (293) \\
\longrightarrow & b_2 b_3 + b_3 b_4 + b_3 b_6 + b_9 b_{10} & 1008/1024 \quad (98\%) \quad (294) \\
\longrightarrow & b_1 b_2 + b_2 b_4 + b_9 b_{10} + b_9 & 1016/1024 \quad (99\%) \quad (295) \\
\longrightarrow & b_1 b_3 - b_2 b_8 + b_7 b_8 + b_8 b_9 + b_8 b_{10} + 2b_8 & 1023/1024 \quad (99\%) \quad (296) \\
\longrightarrow & b_1 b_8 + b_2 b_7 - b_5 b_{10} + b_7 b_8 + b_8 b_9 - b_5 + 2 & 1024/1024(100\%) \quad (297)
\end{aligned}$$

DECOMPOSITION OF DEGREE- k , NOT EXACT- k -OF- n MULTINOMIALS

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + 2b_1 b_2 b_3 + b_1 b_2 b_4 + b_1 b_3 b_4 + b_2 b_3 b_4 : & (k, n) = (4, 4). \quad (298) \\
\longrightarrow & b_1 b_2 + 4b_1 b_3 + b_1 b_4 + b_2 b_3 + b_2 b_4 + b_3 b_4 - b_1 - b_2 - b_3 - b_4 + 1 & 12/16 \quad (75\%) \quad (299) \\
\longrightarrow & b_1 b_2 + b_1 b_3 + 4b_1 b_4 + b_2 b_4 & 16/16(100\%) \quad (300)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + 2b_1 b_2 b_3 + b_1 b_2 b_4 + b_1 b_3 b_4 + b_2 b_3 b_4 : & (k, n) = (4, 4). \quad (301) \\
\longrightarrow & b_1 b_2 + 4b_1 b_3 + b_1 b_4 + b_2 b_3 + b_2 b_4 + b_3 b_4 - b_1 - b_2 - b_3 - b_4 + 1 & 12/16 \quad (75\%) \quad (302) \\
\longrightarrow & 2b_2 b_3 + 3b_2 b_4 + b_3 b_4 & 16/16(100\%) \quad (303)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + 2b_1 b_2 b_3 + b_1 b_2 b_4 + 3b_1 b_3 b_4 + b_2 b_3 b_4 : & (k, n) = (4, 4). \quad (304) \\
\longrightarrow & 2b_1 b_2 + 5b_1 b_4 + b_3 b_4 & 11/16 \quad (69\%) \quad (305) \\
\longrightarrow & -b_1 b_2 + 3b_1 b_3 + 4b_2 b_3 + 2b_2 b_4 - 4b_3 b_4 + 4b_3 - b_4 + 1 & 16/16(100\%) \quad (306)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + 2b_1 b_2 b_3 + b_1 b_3 b_4 : & (k, n) = (4, 4). \quad (307) \\
\longrightarrow & 4b_1 b_3 & 13/16 \quad (81\%) \quad (308) \\
\longrightarrow & 2b_1 b_2 + b_1 b_4 + b_2 b_4 & 16/16(100\%) \quad (309)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 + 2b_1 b_2 b_3 + b_1 b_3 b_4 : & (k, n) = (4, 4). \quad (310) \\
\longrightarrow & 2b_1 b_3 + 2b_3 b_4 & 12/16 \quad (75\%) \quad (311) \\
\longrightarrow & 3b_1 b_2 + b_1 b_4 & 16/16(100\%) \quad (312)
\end{aligned}$$