# Volume 3: List of Multi-run Quadratizations

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## PRODUCT OF POLYNOMIALS

$$f_1 f_2 \dots f_\kappa = \min\left(f_1,f_2,\dots,f_\kappa\right), \qquad f_i\left(b_{k_i}, b_{k_i+1}, \dots, b_{k_{i+1}-1}\right) \geq 0 \quad (1)$$

$$f_1 f_2 \dots f_\kappa = \min\left(\max f_\kappa f_1 f_2 \dots f_{\kappa-1}, f_\kappa - \min f_\kappa + f_1 f_2 \dots f_{\kappa-1}\right), \qquad f_\kappa < 0,\ f_{i<\kappa}\left(b_{k_i}, b_{k_i+1}, \dots, b_{k_{i+1}-1}\right) \geq 0 \quad (2)$$

$b_1b_2b_3b_4 + b_2b_3b_4 - b_3b_4b_5$ : (Example of Eq. 2). (3)

$\longrightarrow 2b_3b_4$ 25/32 (78%) (4)

$\longrightarrow b_1b_2 + b_2 - b_5 - b_3b_4 + 1$ 32/32(100%) (5)

## MONOMIALS

$$b_1b_2b_3\dots b_k = \min\left(b_1b_2\dots b_{k_1}, b_{k_1+1}b_{k_1+2}\dots b_{k_2}, b_{k_2+1}b_{k_2+2}\dots b_{k_3}, \dots, b_{k_n+1}b_{k_n+2}\dots b_k\right) \quad \text{(Example of Eq. 1).} \quad (6)$$

$$b_1b_2b_3\dots b_k = \min\left(b_1, b_2, b_3, \dots, b_k\right) \quad \text{(Example of Eq. 6: Linearization of a degree-}k\text{ monomial).} \quad (7)$$

$$b_1b_2b_3b_4 = \min\left(b_1b_2, b_3b_4\right) \quad \text{(Example of Eq. 6: Quadratization of a degree-4 monomial).} \quad (8)$$

$b_1b_2b_3b_4b_5b_6b_7b_8$ : (9)

$\longrightarrow 3b_a + b_1b_2 + b_1b_3 + b_1b_4 + b_2b_3 + b_2b_4 + b_3b_4 - 2b_a(b_1 + b_2 + b_3 + b_4)$ (10)

$\longrightarrow 3b_a + b_5b_6 + b_5b_7 + b_5b_8 + b_6b_7 + b_6b_8 + b_7b_8 - 2b_a(b_5 + b_6 + b_7 + b_8)$ (11)

$$s_1s_2\dots s_k = \min\left(1 + s_1s_2 - s_3s_4\dots s_k, 1 - s_1s_2 - s_3s_4\dots s_k\right), s_i \in \{x, y, z\} \quad (12)$$

$x_1z_2x_3z_4y_5x_6$ (Example of Eq. 12). (13)

$\longrightarrow 1 + x_1z_2 - x_3z_4y_5x_6$ 48/64 (75%) (14)

$\longrightarrow 1 - x_1z_2 - x_3z_4y_5x_6$ 64/64(100%) (15)

## BINOMIALS OF DEGREE-$k$ TERMS

$$b_1b_2b_3b_4 + b_3b_4b_5b_6 = \min\left(2b_3b_4, b_1b_2 + b_5b_6\right) \quad (k,n) = (4,6). \quad (16)$$

$$b_1b_2b_3b_4 + b_3b_4b_5b_6 = \min_{b_a}\left(b_2b_3 + b_a(1 - b_2 - b_3 + 2b_4) + b_3b_4, b_1b_2 + b_5b_6 + b_5b_a\right) \quad (k,n) = (4,6). \quad (17)$$

$b_1b_2b_3b_4 + b_4b_5b_6b_7$ : $(k,n) = (4,7)$. (18)

$\longrightarrow\ b_3b_4 + b_4b_6 + b_a(b_5 + b_7)$ 89/128 (70%) (19)

$\longrightarrow\ b_1b_2 + b_5b_7 + b_a(1 - b_5 + b_6 - b_7)$ 125/128 (98%) (20)

$\longrightarrow\ b_5b_7 + b_3$ 128/128(100%) (21)

$b_1b_2b_3b_4 + b_4b_5b_6b_7$ : $(k,n) = (4,7)$. (22)

$\longrightarrow\ b_3b_4 + b_4b_6$ 89/128 (70%) (23)

$\longrightarrow\ b_1b_2 + b_6b_7$ 118/128 (92%) (24)

$\longrightarrow\ b_2b_3 - b_5b_6 + b_5b_7 + b_5$ 127/128 (99%) (25)

$\longrightarrow\ b_1b_4 + 2b_5 - b_7 + 1$ 128/128(100%) (26)

$b_1b_2b_3b_4b_5 + b_3b_4b_5b_6b_7$ : $(k,n) = (5,7)$. (27)

$\longrightarrow\ b_1b_5 + b_5b_6 + b_5b_7 + b_6b_7 + b_a(1 - b_5 - 2b_6 - b_7) + b_6$ 188/256 (73%) (28)

$\longrightarrow\ b_3b_4 + b_a(b_4 - b_6) + b_6$ 236/256 (92%) (29)

$\longrightarrow\ b_2b_3 + b_3b_6 - b_4b_6 + b_6b_a + b_6$ 254/256 (99%) (30)

$\longrightarrow\ b_2b_5 + b_5b_7$ 256/256(100%) (31)

$b_1b_2b_3b_4b_5 + b_3b_4b_5b_6b_7$ : $(k,n) = (5,7)$. (32)

$\longrightarrow\ b_2b_3 + b_3b_7$ 85/128 (66%) (33)

$\longrightarrow\ 2b_4b_5$ 121/128 (95%) (34)

$\longrightarrow\ b_1b_2 + b_6b_7 - b_5 + 1$ 128/128(100%) (35)

$b_1b_2b_3b_4b_5b_6 + b_2b_3b_4b_5b_6b_7$ : $(k,n) = (6,7)$. (36)

$\longrightarrow\ b_5b_6 + b_6b_7 + b_a(1 + b_5 - b_6 - b_7)$ 196/256 (77%) (37)

$\longrightarrow\ b_1b_4 + b_2b_4 + b_7b_a$ 238/256 (93%) (38)

$\longrightarrow\ b_1b_3 + b_3b_7 - b_4b_6 + 2b_5b_a - b_6b_7 - b_5 + b_6 + b_7 + b_a + 1$ 252/256 (98%) (39)

$\longrightarrow\ b_2b_6 + b_2 - b_6 + 1$ 256/256(100%) (40)

$b_1b_2b_3b_4b_5b_6 + b_2b_3b_4b_5b_6b_7$ : $(k,n) = (6,7)$. (41)

$\longrightarrow\ 2b_5b_6$ 97/128 (76%) (42)

$\longrightarrow\ b_1b_4 + b_4b_7$ 119/128 (93%) (43)

$\longrightarrow\ b_1b_3 + b_1b_7 + b_2b_3 - b_3b_6 + b_3b_7 - b_4b_5 - b_1 - b_7 + 2$ 127/128 (99%) (44)

$\longrightarrow\ b_1b_2 + b_2b_6$ 128/128(100%) (45)

$b_1b_2b_3b_4b_5 + b_4b_5b_6b_7b_8$ : $(k,n) = (5,8)$. (46)

$\longrightarrow\ b_3b_5 + b_7b_8 + b_a(-1 - b_6 + b_7 + b_8) + b_6 - b_7 - b_8 + 1$ 360/512 (70%) (47)

$\longrightarrow\ b_1b_4 + b_4b_8 + b_a(b_4 + b_6)$ 468/512 (91%) (48)

$\longrightarrow\ b_1b_2 + b_7b_8 + b_a(1 + b_6 - b_7 - b_8)$ 496/512 (97%) (49)

$\longrightarrow\ b_3b_5 + b_5$ 512/512(100%) (50)

$b_1b_2b_3b_4b_5 + b_4b_5b_6b_7b_8$ : $(k,n) = (5,8)$. (51)

$\longrightarrow\ b_2b_5 + b_5b_8$ 169/256 (66%) (52)

$\longrightarrow\ b_1b_4 + b_4b_7 - b_5b_8 + b_8$ 233/256 (91%) (53)

$\longrightarrow\ b_1b_3 + b_6b_7 + b_6b_8 + b_7b_8 - b_6 - b_7 - b_8 + 1$ 252/256 (98%) (54)

$\longrightarrow\ b_2b_3 + b_6b_7$ 256/256(100%) (55)

$b_1b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7b_8$ : $(k,n) = (6,8)$. (56)

$\longrightarrow\ b_1b_6 + b_7b_8 + b_a(1 + b_6 - b_7 - b_8)$ 364/512 (71%) (57)

$\longrightarrow\ b_2b_3 + b_5b_8 - b_6b_8 + b_7b_a - b_7 + b_8 - b_a + 1$ 450/512 (88%) (58)

$\longrightarrow\ b_1b_4 + b_4$ 488/512 (95%) (59)

$\longrightarrow\ b_2b_3 + b_3b_7 - b_6b_8 + b_8 - b_a + 1$ 502/512 (98%) (60)

$\longrightarrow\ b_2b_5 + b_5$ 512/512(100%) (61)

$b_1b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7b_8$ : $(k,n) = (6,8)$. (62)

$\longrightarrow\ 2b_5b_6$ 193/256 (75%) (63)

$\longrightarrow\ b_1b_4 + b_4b_8$ 237/256 (93%) (64)

$\longrightarrow\ b_2b_3 + b_3b_7 - b_4b_6 + b_4b_8 - b_5b_7 - b_5b_8 + b_6b_8 - b_6 + b_7 - b_8 + 2$ 254/256 (99%) (65)

$\longrightarrow\ b_1b_2 + b_7b_8$ 256/256(100%) (66)

$b_1b_2b_3b_4b_5b_6b_7 + b_2b_3b_4b_5b_6b_7b_8$ : $(k,n) = (7,8)$. (67)

$\longrightarrow\ b_6b_7 + b_6b_8 + b_a(1 - b_6 + b_7 - b_8)$ 388/512 (76%) (68)

$\longrightarrow\ b_1b_3 + b_3b_8 + b_a(1 + b_8)$ 470/512 (92%) (69)

$\longrightarrow\ b_2b_4 - b_3b_8 + b_4b_5 + b_a(1 - b_7) + b_8$ 500/512 (98%) (70)

$\longrightarrow\ b_2b_5 + b_2b_8 - b_4b_8 - b_6b_7 + b_6b_8 + b_a(-1 - b_4 - b_7 + b_8) - b_3 + b_7 - b_8 + 4$ 508/512 (99%) (71)

$\longrightarrow\ b_2b_5 - b_7b_8 + b_5 + 1$ 512/512(100%) (72)

$b_1b_2b_3b_4b_5b_6b_7 + b_2b_3b_4b_5b_6b_7b_8$ : $(k,n) = (7,8)$. (73)

$\longrightarrow\ 2b_5b_6$ 193/256 (75%) (74)

$\longrightarrow\ b_1b_4 + b_4b_8$ 235/256 (92%) (75)

$\longrightarrow\ b_2b_3 + b_2b_7 - b_5b_6 + b_6b_8 + b_5 - b_6 - b_8 + 1$ 250/256 (98%) (76)

$\longrightarrow\ b_3b_7 + b_7b_8$ 254/256 (99%) (77)

$\longrightarrow\ b_3b_8 + b_3$ 256/256(100%) (78)

$b_1b_2b_3b_4 + b_5b_6b_7b_8$ : $(k,n) = (4,8)$. (79)

$\longrightarrow\ b_2b_3 + b_6b_8 + b_a(1 - b_6 + b_7 - b_8)$ 390/512 (76%) (80)

$\longrightarrow\ b_1b_4 + b_6b_8 + b_a(1 - b_6 + b_7 - b_8)$ 480/512 (94%) (81)

$\longrightarrow\ b_2b_4 + b_5 - b_a + 1$ 506/512 (99%) (82)

$\longrightarrow\ b_1b_3 - b_6b_a + b_5 + 1$ 512/512(100%) (83)

$b_1b_2b_3b_4 + b_5b_6b_7b_8$ : $(k,n) = (4,8)$. (84)

$\longrightarrow\ b_1b_2 + b_6b_7$ 169/256 (66%) (85)

$\longrightarrow\ b_3b_4 + b_5b_8$ 238/256 (93%) (86)

$\longrightarrow\ b_1b_4 + b_5b_6 + b_5b_7 + b_6b_7 - b_5 - b_6 - b_7 + 1$ 248/256 (97%) (87)

$\longrightarrow\ b_2b_3 + b_6b_7 + b_6b_8 + b_7b_8 - b_6 - b_7 - b_8 + 1$ 254/256 (99%) (88)

$\longrightarrow\ b_1b_2 + b_5b_8$ 256/256(100%) (89)

$b_1b_2b_3b_4b_5 + b_6b_7b_8b_9b_{10}$ : $(k,n) = (5,10)$. (90)

$\longrightarrow\ b_1b_4 + b_7b_9$ 625/1024 (61%) (91)

$\longrightarrow\ b_3b_5 + b_6b_8$ 889/1024 (87%) (92)

$\longrightarrow\ b_2b_5 + b_7b_{10}$ 972/1024 (95%) (93)

$\longrightarrow\ b_2b_4 + b_6b_8$ 999/1024 (98%) (94)

$\longrightarrow\ b_1b_3 + b_9b_{10} + b_9b_a$ 1016/1024 (99%) (95)

$\longrightarrow\ b_1b_5 + b_6b_9$ 1020/1024 (99%) (96)

$\longrightarrow\ b_1b_4 + b_8b_{10}$ 1022/1024 (99%) (97)

$\longrightarrow\ b_2b_3 - b_4b_{10} + b_7b_9 + b_9b_a + 1$ 1024/1024(100%) (98)

$b_1b_2b_3b_4b_5 + b_6b_7b_8b_9b_{10}$ : $(k,n) = (5,10)$. (99)

$\longrightarrow\ b_1b_3 + b_9b_{10}$ 625/1024 (61%) (100)

$\longrightarrow\ b_2b_4 + b_7b_{10}$ 851/1024 (83%) (101)

$\longrightarrow\ b_3b_5 + b_5b_{10} + b_8b_9$ 924/1024 (90%) (102)

$\longrightarrow\ b_1b_2 + b_6$ 972/1024 (95%) (103)

$\longrightarrow\ b_3b_4 + b_8b_9$ 997/1024 (97%) (104)

$\longrightarrow\ b_1b_5 + b_7b_{10}$ 1010/1024 (99%) (105)

$\longrightarrow\ b_2b_3 - b_1b_7 - b_1b_{10} - b_2b_8 - b_2b_{10} + b_3b_5 + b_6b_9 + b_7b_{10} - b_8b_9 + b_9b_{10} - b_3 - b_7 + b_8 + 3$ 1016/1024 (99%) (106)

$\longrightarrow\ b_1b_3 + b_7b_8$ 1020/1024 (99%) (107)

$\longrightarrow\ b_2b_4 + b_2b_6 - b_2b_9 - b_3b_{10} - b_5b_7 + b_7b_{10} + b_9b_{10} - b_{10} + 2$ 1023/1024 (99%) (108)

$\longrightarrow\ b_2b_5 + b_2b_9 + b_6b_8$ 1024/1024(100%) (109)

$b_1b_2b_3b_4b_5b_6 + b_5b_6b_7b_8b_9b_{10}$ : $(k,n) = (6,10)$. (110)

$\longrightarrow\ b_4b_5 + b_5b_9$ 657/1024 (64%) (111)

$\longrightarrow\ b_2b_6 + b_6b_8$ 905/1024 (88%) (112)

$\longrightarrow\ b_1b_3 + b_7b_8$ 982/1024 (96%) (113)

$\longrightarrow\ b_2b_3 + b_a(b_{10} - b_9) + b_9$ 1011/1024 (99%) (114)

$\longrightarrow\ b_2b_4 + b_7b_{10}$ 1020/1024 (99%) (115)

$\longrightarrow\ b_9b_{10} + b_1$ 1023/1024 (99%) (116)

$\longrightarrow\ b_7b_8 + b_4$ 1024/1024(100%) (117)

$b_1b_2b_3b_4b_5b_6 + b_5b_6b_7b_8b_9b_{10}$ : $(k,n) = (6,10)$. (118)

$\longrightarrow\ 2b_5b_6$ 769/1024 (75%) (119)

$\longrightarrow\ b_1b_3 + b_8b_9$ 934/1024 (92%) (120)

$\longrightarrow\ b_2b_4 + b_7b_{10} + b_8b_9 - b_8 - b_9 + 1$ 997/1024 (97%) (121)

$\longrightarrow\ -b_1b_3 + b_1b_9 + b_2b_4 + b_4b_9 + b_5b_8 + b_8b_9 - b_5 - b_8 - b_9 + 2$ 769/1024 (99%) (122)

$\longrightarrow\ b_1b_3 + b_7b_{10} - b_8 - b_9 + 2$ 1014/1024 (99%) (123)

$\longrightarrow\ b_2b_3 + b_8b_9$ 1024/1024(100%) (124)

$b_1b_2b_3b_4b_5b_6b_7 + b_4b_5b_6b_7b_8b_9b_{10}$ : $(k,n) = (7,10)$. (125)

$\longrightarrow\ b_3b_5 + b_5b_8$ 649/1024 (63%) (126)

$\longrightarrow\ b_2b_4 + b_4b_9$ 893/1024 (87%) (127)

$\longrightarrow\ b_1b_7 + b_7b_{10}$ 985/1024 (96%) (128)

$\longrightarrow\ b_1b_6 + b_6b_9 + b_a$ 1015/1024 (99%) (129)

$\longrightarrow\ b_2b_3 + b_8b_{10} + b_a$ 1022/1024 (99%) (130)

$\longrightarrow\ b_1b_3 + b_8b_9$ 1024/1024(100%) (131)

$b_1b_2b_3b_4b_5b_6b_7 + b_4b_5b_6b_7b_8b_9b_{10}$ : $(k,n) = (7,10)$. (132)
$\longrightarrow\ b_3b_7 + b_7b_{10}$ 649/1024 (63%) (133)
$\longrightarrow\ 2b_4b_6$ 937/1024 (92%) (134)
$\longrightarrow\ b_1b_5 + b_5b_8$ 1001/1024 (98%) (135)
$\longrightarrow\ b_1b_2 + b_9b_{10}$ 1019/1024 (99%) (136)
$\longrightarrow\ b_2b_3 + b_8$ 1023/1024 (99%) (137)
$\longrightarrow\ b_3b_7 + b_9b_{10}$ 1024/1024(100%) (138)

$b_1b_2b_3b_4b_5b_6b_7b_8 + b_3b_4b_5b_6b_7b_8b_9b_{10}$ : $(k,n) = (8,10)$. (139)
$\longrightarrow\ b_2b_8 + b_8b_9$ 645/1024 (63%) (140)
$\longrightarrow\ b_1b_3 + b_3b_{10} + b_9b_a$ 887/1024 (87%) (141)
$\longrightarrow\ b_4b_6 + b_5b_6$ 977/1024 (95%) (142)
$\longrightarrow\ b_2b_7 + b_7b_{10}$ 1007/1024 (98%) (143)
$\longrightarrow\ b_1b_4 + b_4b_5 + 2b_9b_a$ 1018/1024 (99%) (144)
$\longrightarrow\ b_1b_5 + b_5b_9$ 1024/1024(100%) (145)

$b_1b_2b_3b_4b_5b_6b_7b_8 + b_3b_4b_5b_6b_7b_8b_9b_{10}$ : $(k,n) = (8,10)$. (146)
$\longrightarrow\ 4b_3b_7$ 768/1024 (75%) (147)
$\longrightarrow\ b_2b_8 + b_8b_9$ 933/1024 (91%) (148)
$\longrightarrow\ 2b_4b_6 + b_8b_9 - b_8 - b_9 + 1$ 1005/1024 (98%) (149)
$\longrightarrow\ b_1b_5 + b_5b_{10} + b_8b_9 - b_8 - b_9 + 1$ 1022/1024 (99%) (150)
$\longrightarrow\ b_1b_2 + b_8b_9 + b_9b_{10} - b_8 - b_9 + 1$ 1024/1024(100%) (151)

$b_1b_2b_3b_4b_5b_6b_7b_8b_9 + b_2b_3b_4b_5b_6b_7b_8b_9b_{10}$ : $(k,n) = (9,10)$. (152)
$\longrightarrow\ b_1b_9 + b_9b_{10} + b_{10}b_a$ 643/1024 (63%) (153)
$\longrightarrow\ b_2b_4 + b_4b_5$ 883/1024 (86%) (154)
$\longrightarrow\ b_3b_7 + b_3b_8$ 973/1024 (95%) (155)
$\longrightarrow\ b_2b_6 + b_6b_8$ 1003/1024 (98%) (156)
$\longrightarrow\ b_2b_5 + b_5b_7 - b_{10}b_a + b_{10}$ 1015/1024 (99%) (157)
$\longrightarrow\ b_1b_8 + b_7b_8$ 1019/1024 (99%) (158)
$\longrightarrow\ b_2b_7 + b_2b_{10} - b_4b_5 - b_{10}b_a + b_{10} + 1$ 1023/1024 (99%) (159)
$\longrightarrow\ b_4b_7 + b_7$ 1024/1024(100%) (160)

$b_1b_2b_3b_4b_5b_6b_7b_8b_9 + b_2b_3b_4b_5b_6b_7b_8b_9b_{10}$ : $(k,n) = (9,10)$. (161)
$\longrightarrow\ 2b_2b_3 - b_8b_9 + b_9$ 577/1024 (56%) (162)
$\longrightarrow\ 3b_8b_9$ 961/1024 (94%) (163)
$\longrightarrow\ 2b_4b_6 - b_8b_9 - b_8b_{10} + b_{10} + 1$ 1009/1024 (99%) (164)
$\longrightarrow\ 2b_5b_7 - b_8b_{10} + b_{10}$ 1021/1024 (99%) (165)
$\longrightarrow\ b_1b_6 + b_{10}$ 1024/1024(100%) (166)

## DEGREE-$k$, EXACT-$k$-OF-$n$ TRINOMIALS

$b_1b_2b_3b_4 + b_2b_3b_4b_5 + b_3b_4b_5b_6$ : $(k,n) = (4,6)$. (167)
$\longrightarrow\ b_1b_4 + 2b_4b_5 + b_7$ 44/64 (69%) (168)
$\longrightarrow\ b_1b_3 + b_2b_3 + b_3b_6 + b_6b_7$ 60/64 (94%) (169)
$\longrightarrow\ b_2b_4 + b_5b_6 + b_2$ 64/64(100%) (170)

$b_1b_2b_3b_4 + b_2b_3b_4b_5 + b_3b_4b_5b_6$ : $(k,n) = (4,6)$. (171)
$\longrightarrow\ b_2b_4 + 2b_4b_5$ 43/64 (67%) (172)
$\longrightarrow\ b_1b_3 + b_2b_3 + b_2b_5 + b_3b_6 - b_4b_5 - b_2 + 1$ 60/64 (94%) (173)
$\longrightarrow\ b_1b_2 + b_2b_5 + b_5b_6$ 64/64(100%) (174)

$b_1b_2b_3b_4 + b_3b_4b_5b_6 + b_5b_6b_7b_8$ : $(k,n) = (4,8)$. (175)
$\longrightarrow\ b_1b_4 + 2b_5b_6$ 159/256 (62%) (176)
$\longrightarrow\ b_2b_3 + b_3b_5 + b_7b_8$ 225/256 (88%) (177)
$\longrightarrow\ b_1b_4 + b_3b_4 - b_5b_7 + b_6b_7 + b_7b_8 - b_6 + 1$ 244/256 (95.3%) (178)
$\longrightarrow\ b_2b_3 + b_6b_8 + b_6$ 253/256 (98.8%) (179)
$\longrightarrow\ b_2b_3 + b_5b_7 + b_5$ 256/256 (100%) (180)

$b_1b_2b_3b_4 + b_3b_4b_5b_6 + b_5b_6b_7b_8$ : $(k,n) = (4,8)$. (181)
$\longrightarrow\ b_2b_4 + 2b_5b_6$ 159/256 (62%) (182)
$\longrightarrow\ b_3b_6 + b_7b_8 + b_3$ 212/256 (83%) (183)
$\longrightarrow\ b_2b_4 - b_5b_7 + b_7b_8 + b_4 + b_7$ 234/256 (91%) (184)
$\longrightarrow\ b_1b_3 + 2b_5b_6$ 253/256 (99%) (185)
$\longrightarrow\ b_7b_8 + b_1 + b_6$ 256/256(100%) (186)

$b_1b_2b_3b_4b_5 + b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7$ : $(k,n) = (5,7)$. (187)
$\longrightarrow\ b_1b_5 + b_5b_6 + b_6b_7 + b_a(-2 - b_5 + 2b_6 + b_7) + b_5 - 2b_6 - b_7 + 2$ 86/128 (67%) (188)
$\longrightarrow\ b_1b_3 + b_3b_4 - b_3b_6 + b_3b_7 + b_a(b_5 + b_7) + b_3$ 112/128 (88%) (189)
$\longrightarrow\ b_1b_4 + b_2b_4 + b_4b_7 + b_5b_7 + b_a(-1 - b_6 - b_7) - b_5 + b_6 + 2$ 124/128 (97%) (190)
$\longrightarrow\ b_2b_4 - 2b_5b_a + b_6b_7 + b_2 + b_5 + 1$ 128/128(100%) (191)

$b_1b_2b_3b_4b_5 + b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7$ : $(k,n) = (5,7)$. (192)
$\longrightarrow\ 2b_4b_5 + b_4b_6$ 81/128 (63%) (193)
$\longrightarrow\ b_1b_3 + b_3b_6 + b_3b_7 - b_4b_5 + b_5$ 111/128 (87%) (194)
$\longrightarrow\ b_1b_2 + b_2b_6 - b_4b_5 + b_6b_7 + b_5$ 122/128 (95%) (195)
$\longrightarrow\ 2b_4b_5 + b_5$ 128/128(100%) (196)

$b_1b_2b_3b_4b_5b_6 + b_2b_3b_4b_5b_6b_7 + b_3b_4b_5b_6b_7b_8$ : $(k,n) = (6,8)$. (197)
$\longrightarrow\ b_1b_6 + 2b_6b_7$ 164/256 (64%) (198)
$\longrightarrow\ b_1b_5 + b_2b_5 - b_3b_6 + b_5b_8 + b_3$ 219/256 (86%) (199)
$\longrightarrow\ b_2b_4 + b_4b_7 + b_4b_8 - b_6 + 1$ 243/256 (95%) (200)
$\longrightarrow\ b_2b_3 + b_3b_8 - b_5b_6 + b_3 + b_6$ 253/256 (99%) (201)
$\longrightarrow\ b_1b_2 + b_2b_6 + b_5b_7 - b_6b_7 + b_7b_8 - b_5 + 1$ 256/256(100%) (202)

$b_1b_2b_3b_4b_5b_6b_7b_8 + b_2b_3b_4b_5b_6b_7b_8b_9 + b_3b_4b_5b_6b_7b_8b_9b_{10}$ : $(k,n) = (8,10)$. (203)

$\longrightarrow\ 3b_5b_8$ 769/1024 (75%) (204)

$\longrightarrow\ 2b_2b_6 + b_4b_6$ 931/1024 (91%) (205)

$\longrightarrow\ b_1b_7 - b_5b_{10} + b_7b_9 + b_9b_{10} - b_6 + b_{10} + 1$ 984/1024 (96%) (206)

$\longrightarrow\ 3b_2b_3 + b_3b_{10} - b_6b_8 + 1$ 1011/1024 (99%) (207)

$\longrightarrow\ b_4b_7 + b_4b_8 - b_3 + b_4 - b_8 + 2$ 1019/1024 (99%) (208)

$\longrightarrow\ b_2b_3 - b_2b_4 - b_3b_4 - b_3b_8 - b_5b_{10} - b_6b_9 + b_7b_8 + b_7b_9 + b_8b_9 + b_7 + 3$ 1023/1024 (99%) (209)

$\longrightarrow\ b_2b_8 + 2b_8b_9$ 1024/1024(100%) (210)

$b_1b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7b_8 + b_5b_6b_7b_8b_9b_{10}$ : $(k,n) = (6,10)$. (211)

$\longrightarrow\ b_5b_6 + b_5b_8 + b_5b_9 + b_8b_{11} + b_9b_{11} + b_{10}b_{11}$ 583/1024 (57%) (212)

$\longrightarrow\ b_1b_2 + b_4b_7 + b_7b_{10} + b_9b_{11} - b_9 - b_{11} + 1$ 815/1024 (80%) (213)

$\longrightarrow\ b_1b_6 + b_5b_6 + b_6 - b_{11} + 1$ 917/1024 (90%) (214)

$\longrightarrow\ b_3b_4 + b_3b_7 + b_8b_9 + b_9b_{11}$ 979/1024 (96%) (215)

$\longrightarrow\ b_2b_4 + b_4b_8 + b_8b_9 - b_9b_{11} + b_9 - b_{11} + 1$ 1007/1024 (98%) (216)

$\longrightarrow\ b_1b_3 + b_7b_{10} + b_{10}b_{11} + b_3$ 1016/1024 (99%) (217)

$\longrightarrow\ b_1b_4 + b_4b_8 + b_7b_{10} + b_9b_{11} + b_{10}b_{11} - b_9 - b_{11} + 1$ 1021/1024 (99%) (218)

$\longrightarrow\ b_1b_3 - b_2b_{11} + b_7b_8 + b_8b_9 - b_{10}b_{11} - b_{11} + 3$ 1024/1024(100%) (219)

$b_1b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7b_8 + b_5b_6b_7b_8b_9b_{10}$ : $(k,n) = (6,10)$. (220)

$\longrightarrow\ 2b_3b_4 + b_7b_{10}$ 591/1024 (58%) (221)

$\longrightarrow\ 2b_3b_5 + b_5b_6$ 847/1024 (83%) (222)

$\longrightarrow\ b_1b_2 + b_7b_8 + b_8b_9$ 951/1024 (93%) (223)

$\longrightarrow\ 3b_5b_6$ 995/1024 (97%) (224)

$\longrightarrow\ b_1b_3 + b_3b_4 + b_9b_{10}$ 1009/1024 (99%) (225)

$\longrightarrow\ b_1b_2 + b_5b_7 + b_7b_{10}$ 1018/1024 (99%) (226)

$\longrightarrow\ 2b_1b_4 - b_1b_{10} + b_2b_4 + b_4b_5 + b_4b_{10} + b_5b_8 - b_6b_8 + b_8b_9 + b_7(b_{10} - b_6 - b_5 - b_1) + 3$ 1023/1024 (99%) (227)

$\longrightarrow\ b_2b_8 + b_3b_6 + b_6b_8$ 1024/1024(100%) (228)

$b_1b_2b_3b_4 + b_4b_5b_6b_7 + b_7b_8b_9b_{10}$ : $(k,n) = (4,10)$. (229)

$\longrightarrow\ b_2b_3 + b_6b_7 + b_7b_9 + 2b_9b_a$ 581/1024 (57%) (230)

$\longrightarrow\ b_2b_4 + b_4b_6 + b_a(b_9 - b_{10}) + b_{10}$ 823/1024 (80%) (231)

$\longrightarrow\ b_1b_3 + b_5b_6 + b_8b_9 + b_a(b_9 - b_{10}) - b_9 + 1$ 930/1024 (91%) (232)

$\longrightarrow\ b_1b_4 + b_4b_5 + b_4b_{10} + b_8b_{10} + b_a(1 - b_7 + b_9)$ 978/1024 (96%) (233)

$\longrightarrow\ b_1b_4 + b_7b_8 + b_a(1 + b_9) + b_7$ 1000/1024 (98%) (234)

$\longrightarrow\ b_2b_3 + b_a(b_9 - b_{10}) + b_5 + b_{10}$ 1015/1024 (99%) (235)

$\longrightarrow\ b_1b_3 + b_6 + b_{10}$ 1020/1024 (99%) (236)

$\longrightarrow\ b_5b_6 + b_2 + b_8 + b_a$ 1024/1024(100%) (237)

$b_1b_2b_3b_4 + b_4b_5b_6b_7 + b_7b_8b_9b_{10}$ : $(k,n) = (4,10)$. (238)
$\longrightarrow\ b_3b_4 + b_4b_6 + b_9b_{10}$ 581/1024 (57%) (239)
$\longrightarrow\ b_1b_2 + b_5b_7 - b_8b_9 + b_9b_{10} + b_9$ 759/1024 (74%) (240)
$\longrightarrow\ b_5b_6 + b_8b_9 + b_1 + b_8$ 842/1024 (82%) (241)
$\longrightarrow\ b_2b_4 + b_7b_{10} - b_8b_9 + b_7 + b_8$ 935/1024 (91%) (242)
$\longrightarrow\ b_2b_4 + b_4b_6 - b_8b_9 + b_8b_{10} - b_7 + b_8 + 1$ 969/1024 (95%) (243)
$\longrightarrow\ b_1b_3 + b_3b_4 + b_5b_7 + b_7b_9 - b_8b_9 + b_9$ 992/1024 (97%) (244)
$\longrightarrow\ b_2b_3 + b_3b_5 + b_3b_{10} + b_4b_8 + b_5b_6 - b_4 + 1$ 1004/1024 (98%) (245)
$\longrightarrow\ b_1b_3 + b_6b_7 + b_9b_{10}$ 1013/1024 (99%) (246)
$\longrightarrow\ b_1b_9 + b_7b_8 - b_8b_9 - b_9b_{10} + b_1 + b_7 + b_8 + b_9$ 1019/1024 (99%) (247)
$\longrightarrow\ b_2b_3 + b_5b_6 - b_8b_9 + b_9b_{10} + b_9$ 1022/1024 (99%) (248)
$\longrightarrow\ -b_1b_5 + b_1b_8 + b_3b_7 + b_3 + b_7 + 1$ 1023/1024 (99%) (249)
$\longrightarrow\ b_1b_5 - b_1b_{10} + b_2 + b_8 - b_{10} + 2$ 1024/1024(100%) (250)

$b_1b_2b_3b_4b_5 + b_3b_4b_5b_6 + b_4b_5b_6b_7b_8$ : $(k,n) = (5,8)$. (251)
$\longrightarrow\ b_1b_3 + b_3b_4 + b_6b_8 - b_6b_9 + b_7b_9 - b_8b_9 + b_9$ 156/256 (61%) (252)
$\longrightarrow\ b_1b_5 + b_5b_7 + b_7b_9 + b_8b_9 + b_5 - b_7 - b_9 + 1$ 202/256 (79%) (253)
$\longrightarrow\ b_2b_4 + b_6b_8 + b_6b_9 - b_7b_9 + b_8b_9 + b_7 - b_8 - b_9 + 1$ 230/256 (90%) (254)
$\longrightarrow\ b_2b_4 + b_4b_8 + b_4 - b_9 + 1$ 246/256 (96%) (255)
$\longrightarrow\ b_1b_5 + 2b_6$ 252/256 (98%) (256)
$\longrightarrow\ b_2b_5 + b_7b_8 + b_5$ 256/256(100%) (257)

$b_1b_2b_3b_4b_5 + b_3b_4b_5b_6 + b_4b_5b_6b_7b_8$ : $(k,n) = (5,8)$. (258)
$\longrightarrow\ b_4b_5 + 2b_5b_6$ 165/256 (64%) (259)
$\longrightarrow\ b_2b_4 + b_3b_4 + b_4b_8 - b_5b_7 + b_7$ 215/256 (84%) (260)
$\longrightarrow\ b_2b_3 + b_3b_6 - b_4b_5 - b_5b_7 + b_7b_8 + b_5 + b_7$ 242/256 (95%) (261)
$\longrightarrow\ b_1b_3 + b_5b_6 + b_6b_7$ 254/256 (99%) (262)
$\longrightarrow\ b_1b_2 + b_5b_6 + b_6b_8$ 256/256(100%) (263)

## DEGREE-$k$, EXACT-$k$-OF-$n$ QUADRINOMIALS

$b_1b_2b_3 + b_1b_2b_4 + b_1b_3b_4 + b_2b_3b_4$ : $(k,n) = (3,4)$. (264)
$\longrightarrow\ 2b_1b_2 + b_1b_3 + 2b_1b_4 + b_2b_3 + 2b_2b_4 + b_3b_4 - 2b_1 - 2b_2 - b_3 - 2b_4 + 3$ 13/16 (81%) (265)
$\longrightarrow\ 2b_1b_3 + b_2b_3 + b_2$ 16/16(100%) (266)

$b_1b_2b_3b_4b_5b_6b_7 + b_2b_3b_4b_5b_6b_7b_8 + b_3b_4b_5b_6b_7b_8b_9 + b_4b_5b_6b_7b_8b_9b_{10}$ : $(k,n) = (7,10)$. (267)
$\longrightarrow\ 4b_4b_5$ 769/1024 (75%) (268)
$\longrightarrow\ b_2b_6 + 2b_3b_6 + b_6b_9$ 915/1024 (89%) (269)
$\longrightarrow\ b_1b_7 + b_5b_7 + b_6b_7 + b_7b_{10}$ 974/1024 (95%) (270)
$\longrightarrow\ b_1b_2 + b_2b_8 + b_7b_8 + b_9b_{10}$ 995/1024 (97%) (271)
$\longrightarrow\ b_2b_3 + b_3b_4 + b_3b_6 + b_9b_{10}$ 1008/1024 (98%) (272)
$\longrightarrow\ b_1b_2 + b_2b_4 + b_9b_{10} + b_9$ 1016/1024 (99%) (273)
$\longrightarrow\ b_1b_3 - b_2b_8 + b_7b_8 + b_8b_9 + b_8b_{10} + 2b_8$ 1023/1024 (99%) (274)
$\longrightarrow\ b_1b_8 + b_2b_7 - b_5b_{10} + b_7b_8 + b_8b_9 - b_5 + 2$ 1024/1024(100%) (275)

## DEGREE-$k$, NOT EXACT-$k$-OF-$n$ MULTINOMIALS

$b_1b_2b_3b_4 + 2b_1b_2b_3 + b_1b_2b_4 + b_1b_3b_4 + b_2b_3b_4$ : $(k, n) = (4, 4)$. (276)
$\longrightarrow\ b_1b_2 + 4b_1b_3 + b_1b_4 + b_2b_3 + b_2b_4 + b_3b_4 - b_1 - b_2 - b_3 - b_4 + 1$ 12/16 (75%) (277)
$\longrightarrow\ b_1b_2 + b_1b_3 + 4b_1b_4 + b_2b_4$ 16/16(100%) (278)

$b_1b_2b_3b_4 + 2b_1b_2b_3 + b_1b_2b_4 + b_1b_3b_4 + b_2b_3b_4$ : $(k, n) = (4, 4)$. (279)
$\longrightarrow\ b_1b_2 + 4b_1b_3 + b_1b_4 + b_2b_3 + b_2b_4 + b_3b_4 - b_1 - b_2 - b_3 - b_4 + 1$ 12/16 (75%) (280)
$\longrightarrow\ 2b_2b_3 + 3b_2b_4 + b_3b_4$ 16/16(100%) (281)

$b_1b_2b_3b_4 + 2b_1b_2b_3 + b_1b_2b_4 + 3b_1b_3b_4 + b_2b_3b_4$ : $(k, n) = (4, 4)$. (282)
$\longrightarrow\ 2b_1b_2 + 5b_1b_4 + b_3b_4$ 11/16 (69%) (283)
$\longrightarrow\ -b_1b_2 + 3b_1b_3 + 4b_2b_3 + 2b_2b_4 - 4b_3b_4 + 4b_3 - b_4 + 1$ 16/16(100%) (284)

$b_1b_2b_3b_4 + 2b_1b_2b_3 + b_1b_3b_4$ : $(k, n) = (4, 4)$. (285)
$\longrightarrow\ 4b_1b_3$ 13/16 (81%) (286)
$\longrightarrow\ 2b_1b_2 + b_1b_4 + b_2b_4$ 16/16(100%) (287)

$b_1b_2b_3b_4 + 2b_1b_2b_3 + b_1b_3b_4$ : $(k, n) = (4, 4)$. (288)
$\longrightarrow\ 2b_1b_3 + 2b_3b_4$ 12/16 (75%) (289)
$\longrightarrow\ 3b_1b_2 + b_1b_4$ 16/16(100%) (290)