1 Terminology

Directed tree: A directed graph that would be a tree if we ingored the directions of the edges.

Arborescence: A directed tree with a "root" such that every node of it has a unique path to it from the root. There's always exactly one root.

Spanning subgraph of $D(V, A)$: A subgraph $D(V, B)$ such that $B \subseteq A$.

Branching system of $D(V, A)$ with k specified root nodes: A spanning subgraph of D created by edge-disjoint arborescences rooted at the k root nodes. Edge-disjoint means none of these arborescences share any edges. Every node has at most k incoming edges (at most one from each arborescence).

Optimal branching system of a weighted digraph with k specified root nodes: the branching system with the lowest total weight.

2 Algorithm

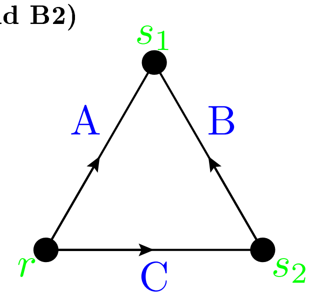
To find an OBS we need to find a min-cost set of edges which is both a basis of matroid M_1 and of M_2:

- M_1=(E, J_1) in which $j \in J_1$ has no edges entering any root nodes, and at most k edges entering each other node of the graph.
- M_2=(E, J_2) in which $j \in J_2$ can be partitioned into at most k **arborescences**.

3 Examples:

3.1 3-nodes

3.1.1 1 root (error in the graph! s1->s2 should be going in the opposite direction! ... Also it seems that J1 also should contain subsets of B1 and B2)

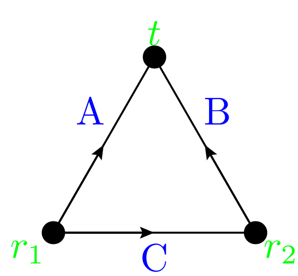


The branchings systems are:

$$B_1 = \{A, C\} \tag{3.1}$$
$$B_2 = \{A, B\} \tag{3.2}$$

We also have that $J_1 = J_2 = \{B_1, B_2\}$.

3.1.2 2 roots



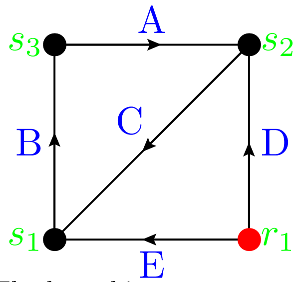
.
. The only branching system is:

$$B = \{A\}. \tag{3.3}$$

We also have that $J_1 = J_2 = B$.

3.2 4-nodes:

3.2.1 1 root



The branching systems are:

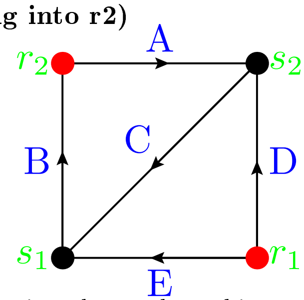
$$B_1 = \{E, B, A\}, \tag{3.4}$$
$$B_2 = \{E, B, D\}, \tag{3.5}$$
$$B_3 = \{D, C, B\}. \tag{3.6}$$

We also have that:

$$J_1 = \{B_1, B_2, B_3, \{A, C, B\}\} \tag{3.7}$$
$$J_2 = \{B_1, B_2, B_3\}. \tag{3.8}$$

Since $J_2 \subset J_1$,we know that $J_1 \cap J_2 = J_2$, **so the OBS will be the min-cost branching system in** J_2**.**

3.2.2 2 roots (seems wrong since B is going into r2)



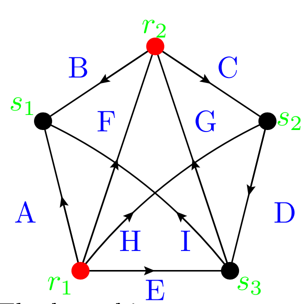
There is only one branching system:

$$B = \{E, B, D, C\}, \tag{3.9}$$

We also have that: $J_1 = J_2 = B$.

3.3 5-nodes:

3.3.1 2 roots (seems totally wrong)



The branching systems are:

$$B_1 = \{B, C, A, H, D, E\}, \tag{3.10}$$
$$B_2 = \{B, C, D, E, H, I\}, \tag{3.11}$$
$$B_3 = \{C, D, I, A, H, E\}. \tag{3.12}$$

We also have that: $J_1 = J_2 = \{B_1, B_2, B_3\}$.