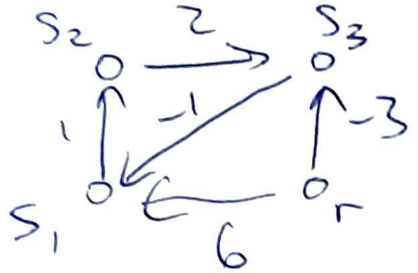
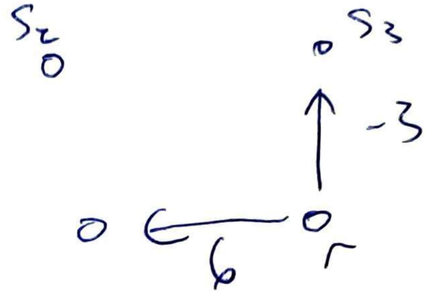
4 node system:

Let roots $= \{r, s_2\}$.



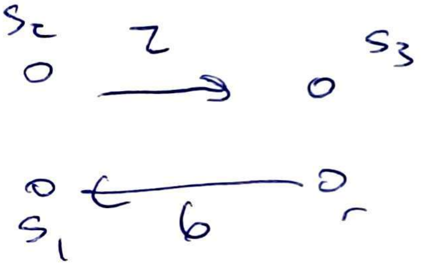
The branchings are:

1.



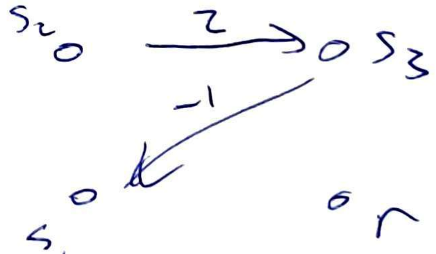
weight $= 3$.

2.



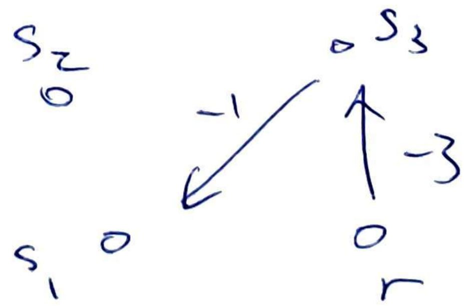
weight $= 8$

3.



weight $= 1$

4.



weight $= -4$

In all cases, $k = 2$. (2 roots).

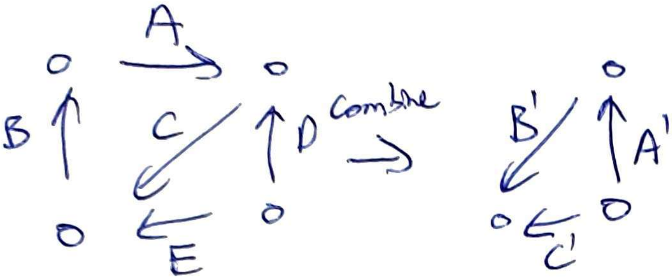
Clearly 4 is the answer with weight $= -4$.

Constructing M_1:

At most k of them at any node, i.e. 2 of them.

So,

~~$S_1 = \{A', B', C'\}$, $M_1 = \{E(G), \{S_1\}\}$.~~



~~$S_1: \{A', B', C'\}$~~

$c(A') = \min(A, D) = -3$ $c(B') = c(C) = -1$

$c(C') = c(E) = 6$

☆ Note: when reducing a set $\{r(i) : i \in [1, k]\}$, if 2 edges are going towards the same node, take the minimum of those edges.

~~Using independence oracles instead?~~

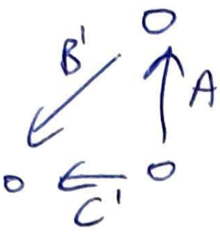
$\{\}$ counts. $\{A'\}$, $\{B'\}$, $\{C'\}$.

$M_1 = \{\{\}, \{A'\}, \{B'\}, \{C'\}, \{A', B'\},$
$\{A', B', C'\}, \{A', C'\}, \{B', C'\}\}$.

$M_2 = M_1$?

Constructing M_2:

Taking



Clearly $\{A', B', C'\}$ works, and so, since this is every edge, the matroid is simply $\{A', B', C'\}$.

Matroid intersection:

between $M_1 = \{E(G), \{\{A', B', C'\}\}\}$ and

$M_2 = \{E(G), \{\{A', B', C'\}\}\}$.

① $k=0$ $\quad X_0 = \emptyset \quad c_1 = \begin{bmatrix}-3\\-1\\6\end{bmatrix} \quad c_2 = \begin{bmatrix}0\\0\\0\end{bmatrix}$.

② $y \in \{A', B', C'\}$.

Compute:

$C_1(\emptyset, \overset{A'}{\cancel{y}}) = \{x \in \{A'\} : \{A'\} \notin \{A', B', C'\}, \ldots\} = \emptyset$

similarly,

$C_1(\emptyset, B') = C_1(\emptyset, C') = C_2(\emptyset, A') = \cdots = \emptyset$.

③ $A^{(1)} = \emptyset$. $A^{(2)} = \emptyset$ (All empty).

$S = \emptyset = T$ since $E \setminus X_0 = \emptyset$.

④ $m_1 = -\infty$, $m_2 = -\infty$ (not def'n).

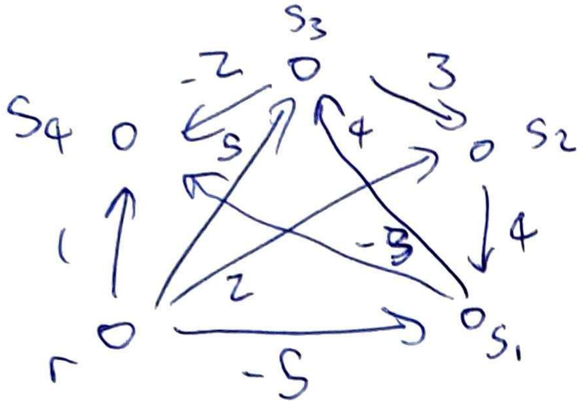
$\bar{S} = \bar{T} = \emptyset$. $\bar{A}^{(1)} = \bar{A}^{(2)} = \emptyset$.

$\bar{G}$ = empty edges,

$R = \emptyset$. $\Rightarrow$ $\epsilon = \infty$. ???

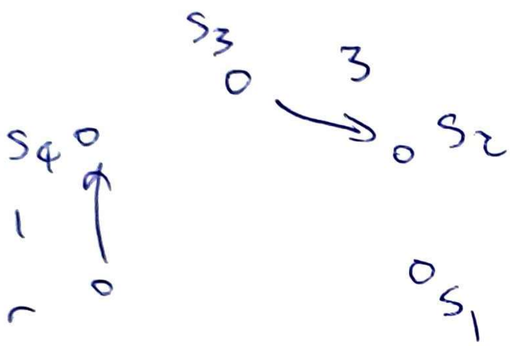
~~Xa~~ $F_1 =$

5 node system!



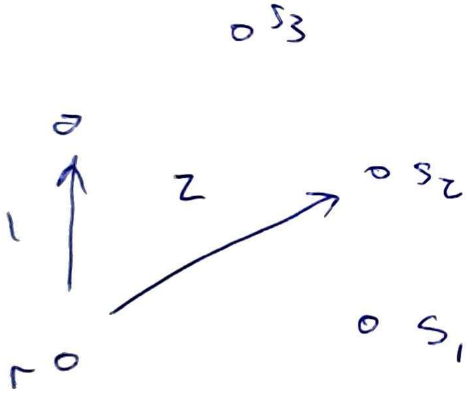
Let's choose $\{r, s_1, s_3\}$ to be our nodes.

1.



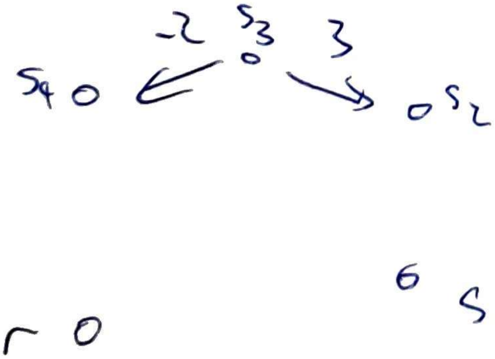
weight = 4.

2.



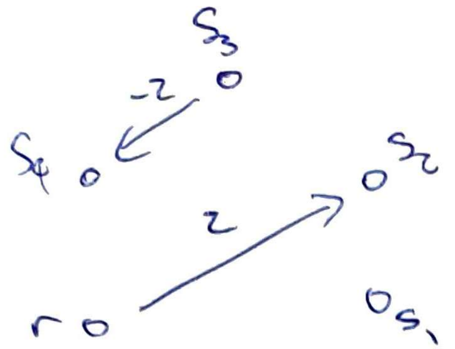
weight = 3

3.



weight = 1

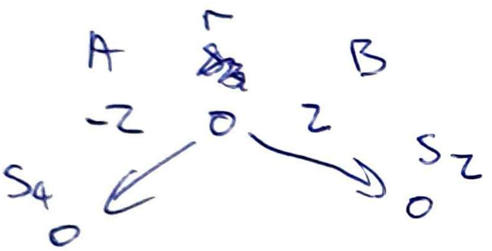
4.



weight = 0.

Clearly 4. is the best branching. $k=3$.

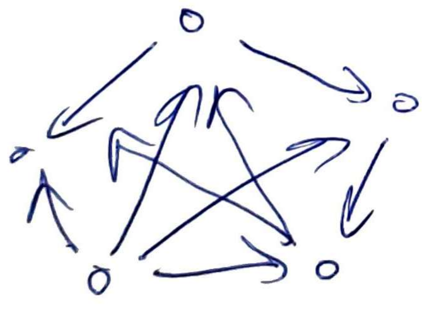
Reducing M:



$M_1 = \{E(G), \{\}, \{A\}, \{B\}, \{A,B\}\}$.

M_2: $\{\{\}, \{A\}, \{B\}, \{A, B\}\}$?

↳ maybe consider:



?

$k = 3$.