Quadratic Envelopes: A method that can quadratize with a negative number of auxiliary variables

Nike Dattani[1] and Andreas Soteriou[2]

[1]*Harvard-Smithsonian Center for Astrophysics, USA*
[2]*Surrey University, Department of Mathematics, UK*

We present a quadratization method that, for all of the many cases considered, provides quadratizations that require fewer auxiliary variables, less connectivity between the variables, and *much* smaller coefficients than any previously known quadratization method. The quadratizations presented in this work, are also obtainable much more quickly (in terms of computation runtime) than quadratizatons for functions of similar complexity have traditionally been in the past. The idea is that many *very* simple quadratic functions might *almost* reproduce a degree-k function perfectly, but each of these *almost* perfect quadratic functions can compensate for each others imperfections, hence leading to a perfect quadratization when all such quadratic functions are considered collectively. This collection is called a *quadratic envelope*. We are able to quadratize positive monomials with quadratic functions that have exponentially smaller coefficients than in [Boros *et al.* (2018)], while also having far less connectivity between the variables of the quadratics, in addition to requiring fewer auxiliary variables. We will also show examples in which an envelope of quadratic functions, each with *fewer* total variables than the original degree-k function, collectively can be made to *exactly* reproduce the degree-k function.

I. INTRODUCTION

II. RESULTS

2-runs/0-aux case:

$$b_1 b_2 b_3 b_4 = \min\left(b_1 b_2, b_3 b_4\right) \tag{1}$$

3-runs/0-aux case to be applied in Computer Vision and LHZ lattice:

$$b_1 b_2 b_3 b_4 + b_3 b_4 b_5 b_6 = \min\left(b_2 b_3 + b_3 b_6, b_1 b_4 + b_4 b_5, b_1 b_2 + b_5 b_6 - b_3 - b_4 + 2\right) \tag{2}$$

Linearization:

$$b_1 b_2 b_3 \dots b_k = \min\left(b_1, b_2, b_3, \dots, b_k\right) \tag{3}$$

3-runs/0-aux case:

$$b_1 b_2 b_3 b_4 + b_2 b_3 b_4 b_5 + b_3 b_4 b_5 b_6 : \qquad (k, n) = (4, 6). \tag{4}$$

$$\longrightarrow \; 3 b_3 b_4 + b_3 b_5 + b_4 b_5 - b_3 - b_4 - b_5 + 1 \tag{5}$$

$$\longrightarrow \; b_1 b_4 + b_3 b_5 + b_4 b_5 \tag{6}$$

$$\longrightarrow \; b_1 b_2 + b_2 b_6 + b_3 b_5 + b_5 b_6 + b_2 - b_3 - b_4 - b_5 + 2 \tag{7}$$

III. EXAMPLES

IV. DISCUSSION

V. ACKNOWLEDGMENTS