“Exrtaordinarily compact and efficient quadratizations” or “Multi-run quadratizations” or “Quadratization by minimizing over multiple functions”

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We present a quadratization method that, for all of the many cases considered, provides quadratizations that require fewer auxiliary variables, less connectivity between the variables, and *much* smaller coefficients than any previously known quadratization method. The quadratizations presented in this work, are also obtainable much more quickly (in terms of computation runtime) than quadratizatons have traditionally in the past for functions of similar complexity. The idea is that many *very* simple quadratic functions might *almost* reproduce a degree-k function perfectly, but each of these *almost* perfect quadratic functions can compensate for each others imperfections, hence leading to a perfect quadratization when all such quadratic functions are considered collectively. We are able to quadratize positive monomials with quadratic functions that have exponentially smaller coefficients than in [Boros *et al.* (2018)], while also having far less connectivity between the variables of the quadratics, and with up to a factor of 2 improvement in terms of the efficiency.

I. INTRODUCTION

II. RESULTS

2-runs/0-aux case:

$$b_1b_2b_3b_4 = \min\left(b_1b_2, b_3b_4\right) \tag{1}$$

3-runs/0-aux case to be applied in Computer Vision and LHZ lattice:

$$b_1b_2b_3b_4 + b_3b_4b_5b_6 = \min\left(b_2b_3 + b_3b_6, b_1b_4 + b_4b_5, b_1b_2 + b_5b_6 - b_3 - b_4 + 2\right) \tag{2}$$

Linearization:

$$b_1b_2b_3 \ldots b_k = \min\left(b_1, b_2, b_3, \ldots, b_k\right) \tag{3}$$

3-runs/0-aux case:

$$b_1b_2b_3b_4 + b_2b_3b_4b_5 + b_3b_4b_5b_6 : \qquad (k,n) = (4,6). \tag{4}$$

$$\longrightarrow\ 3b_3b_4 + b_3b_5 + b_4b_5 - b_3 - b_4 - b_5 + 1 \tag{5}$$

$$\longrightarrow\ b_1b_4 + b_3b_5 + b_4b_5 \tag{6}$$

$$\longrightarrow\ b_1b_2 + b_2b_6 + b_3b_5 + b_5b_6 + b_2 - b_3 - b_4 - b_5 + 2 \tag{7}$$

III. EXAMPLES

IV. DISCUSSION

V. ACKNOWLEDGMENTS