Pegasus: The second connectivity graph for large-scale quantum annealling hardware

Nike Dattani*
Harvard-Smithsonian Center for Astrophysics

Szilard Szalay†
Wigner Institute

Nicholas Chancellor‡
Durham University

Pegasus is a graph which offers substantially increased connectivity between the qubits of quantum annealing hardware compared to the graph Chimera. It is the first fundamental change in the connecctivity graph of quantum annealers built by D-Wave since Chimera was introduced in 2011 for D-Wave's first commercial quntum annealer. In this article we describe an algorithm which defines the connectivity of Pegasus and we provide what we believe to be the best way to graphically visualize Pegasus in 2D and 3D in order to see which qubits couple to each other. As Supplemental Material, we provide open source codes for generating Pegasus graphs.

The 128 qubits of the first commercial quantum annealer (D-Wave One, released in 2011) were connected by a graph called Chimera, which is rather easy to describe: A 2D array of $K_{4,4}$ graphs, with one partition of each $K_{4,4}$ being also connected to the same coresopnding partition on the $K_{4,4}$ unit cell above it, and the other partition being connected to the same corresponding partition on the $K_{4,4}$ to the right of it (see Figure 1). The degree of the graph is six, since each qubit couples to four qubits within its $K_{4,4}$ unit cell, and to one qubit in a $K_{4,4}$ above it and one qubit in a $K_{4,4}$ to the right of it. All commercial quantum annealers built to date follow this graph connectivity, with just larger and larger numbers of $K_{4,4}$ unit cells (See Table 1).

Figure 1. The Chimera graph, with open edges to show that the pattern repeats.

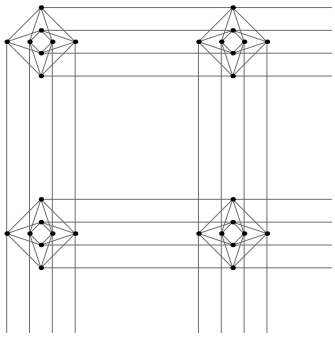


Table I. Chimera graphs in all commercial quantum computers to date.

	Array of $K_{4,4}$ unit cells	Total # of qubits
D-Wave One	4×4	128
D-Wave Two	8×8	512
D-Wave 2X	12×12	1152
D-Wave 2000Q	16×16	2048

In 2018, D-Wave announced the contruction of a (not yet commercial) quantum annealer with a greater connectivity than Chimera offers, and a publicly available program (NetworkX) which allows users to generate Pegasus graphs of arbitrary size. However, no explicit description of the graph connectivity in Pegasus has been published yet, so we have had to apply the process of reverse engineering to determine it, and the following section describes our algorithm for generating Pegasus.

I. ALGORITHM FOR GENERATING PEGASUS

Start with K layers of Chimera graphs, each being an $I \times J$ array of $K_{4,4}$ unit cells (therefore we have an $I \times J \times K$ array of $K_{4,4}$ unit cells). The indices (i, j, k) will be used to decribe the location of each unit cell along the indices corresponding to the dimension picked from (I, J, K). Each $K_{4,4}$ cell has two partitions, labeled $l \in \{1, 2\}$, so that there's 4 qubits (vertices) for every (i, j, k, l). We will arbitrarily label these 4 qubits using two more labels: $(m, n) \in \{1, 2\}^2$. Therefore every qubit is associated with 5 indices: (i, j, k, l, m, n), with their ranges and descriptions given in Table 2.

* n.dattani@cfa.harvard.edu
† szalay.szilard@wigner.mta.hu
‡ nicholas.chancellor@durham.ac.uk

Figure 2. The Pegasus graph, with open edges to show that the pattern repeats.

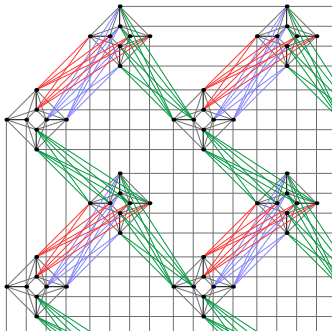


Table II. Indices used to describe each qubit (vertex) in Pegasus

i	1 to I	Row within a Chimera layer
j	1 to J	Column within a Chimera layer
k	1 to K	Chimera layer
l	1,2	Bi-partition within $K_{4,4}$
m	1,2	Top partition within each bi-partition of $K_{4,4}$
n	1,2	Bottom partition within each bi-partition of $K_{4,4}$

We then have the connections between $K_{4,4}$ cells of a particular layer, which would normally appear in Chimera, as in Figure 1: [insert mathematical description here].

Pegasus first adds connections within each $K_{4,4}$ cell (drawn in black color in Figure 2). The rest of the new connections in Pegasus come from connecting $K_{4,4}$ cells between different layers of $I \times J$ Chimera graphs. The qubits of $K_{4,4}$ will be connected to three different $K_{4,4}$ cells from a specific Chimera graph (i, j) on a different Chimera layer, and each pair of connected $K_{4,4}$ cells will have eight different connecctions in the form of $K_{2,2}$:[insert mathematical description here].

II. ALTERNATIVE GRAPHICAL REPRESENTATIONS OF PEGASUS

There are many ways that Pegasus can be drawn, so we show some of these in Figures 3 and 4.

III. COMPARISON TO CHIMERA

A. Graph degree

As the yellow vertex in Fig. ? indicates, the vertices of Pegasus (except for vertices on an edge) have a degree 15, which is 2.5 times larger than the maimum degree achieved in Chimera.

B. Non-planarity

We note that certain binary optimization problems forming planar graphs can be solved on a classical computer with a number of operations that scales polynomially with the number of binary variables, with the blossom algorithm [1, 2]. Therefore it is important that the qubits of a quantum annealer are connected by a non-planar graph. The $K_{4,4}$ "unit cells" of Chimera are already sufficient to make all commercial D-Wave annealers non-planar. However, if each $K_{4,4}$ unit cell of Chimera physical qubits were to encode one logical qubit (in for example, an extreme case of minor embedding), then Chimera would be planar. While all red, blue, and green lines added in Pegasus are of the form $K_{2,4}$, which itself is planar; these $K_{2,4}$ lines connect unit cells of different planes of chimeras in a non-planar way, such that even if each unit cell were to represent one logical qubit, *these logical qubits would still form a non-planar graph in Pegasus.* This should expand the number of binary optimization problems that can be embedded onto a D-Wave annealer, and cannot be solved on a classical computer with a number of operations that scales polynomially with the number of binary variables.

C. Embedding of quadratization gadgets

We have written an entire paper on the embedding of quadratization gadgets onto Chimera and Pegasus. One highlight of that work is the fact that *all* quadratization gadgets for single cubic terms which require one auxiliary qubit, can be embedded onto Pegasus with no further auxiliary qubits because Pegasus contains K_4,which means that all three logical qubis and the auxiliary qubit can be connected in any way, without any minor-embedding.

[1] J. Edmonds, Canadian Journal of Mathematics **17**, 449 (1965).

[2] J. Edmonds, *JOURNAL OF RESEARCH of the Na ti onal Bureau of Standards-B. Mathematics and Mathemat-*

Figure 3.

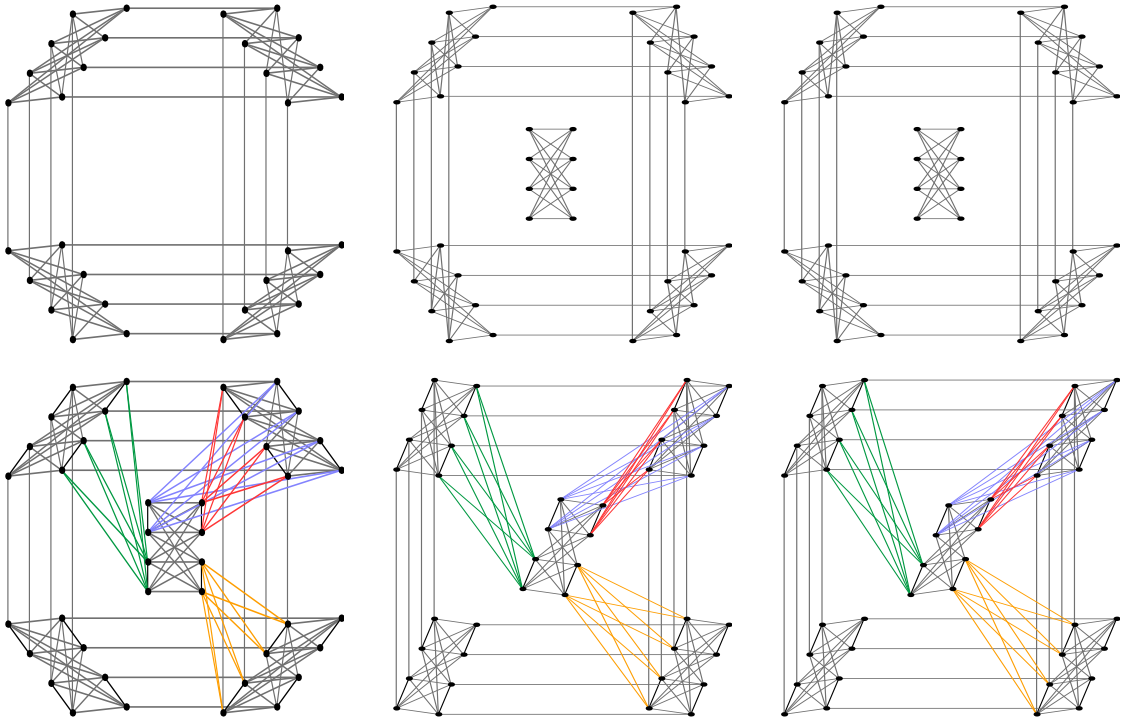
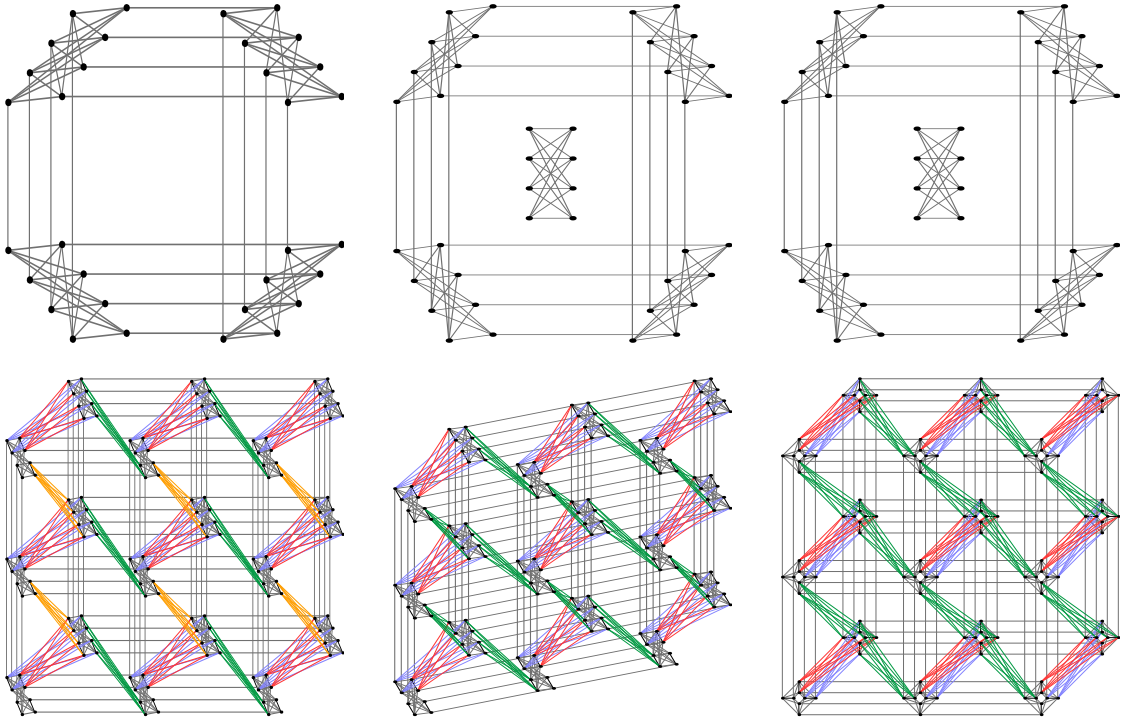


Figure 4.



ical Physics, Tech. Rep. 2 (1965).