$$-b_1b_2b_3b_4-b_1b_2b_3b_5-b_1b_2b_4b_5-b_1b_3b_4b_5-b_2b_3b_4b_5-b_1b_2b_3b_4b_5 \longrightarrow b_a(19-5b_1-5b_2-5b_3-5b_4-5b_5)$$

$$-b_1b_2b_3b_5-b_1b_2b_4b_5-b_1b_3b_4b_5-b_2b_3b_4b_5-b_1b_2b_3b_4b_5 \longrightarrow b_a(16-4b_1-4b_2-4b_3-4b_4-5b_5)$$

$$-b_1b_2b_4b_5 - b_1b_3b_4b_5 - b_2b_3b_4b_5 - b_1b_2b_3b_4b_5 \longrightarrow b_a(13-3b_1-3b_2-3b_3-4b_4-4b_5)$$

$$-b_1b_3b_4b_5 - b_2b_3b_4b_5 - b_1b_2b_3b_4b_5 \longrightarrow b_a(10-2b_1-2b_2-3b_3-3b_4-3b_5)$$

$$-b_1b_2b_3b_5 - b_1b_2b_4b_5 - b_1b_3b_4b_5 - b_2b_3b_4b_5 \longrightarrow b_a(12-3b_1-3b_2-3b_3-3b_4-4b_5)$$

$$-b_1b_2b_4b_5 - b_1b_3b_4b_5 - b_2b_3b_4b_5 \longrightarrow b_a(9-2b_1-2b_2-2b_3-3b_4-3b_5)$$

$$-b_1b_3b_4b_5 - b_2b_3b_4b_5 \longrightarrow b_a(6-b_1-b_2-2b_3-2b_4-2b_5)$$

All of these are examples of a general 5-var/1-aux theorem for negative coefficients:

If $a_i \le 0$ and $a_0 + a_i + a_j + a_k \le 0$ for all distinct i, j, k, then

$$a_0b_1b_2b_3b_4b_5 + a_1b_2b_3b_4b_5 + a_2b_1b_3b_4b_5 + a_3b_1b_2b_4b_5 + a_4b_1b_2b_3b_5 + a_5b_1b_2b_3b_4$$
$$\longrightarrow b_a\left(a_0(b_1+b_2+b_3+b_4+b_5-4) + a_1(b_2+b_3+b_4+b_5-3) + \cdots + a_5(b_1+b_2+b_3+b_4-3)\right)$$

This can be generalized: Let $n \in \mathbb{Z}^+,\ a_0, a_1, \cdots a_n \le 0$. Then

$$a_0 \prod_{i=1}^{n} b_i + \sum_{i=1}^{n}\left(a_i \prod_{j\neq i} b_j\right) \longrightarrow b_a\left(a_0\left(\sum_{i=1}^{n} b_i - (n-1)\right) + \sum_{i=1}^{n} a_i\left(\sum_{j\neq i} b_j - (n-2)\right)\right)$$

We also found some more quadratizations:

$$-b_1b_4b_5 - b_2b_4b_5 - b_3b_4b_5 \longrightarrow b_a(6-b_1-b_2-b_3-3b_4-3b_5)$$

$$-b_3b_4b_5 - b_1b_3b_4b_5 - b_2b_3b_4b_5 \longrightarrow b_a(8-b_1-b_2-3b_3-3b_4-3b_5)$$

These are examples of:

$$\min\left(\sum a_ib_i + d, 0\right) \longrightarrow b_a\left(\sum a_ib_i + d\right)$$