VITA with Trotter and Strang Splitting

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Trotter splitting: $e^{(A+B)t} = e^{At}e^{Bt} + \mathcal{O}\left(t^2\right)$
Strang splitting: $e^{(A+B)t} = e^{A\frac{1}{2}t}e^{Bt}e^{A\frac{1}{2}t} + \mathcal{O}\left(t^3\right)$

Ansatz converging slowly with decreasing t: $e^{\frac{\mathrm{i}}{\hbar}Ht} = \prod e^{A\mathrm{d}t}e^{B\mathrm{d}t} + \mathcal{O}\left(\mathrm{d}t\right)$
Ansatz converging more rapidly with decreasing t: $e^{\frac{\mathrm{i}}{\hbar}Ht} = \prod e^{\frac{\mathrm{d}t}{2}A}e^{\mathrm{d}tB}e^{\frac{\mathrm{d}t}{2}A} + \mathcal{O}\left(\mathrm{d}t^2\right)$

Ansatz converging slowly: $\prod e^{\alpha_p A}e^{\alpha_p B} + \mathcal{O}\left(P \max_p \alpha_p^2\right)$
Ansatz converging more rapidly: $\prod e^{\alpha_p \frac{1}{2}A}e^{\alpha_p B}e^{\alpha_p \frac{1}{2}A} + \mathcal{O}\left(P \max_p \alpha_p^3\right)$

In the last equations, the error comes from the fact that each Trotter/Strang application introduces an error of α_p^2 or α_p^3, and we have P applications of Trotter/Strang splittings so the total error goes as $P\alpha_p^2$ or $P\alpha_p^3$. The error will be dominated by the biggest α_p so that's why we have the "maximum" function.

As long as α_p is small (which has to be the case for either of these to converge), α_p^3 will be much smaller than α_p^2, and hence the total error will be smaller with Strang splitting compared with Trotter.

This can be generalized to include both α_p and β_p.